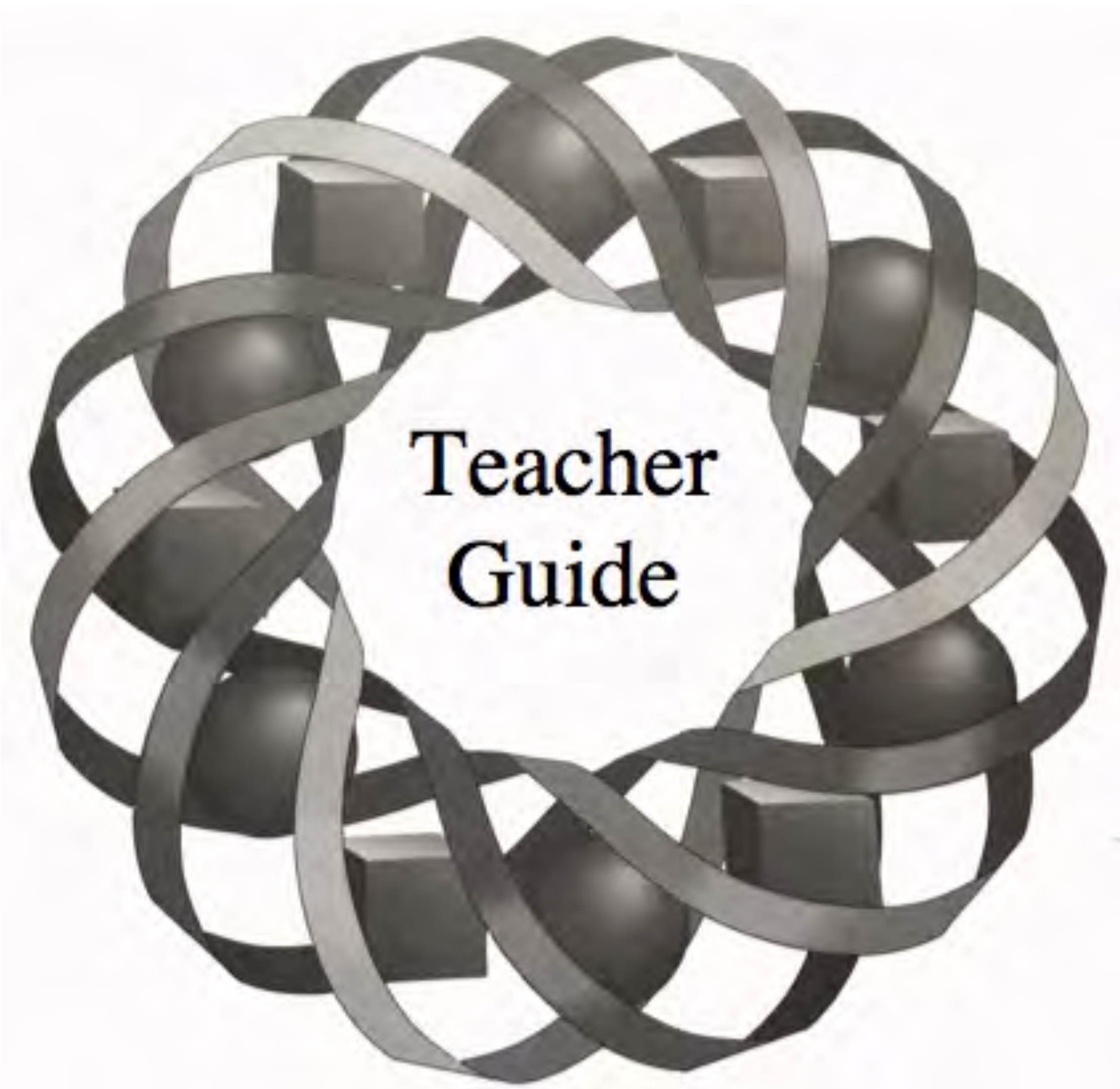
SIMMS Integrated Mathematics:

A Modeling Approach Using Technology



Level 1 Volumes 1-3

LEVEL 1 VOLUMES 1-3

Teacher Guide
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About *Integrated Mathematics: A Modeling Approach Using Technology*

The Need for Change

In recent years, many voices have called for the reform of mathematics education in the United States. Teachers, scholars, and administrators alike have pointed out the symptoms of a flawed system. From the ninth grade onwards, for example, about half of the students in this country's mathematical pipeline are lost each year (National Research Council, 1990, p. 36). Attempts to identify the root causes of this decline have targeted not only the methods used to instruct and assess our students, but the nature of the mathematics they learn and the manner in which they are expected to learn. In its *Principles and Standards for School Mathematics*, the National Council of Teachers of Mathematics addressed the problem in these terms:

> When students can connect mathematical ideas, their understanding is deeper and more lasting. They can see mathematical connections in the rich interplay among mathematical topics, in contexts that relate mathematics to other subjects, and in their own interests and experience. Through instruction that emphasizes the interrelatedness of mathematical ideas, students not only learn mathematics, they also learn about the utility of mathematics. (p. 64)

Some Methods for Change

Among the major objectives of the *Integrated Mathematics* curriculum are:

- offering a 9–12 mathematics curriculum using an integrated inter-disciplinary approach for *all* students.
- incorporating the use of technology as a learning tool in all facets and at all levels of mathematics.
- offering a *Standards*-based curriculum for teaching, learning, and assessing mathematics.

The *Integrated Mathematics* Curriculum

An integrated mathematics program "consists of topics chosen from a wide variety of mathematical fields. . . [It] emphasizes the relationships among topics within mathematics as well as between mathematics and other disciplines" (Beal, et al., 1992; Lott, 1991). In order to create innovative, integrated, and accessible materials, *Integrated Mathematics: A Modeling Approach Using Technology* was written, revised, and reviewed by secondary teachers of mathematics and science. It is a complete, *Standards*-based mathematics program designed to replace all currently offered secondary mathematics courses, with the possible exception of advanced placement classes, and builds on middle-school reform curricula.

The *Integrated Mathematics* curriculum is grouped into six levels. All students should take at least the first two levels. In the third and fourth years, *Integrated Mathematics* offers a choice of courses to students and their parents, depending on interests and goals. A flow chart of the curriculum appears in Figure 1.

Each year-long level contains 14–16 modules. Some must be presented in sequence, while others may be studied in any order. Modules are further divided into several activities, typically including an exploration, a discussion, a set of homework assignments, and a research project.

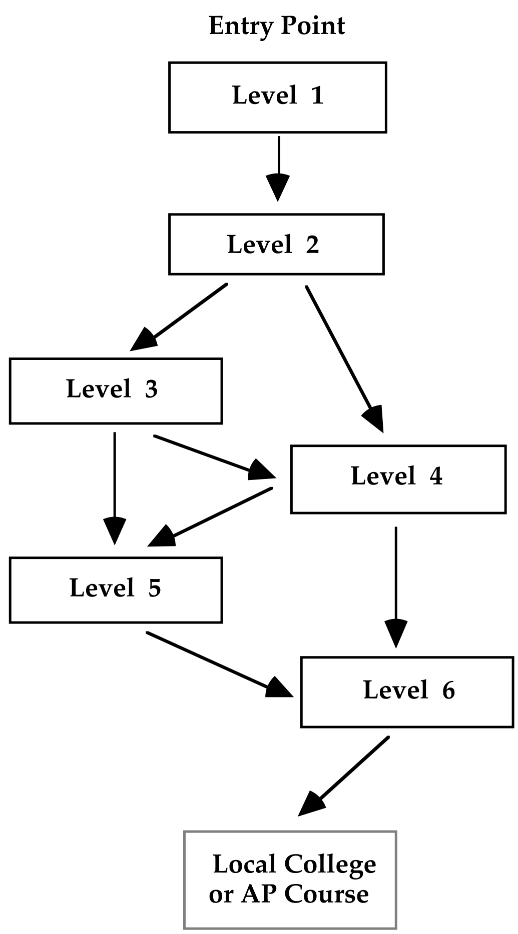

Figure 1: ***Integrated Mathematics*** **course sequence**

Assessment materials—including alternative assessments that emphasize writing and logical argument—are an integral part of the curriculum. Suggested assessment items for use with a standard rubric are identified in all teacher editions.

Level 1: a first-year course for ninth graders (or possibly eighth graders)

Level 1 concentrates on the knowledge and understanding that students need to become mathematically literate citizens, while providing the necessary foundation for those who wish to pursue careers involving mathematics and science. Each module in Level 1, as in all levels of the curriculum, presents the relevant mathematics in an applied context. These contexts include the properties of reflected light, population growth, and the manufacture of cardboard containers. Mathematical content includes data collection, presentation, and interpretation; linear, exponential, and step functions; and three-dimensional geometry, including surface area and volume.

Level 2: a second-year course for either ninth or tenth graders

Level 2 continues to build on the mathematics that students need to become mathematically literate citizens. While retaining an emphasis on the presentation and interpretation of data, Level 2 introduces trigonometric ratios and matrices, while also encouraging the development of algebraic skills. Contexts include pyramid construction, small business inventory, genetics, and the allotment of seats in the U.S. House of Representatives.

Levels 3 and 4: options for students in the third year

Both levels build on the mathematics content in Level 2 and provide opportunities for students to expand their mathematical understanding. Most students planning careers in math and science will choose Level 4. While Level 3 also may be suitable for some of these students, it offers a slightly different mixture of context and content.

Contexts in Level 4 include launching a new business, historic rainfall patterns, the pH scale, topology, and scheduling. The mathematical content includes rational, logarithmic, and circular functions, proof, and combinatorics.

In Level 3, contexts include nutrition, surveying, and quality control.

Mathematical topics include linear programming, curve-fitting, polynomial functions, and sampling.

Levels 5 and 6: options for students in the fourth year

Level 6 materials continue the presentation of mathematics through applied contexts while embracing a broader mathematical perspective. For example, Level 6 modules explore operations on functions, instantaneous rates of change, complex numbers, and parametric equations.

Level 5 focuses more specifically on applications from business and the social sciences, including hypothesis testing, Markov chains, and game theory.

More About Level 1

"Reflect on This," a module based on the physical properties of reflected light, sets the tone for the use of technology and cooperative groups. "So You Want to Buy a Car" is designed to introduce the spreadsheet functions and data analysis techniques used throughout the year.

The modules "Yesterday's Food Is Walking and Talking Today," "Under the Big Top but Above the Floor," and "What Will We Do When the Well Runs Dry?" involve linear functions. "A New Look at Boxing," "Oil: Black Gold," "Digging into 3-D," and "Are You a Small Giant?" have primarily geometric themes.

"Skeeters Are Overrunning the World" examines exponential growth; "I'm Not So Sure Anymore' and "AIDS: The Preventable Epidemic" explore probability; "One Step Beyond" introduces step functions; and "From Rock Bands to Recursion" works with arithmetic and geometric series. "Going in Circuits" introduces graph theory.

The Teacher Edition

To facilitate use of the curriculum, the teacher edition contains these features:

Overview /Objectives/Prerequisites

Each module begins with a brief overview of its contents. This overview is followed by a list of teaching objectives and a list of prerequisite skills and knowledge.

Time Line/Materials & Technology Required

A time line provides a rough estimate of the classroom periods required to complete each module. The materials required for the entire module are listed by activity. The technology required to complete the module appears in a similar list.

Assignments/Assessment Items/Flashbacks

Assignment problems appear at the end of each activity. These problems are separated into two sections by a series of asterisks. The problems in the first section cover all the essential elements in the activity. The second section provides optional problems for extra practice or additional homework.

Specific assignment problems reconmended for assessment are preceded by a single asterisk in the teacher edition. Each module also contains a Summary Assessment in the student edition and a Module Assessment in the teacher edition, for use at the teacher's discretion. In general, Summary Assessments offer more open-ended questions, while Module Assessments take a more traditional approach. To review prerequisite skills, each module includes brief problem sets called "Flashbacks." Like the Module Assessment, they are designed for use at the teacher's discretion.

Technology in the Classroom

The *Integrated Mathematics* curriculum takes full advantage of the appropriate use of technology. In fact, the goals of the curriculum are impossible to achieve without it. Students must have ready access to the functionality of a graphing utility, a spreadsheet, a geometry utility, a statistics program, a symbolic manipulator, and a word processor. In addition, students should have access to a science interface device that allows for electronic data collection from classroom experiments, as well as a telephone modem.

In the student edition, references to technology provide as much flexibility as possible to the teacher. In the teacher edition, sample responses refer to specific pieces of technology, where applicable.

Professional Development

A program of professional development is recommended for all teachers planning to use the curriculum. The *Integrated Mathematics* curriculum encourages the use of cooperative learning, considers mathematical topics in a different order than in a traditional curriculum, and teaches some mathematical topics not previously encountered at the high-school level.

In addition to incorporating a wide range of context areas, *Integrated Mathematics* invites the use of a variety of instructional formats involving heterogeneous classes. Teachers should learn to use alternative assessments, to integrate writing and communication into the mathematics curriculum, and to help students incorporate technology in their own investigations of mathematical ideas.

Approximately 30 classroom teachers and 5 university professors are available to present inservice workshops for interested school districts. Please contact Kendall Hunt Publishing Company for more information.

Student Performance

During the development of *Integrated Mathematics*, researchers conducted an annual assessment of student performances in pilot schools. Each year, two basic measures—the PSAT and a selection of open-ended tasks—were administered to two groups: students in classes using *Integrated Mathematics* and students in classes using other materials. Students using *Integrated Mathematics* materials typically had access to technology for all class work. During administration of the PSAT, however, no technology was made available to either group. Student scores on the mathematics portion of this test indicated no significant difference in performance.

During the open-ended, end-of-year test, technology was made available to both groups. Analysis of student solutions to these tasks showed that students using *Integrated Mathematics* were more likely to provide justification for their solutions and made more and better use of graphs, charts, and diagrams. They also demonstrated a greater variety of problem-solving strategies and were more willing to attempt difficult problems.

References

Beal, J., D. Dolan, J. Lott, and J. Smith. *Integrated Mathematics: Definitions, Issues, and Implications; Report and Executive Summary*. ERIC Clearing-house for Science, Mathematics, and Environmental Education. The Ohio State University, Columbus, OH: ED 347071, January 1990, 115 pp.

Lott, J., and A. Reeves. "The Integrated Mathematics Project," *Mathematics Teacher* 84 (April 1991): 334–35.

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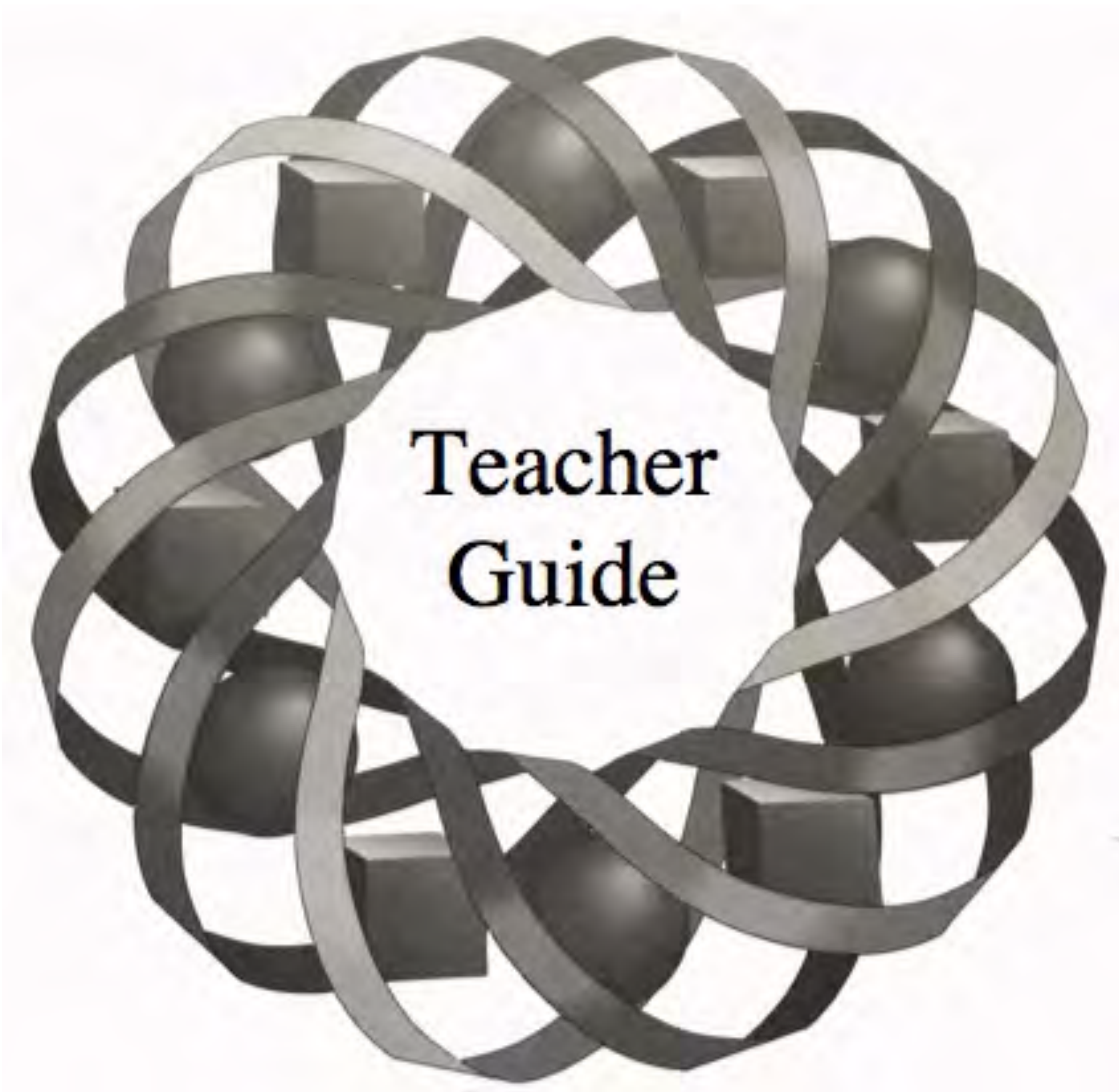
Reflect on This



What do you see when you look in the mirror? This module takes a peek at some concepts in both physics and geometry—through the looking glass.

Randy Carspecken • Bonnie Eichenberger • Darlene Pugh • Terry Souhrada

Teacher Edition

Reflect on This

Overview

In this module, students investigate reflections in a plane using mirrors (or other reflective materials) and a geometry utility. The properties of reflections are used to find shortest paths, predict pool shots, and design miniature golf holes.

Objectives

In this module, students will:

- use paper-folding to model reflections in a line
- use congruent, complementary, and supplementary angle relationships to make conjectures
- use congruent segments and the shortest distance between two points to make conjectures about the paths traveled by light rays
- examine the perpendicular bisector relationships created by reflections
- explore the relationships between the coordinates of a point and the coordinates of its image under a reflection in the x- or y-axis
- be introduced to the relationship between theorems and conjectures.

Prerequisites

For this module, students should know:

- the geometric notation for points, segments, lines, and angles
- how to measure and draw angles with a protractor
- the definition of an isosceles triangle
- the concepts of distance, congruence, collinearity, perpendicularity, and parallelism
- how to make scale drawings
- how to plot points on a coordinate plane.

Time Line

Activity	Intro.	1	2	3	4	Summary Assessment	Total
Days	1	1	2	3	2	2	11

Materials Required

Materials	Activity					
	Intro.	**1**	**2**	**3**	**4**	**Summary Assessment**
straightedge		X	X	X	X	
protractor		X	X			
mirrors	X	X	X		X	
tinted plastic reflectors (such as MIRAs™ or Reflectas™)				X		
fiberboard blocks			X			
pushpins			X			
colored paper or commercial confetti	X	X				
tape	X	X			X	
graph paper			X	X	X	
rubber bands			X			
plain white paper	X	X				
scissors	X					

Technology

Software	Activity					
	Intro.	**1**	**2**	**3**	**4**	**Summary Assessment**
geometry utility				X		

Reflect on This

Introduction (page 3)

Students build model kaleidoscopes using hinged mirrors, colored paper, and confetti. This model provides a preview of the types of reflections students will study throughout the module. **Note:** You also may wish to show students some actual kaleidoscopes.

Materials List

- mirrors (about $10 \text{ cm} \times 12 \text{ cm}$; two per group)
- tape (one roll per group)
- colored paper (several sheets per group) or commercial confetti
- plain white paper (one sheet per group)
- scissors (one pair per group)

Exploration (page 3)

a–d. Students build a model kaleidoscope.

e. As students open and close the mirrors, the number of reflections seen will vary. Students should observe images that consist of one or more isosceles triangles.

Discussion (page 3)

a. The patterns consist of one or more isosceles triangles, with each overall pattern forming a regular polygon.

b. As the mirrors are opened and closed, the number of reflections seen changes.

c. As the measure of the hinge angle decreases, the number of reflections seen increases.

(page 4)

Activity 1

Students use multiple reflections to create regular polygons and determine the relationship between the number of sides in a regular polygon and the measure of its central angles.

Materials List

- mirrors (about $10 \text{ cm} \times 12 \text{ cm}$; two per group)
- tape (one roll per group)
- colored paper (several sheets per group)
- plain white paper (one sheet per group)
- protractor (one per group)
- straightedge (one per group)

Exploration

(page 4)

a–b. Students should discover that a hinge angle of 120° is needed to form a triangle.

c. Sample table:

Polygon	No. of Sides	Measure of Hinge Angle
triangle	3	120°
quadrilateral	4	90°
pentagon	5	72°
hexagon	6	60°
heptagon	7	$51\frac{3}{7}^\circ$
octagon	8	45°
nonagon	9	40°
decagon	10	36°

d. Students try to discover a relationship between the number of sides in a polygon and the measure of the hinge angle. All of the following relationships are true:

- (number of sides)(measure of hinge angle) = 360°
- $\dfrac{360^\circ}{\text{number of sides}}$ = measure of hinge angle
- $\dfrac{360^\circ}{\text{measure of hinge angle}}$ = number of sides

Discussion

(page 5)

a. As the measure of the hinge angle becomes smaller, the number of reflections seen increases. Hence, the number of sides seen also increases.

b. The polygons formed by the reflections are always regular polygons because a reflection in a flat mirror preserves all length and angle measures of the original object. In other words, the length of the reflected segment is always the same as the length of the original segment. Likewise, the measure of the reflected angle is always the same as the measure of the original angle. Mirrors that are not flat do not always preserve length and angle measures.

c.
1. The product of the number of sides in the polygon and the measure of its central angle is always 360°.
2. This relationship may be expressed as follows, where *n* represents the number of sides and *a* represents the measure of the central angle:

$$n \bullet a = 360^\circ \text{ or } \frac{360^\circ}{a} = n \text{ or } \frac{360^\circ}{n} = a$$

d. Every regular polygon has the same number of central angles as its number of sides.

e. The congruent isosceles triangles in Figure **5** are ΔAOB, ΔAOC, and ΔBOC.

f. The vertex angle of any one of the isosceles triangles is congruent to a central angle of the polygon.

g. The measure of each base angle of the isosceles triangles is equal to one-half the measure of an interior angle of the polygon.

Assignment

(page 6)

***1.1**
- **a.** 30°
- **b.** 20°
- **c.** $17\frac{1}{7}^\circ \approx 17.14^\circ$
- **d.** $360^\circ/n$

***1.2**
- **a.** 9 sides
- **b.** 18 sides
- **c.** 15 sides
- **d.** If the measure of the central angle is 0°, a polygon is not formed. (This is the limiting case; the limit is a circle, not a polygon.)

1.3 Answers may vary. Some sample responses are given below.

a. Reflect a point in a mirror. Draw one line along one mirror and another line that seems to connect the point and its reflection.

b. Open the hinged mirrors until a triangle appears (as in Part **b** of the exploration). Since the hinge angle formed has a measure of 120°, tracing the angle formed by the two mirrors gives the desired angle.

c. Draw a segment 5 cm long. Open the hinged mirrors until the segment is reflected into a regular hexagon. Sketch the isosceles triangle that is formed by the central angle and the segment. Keeping the hinge angle constant and the vertex of the hinge angle on the vertex of the isosceles triangle, rotate the mirrors left (or right) until only one mirror is on a side of the isosceles triangle. Sketch the central angle that is formed by the mirrors. Connect the endpoints of the angle to form the second side of the hexagon. Continue this process until all six sides have been sketched.

* * * * *

1.4 **a.** 72°

b. 45°

c. 36°

d. $360^\circ/n$

1.5 **a.** 54°

b. 67.5°

c. 72°

d. $\dfrac{180^\circ-(360^\circ/n)}{2}$

1.6 **a.** **1.** 108°

2. 135°

3. 144°

4. $180^\circ-\dfrac{360^\circ}{n}$

b. **1.** 540°

2. 1080°

3. 1440°

4. $180^\circ(n)-360^\circ$ or $180^\circ(n-2)$

1.7 **a.** The area of a regular pentagon is 5 times the area of one of the congruent isosceles triangles.

b. The area of a regular octagon is 8 times the area of one of the congruent isosceles triangles.

c. The area of a regular decagon is 10 times the area of one of the congruent isosceles triangles.

d. The area of a regular *n*-gon is *n* times the area of one of the congruent isosceles triangles.

* * * * * * * * * *

(page 8)

Activity 2

Students discover that the paths of light rays reflecting off a surface form congruent angles. The conventions introduced here for naming these angles are used for the remainder of the module. Students also discuss experimental error and its implications when making conclusions.

Materials List

- graph paper (one sheet per group; a blackline master appears at the end of the teacher edition FOR THIS MODULE)
- half-inch fiberboard blocks (about 12 cm ∞ 12 cm; one per group)
- pushpins (two per group)
- rubber bands (one per group)
- flat mirrors (about 10 cm ∞ 12 cm; one per group)
- protractors (one per group)
- straightedge (one per group)

Exploration

(page 8)

a–c. Students draw axes, label points, and position the graph paper as indicated in Figure **7**.

d–e. Students choose points *P* and *Q* accordingly, label them, find the reflection of point *P* in the mirror, and determine the placement of point *C*.

f. Students use a rubber band to check the placement of point *C*.

g. Students draw the path of light from *Q* to the mirror to *P*, then measure the incoming and outgoing angles. Students should discover that these angles are congruent. **Note:** Remind students to save a copy of the graph for use in the assignment and in Activity **3**.

Teacher Note

You may wish to ask students to collect the class data for the angle measures in Part **g** and organize them in a table for the discussion.

Discussion

(page 10)

a. Since students look from point P to the mirror, the light rays travel from Q to the mirror, then from the mirror to the eye. Therefore, $\angle QCB$ is the incoming angle, while $\angle PCA$ is the outgoing angle.

b. Student data should suggest that the measures of the incoming and outgoing angles are congruent.

c. Due to inaccuracies in measurement, some pairs of angle measures may appear to contradict the conjecture.

d. Sample response: Errors might occur while aligning the mirror on the x-axis, locating the position of point C, and measuring the angles with a protractor. Such errors could make the relationship between the angle measures harder to discover.

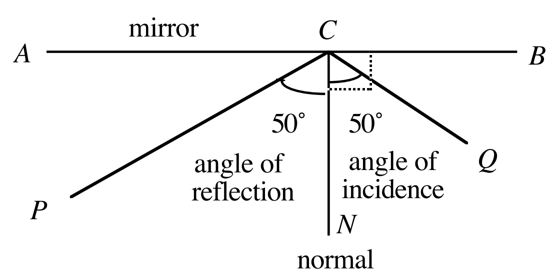
e. Answers will vary. Some students may suggest using technology to test their conjecture.

f. The incoming angle is the complement of both the angle of incidence and the angle of reflection. The same is true of the outgoing angle.

Assignment

(page 11)

2.1 **a.** Sample response:



b. **1.** The angle of incidence is congruent to the angle of reflection.

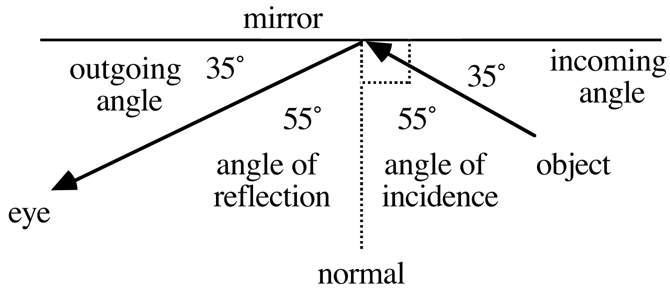
2. The incoming angle and angle of incidence are complementary.

3. The outgoing angle and angle of reflection are complementary.

c. Since the normal is perpendicular to the mirror, the sum of the measures of the incoming angle, $\angle BCQ$, and the angle of incidence, $\angle QCN$, is 90°, illustrative that the two angles are complimentary. Likewise, the sum of the measures of the outgoing angle, $\angle ACP$, and the angle of reflection, $\angle PCN$, is 90°, therefore these two angles are complimentary.

Since $m\angle BCQ + m\angle QCN = 90^\circ$ and $m\angle BCQ = m\angle ACP$, then $m\angle ACP + m\angle QCN = 90^\circ$. Also, since $m\angle ACP + m\angle PCN = 90^\circ$, then $m\angle QCN = m\angle PCN$, showing the angle of incidence is congruent to the angle of reflection.

***2.2** **a–c.** Sample response:



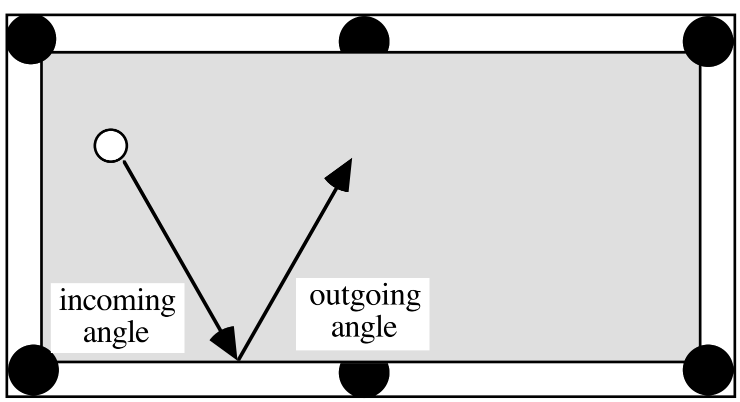
***2.3** **a.** Student diagrams should resemble the sample given in Problem **2.2**.

b. Sample response: When a ray of sunlight hits the watch, it reflects off the watch crystal. The angle at which the ray hits the watch (incoming angle) is congruent to the angle at which the ray reflects off the watch (outgoing angle). The angle of incidence is congruent to the angle of reflection. The incoming angle and angle of incidence are complementary, as are the outgoing angle and angle of reflection.

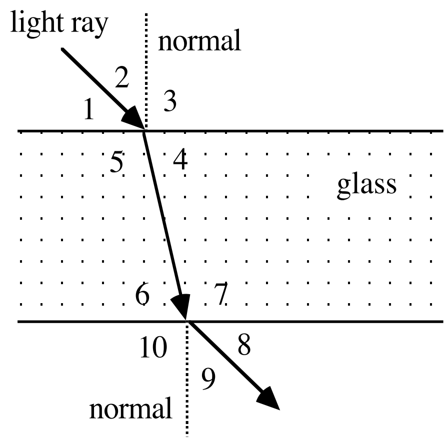
* * * * *

2.4 Sample response: As the driver adjusts the mirror, the incoming and outgoing angles for light rays change. This changes the area to the side or behind the car that the driver can see.

2.5 Sample response: When the ball hits the side rail, it will bounce off with an outgoing angle congruent to its incoming angle. This path will not put the ball in the upper right corner pocket, as shown in the diagram below.



2.6 a–b. Sample response:



c. If the angles are numbered as shown in the diagram above, $\angle 1 \cong \angle 8$, $\angle 2 \cong \angle 9$, $\angle 4 \cong \angle 6$, $\angle 5 \cong \angle 7$, and $\angle 3 \cong \angle 10$.

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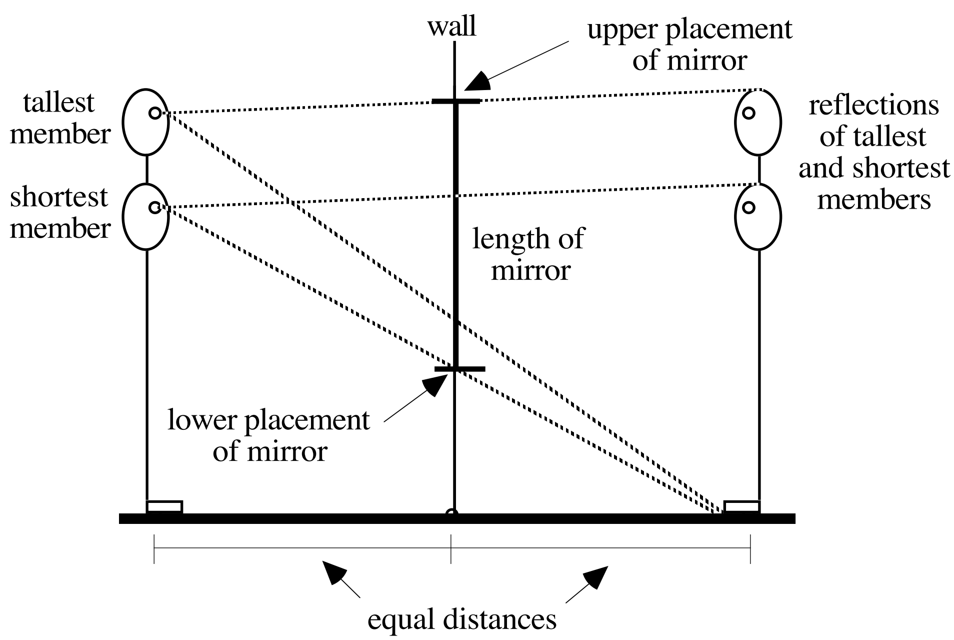
Research Project (page 13)

1. The length of the smallest mirror needed depends on the heights of the tallest and shortest members of the family. Since the tallest person must be able to see the reflection of the top of her head, the top of the mirror must be placed at this point of reflection, as shown in the following diagram. Likewise, since the shortest person must be able to see the reflection of his feet, the bottom of the mirror must be placed at this point of reflection. The mirror length can be determined by finding the distance between these two points of reflection.

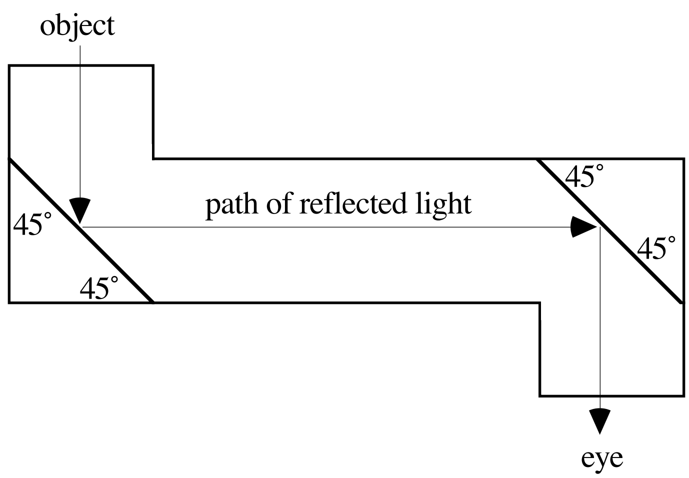


2. Student reports should include the following information.

a. A periscope is an apparatus with a tube and mirrors.

b. Periscopes are used to see an otherwise obstructed field of view. For example, a submarine commander might use a periscope to see an object above the water, while someone in the back of a crowded concert hall might use a periscope to see the performers.

c–d. The mirrors in a periscope are placed so that incoming light reflects off each mirror and into the eye. A simple periscope may be made using mirrors and cardboard. Sample diagram:



(page 13)

Activity 3

Students use a tinted plastic reflector and a coordinate grid to find the image of a reflection in a line. This process is then modeled on a geometry utility.

Materials List

- tinted plastic reflectors (such as MIRAs™ or Reflectas™ ; one per group)
- straightedge (one per group)

Technology

- geometry utility

Exploration 1 (page 13)

a–c. Students make additions to their sketches from the exploration in Activity **2**.

d.

1. Students should observe that the distance from the *x*-axis to Q is the same as the distance from the *x*-axis to Q'.

2. Students should discover that the distance from the *x*-axis to P is the same as the distance from the *x*-axis to P'.

e. The distance from the line of reflection to the object is the same as the distance from the line of reflection to the image.

Discussion 1 (page 14)

a. The pinholes should correspond with the locations of P' and Q' found using the reflector. (Some possible sources of error include the placement of the reflector, the thickness of the reflector, and the subjectivity involved in judging the position of the image.)

b. The two distances should be equal.

c.

1. Both segments should intersect at point C.

2. Sample response: This indicates that reflected light rays travel in the same way, regardless of direction. In other words, light travels from P to Q along the same path that it travels from Q to P.

d. Although some errors in measurement may occur, the class data should support the conjecture. You may wish to discuss how close two measurements should be to be considered equal in this situation.

Exploration 2 (page 15)

Students use a geometry utility to continue their investigations of reflections in a line. **Note:** For best results, students should set the measurement preferences on their geometry utilities so that angles are measured to the nearest degree and lengths to the nearest 0.1 units.

a. Student constructions should resemble the one shown in Figure **11**.

b. From Step **9**, students should observe that $m\angle QEB = 90^\circ$. From Step **11**, they should observe that $EQ = EQ'$. This should lead to the conjecture that $\overline{AB}$ is the perpendicular bisector of $\overline{QQ'}$.

c. Students should note that, even as the lengths of the segments and positions of the points change, the relationships among them do not. For example, the measures of the incoming and outgoing angles remain congruent and $\overline{AB}$ remains the perpendicular bisector of $\overline{QQ'}$.

d. Students should discover that the shortest total distance occurs when *S* and *C* are concurrent.

Discussion 2

(page 17)

a. **1.** Students should conjecture that $\overline{AB}$ is the perpendicular bisector of $\overline{PP'}$.

2. Yes. Since $\overline{DP} \cong \overline{DP'}$ and $\angle PDA$ is a right angle, $\overline{AB}$ is the perpendicular bisector of $\overline{PP'}$.

b. Sample response: Since the shortest total distance occurred when *S* was at the same position as *C*, the path with the shortest total distance always passes through the reflection point on the mirror line.

c. **1.** Since $m\angle QEB$ is 90°, the *x*-coordinates of *Q* and Q' are the same.

2. Since $\overline{EQ}$ and $\overline{EQ'}$ are congruent, the *y*-coordinates of *Q* and Q' have the same absolute value, but are opposites.

3. $\overline{DP}$ and $\overline{DP'}$

d. **1–2.** The *x*-coordinates are the same, while the *y*-coordinates are opposites.

3. Yes, since the mirror was placed along the *x*-axis.

e. **1.** Under a reflection in the *x*-axis, the image of a point (x, y) has coordinates $(x, -y)$.

2. Under a reflection in the *y*-axis, the image of a point (x, y) has coordinates $(-x, y)$.

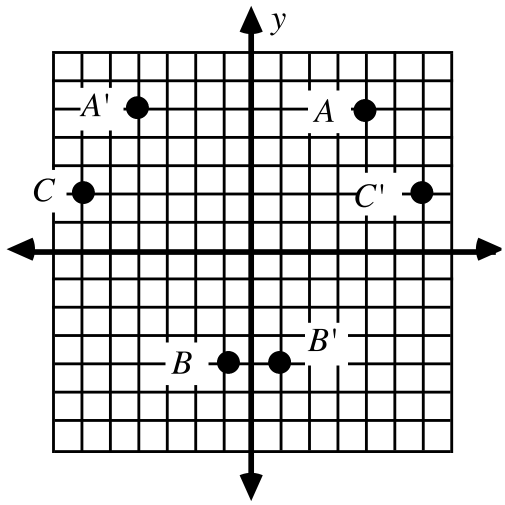
f. **1.** Student measurements should uphold the conjecture that the incoming and outgoing angles are congruent.

2. Yes, this conjecture becomes a theorem only if proved in general for all cases. One approach might be to demonstrate that ΔPDC is similar to ΔQEC, then show that $\angle PCD \cong \angle QCE$.

Assignment

(page 18)

***3.1** **a–c.** Answers will vary. The following sample graph shows the points $A(4,5)$, $B(-1,-4)$, and $C(-6,2)$. The coordinates of the corresponding image points are $A'(-4,5)$, $B'(1,-4)$, and $C'(6,2)$.

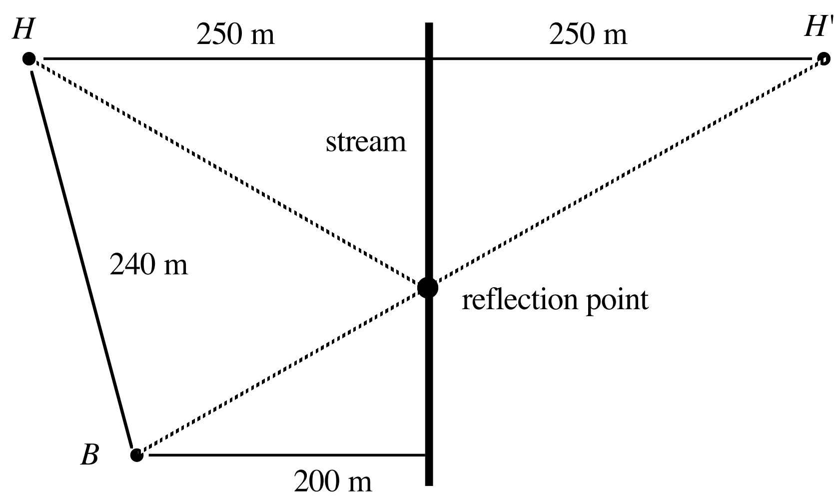


d. The coordinates of D' are $(-x,y)$.

3.2 **a.** In a plane, the shortest path between two points is always the line segment connecting the points.

b. Sample response: The lengths of the paths are equal. The distance from *P* to *C* is the same in both cases, and the distance from *Q* to *C* is the same as the distance from Q' to *C*. This can be seen by folding the graph paper along the line of reflection.

***3.3** **a–b.** Sample response:



c. Sample response: To find the shortest distance, you must reflect the house in a line that represents the bank of the stream, then connect the image of the house to the barn. The point where the segment intersects the stream is the point of reflection, where the incoming angle of the path equals the outgoing angle of the path. The path that connects the house to the point of reflection, then to the barn, represents the shortest path.

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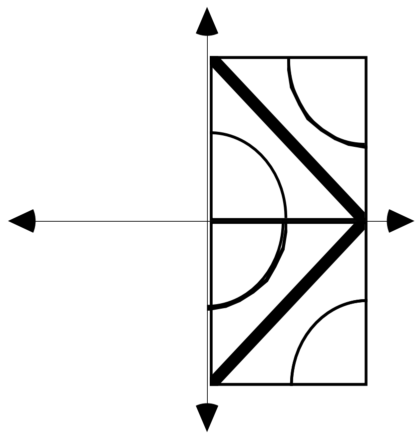
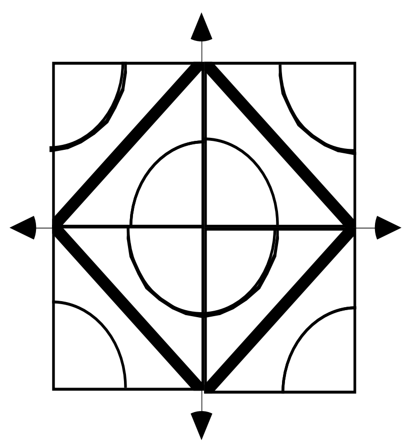
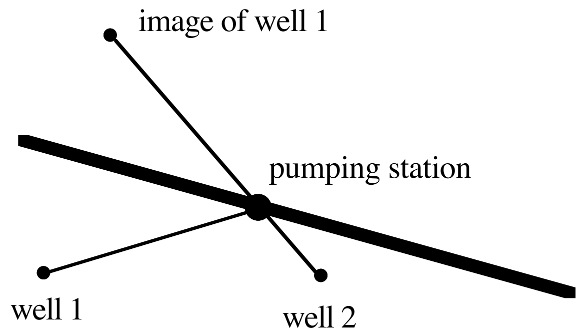
3.4 **a.** A reflection in the *x*-axis results in the graph below:



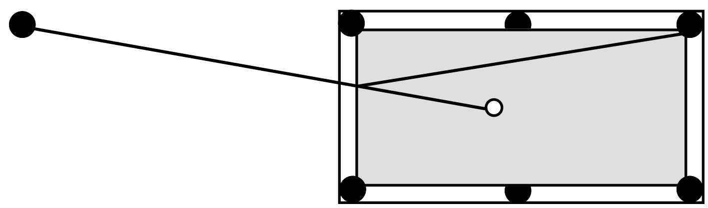
b. The reflection of the graph in Part **a** in the *y*-axis produces the following graph:



3.5 The point identified in the diagram below is the best location for the pumping station. This location provides the shortest total distance from the pumping station to both wells.

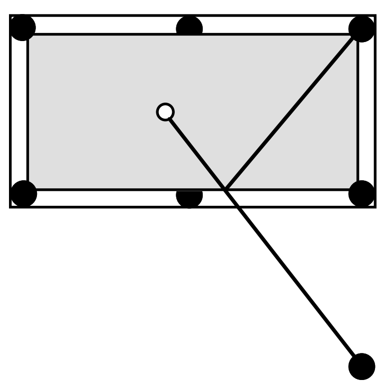


3.6 **a.** Answers may vary. The following diagram shows one possible shot. The paths of other shots may be determined by reflecting the corresponding pocket in a similar manner.

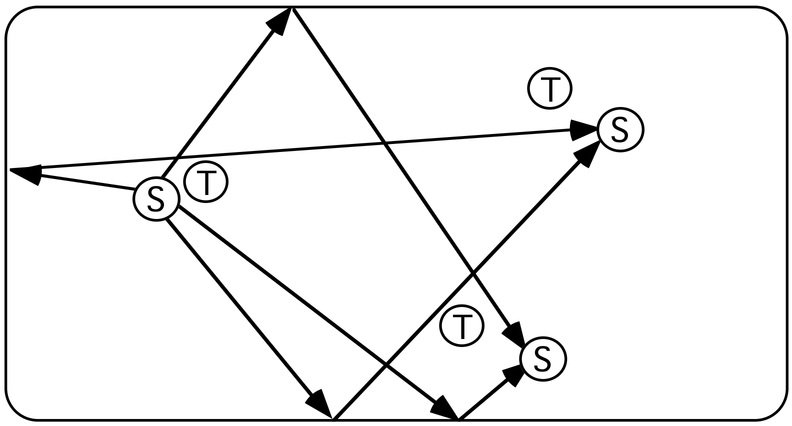


b. Sample response: The segment connecting the ball to the image of the pocket locates the point of reflection on the side rail. This is the point where the cue ball needs to be banked.

c. Another path to the same pocket may be determined by reflecting the pocket in a different side than the one in Part **a**. Sample response:



3.7 The diagram below shows the paths of four successful passes.



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(page 20)

Activity 4

In this activity, students investigate double reflections using mirrors and a geometry utility.

Materials List

- straightedge (one per group)
- flat mirrors (two per group)
- tape (one roll per group)
- graph paper (one sheet per group)

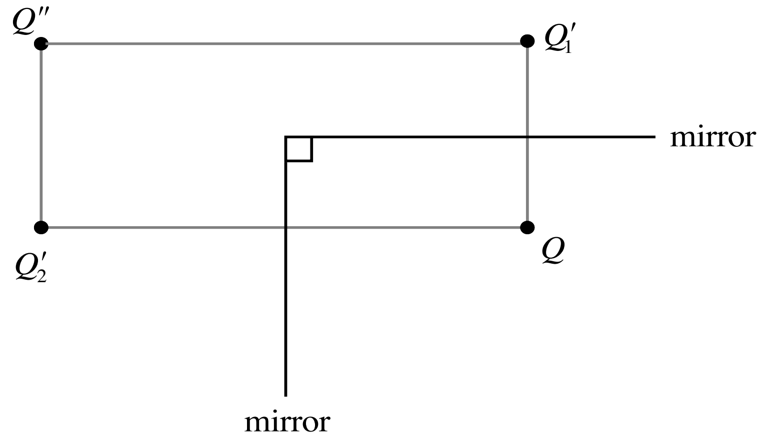
Exploration 1

(page 20)

Students explore the double reflection of an object using hinged mirrors.

a–c. Students may predict that they will see two virtual images, one from each mirror. However, there are actually three images—two from single reflections and one from a double reflection of point *Q*.

d. Student diagrams should resemble the figure below. The order of reflection does not affect the position of Q''.



Note: Students should save their diagrams for use in Exploration **2**.

Discussion 1

(page 21)

a. Although most students will have predicted two virtual images, they should have seen three virtual images in the two mirrors.

b. Two of the virtual images are each produced by a single reflection in one of the two mirrors. The third virtual image is the result of a double reflection involving both mirrors.

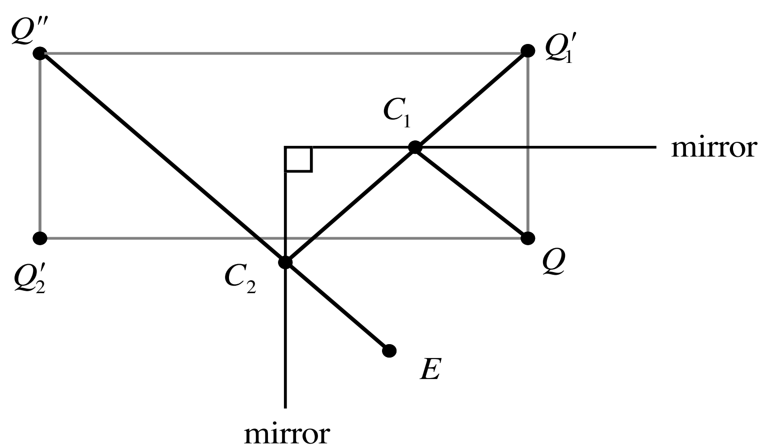
c. The image of a triple reflection could be labeled Q'''.

d. Kaleidoscopes use two or more mirrors to produce many images and create elaborate designs.

Exploration 2

(page 22)

a–c. Students construct the path that light follows in a double reflection. Their final diagrams should resemble the one shown below:



Discussion 2

(page 22)

a. The path represents light traveling in a double reflection from point Q to point E.

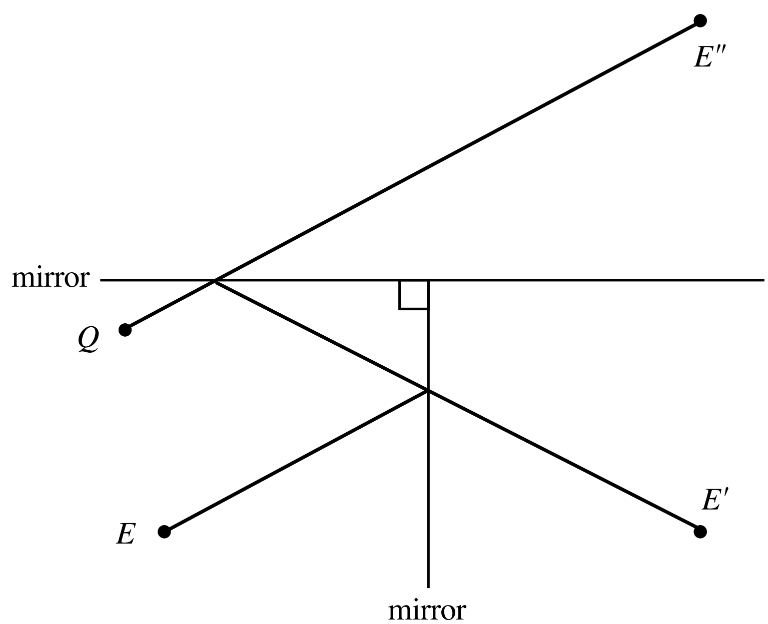
b. Points C_1 and C_2 represent the points of reflection in the two mirrors.

c. The virtual image appears where it does in the mirror because the light ray involved in the double reflection comes from that direction toward your eye (Q'', C_2, and E are all collinear).

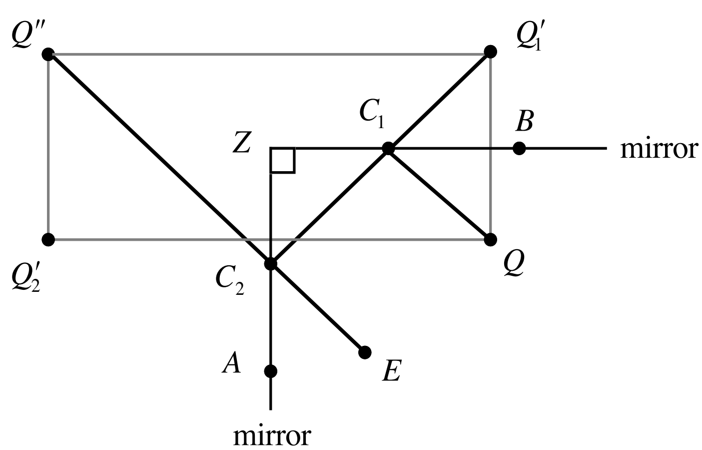
Assignment

(page 23)

4.1 Sample response:



4.2 **a.** Sample diagram: Q''



b. Answers will vary. Sample response: The measure of the incoming angle $\angle ZC_2C_1$, to the nearest degree, is 37°.

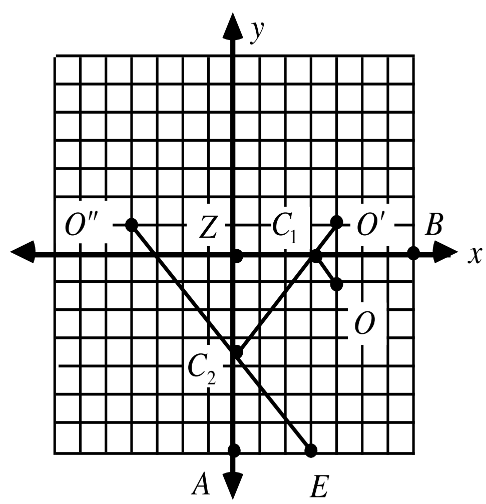
c. Since $m\angle C_2ZC_1$ is 90° and the sum of the angles in ΔC_2ZC_1 is 180°, $\angle ZC_1C_2$ and $\angle ZC_2C_1$ are complements. For the sample response in Part **a**, $m\angle ZC_1C_2 = 90° - 37° = 53°$. The remaining two angles are congruent to their companion outgoing or incoming angles.

d. Sample response: Incoming angle $\angle BC_1Q$ and outgoing angle $\angle ZC_1C_2$ both measure 53°; incoming angle $\angle ZC_2C_1$ and outgoing angle $\angle AC_2E$ both measure 37°.

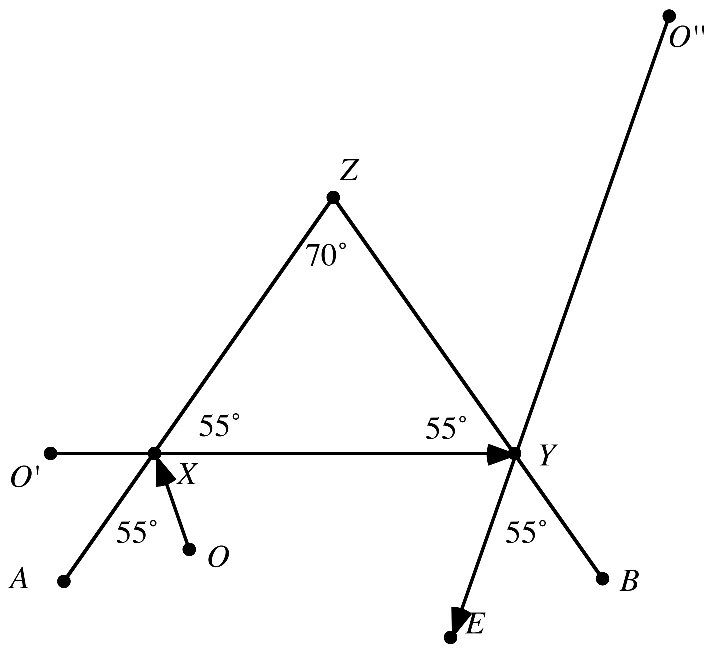
e. **1.** The sum of the four angles is 180°.

2. When the two mirrors are hinged at 90°, the path of light in a double reflection—like the path of a pool ball banked off two sides—reverses its direction.

4.3 In the following diagram, the path of light follows $\overline{OC_1}$, $\overline{C_1C_2}$, and $\overline{C_2E}$. Incoming angle $\angle BC_1O$ is congruent to outgoing angle $\angle ZC_1C_2$; both angles measure 49° (to the nearest degree). Incoming angle $\angle C_1C_2Z$ is congruent to outgoing angle $\angle AC_2E$; both angles measure 41° (to the nearest degree).



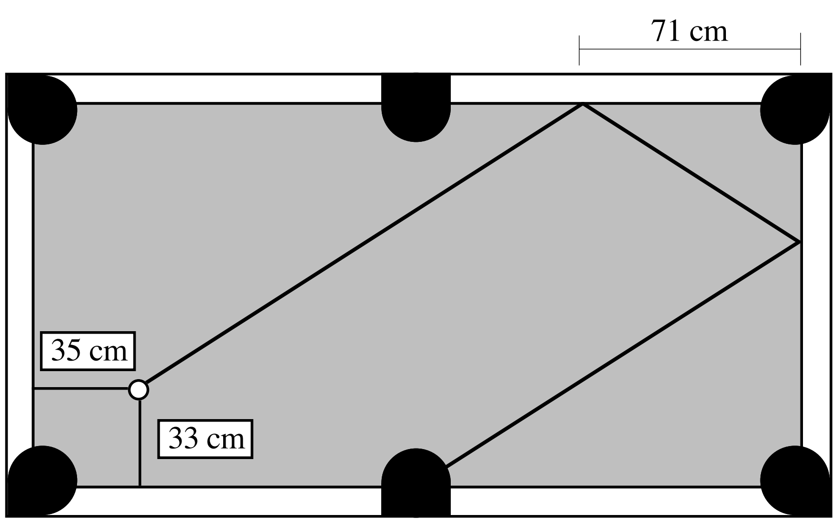
***4.4** **a–e.** Responses will vary, depending on the locations of points *O* and *E*. Sample response:



f. In the sample drawing above, $\angle YXZ$ and $\angle BYE$ are incoming angles; $\angle AXO$ and $\angle XYZ$ are outgoing angles.

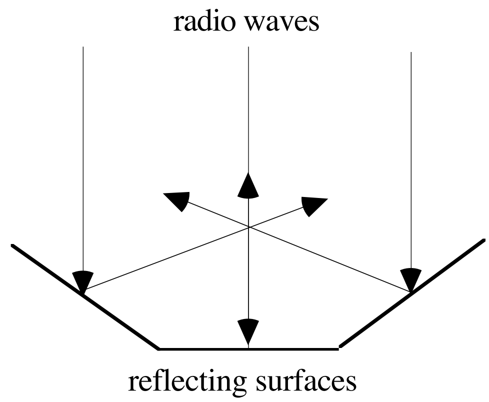
g. The sum of the measures of the four angles is 220˚. Unless students stipulate that the hinge angle must measure 90˚ in their responses to Problem **4.2**, this sum should contradict their conjectures.

***4.5** By creating their scale drawings near the center of a sheet of graph paper, students can use paper folding to locate reflections. Sample response:

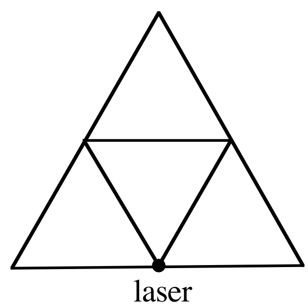


* * * * *

4.6 As shown in the diagram below, the outgoing angle for each wave is congruent to the incoming angle. The middle wave reflects back along its original path.



4.7 **a.** The following diagram shows the only possible path.



b. The measure of each incoming and outgoing angle is 60˚. The measure of each angle of incidence and angle of reflection is 30˚.

c. The four small triangles are congruent equilateral triangles. They are similar to the original triangle.

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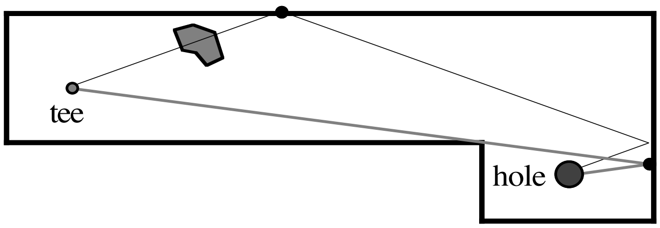
(page 26)

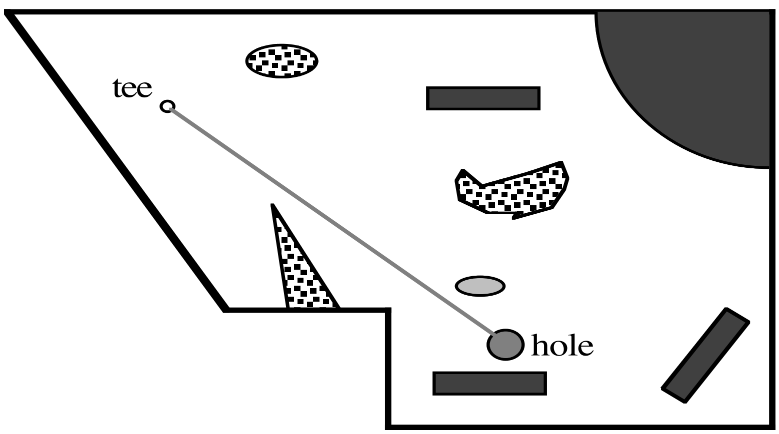
Summary Assessment

1. Some sample responses are shown below.

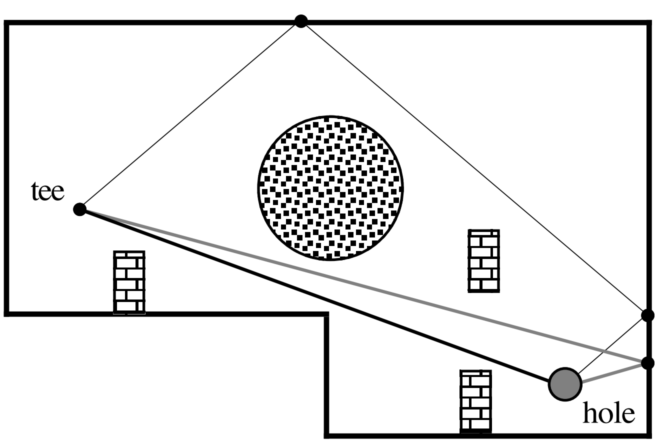
a. This hole looks simple, but a hole-in-one is impossible.



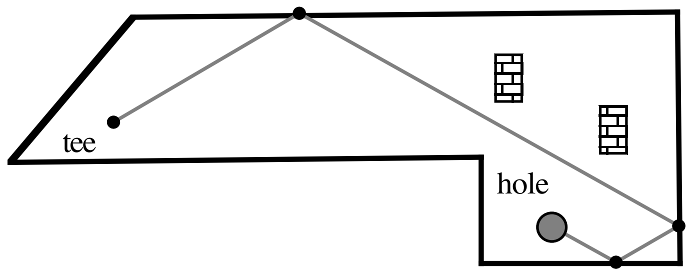
b. This hole looks difficult, but actually has a simple path for a hole-in-one.



c. This hole has at least three possible paths for a hole-in-one.



d. This hole requires a player to bank the ball off exactly three walls to get a hole-in-one.



2. Students test each other's designs by trying to sketch the path of a hole-in-one.

3. Students may have used one or more of the following methods when creating their designs:

- Determine the placement of the ball in the tee area. Choose a starting angle and direction for the shot. Given this angle and direction, determine the ball's path using incoming angles and outgoing angles. Place the hole in a location that aligns with the ball's path.
- Determine the placement of the ball in the tee area and the placement of the hole. Using trial-and-error and incoming and outgoing angles, determine a path from the ball to the hole.
- Determine the placement of the ball in the tee area and the placement of the hole. Using lines of reflection, images, preimages, and points of reflection, determine a path from the ball to the hole.

Module Assessment

1. **a.** Using a protractor and straightedge, draw a diagram of a light ray reflecting off a flat surface with an incoming angle of 20°.

b. Draw and label a normal to the surface.

c. Label and measure all the angles in the diagram.

2. Given points $A(3,-2)$ and $B(-2,1)$, determine the coordinates of A' and B' for each of the following:

a. a reflection in the *x*-axis

b. a reflection in the *y*-axis.

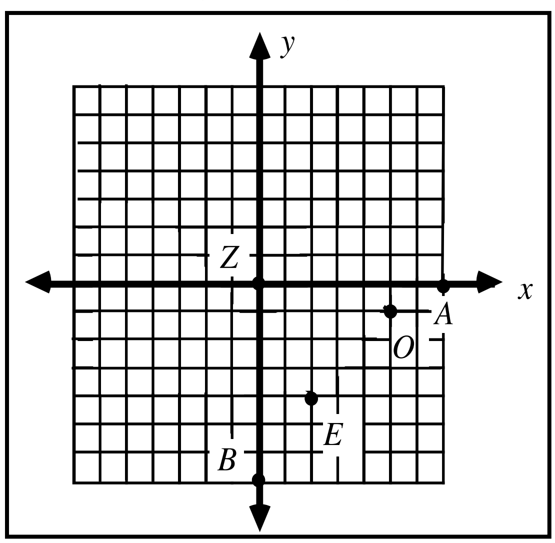
3. Mr. Ned is an intelligent, but lazy, horse. From his place in the pasture, he wants to walk to the stream for a drink, then back to the barn for a nap.

a. Draw a model that shows the shortest route Mr. Ned could take.

b. Explain why the route you identified in Part **a** is the shortest possible.

c. Identify a segment in your model from Part **a** that is a perpendicular bisector and describe its geometric features.

4. In the following diagram, point *O* represents an object and point *E* represents the eye. **Note:** The intersection of the *x*-axis and the *y*-axis of a Cartesian coordinate system always forms an angle of 90°.



a. If mirrors were placed on both axes in the diagram, determine the path a light ray would travel from *O* to *E* in a double reflection.

b. Label the points of reflection on the *x*- and *y*-axes C_1 and C_2, respectively.

c. Label the two incoming and two outgoing angles and give their measures.

d. Identify all the congruent angles and all the complementary angles.

e. Describe the relationship between the path of the light ray from O to C_1 and the path of the light ray from C_2 to E.

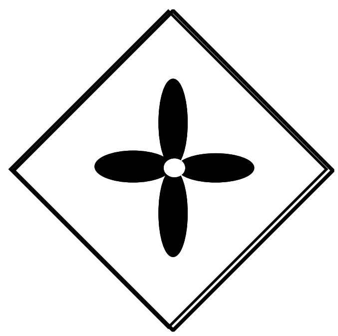
5. Find the measure of the central angle for each of the following:

a. a regular pentagon

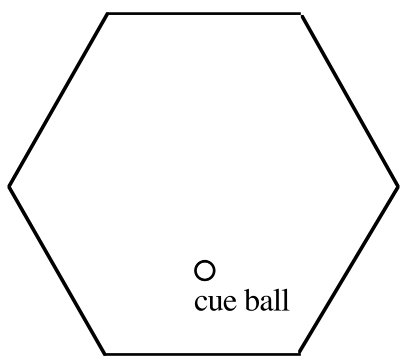
b. a regular decagon

c. a regular n-gon.

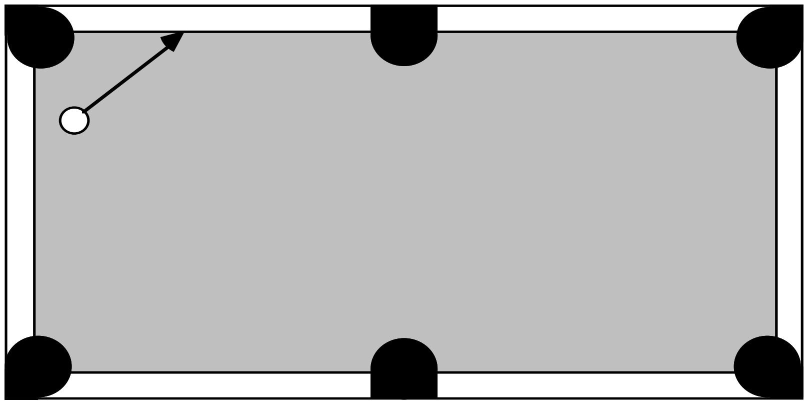
6. Describe how the reflections in two hinged mirrors could be used to create the pattern below.



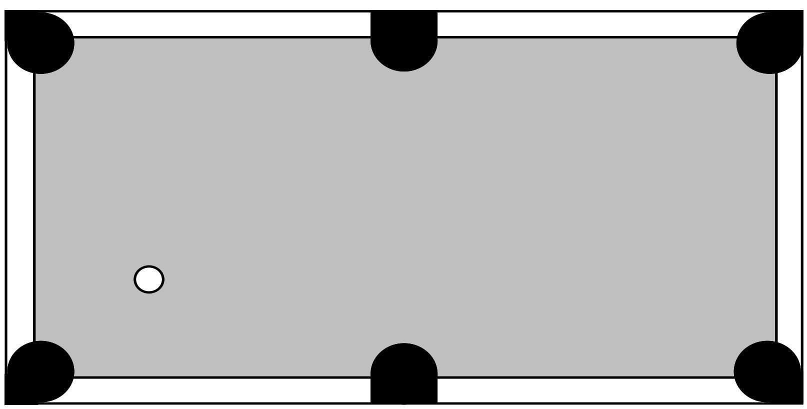
7. The diagram below shows a pool table shaped like a regular hexagon. Draw the path of a shot in which the ball bounces off exactly two side rails before returning to its original position.



8. The following diagram shows the path of a pool ball hit toward the side rail. If the ball is hit with enough force, will it eventually roll into one of the pockets? Justify your response, including a drawing.

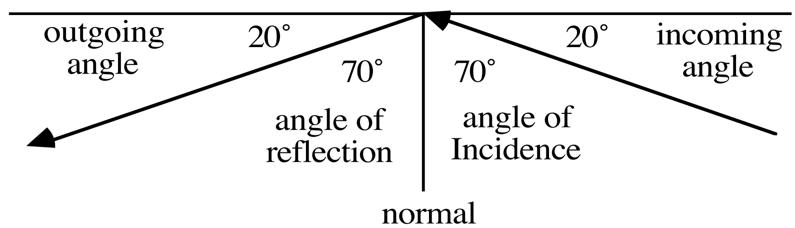


9. The diagram below shows the location of a ball on a pool table. On a copy of this diagram, sketch the path of the ball that bounces off two side rails before falling into the upper right-hand corner pocket.



Answers to Module Assessment

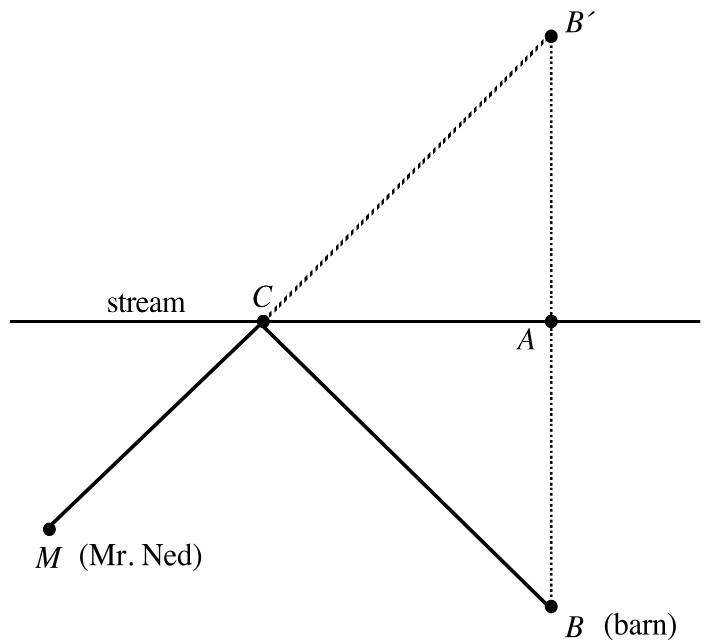
1. **a–c.** Sample response:



2. **a.** A' (3,2) and B' (–2,–1)

b. A' (–3,–2) and B' (2,1)

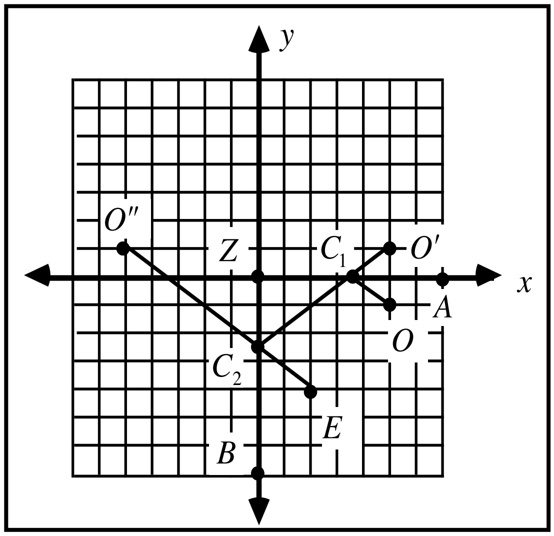
3. **a.** Sample response:



b. Sample response: In this case, the stream is the mirror line, B' is the image of the barn reflected in the stream, and *C* is the reflection point. The path with the shortest total distance always passes through the reflection point on the mirror line.

c. In the sample diagram above, the segment that represents the stream is a perpendicular bisector of segment $\overline{BB'}$. The perpendicular bisector divides $\overline{BB'}$ into two congruent segments, $\overline{BA}$ and $\overline{AB'}$. It also intersects $\overline{BB'}$ at a 90° angle. Therefore, $\angle CAB$ and $\angle CAB'$ both measure 90°.

4. **a–b.** Sample response:



c. The two incoming angles are $\angle AC_1O$ and $\angle C_1C_2Z$. Their measures are 36° and 54°, respectively. The two outgoing angles are $\angle C_2C_1Z$ and $\angle BC_2E$. Their measures are also 36° and 54°, respectively.

d. Since an incoming angle is congruent to its outgoing angle, $\angle AC_1O$ is congruent to $\angle C_2C_1Z$ are congruent, and $\angle BC_2E$ is congruent to $\angle C_1C_2Z$.

Since $m\angle C_2ZC_1$ is 90° and the sum of the angles of a triangle is 180°, $\angle C_2C_1Z$ and $\angle C_1C_2Z$ are complementary. Because $\angle C_2C_1Z$ is congruent to $\angle AC_1O$, $\angle AC_1O$ and $\angle C_1C_2Z$ are also complementary. Similarly, $\angle C_2C_1Z$ and $\angle BC_2E$ are complementary, as well as $\angle AC_1O$ and $\angle BC_2E$.

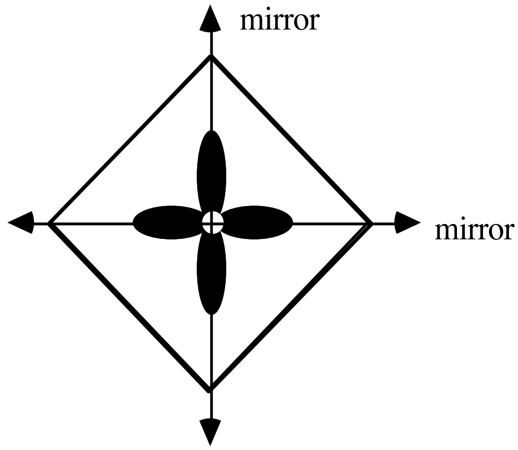
e. The path of the light ray from O to C_1 is parallel to the path of the light ray from C_2 to E., but their directions are opposite.

5. **a.** 72°

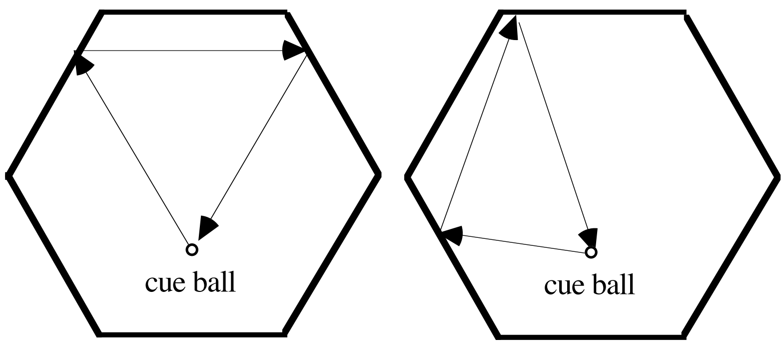
b. 36°

c. 360°/n

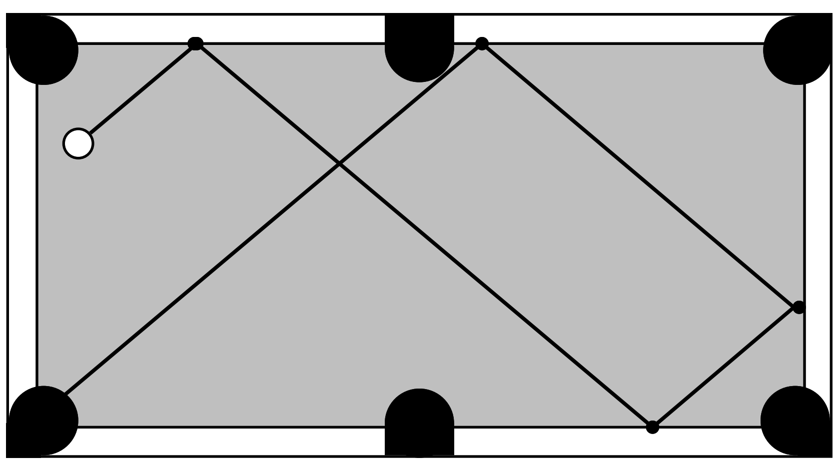
6. As shown in the diagram below, the two mirrors should be placed along two perpendicular axes, with their intersection at the center of the pattern:



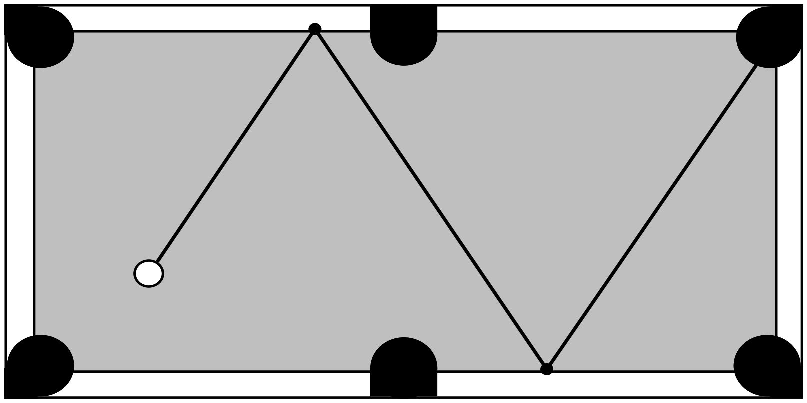
7. Answers may vary. The diagram below shows two possible paths.



8. The ball will fall in the lower left-hand corner pocket after four banks, as shown in the diagram below.



9. One possible double-bank shot is shown below.



Selected References

Iowa Academy of Science. *Physics Resources and Instructional Strategies for Motivating Students*. Cedar Falls, IA: University of Iowa, 1985.

Murphy, J., and R. Smoot. *Physics Principles and Problems*. Columbus, OH: Charles E. Merrill Publishing Co., 1977.

Flashbacks

Activity 1

1.1 Make a sketch and write a short definition for each of the following geometric figures:

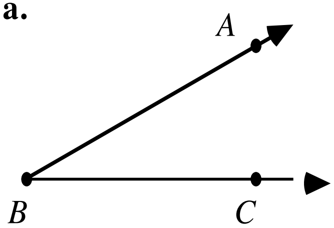
- **a.** an isosceles triangle
- **b.** a square
- **c.** a pentagon
- **d.** an octagon.

1.2 Draw an angle with each of the following measures:

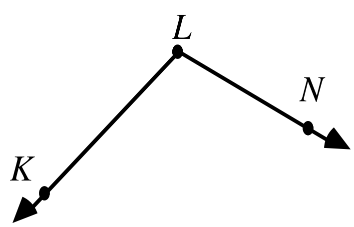
- **a.** 35˚
- **b.** 90˚
- **c.** 120˚

1.3 Estimate the measures of the two angles below. Use a protractor to check your estimates.

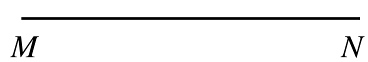
a.



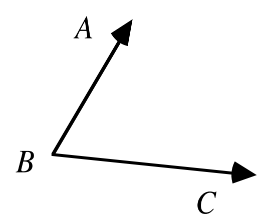
b.



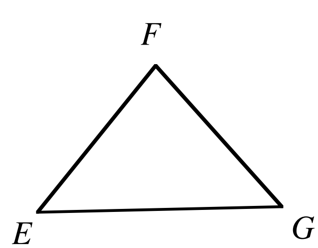
1.4 **a.** Create and name a segment that is congruent to the segment below:



b. Create and name an angle that is congruent to the angle below:



c. Create and name a figure that is congruent to the figure below:



Activity 2

2.1 Draw and label an *x*-axis and a *y*-axis on a sheet of graph paper.

a. Plot and label the following ordered pairs on your Cartesian coordinate system: $A(0,0)$, $B(5,3)$, $C(-7,2)$, $D(4,-4)$, and $E(-1,6)$.

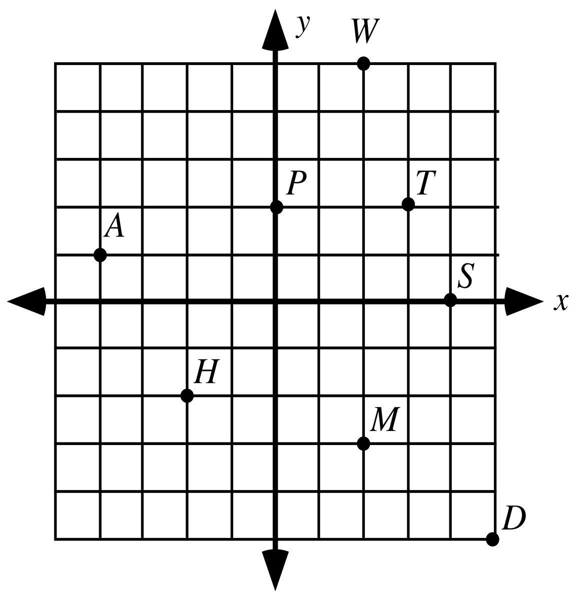
b. Find a point that is collinear to each of the following pairs of points:

1. (2,4) and (2,–1)

2. (–4,5) and (3,5)

3. (3,4) and (–2,–1).

2.2 Find the coordinates of each point on the following graph if each grid mark represents 1 unit.



2.3 **a.** Use a protractor and straightedge to draw a pair of complementary angles.

b. Use a protractor and straightedge to draw a pair of supplementary angles.

Activity 3

3.1 **a.** Draw a segment with endpoints (8,5) and (8,–4) on a sheet of graph paper.

b. Draw a segment with one endpoint at (6,3) that intersects the *x*-axis and is parallel and congruent to the segment in Part **a**.

c. Write the coordinates of the points where these two segments intersect the *x*-axis.

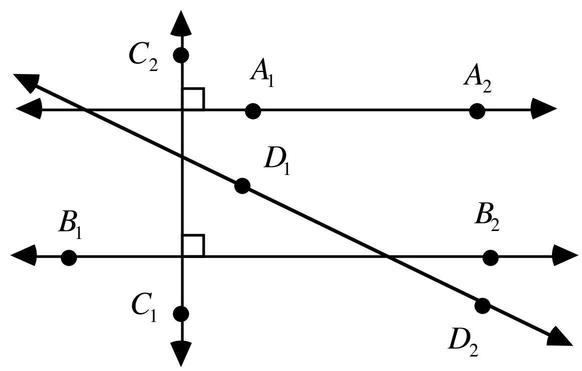
3.2 **a.** Draw a segment with endpoints $(2,-10)$ and $(-4,-10)$.

b. Draw a segment with one endpoint at $(-2,-5)$ that intersects the segment in Part **a** and is perpendicular and congruent to it.

Activity 4

4.1 What is the measure of the central angle for a regular polygon with 14 sides?

4.2 Use the diagram below to complete Parts **a** and **b.**



a. Name a pair of perpendicular line segments in the diagram.

b. Name a pair of parallel line segments in the diagram.

4.3 Determine the coordinates of the image when the point *A*(3,5) is reflected:

a. in the *x*-axis

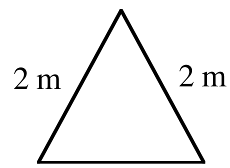
b. in the *y*-axis

c. in the *x*-axis, then in the *y*-axis.

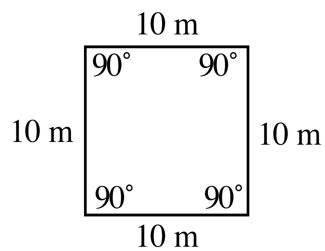
Answers to Flashbacks

Activity 1

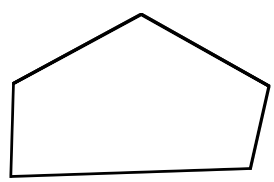
1.1 **a.** An isosceles triangle is a triangle with at least two sides of equal length. The angle between the two sides of equal length is the vertex angle, while the other two angles are the base angles.



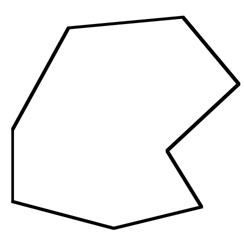
b. A square is a quadrilateral with all four sides congruent and four right angles.



c. A pentagon is a closed geometric figure with five sides.

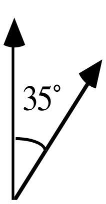


d. An octagon is a closed geometric figure with eight sides.

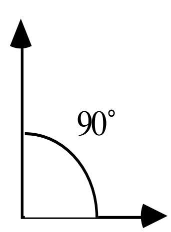


1.2 Sample response:

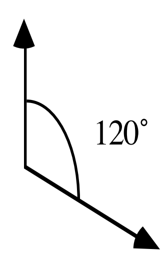
a.



b.



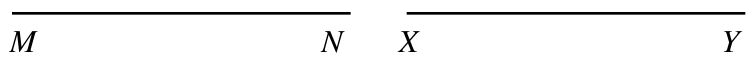
c.



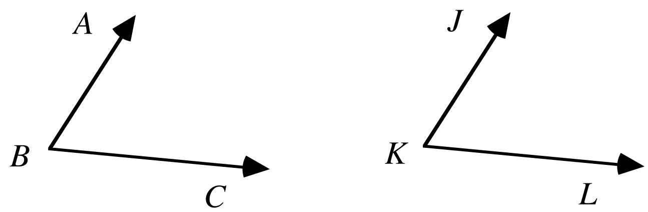
1.3 **a.** $m\angle ABC = 29^\circ$

b. $m\angle NLK = 106^\circ$

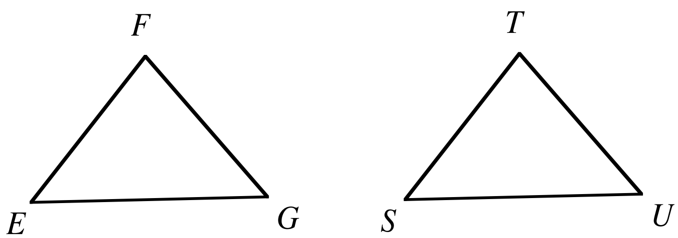
1.4 **a.** Any segment that is the same length as $\overline{MN}$ is a congruent segment. Sample response:



b. Any angle that has the same degree measure as $\angle ABC$ is a congruent angle. Sample response:

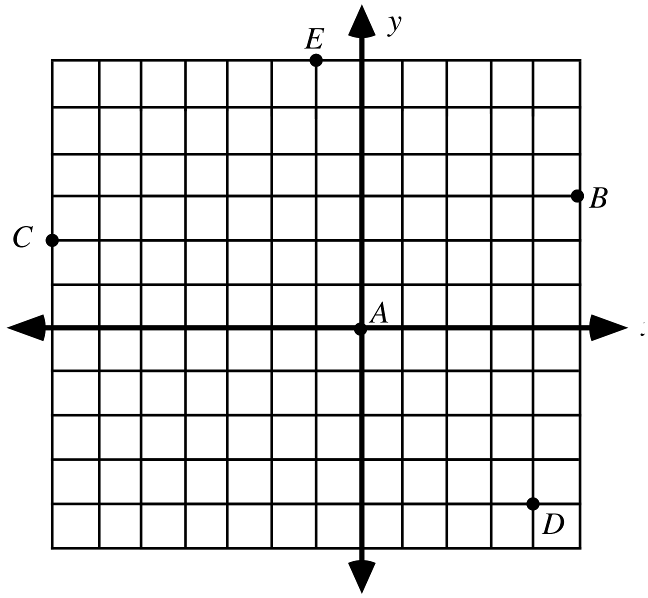


c. A triangle is congruent to ΔEFG if corresponding angles are congruent and corresponding sides are congruent. Sample response:



Activity 2

2.1 **a.** In the following sample graph, each grid mark represents 1 unit.



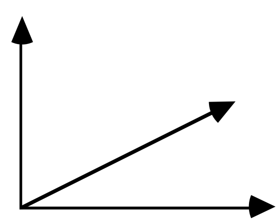
b. **1.** In general, any ordered pair $(2, y)$ is collinear to (2,4) and (2,–1).

2. In general, any ordered pair $(x, 5)$ is collinear to (–4,5) and (3,5).

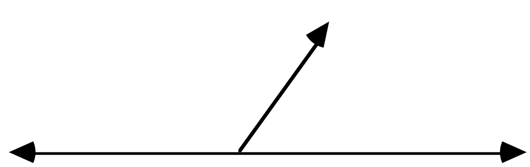
3. In general, any ordered pair $(x, x + 1)$ is collinear to (3,4) and (–2,–1).

2.2 The ordered pairs are $A(-4,1)$, $D(5,-5)$, $H(-2,-2)$, $M(2,-3)$, $P(0,2)$, $S(4,0)$, $T(3,2)$, and $W(2,5)$.

2.3 **a.** The sum of the measures of the two angles must equal 90˚. When placed adjacent to each other, with a common side, the two angles must form a right angle. Sample response:

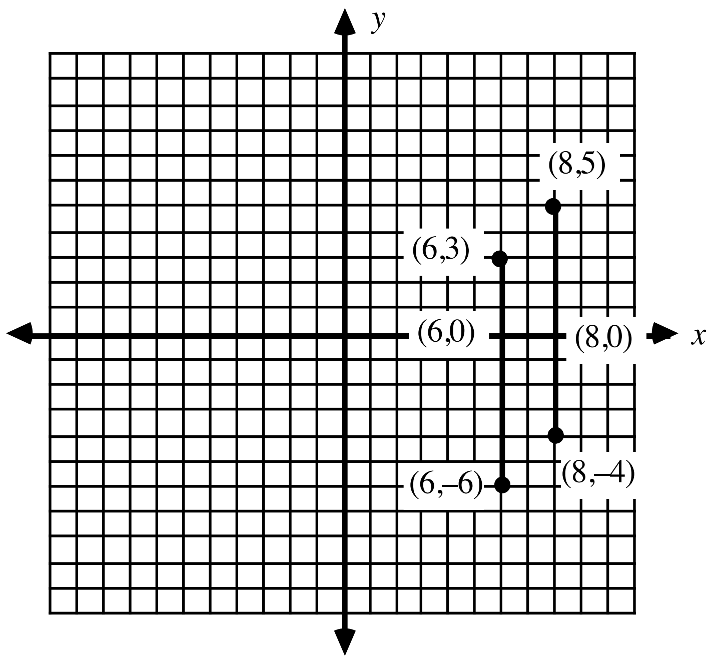


b. The sum of the measures of the two angles must equal 180˚. When placed adjacent to each other, with a common side, the two angles must form a straight line. Sample response:

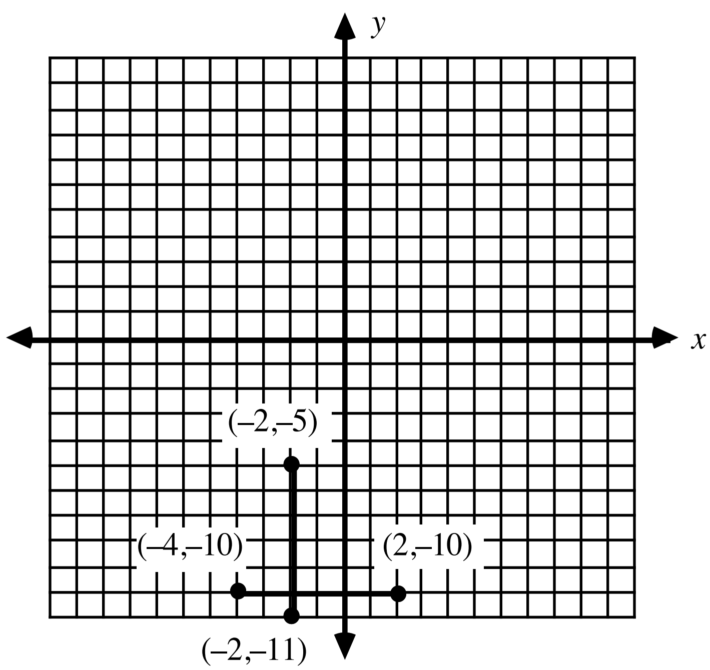


Activity 3

3.1 a–c. In the following graph, each grid mark represents 1 unit.



3.2 a–b. In the following graph, each grid mark represents 1 unit.



Activity 4

4.1 The measure of the central angle for a regular polygon with 14 sides is $25\frac{5}{7}^{\circ} \approx 25.71^{\circ}$.

4.2 **a.** One pair of perpendicular line segments is $\overline{C_1C_2}$ and $\overline{B_1B_2}$.

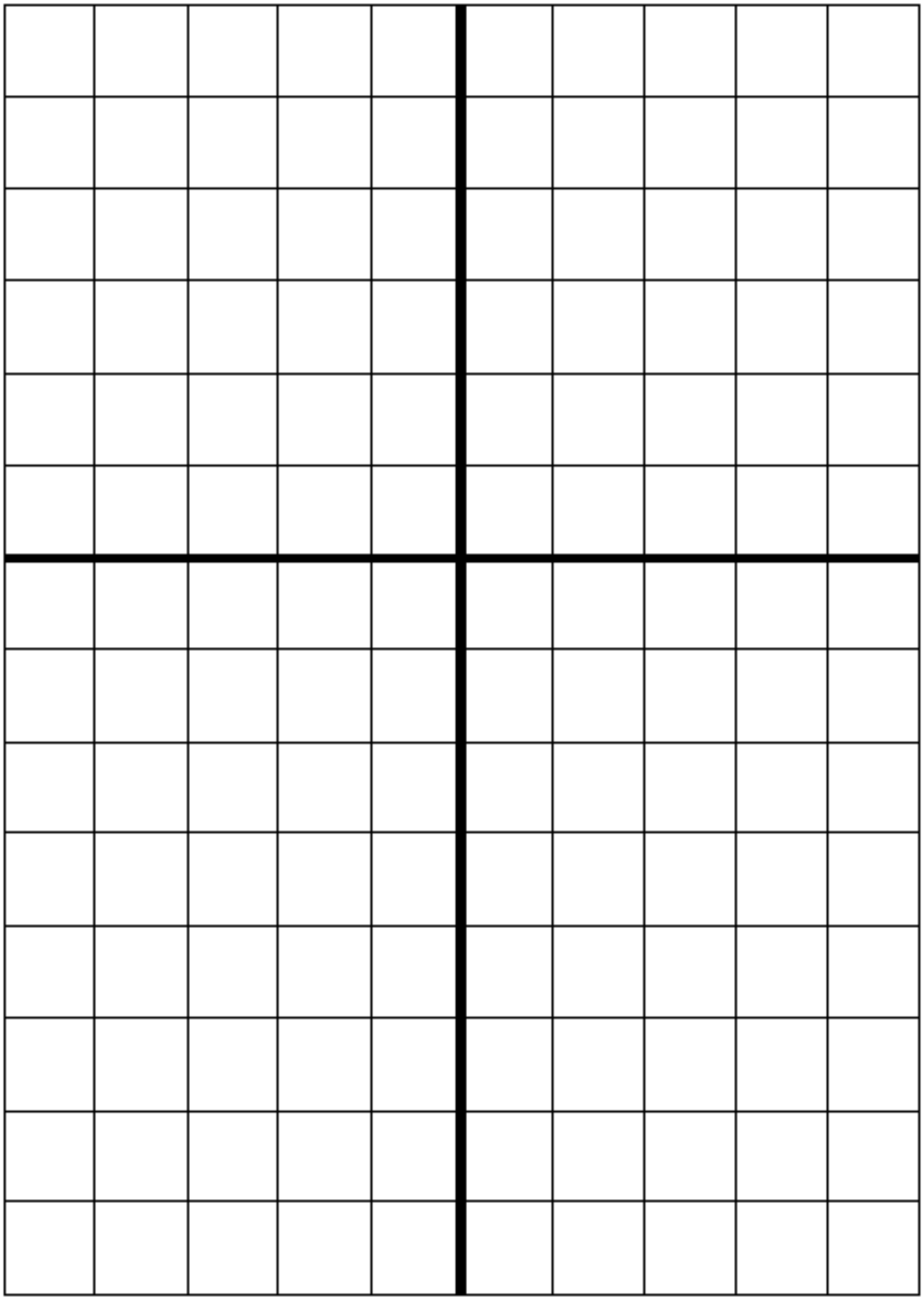
b. One pair of parallel line segments is $\overline{A_1A_2}$ and $\overline{B_1B_2}$.

4.3 **a.** (3,–5)

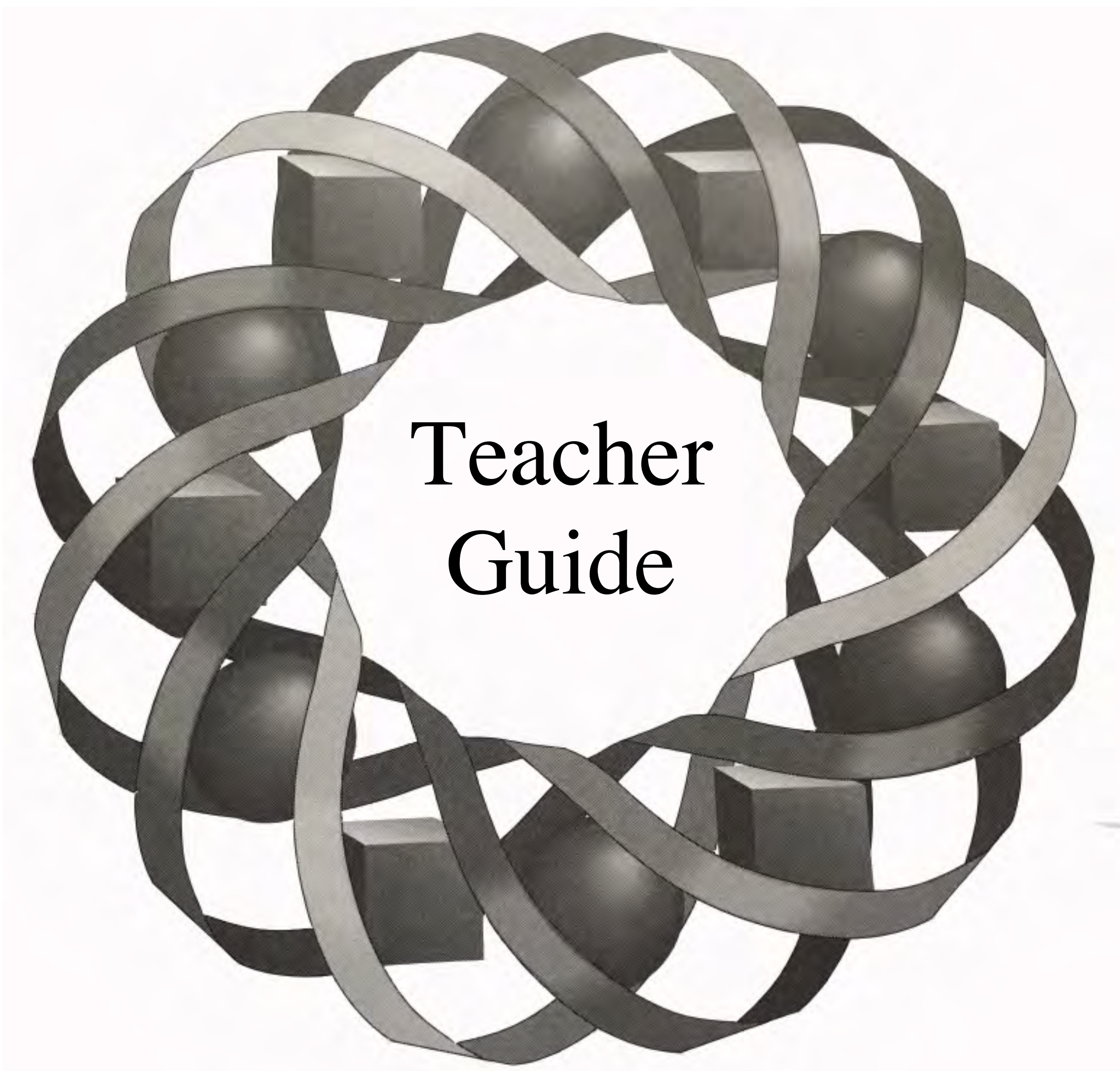
b. (–3,5)

c. (–3,–5)

Template for Cartesian Coordinate System



So You Want to Buy a Car



Choosing a new car is never easy. In this module, you'll use mathematics and technology to analyze some common concerns of today's car buyers.

Ted Dreith • Anne Merrifield • Dean Preble • Deanna Turley

Teacher Edition

So You Want to Buy a Car

Overview

This module uses the process of choosing a car to present the mathematical concepts of ordering numbers, graphing and interpreting data, trends in data, correlation, amount of change, and percent change. Correlations are referred to as positive or negative associations. Most of the graphing and data interpretation is facilitated by the use of a spreadsheet, graphing utility, or graphing calculator.

Objectives

In this module, students will:

- develop an understanding of simple spreadsheet functions
- use a spreadsheet to organize data using tables and graphs
- use a spreadsheet to interpret the relationship between paired sets of data
- use a spreadsheet to create histograms and scatterplots
- interpret histograms and scatterplots
- recognize positive and negative associations within scatterplots.
- determine percent increase and percent decrease.

Prerequisites

For this module, students should know:

- how to plot ordered pairs on a coordinate plane
- how to interpret bar graphs
- how to calculate percentages.

Time Line

Activity	1	2	3	4	Summary Assessment	Total
Days	2	2	2	2	1	9

Materials Required

Materials	Activity				
	1	**2**	**3**	**4**	**Summary Assessment**
graph paper	X				

Technology

Software	Activity				
	1	**2**	**3**	**4**	**Summary Assessment**
spreadsheet	X	X	X	X	X
graphing utility	X	X	X	X	X

Teacher Note

Throughout this module, students use technology to sort data and represent information graphically. These graphical representations include scatterplots, connected line graphs, and histograms.

Activity **2** is designed to give students additional practice with a spreadsheet. To save time, you may wish to treat this as an optional activity. You also may wish to create spreadsheets that contain the appropriate data and distribute them to students before each exploration. This allows students to focus on creating graphs and manipulating (rather than entering) data.

Teacher Edition

So You Want to Buy a Car

Introduction

Students consider the decisions involved in choosing a car. They design a strategy for selecting a class list of 10 important characteristics and evaluate its results.

Exploration

(page 33)

a. Each student should list 10 factors. Some possible considerations include price, color, model, make, body type (2-door, 4-door, hatchback), engine size, transmission (standard or automatic), drive options (front-wheel, rear-wheel, or four-wheel), place of manufacture (foreign or domestic), comfort, warranty, and dependability, as well as the quality and availability of sound systems, air conditioning, and other option packages.

b. Students should share their individual lists with classmates and note the range of factors identified as important.

c. The process used to determine a Top 10 will vary. Any strategy that attempts to ensure fairness is acceptable. Students may suggest working in small groups first, agreeing on group lists, then agreeing on a class list. Or, they may wish to proceed directly to a class list.

Teacher Note

When discussing student lists, it is important for all members of the class to understand the meaning of each factor. Particularly knowledgeable students may wish to share their expertise with the rest of the class.

Discussion

(page 33)

a. Students may disagree on how well the list reflects their personal preferences. A class list does not necessarily represent the opinion of any particular individual—it represents the opinion of the entire class. If students feel that the process they used was unfair, they should revise their system and develop a new list.

b. Encourage students to discuss the advantages and disadvantages of the method they used. They should realize that no method is likely to be completely fair to all members. However, an effort to consider as many individual opinions as possible may be acceptable to the majority of the class.

(page 33)

Activity 1

Students investigate use of a spreadsheet to sort, manipulate, and graph data.

Materials List

- graph paper (optional)

Technology

- spreadsheet
- graphing utility

Teacher Note

You may wish to use graph paper to introduce spreadsheet organization. In the following exploration, students use the capabilities of a spreadsheet to sum rows or columns, find the mean for rows or columns, sort data, and create bar graphs.

Each student should receive the opportunity to enter both data and formulas in a spreadsheet.

Exploration

(page 33)

a. Each student arranges the 10 factors in order of importance and assigns rank numbers.

b. To save class time, students should build their spreadsheets using the rankings for a small group rather than the information for the entire class. (See sample spreadsheet given in Part **d** of the exploration.)

c. **1–2.** Students should refer to their spreadsheet manuals for the proper way to indicate cells to be added. In most spreadsheets, an equal sign (=) must precede the formula. To find the sum of cells B2, C2, and D2, for example, you might enter the following: $= B2 + C2 + D2$.

3–4. Students use the spreadsheet's built-in sum function by entering the appropriate code or command.

5. After displaying the sum for each factor, students should rank them in order of importance by assigning 1 to the lowest total and 10 to the highest.

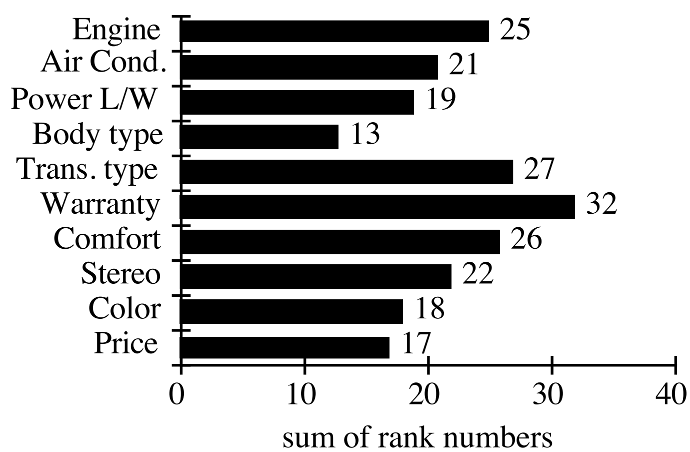
d. Students should refer to their manuals for instructions on how to use this function. **Note:** If a built-in function is not available, the mean may be found by using a formula that adds cells, then divides by the number of cells. Sample spreadsheet:

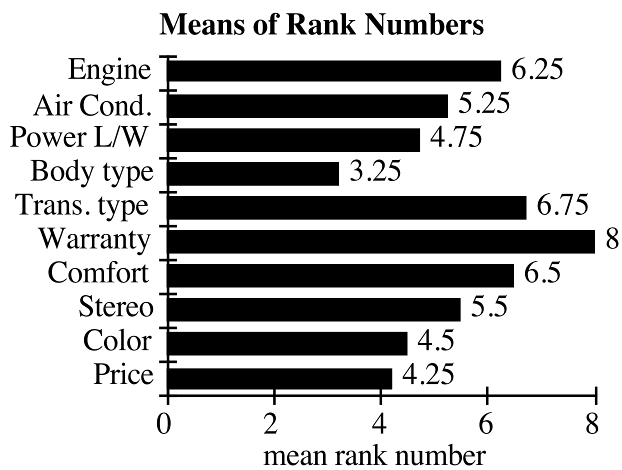
	A	**B**	**C**	**D**	**E**	**F**	**G**
1		Marta	Rico	Ryan	Pam	Sum	Mean
2	Price	5	3	1	8	17	4.25
3	Color	7	5	4	2	18	4.5
4	Stereo	2	2	8	10	22	5.5
5	Comfort	10	9	2	5	26	6.5
6	Warranty	9	10	6	7	32	8
7	Trans. type	3	6	9	9	27	6.75
8	Body type	4	1	7	1	13	3.25
9	Power L/W	1	4	10	4	19	4.75
10	Air Cond.	8	7	3	3	21	5.25
11	Engine	6	8	5	6	25	6.25

After displaying the mean for each factor, students should rank them in order of importance by assigning 1 to the lowest mean and 10 to the highest. These rankings should be the same as those found using the sums.

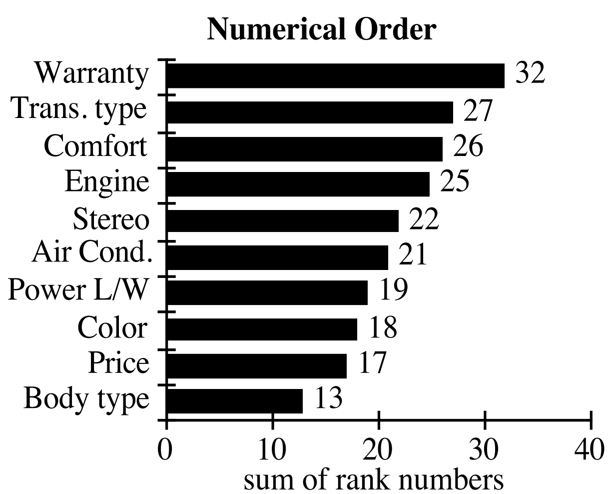
e. The following sample graphs are based on the data in the sample spreadsheet given in Part **d**.

Sums of the Rank Numbers





f. Students should refer to their manuals for instructions on how to use this function. The following sample graph shows the 10 factors given in the sample spreadsheet in decreasing order of the sum of the rank numbers.



Discussion

(page 35)

a. One of the primary benefits is the speed at which calculations can be made on large amounts of information. The graphing capabilities also simplify the process of displaying information.

b. Sample response: Entering the data can be time consuming. When first becoming familiar with the spreadsheet, the time factor involved in completing tasks could be a drawback.

c. The shapes of the graphs are similar because the rankings are the same for both the sum and the mean of the rank numbers. The differences in the scale on the x-axis are due to the different numerical values.

d. When interpreting tables and graphs in which the data has been sorted, the rankings may be easier to visualize.

Teacher Note

Although some of problems in the assignment may be easily completed without the use of technology, they were designed to assess spreadsheet skills.

Assignment

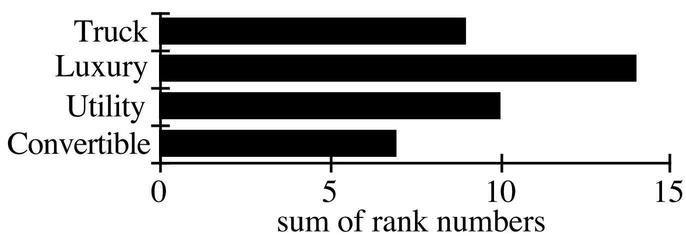
(page 36)

***1.1** **a.** Sample spreadsheet.

Salesperson	Convertible	Utility	Luxury	Truck
A	1	4	3	2
B	3	2	4	1
C	1	3	4	2
D	2	1	3	4
SUM	7	10	14	9
MEAN	1.75	2.5	3.5	2.25

b. The following sample graph shows the sum of the rank numbers for each model.

Sum of the Rank Numbers for Each Model

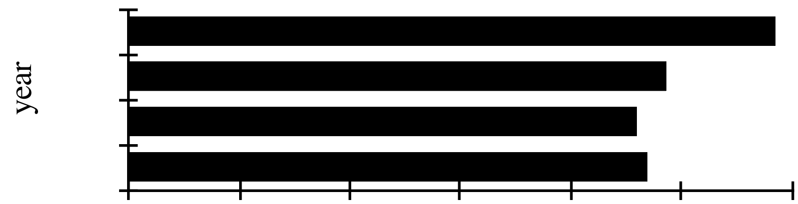


c. Since the sums of the rank numbers for the convertible and the truck are the two lowest, students should recommend these two models for the showroom.

1.2 **a–b.** The total U.S. exports for each year appear in the right-hand column of the table below.

Year	Japan	Canada	Other	Sum
1989	327	6824	2274	9425
1990	531	6232	2465	9228
1991	496	6195	3070	9761
1992	694	5928	5091	11713

c. Sample graph:



d. Sample response: The information from the spreadsheet and the graph shows a general increase in new car exports from 1989 to 1992. From 1989 to 1990, however, there was a slight decline in new car exports.

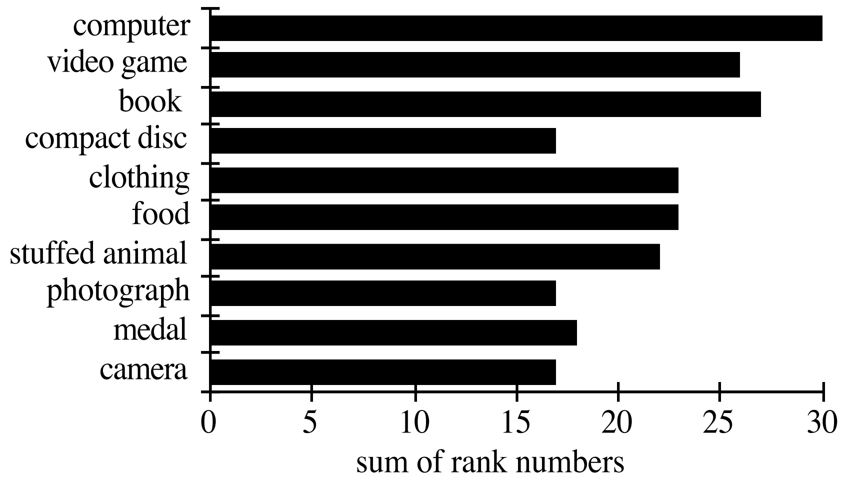
* * * * *

1.3 a–c. A sample spreadsheet is shown below.

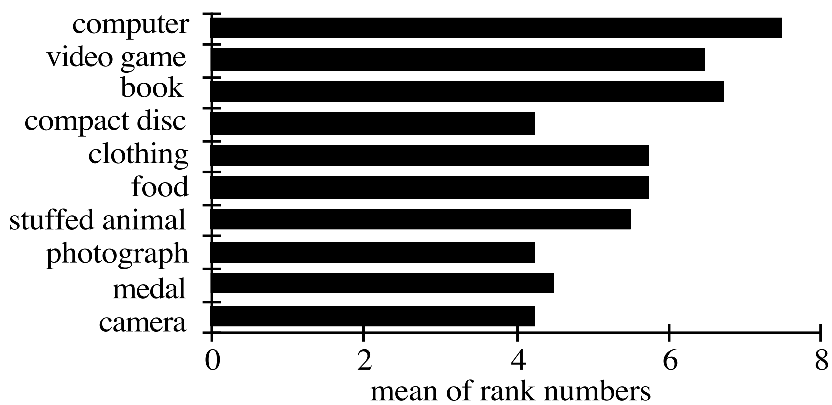
Item	**Student**				**Sum**	**Mean**
	A	**B**	**C**	**D**		
compact disc	1	3	5	8	17	4.3
medal	4	5	7	2	18	4.5
photograph	8	2	2	5	17	4.3
stuffed animal	2	9	1	10	22	5.5
food	6	6	10	1	23	5.8
clothing	9	1	9	4	23	5.8
camera	7	4	3	3	17	4.3
book	10	7	4	6	27	6.8
video game	3	8	8	7	26	6.5
computer	5	10	6	9	30	7.5

d. The sample bar graphs shown below were created using the information given in the above spreadsheet. Both graphs display the same rankings for the items. The graphs differ in the numerical scaling of the horizontal axis.

Sum of Rank Numbers for Favorite Items



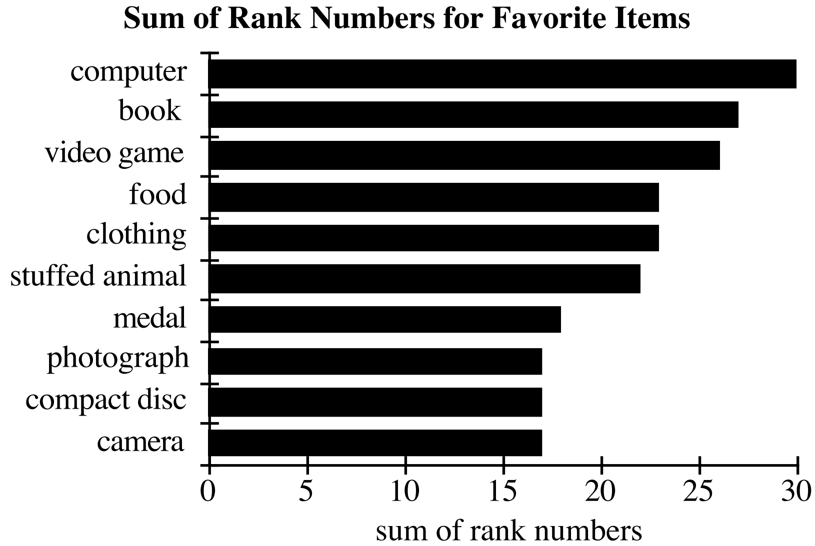
Mean of Rank Numbers for Favorite Items



e. When the data has been sorted alphabetically, the resulting list can provide a quick way of identifying the sum and mean of the rank numbers for a specific item.

f. Sample response: Sorting the data numerically can make it easier to see the rankings of the items.

g. The following sample graph shows the data sorted numerically by the sum of the rank numbers.



* * * * * * * * * *

(page 37)

Activity 2

In this activity, students examine the difference between subjective and objective information and use the results of several handling and performance tests to rank new cars.

Materials List

- none

Technology

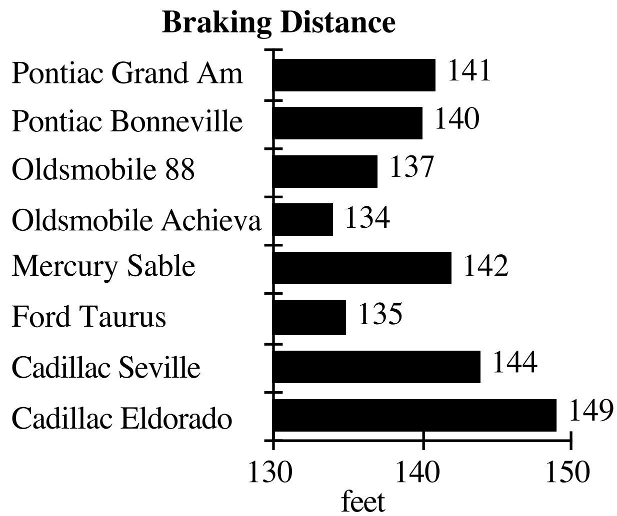
- spreadsheet
- graphing utility

Teacher Note

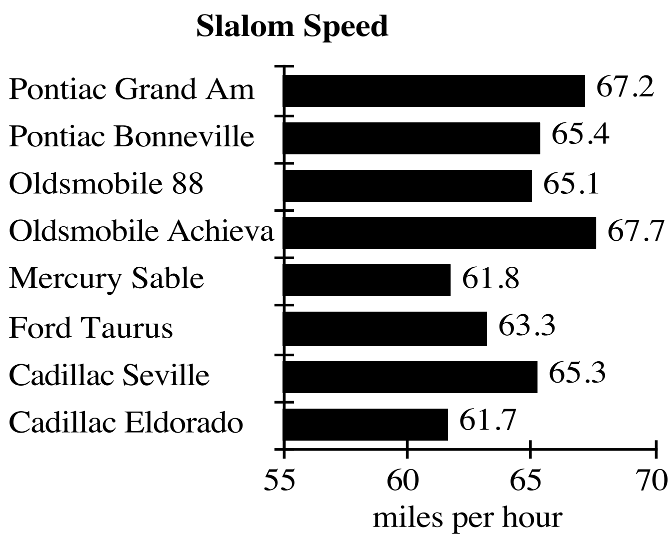
Students should read the explanations of the information given in Tables **2** and **3** before beginning the exploration.

Exploration
(page 39)

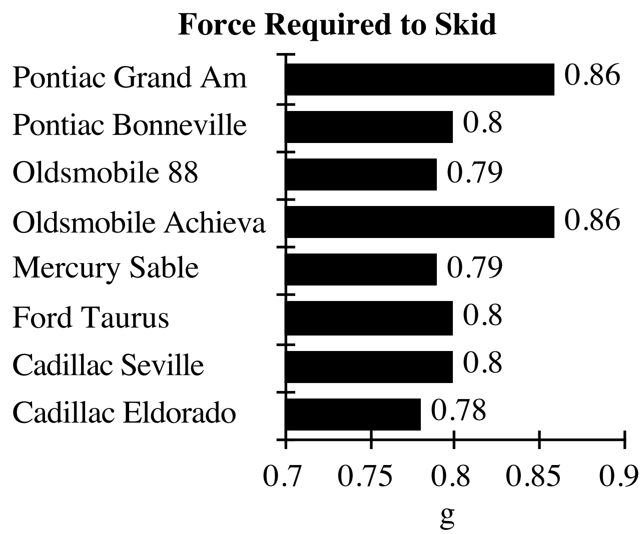
a. When braking, a car should stop in the shortest possible distance. In the braking test, a shorter bar indicates a better test result. Sample graph:



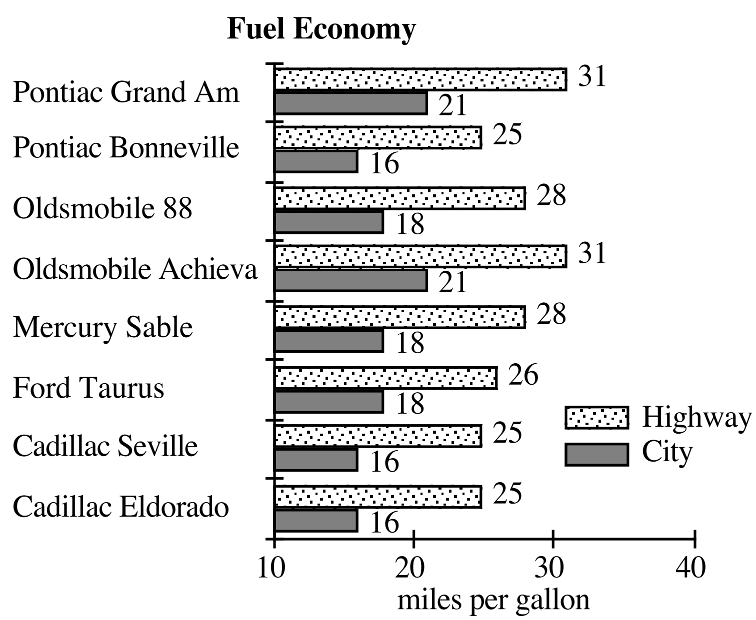
Cars that can negotiate turns at a higher speed are more maneuverable. In the slalom test, a longer bar indicates a better test result.



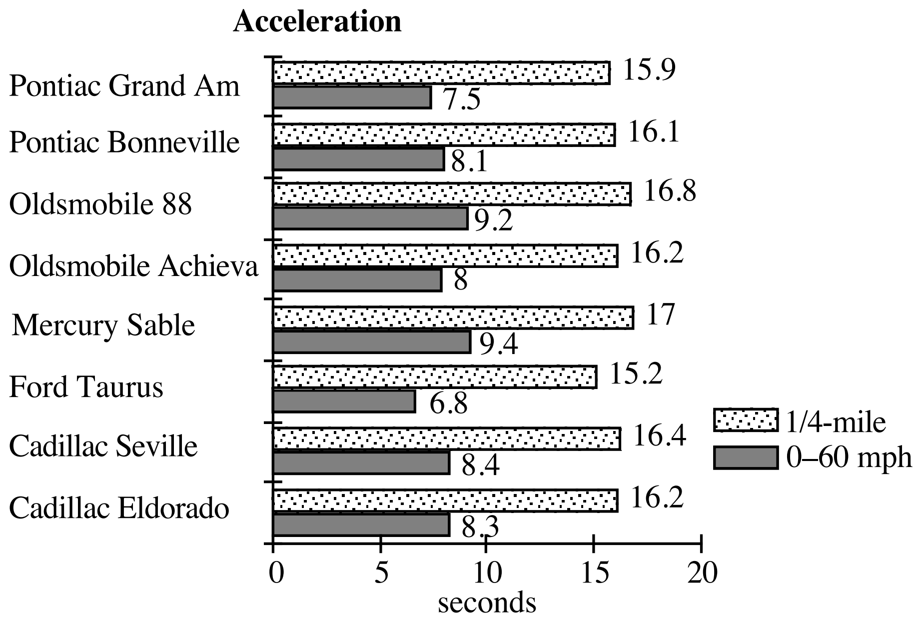
If a greater force is required to cause a car to skid, then that car is safer to drive. In the skid test, a longer bar indicates a better test result.



b. **1.** A car that travels more miles per gallon is more economical to drive. For the fuel economy test, a longer bar indicates a better result. Sample graph:



2. When accelerating to 60 mph or accelerating over a quarter mile, faster cars will produce shorter times. For the acceleration tests, a shorter bar indicates a better result. Sample graph:



c. Answers will vary. The sample response given below was determined by ranking the cars from 1 through 8 according to the results of each test. The individual rank numbers were then added and the car with the smallest sum assigned the final rank of 1.

Model	**Rank Number**			**Sum**	**Final Rank**
	Braking	Slalom	Skid		
Cadillac Eldorado	8	8	4	20	8
Cadillac Seville	7	4	2	13	6
Ford Taurus SHO	2	6	2	10	4
Mercury Sable LS	6	7	3	16	7
Oldsmobile Achieva	1	1	1	3	1
Oldsmobile 88 Royale	3	5	3	11	5
Pontiac Bonneville SSEi	4	3	2	9	3
Pontiac Grand Am GT	5	2	1	8	2

d. Answers will vary. The sample ranking below was determined as described in Part **c** above.

Model	**Rank Number**				**Sum**	**Final Rank**
	City	Highway	0–60	1/4–mi		
Eldorado	3	4	5	4	16	5
Seville	3	4	6	5	18	7
Taurus	2	3	1	1	7	2
Sable	2	2	8	7	19	8
Achieva	1	1	3	4	9	3
Olds 88	2	2	7	6	17	6
Bonneville	3	4	4	3	14	4
Grand Am	1	1	2	2	6	1

e. Two possible solutions are given below. The first uses the overall rank numbers for performance and handling determined in Parts **c** and **d** to rank the cars, while the second uses the sum of the rank numbers for each individual test.

1. In the following sample response, the Pontiac Grand Am is the 1992 Car of the Year.

Model	**Overall Rank Numbers**		**Sum**	**Final Rank**
	Performance	Handling		
Cadillac Eldorado	5	8	13	6
Cadillac Seville	7	6	13	6
Ford Taurus SHO	2	4	6	3
Mercury Sable LS	8	7	15	8
Oldsmobile Achieva	3	1	4	2
Oldsmobile 88 Royale	6	5	11	5
Pontiac Bonneville	4	3	7	4
Pontiac Grand Am GT	1	2	3	1

2. In the following sample response, the Oldsmobile Achieva is the 1992 Car of the Year.

Model	**Sum of Rank Numbers**		**Sum**	**Final Rank**
	Performance	Handling		
Cadillac Eldorado	16	20	36	8
Cadillac Seville	18	13	31	6
Ford Taurus SHO	7	10	17	3
Mercury Sable LS	19	16	35	7
Oldsmobile Achieva	9	3	12	1
Oldsmobile 88 Royale	17	11	28	5
Pontiac Bonneville	14	9	23	4
Pontiac Grand Am GT	6	8	14	2

Discussion (page 39)

a. Sample response: The cars were ranked 1 through 8 on the results of each test. The rank numbers were then totaled. The car with the lowest total was assigned the final rank of 1.

b. Answers will vary. Students may have used the sum of the rank numbers for the two major categories, or the sum of the rank numbers for each individual test. See response to Part **e** of the exploration.

c. Answers will vary, depending on the response to Part **b** above.

d. See response to Part **e** of the exploration. Students are not likely to agree with *Motor Trend*'s selection.

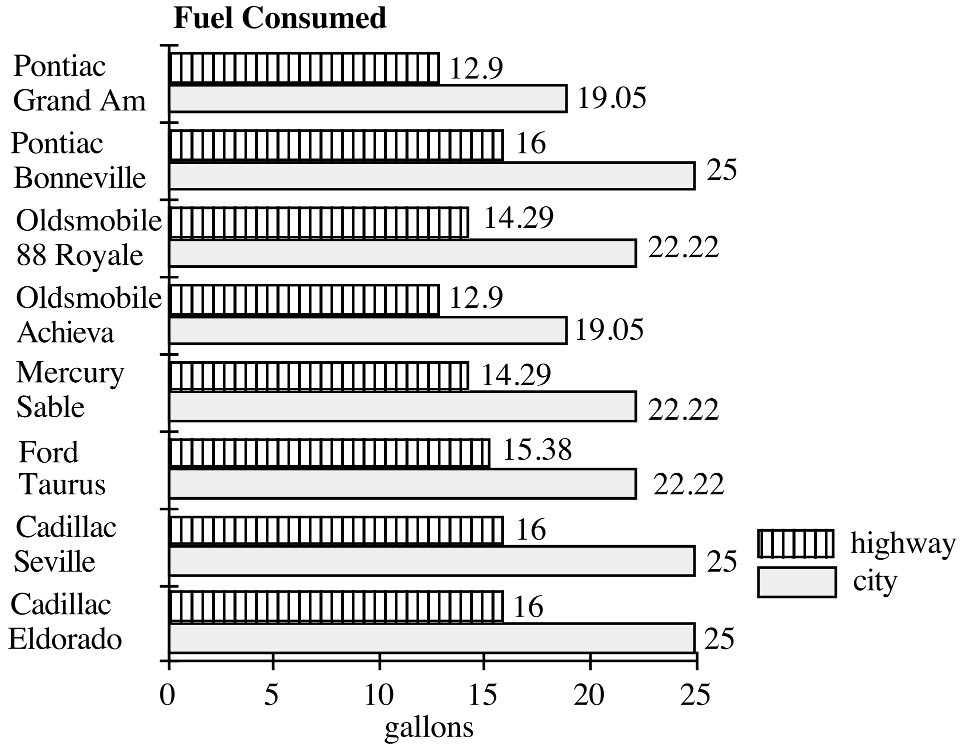
e. The *Motor Trend* selection was based on six subjective factors and two objective factors. The Cadillac Seville must have ranked higher than either the Pontiac Grand Am or the Oldsmobile Achieva in several subjective categories.

Assignment (page 40)

2.1 **a.** Students should use technology with spreadsheet capabilities. The following table shows the amount of fuel consumed for 400 miles of city driving and 400 miles of highway driving.

Model	City		Highway	
	mpg	gallons	mpg	gallons
Cadillac Eldorado	16	25.00	25	16.00
Cadillac Seville	16	25.00	25	16.00
Ford Taurus	18	22.22	26	15.38
Mercury Sable	18	22.22	28	14.29
Oldsmobile Achieva	21	19.05	31	12.90
Oldsmobile 88 Royale	18	22.22	28	14.29
Pontiac Bonneville	16	25.00	25	16.00
Pontiac Grand Am	21	19.05	31	12.90

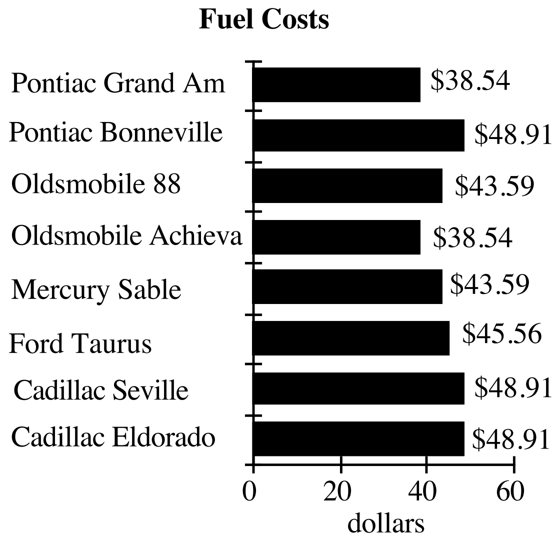
b. Sample graph:



***2.2** **a.** The following sample response uses a fuel price of $1.30 per gallon.

Model	**City**		**Highway**		**Total**	**Cost**
	mpg	**gallons**	**mpg**	**gallons**		
Eldorado	16	15.63	25	22.00	37.63	$48.91
Seville	16	15.63	25	22.00	37.63	$48.91
Taurus	18	13.89	26	21.15	35.04	$45.56
Sable	18	13.89	28	19.64	33.53	$43.59
Achieva	21	11.90	31	17.74	29.65	$38.54
Olds 88	18	13.89	28	19.64	33.53	$43.59
Bonneville	16	15.63	25	22.00	37.63	$48.91
Grand Am	21	11.90	31	17.74	29.65	$38.54

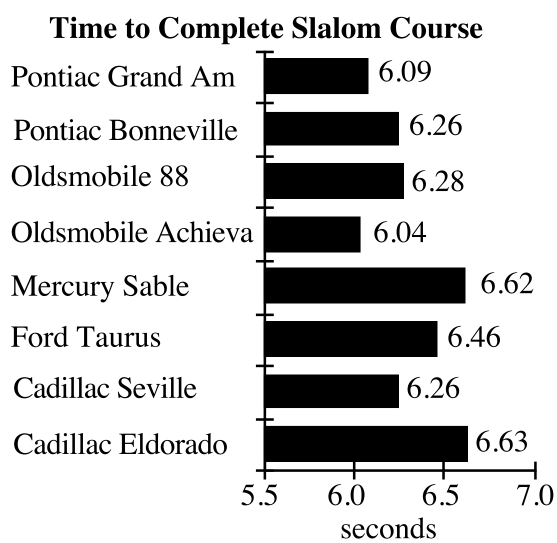
b. Sample graph:



2.3 The table below shows the time required for each car to complete the course.

Model	Speed (mph)	Time (sec)
Cadillac Eldorado	61.7	6.63
Cadillac Seville	65.3	6.26
Ford Taurus SHO	63.3	6.46
Mercury Sable LS	61.8	6.62
Oldsmobile Achieva	67.7	6.04
Oldsmobile 88	65.1	6.28
Pontiac Bonneville	65.4	6.26
Pontiac Grand Am	67.2	6.09

Sample graph:



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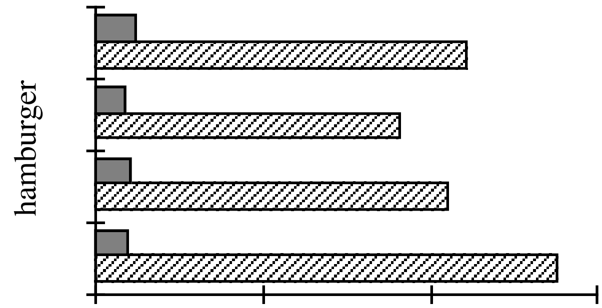
2.4 **a.** Sample graph:

Sodium and Cholesterol Content of Hamburgers



b. Sample response: Since nutritionists recommend a lower intake of sodium and cholesterol, shorter bars would indicate healthier hamburgers.

c. For calories and fat, shorter bars would indicate better hamburgers. For protein, a longer bar would indicate a better nutritional value.

d. In the following sample response, a final rank of 1 indicates the most nutritious hamburger.

Burger	**Rank Numbers**					**Sum**	**Final Rank**
	Cal.	Fat	Chol.	Sod.	Pro.		
A	1	1	4	3	4	13	2
B	2	2	1	1	3	9	1
C	3	3	3	2	2	13	2
D	4	4	2	4	1	15	4

e. Answers will vary. Students may mention taste, freshness, and quality of the meat.

* * * * * * * * * *

Student reports should include a description of the selection process, along with a detailed explanation of the calculations involved in determining the cost per mile.

A 1995 study by the American Automobile Association determined the following costs per mile based on 10,000 miles of driving per year.

Model	Cost per Mile
subcompact car	$0.35
midsize car	$0.39
full-size car	$0.50
compact pickup	$0.33
minivan	$0.41

The study used a subcompact Ford Escort, a midsize Ford Taurus, a full-size Chevrolet Caprice, a compact Ford Ranger pickup, and a Dodge Caravan as its benchmark vehicles. Each vehicle was equipped with automatic transmission, air conditioning, power steering, power antilock disc brakes, driver's-side airbag, cruise control, tilt steering, tinted glass, and rear window defogger.

Students can determine the costs of regular repairs by consulting automobile owner's manuals. In the following sample maintenance chart, the letter C indicates that a change is recommended, while the letter I indicates that an inspection is recommended.

Maintenance Intervals	**No. of months or miles**					
months		7.5	15	22.5	30	37.5
miles	2,000	7,500	15,000	22,500	30,000	37,500
Maintenance Operation						
engine oil	C	C	C	C	C	C
oil filter		C	C	C	C	C
air cleaner filter					C	
drive belts					I	
spark plugs					C	
idle speed		I		I		I
fuel lines					I	
fuel filter					C	
cooling system			I			I
engine coolant					C	
brake fluid					C	
transmission oil					C	
wheel lubrication					C	
rear axle oil	I				C	

Note: Tire rotation is recommended at least every 3,750 miles.

Although students should be encouraged to locate their own sources of information, you may wish to supply some of the following data. The figures shown below may be found in any mortgage table.

Monthly payments per $1000 borrowed

Interest Rate	Term (years)				
	1	**1.5**	**2**	**2.5**	**3**
7.0%	86.53	58.69	44.78	36.44	30.88
7.5%	86.76	58.92	45.00	36.66	31.11
8.0%	86.99	59.15	45.23	36.89	31.34
8.5%	87.22	59.37	45.46	37.12	31.57
9.0%	87.46	59.60	45.69	37.35	31.80
9.5%	87.69	59.83	45.92	37.58	32.04
10.0%	87.92	60.06	46.15	37.82	32.27
10.5%	88.15	60.29	46.38	38.05	32.51
11.0%	88.39	60.52	46.61	38.28	32.74
11.5%	88.62	60.76	46.85	38.52	32.98
12.0%	88.85	60.99	47.08	38.75	33.22
12.5%	89.09	61.22	47.31	38.99	33.46
13.0%	89.32	61.45	47.55	39.23	33.70
13.5%	89.56	61.69	47.78	39.46	33.94
14.0%	89.79	61.92	48.02	39.70	34.18

(page 41)

Activity 3

In this activity, students analyze the relationships between two sets of data. Using technology, they create graphs of the data and look for trends.

Materials List

- none

Technology

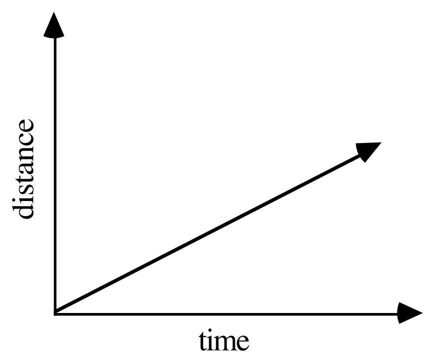
- spreadsheet

Teacher Note

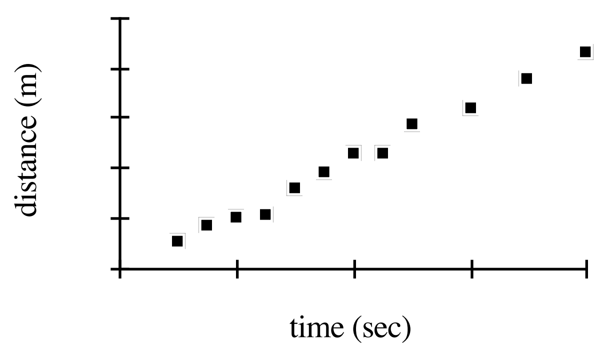
The following exploration allows students to review some concepts involving coordinate planes. The definitions given for positive and negative associations in the mathematics note provide an introduction to correlation.

Exploration (page 43)

a. **1.** Sketches will vary. Students should recognize that as time increases, the distance traveled increases. If a car's speed is constant, the graph is linear. If speed varies, the graph is curved. In the following sample graph, the speed is assumed to be constant.



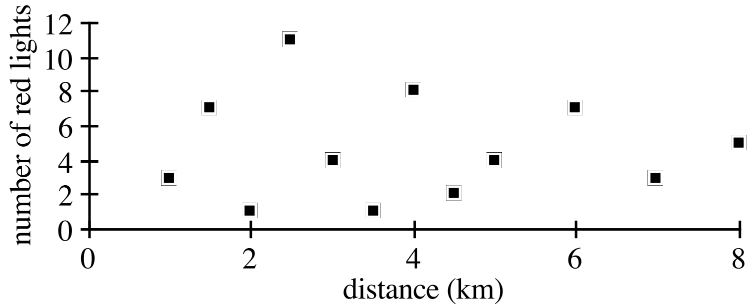
2. Sample response: The graphs have similar shapes, except that the sketch from Step **1** is a ray while the scatterplot is a set of points.



3. Sample response: The longer the car is on the obstacle course, the greater the distance traveled.

4. In 9 sec, the car will have traveled about 250 m.

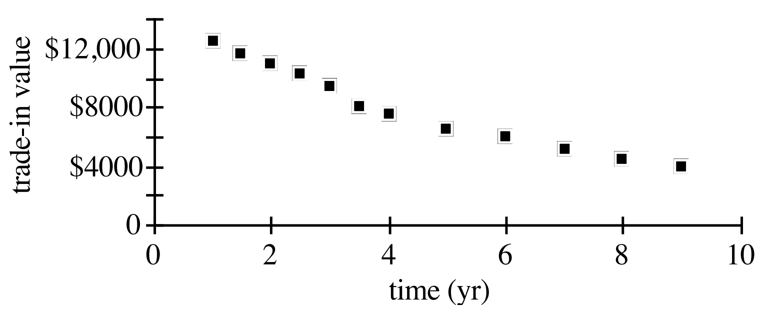
b. **1.** Sample graph:



2. Sample response: There doesn't appear to be a relationship between the distance traveled and the number of red lights encountered.

3. Since there is no apparent relationship, it is not possible to make predictions from the graph.

c. **1.** Sample graph:



2. Sample response: As a vehicle's age increases, its value decreases.

3. When the car is 10 years old, its value will be approximately $3500.

Discussion (page 44)

a. Sample response: In the first graph, as the *x*-values increase, so do the *y*-values. The second graph shows no definite relationship between the two quantities. In the third graph, as the *x*-values increase, the *y*-values decrease.

b. Yes, the graphs would show the same relationships even if the axes on which the quantities were graphed were interchanged.

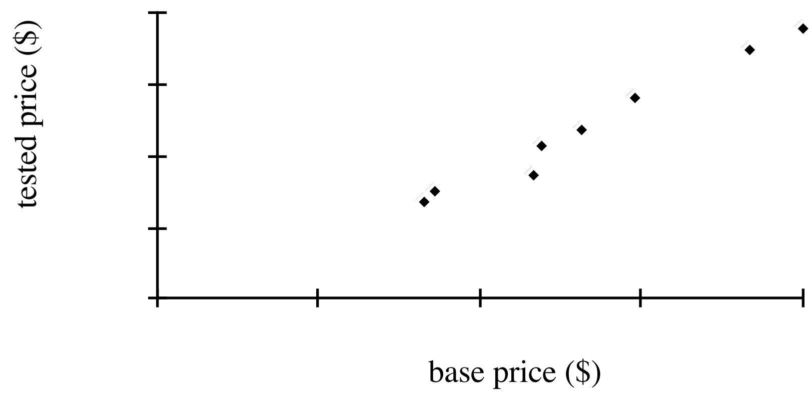
c. The association shown by the graph in Part **a** of the exploration is positive. The graph in Part **b** shows neither a positive nor a negative association. The association in the graph in Part **c** is negative.

d. Sample response: Scatterplots can be used to make predictions when the graph shows a positive or negative association between the quantities.

Assignment

(page 45)

3.1 **a.** Sample scatterplot (scales on *x*- and *y*-axes are different):



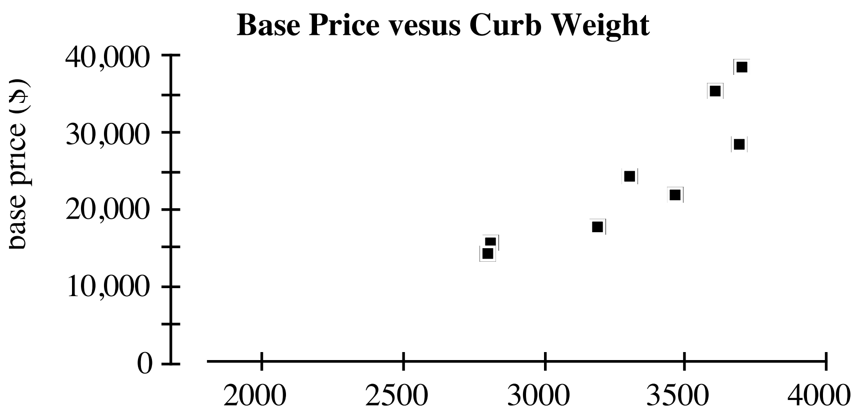
b. There appears to be a positive association between the two sets of data.

c. Sample response: Manufacturers may want tested cars to come equipped with several appealing options.

3.2 **a.** Answers may vary. Some students may suggest that a heavier car will cost more because it requires more material.

b. Students may choose either price, or decide to average the two. Explanations will vary. Sample response: I would use the tested price because it is the price of the actual model used in the tests.

c. The sample scatterplot below shows a moderately positive association between base price and curb weight for the eight cars.



d. Using the sample graph given in Part **c**, a 4000-lb car would have a base price of about $42,000.

3.3 The following sample responses assume that curb weight and base price have a positive association.

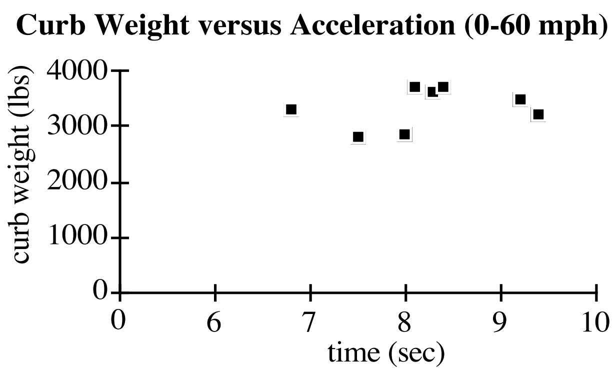
a. This statement is true. As a car's weight increases, so does its price.

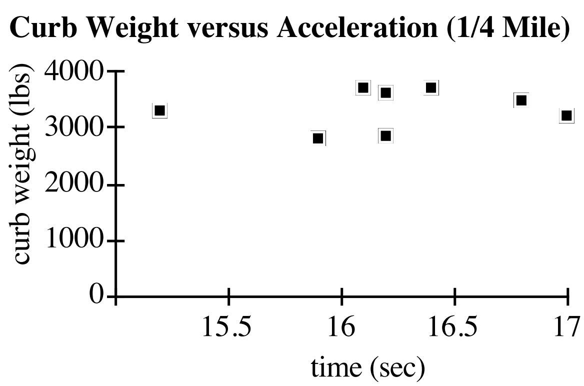
b. This statement is false. For example, the Pontiac Bonneville is more expensive than the Oldsmobile Achieva and weighs more, not less.

c. This statement is false. For example, the Ford Taurus is heavier than the Pontiac Grand Am and costs more, not less.

d. This statement is true. As indicated by the graph, *x*-values that are farther from the origin have corresponding *y*-values that are also farther from the origin.

***3.4** Students should create a scatterplot of the relationship between acceleration (either 0–60 mph or 1/4-mile) and curb weight. In either case, the association appears to be neither positive nor negative. Light cars do not accelerate better or worse than heavy cars.





3.5 **a.** Sample response: The association between the price of a new car and the size of the monthly payments is positive. As the price of the car increases, the size of the monthly payments increases.

b. Sample response: The association between a car's curb weight and its gas mileage is negative. As the car's weight increases, its gas mileage decreases.

c. Sample response: There is neither a positive nor a negative association between the braking distance of a car and its highway gas mileage.

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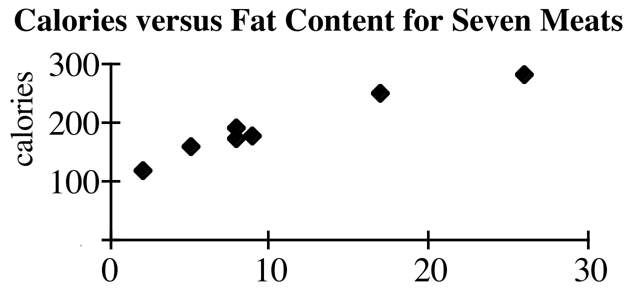
3.6 **a.** Sample graph:



b. There appears to be a positive association between the fat content and the number of calories in meat.

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(page 47)

Activity 4

In this activity, students look for trends in the trade-in values of used cars over time. They then explore amount of depreciation and percent decrease.

Materials List

- none

Technology

- spreadsheet

Teacher Note

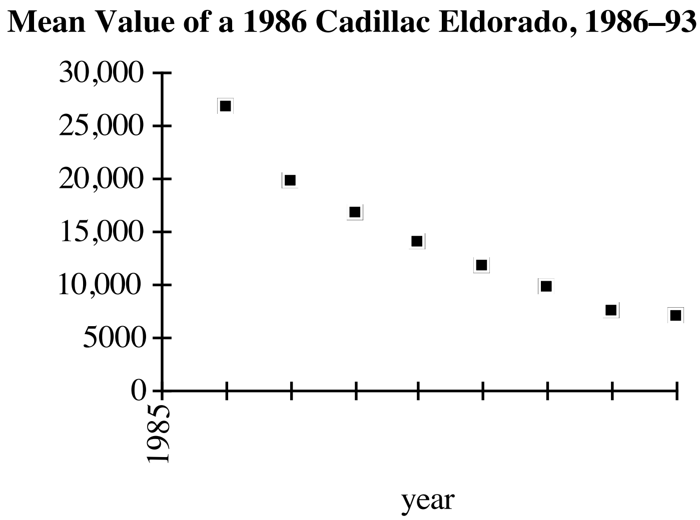
The high and low automobile values used in this activity were taken from the *N.A.D.A. Official Used Car Guide*. The *N.A.D.A. Guide* does not include high and low values for new cars. It lists only the manufacturer's suggested retail price.

Exploration

(page 48)

a. Students may use the spreadsheet's built-in function to calculate the mean or enter a formula that adds the high and low values then divides by 2.

b. **1.** Sample scatterplot:

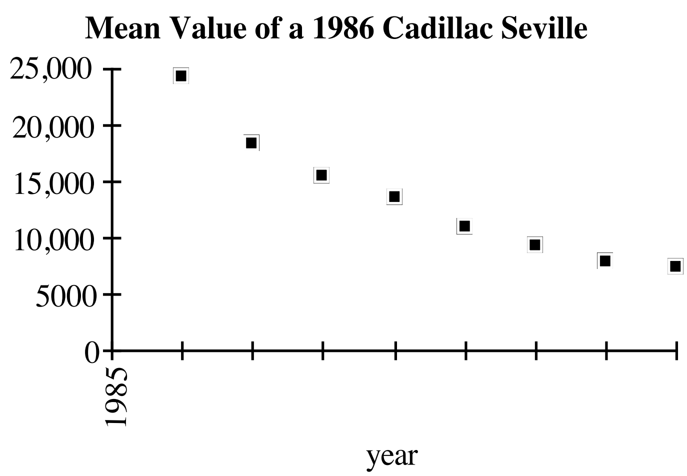


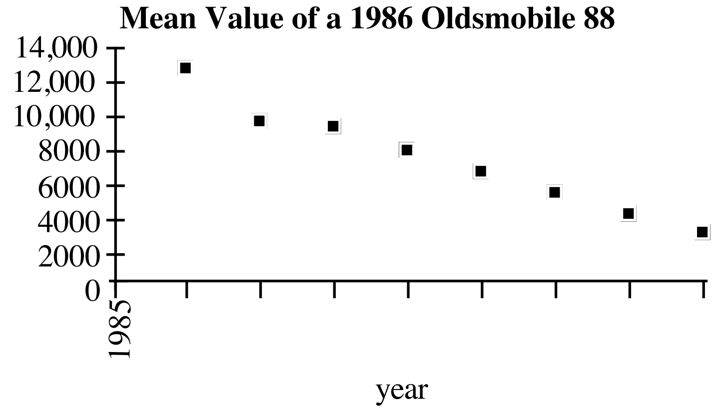
2. Cars lose most of their value during the first few years of ownership. The loss in value decreases as the car gets older. The scatterplot shows a steep decline in the first few years before gradually leveling out.

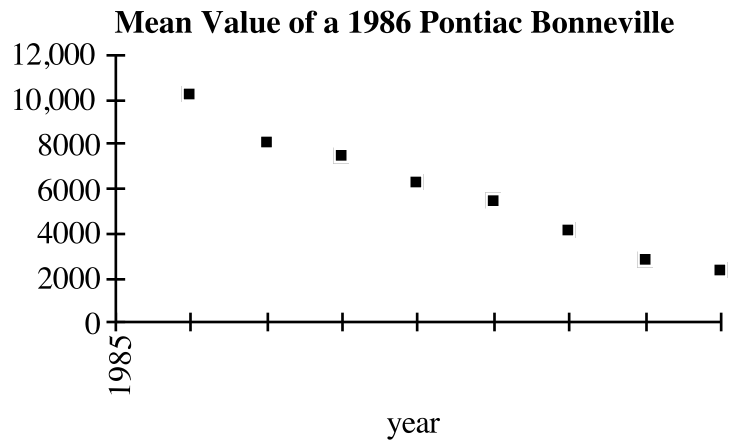
3. The car's value decreases with time.

4. The mean value of a 1986 Eldorado in 1995 is approximately $6000.

c. The scatterplots for the other three cars reveal similar associations.







d. All four graphs show a decrease in value as age increases and a larger first-year drop than in the next six years. The most obvious difference appears in the scaling on the vertical axis. (Some spreadsheet programs scale the axes automatically. You may want to discuss how this feature can affect interpretation.) Students may also note that the graphs of the less expensive cars tend to be more linear than the graphs of the more expensive models.

e. **1.** One possible formula subtracts a cell's value from the value in the preceding cell. In the following tables, the means and decreases are rounded to the nearest dollar.

Cadillac Eldorado

Year	High	Low	Mean	Decrease
1986	26756		26756	
1987	21925	17475	19700	7056
1988	18700	14775	16738	2963
1989	15675	12200	13938	2800
1990	13375	10175	11775	2163
1991	11250	8250	9750	2025
1992	8775	6300	7538	2213
1993	8125	5675	6900	638

Cadillac Seville

Year	High	Low	Mean	Decrease
1986	24251		24251	
1987	20550	16275	18413	5839
1988	17475	13725	15600	2813
1989	15300	11900	13600	2000
1990	12600	9550	11075	2525
1991	10775	7925	9350	1725
1992	8575	6525	7550	1800
1993	8675	6125	7400	150

Oldsmobile 88

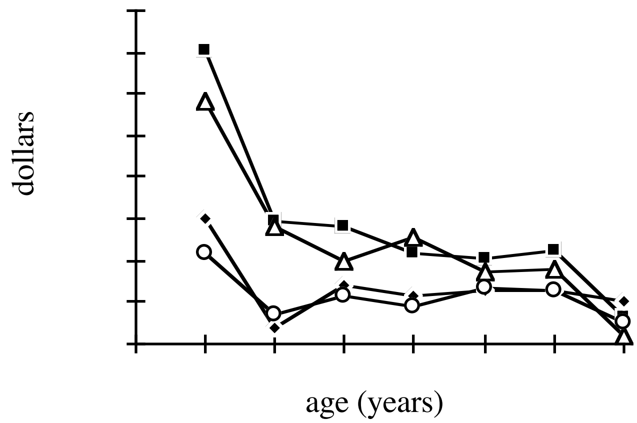
Year	High	Low	Mean	Decrease
1986	12760		12760	
1987	10925	8550	9738	3023
1988	10525	8225	9375	363
1989	8975	6975	7975	1400
1990	7875	5825	6850	1125
1991	6500	4600	5550	1300
1992	5125	3475	4300	1250
1993	4025	2500	3263	1038

Pontiac Bonneville

Year	High	Low	Mean	Decrease
1986	10249		10249	
1987	9100	7075	8088	2162
1988	8375	6475	7425	663
1989	7125	5425	6275	1150
1990	6325	4500	5413	863
1991	4850	3250	4050	1363
1992	3400	2100	2750	1300
1993	2900	1625	2263	488

2. The sample graph below shows the decrease for all four models.

Decrease in Mean Trade-In Values



f. The total depreciation for each car is shown in the following table.

Car	Value in 1986	Mean Value in 1993	Total Depreciation
Cadillac Eldorado	$26,756	$6,900	$19,856
Cadillac Seville	$24,251	$7,400	$16,851
Oldsmobile 88	$12,760	$3,263	$9,497
Pontiac Bonneville	$10,249	$2,263	$7,986

g. Students should recognize that depreciation represents a significant portion of the cost of owning a vehicle. The amount of depreciation varies from $0.28 per mile for the Cadillac Eldorado to $0.11 per mile for the Pontiac Bonneville.

Car	Total Depreciation	Depreciation per Mile
Cadillac Eldorado	$19,856	$0.28
Cadillac Seville	$16,851	$0.24
Oldsmobile 88	$9,497	$0.14
Pontiac Bonneville	$7,986	$0.11

Discussion

(page 49)

a. **1.** Since the cars were new in 1986 (age 0), the amount of decrease cannot be calculated.

2. The data points for each car are indicated by different shapes.

3. The amount of decrease is not the same from year to year. The difference in the amount is indicated by the vertical distance between consecutive points. The greater the distance, the greater the decrease in value.

4. No, this indicates that the loss in value increased from year 2 to year 3. In this case, the trade-in value actually dropped faster.

b. Between 1990 and 1991, the Cadillac Eldorado had the greatest decrease in value. Students should understand how to answer this question using both tables and graphs. Some students may feel that tables are easier to use because of their familiarity with them.

c. **1.** Depreciation can be expressed as a proportion of the original value or as a percentage of the original value.

2. Students may feel that using a proportion or percentage of the original value is a more reasonable way to compare depreciation because more expensive vehicles have more value to lose.

d. **1.** The Pontiac Bonneville loses $7986 in mean value over this period.

2. No, the Bonneville's depreciation represents 78% of its original value. Over the same period, the Cadillac Seville depreciates $16,851, or 61% of its original value.

e. Student responses will vary, depending on the way in which depreciation is approached. The Pontiac Bonneville has the smallest amount of depreciation in terms of dollars. The Cadillac Seville has the smallest percent decrease in value of the four vehicles.

Assignment

(page 50)

4.1 **a.** **1.** 85%

2. 15%

b. The percentage calculated in Step **2** (15%) represents the percent decrease because it compares the loss to the previous year's value.

c. The sum of the two percentages is 100%. Since 15% represents the percentage of value lost and 85% represents the percentage remaining, together they represent the entire previous value—or 100%.

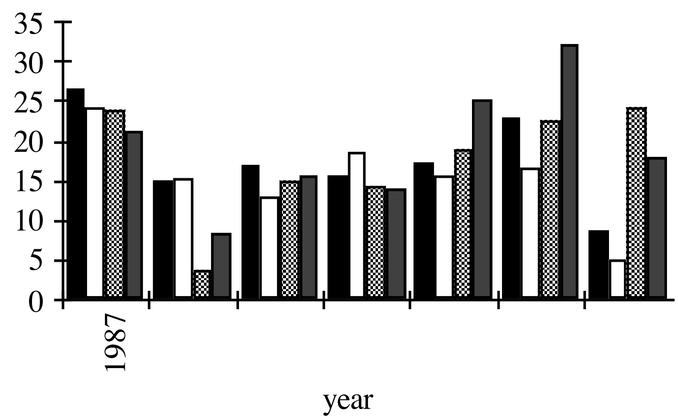
4.2 **a.** The following tables show the percent decrease in mean value for each year.

	Eldorado		Seville	
Year	**Mean Value**	**Percent Decrease**	**Mean Value**	**Percent Decrease**
1986	26756		24251	
1987	19700	26	18413	24
1988	16738	15	15600	15
1989	13938	17	13600	13
1990	11775	16	11075	19
1991	9750	17	9350	16
1992	7538	23	7550	19
1993	6900	8	7400	2

	Oldsmobile		Pontiac	
Year	**Mean Value**	**Percent Decrease**	**Mean Value**	**Percent Decrease**
1986	12760		10249	
1987	9738	24	8088	21
1988	9375	4	7425	8
1989	7975	15	6275	15
1990	6850	14	5413	14
1991	5550	19	4050	25
1992	4300	23	2750	32
1993	3263	24	2263	18

b. Students should experiment to determine which type of graph they feel most effectively displays the data. The sample response below uses a bar graph.

Percent Decrease in Value



c. All of the cars have a large percent decrease in the first year. In the third year, the percent decrease is greater than the previous year for three of the four cars. The more expensive cars (Eldorado and Seville) tend to have more consistent decreases over the 7-yr period.

d. The Pontiac Bonneville had the greatest percent decrease—over 30% between 1991 and 1992. Two other cars, the Eldorado and the Seville, both had larger amounts of depreciation.

e. Between 1988 and 1989, the Seville had an amount of depreciation of $2000. In that same year, both the Bonneville and the Oldsmobile 88 had greater percent decreases.

f. Students may argue that either method is reasonable. Their responses should illustrate an understanding of both terms.

***4.3** **a.** The table below shows the percent decrease in value for each from 1986 to 1993.

Car	Percent Decrease
Eldorado	74.21
Seville	69.45
Olds 88	74.43
Bonneville	77.92

b. Using percent decrease produces the following ranking, where 1 represents the car with the least depreciation: 1) Seville, 2) Eldorado, 3) Oldsmobile 88, 4) Bonneville. Using amount of depreciation produces the following ranking: 1) Bonneville, 2) Oldsmobile 88, 3) Seville, 4) Eldorado.

c. Based on the sum of the rank numbers, the Eldorado is the car that best holds its value over time.

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4.4 **a.** Since the volume of the original bottle was 200 mL and the volume of the new bottle is 300 mL, the percent increase is:

$$\frac{300 - 200}{200} = 50\%$$

b. Sample response: The advertisement should be changed to mention the percent increase. Using percent increase, the company can claim that the bottle contains 50% more rather than 33% free. The larger percentage will be more attractive to consumers.

4.5 **a.** The percent increase can be calculated as follows:

$$\frac{1100 - 1000}{1000} = 10\%$$

b. The percent increase can be calculated as follows:

$$\frac{1500 - 1000}{1000} = 50\%$$

4.6 **a.** Sample spreadsheet:

Year	Previous Balance	Interest	New Balance
1	1000.00	100.00	1100.00
2	1100.00	110.00	1210.00
3	1210.00	121.00	1331.00
4	1331.00	133.10	1464.10
5	1464.10	146.41	1610.51
6	1610.51	161.05	1771.56
7	1771.56	177.16	1948.72
8	1948.72	194.87	2143.59
9	2143.59	214.36	2357.95
10	2357.95	235.79	2593.74
11	2593.74	259.37	2853.12
12	2853.12	285.31	3138.43
13	3138.43	313.84	3452.27
14	3452.27	345.23	3797.50
15	3797.50	379.75	4177.25

b. The percent increase can be calculated as follows:

$$\frac{(4177.25 - 1000)}{1000} \approx 3.18 = 318\%$$

c. Since the balance increases from year to year, the amount of interest earned each year also increases. Therefore, the percent increase is more than $15 \bullet 10\%$.

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Answers to Summary Assessment

(page 53)

1. a–b. Students may use either a spreadsheet or calculator to complete this problem. In the following sample spreadsheet, the "total" represents the increase in value after 10 years. The "percent" represents this change as a percentage of the original value.

Jaguar XE-6		
Year	Value ($)	Increase ($)
0	22000.00	
1	23100.00	1100.00
2	24255.00	1155.00
3	25467.75	1212.75
4	26741.14	1273.39
5	28078.19	1337.06
6	29482.10	1403.91
7	30956.21	1474.11
8	32504.02	1547.81
9	34129.22	1625.20
10	35835.68	1706.46
	Total =	**13835.68**
	Percent =	**63%**

Mercedes Sedan		
Year	Value ($)	Increase ($)
0	31000.00	
1	32240.00	1240.00
2	33529.60	1289.60
3	34870.78	1341.18
4	36265.62	1394.83
5	37716.24	1450.62
6	39224.89	1508.65
7	40793.89	1569.00
8	42425.64	1631.76
9	44122.67	1697.03
10	45887.57	1764.91
	Total =	**14887.57**
	Percent =	**48%**

Ford Model A		
Year	Value ($)	Increase ($)
0	10000.00	
1	10800.00	800.00
2	11664.00	864.00
3	12597.12	933.12
4	13604.89	1007.77
5	14693.28	1088.39
6	15868.74	1175.46
7	17138.24	1269.50
8	18509.30	1371.06
9	19990.05	1480.74
10	21589.25	1599.20
	Total =	**11589.25**
	Percent =	**116%**

The following tables compare the changes in value for the three cars over a 20-year period.

	Jaguar XE-6		Mercedes Sedan	
Year	Value ($)	Increase ($)	Value ($)	Increase ($)
11	$37627.47	$1791.79	$47723.08	$1835.51
12	$39508.84	$1881.37	$49632.00	$1908.92
13	$41484.29	$1975.44	$51617.28	$1985.28
14	$43558.50	$2074.21	$53681.97	$2064.69
15	$45736.42	$2177.92	$55829.25	$2147.28
16	$48023.25	$2286.82	$58062.42	$2233.17
17	$50424.41	$2401.16	$60384.92	$2322.50
18	$52945.63	$2521.22	$62800.32	$2415.40
19	$55592.91	$2647.28	$65312.33	$2512.01
20	$58372.56	$2779.65	$67924.82	$2612.49
	Total =	**$36372.56**	**Total =**	**$36924.82**
	Percent =	**165.33%**	**Percent =**	**119.11%**

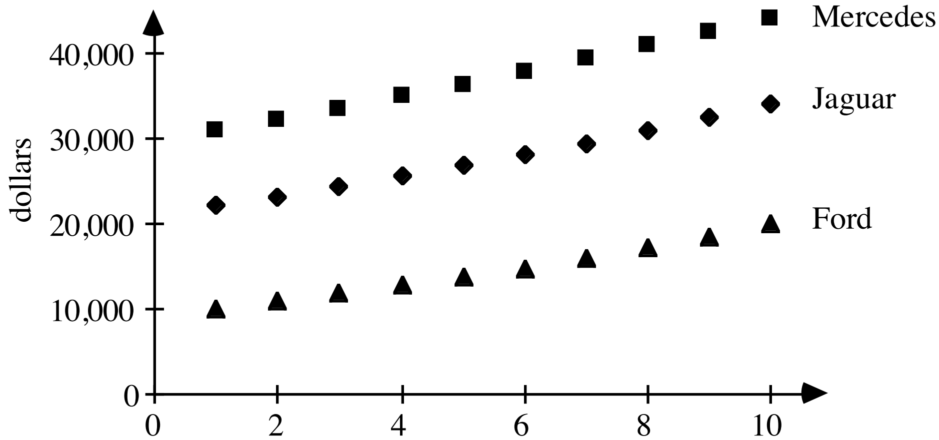
	Model A	
Year	Value ($)	Increase ($)
11	$23316.39	$1727.14
12	$25181.70	$1865.31
13	$27196.24	$2014.54
14	$29371.94	$2175.70
15	$31721.69	$2349.75
16	$34259.43	$2537.74
17	$37000.18	$2740.75
18	$39960.20	$2960.01
19	$43157.01	$3196.82
20	$46609.57	$3452.56
	Total =	**$36609.57**
	Percent =	**366.10%**

c. After either 10 or 20 years, the Mercedes will produce the greatest profit, but the Ford will have the greatest percent increase.

2. Answers will vary, depending on whether students consider total profit or percent increase. Students who use percent increase should recommend the Model A as the best investment for both time periods.

Those who consider only the amount of the profit may recommend the Mercedes for both time periods. After 20 years, however, the profits for the three cars are within $600 of each other. The value of the Mercedes increases $36,924.82, the Ford $36,609.57, and the Jaguar $36,372.56.

3. Students should present their spreadsheets and at least one graph to support their recommendations. Sample graph:



Module Assessment

1. The table below shows information on engine size and fuel economy.

Engine Size (L)	**Fuel Economy (km/L)**
1.6	20
2.0	18
3.2	17
4.3	12
6.2	8

a. Use a graphing utility to create a scatterplot of this data.

b. Based on your scatterplot, what type of association exists between engine size and fuel economy?

c. Based on your scatterplot, write a true statement about engine size and fuel economy.

2. The value of a $30,000 car is expected to decrease 10% by next year.

a. What will be the car's value next year?

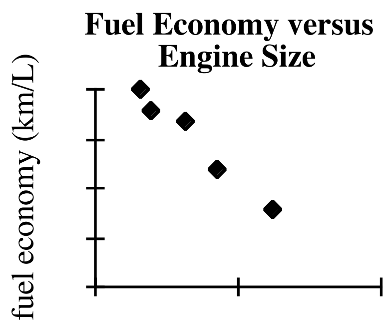
b. How much will the car be worth if it loses another 10% of its value in the following year?

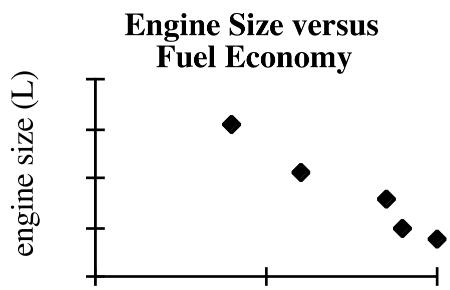
c. Explain why the amount of decrease in the car's value changes from year to year, even though the percent decrease remains the same.

d. Assume that the car loses 10% of its value each year for the next 10 yr. Create a scatterplot of the car's values over this period.

Answers to Module Assessment

1. **a.** Students may graph engine size on either the horizontal or the vertical axis. Both graphs are shown below.





b. There is a negative association between engine size and fuel economy.

c. Sample responses:

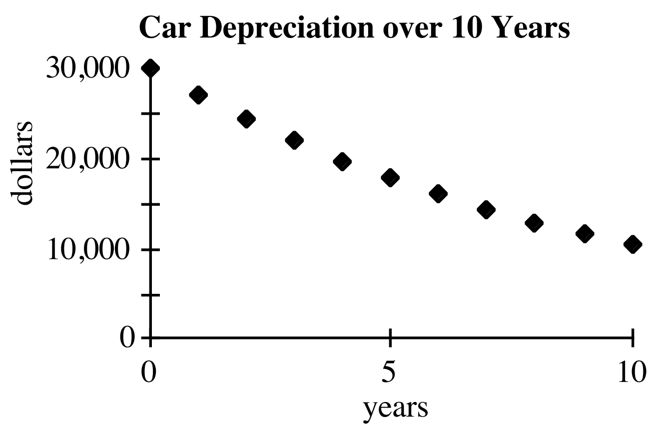
- As engine size increases, fuel economy decreases.
- As engine size decreases, fuel economy increases.
- As fuel economy decreases, engine size increases.
- As fuel economy increases, engine size decreases.

2. **a.** After the first year, the decrease is $0.10 \cdot \$30,000 = \$3000$. The car's value is $\$30,000 - \$3000 = \$27,000$.

b. After the second year, the decrease is $0.10 \cdot \$27,000 = \$2700$. The car's value is $\$27,000 - \$2700 = \$24,300$.

c. Sample response: The amount of decrease per year is calculated using the previous year's value, which changes from $30,000 to $27,000.

d. Sample graph:



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Swan, T. "Fast and Fancy." *Popular Mechanics* (February 1992): 39–42.

U.S. Department of Agriculture. *Nutritive Value of Foods*. Washington, DC: U.S. Government Printing Office, 1981.

U.S. Bureau of the Census. *Statistical Abstract of the United States: 1993*. Washington, DC: U.S. Government Printing Office, 1993.

Weaver, L., ed. *N.A.D.A. Official Used Car Guide*. McLean, VA: National Automobile Dealers Used Car Guide Co., 1992.

Other useful resources include: *Automobile Magazine, Car and Driver, Consumer Reports, Fodor's Used Car Guide,* and *Road and Track*.

Flashbacks

Activity 1

1.1 Consider the numbers 0.24, 0.45, 0.32, 1.60, 1.62, 0.452, 0.35, 2.33, 2.37, and 0.318.

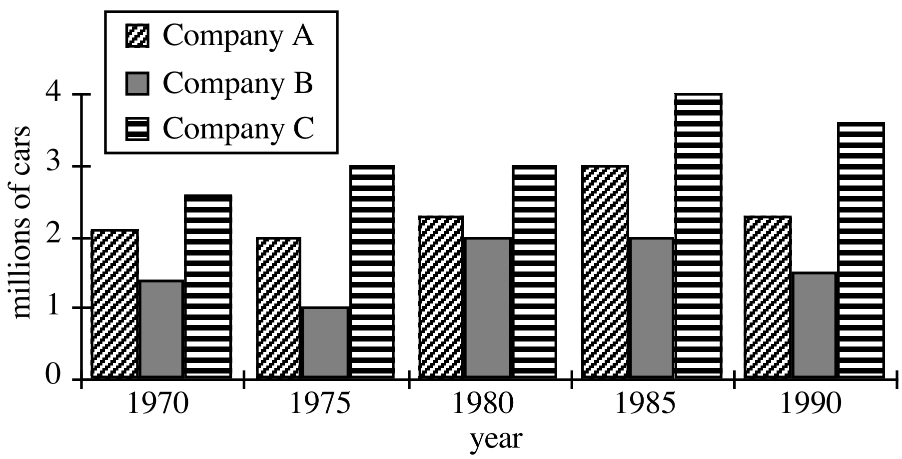
a. Find the mean of these numbers.

b. Find the median of these numbers.

c. Find the mode of these numbers.

1.2 The following bar graph shows the number of cars produced by three different companies in selected years.

Car Production in Selected Years



a. Which manufacturer produces the most vehicles?

b. **1.** During which year did the three companies produce the largest number of vehicles?

2. Approximately how many cars were produced during that year?

c. Compare the number of vehicles produced by company A and company B to the number produced by company C in 1975.

Activity 2

2.1 A classroom contains 24 women and 18 men.

a. What fraction of the people in the room are women?

b. What fraction of the people in the room are men?

2.2 A bag contains 18 red marbles, 20 blue marbles, and 14 green marbles.

a. What fraction of the marbles are blue?

b. What fraction of the marbles are green or red?

2.3 When choosing the college she would attend, Mary considered the following characteristics: location, cost, available scholarships, extracurricular activities, student population, and entrance requirements.

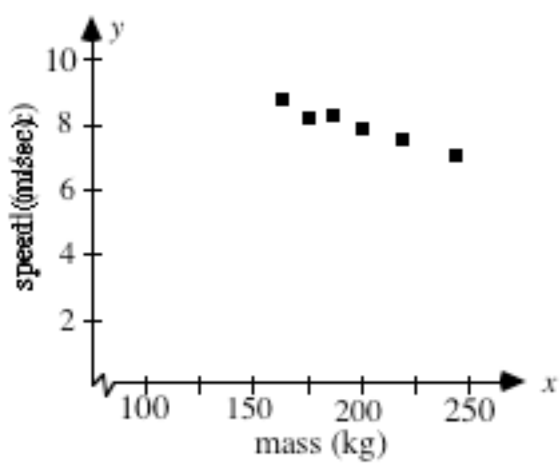
a. If each of these concerns is equally important to Mary, what fraction of her decision is based on each one?

b. Which two of these characteristics would be most important in your selection of a college? What fraction of all the characteristics listed are represented by these two?

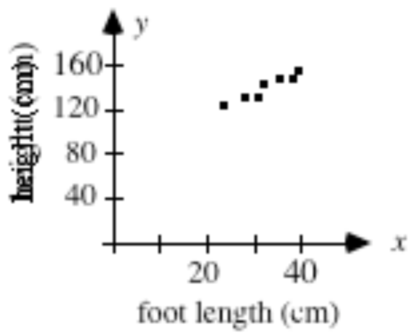
Activity 3

3.1 Plot the following points on a rectangular coordinate system: $A(1,5)$, $B(-1,2)$, $C(5,0)$, $D(-6,3)$, $E(8,-8)$, $F(0,10)$, $G(6,6)$.

3.2 The following graph shows the mass of six runners and their respective speeds during a short sprint. In this case, mass is measured in kilograms, while speed is measured in meters per second. Use this graph to describe the relationship between a runner's mass and speed.



3.3 The following graph shows the foot length and height, in centimeters, of seven people. Use this graph to describe the relationship between foot length and height.



Activity 4

4.1 A bag contains 18 red marbles, 20 blue marbles, and 14 green marbles.

a. What percentage of the marbles are red?

b. What percentage of the marbles are green or red?

4.2 Community Airport offers a total of 120 flights each day. Of these flights, 25% are operated by Wide World Airlines. How many flights does Wide World Airlines offer each day?

4.3 A portable stereo worth $60 last year is worth $45 this year.

a. By how much did the value of the stereo decrease?

b. What percentage of the stereo's original value is represented by the amount of the decrease?

4.4 A 1968 Mustang automobile worth $24,000 last year is worth $26,500 this year.

a. By how much did the value of the car increase?

b. What percentage of last year's value is represented by the amount of the increase?

Answers to Flashbacks

Activity 1

1.1 **a.** The mean is 1.005.

b. The median is 0.451.

c. There is no mode since each number occurs only once.

1.2 **a.** Company C produces the largest number of vehicles.

b. **1.** 1985

2. approximately 9 million vehicles

c. Company C produced approximately 1.5 times as many cars as Company A and approximately 3 times as many as Company B.

Activity 2

2.1 **a.** The ratio of women to the total number of people in the room is:

$$\frac{24}{24+18}=\frac{4}{7}$$

b. The ratio of men to the total number of people in the room is:

$$\frac{18}{24+18}=\frac{3}{7}$$

2.2 **a.** The ratio of blue marbles to the total number of marbles is:

$$\frac{20}{52}=\frac{5}{13}$$

b. The ratio of marbles that are either green or red to the total number of marbles in the bag is:

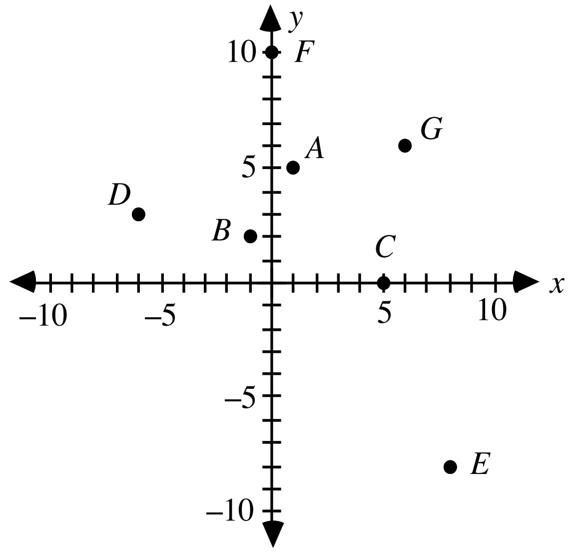
$$\frac{32}{52}=\frac{8}{13}$$

2.3 **a.** Each characteristic accounts for $1/6$ of Mary's decision.

b. Student selections will vary. Two characteristics represent $1/3$ of the total number listed.

Activity 3

3.1 Sample graph:



3.2 As the mass of a runner increases, speed decreases.

3.3 As foot length increases, height increases.

Activity 4

4.1 **a.** Approximately 35% of the marbles are red.

b. Approximately 62% of the marbles are green or red.

4.2 The number of flights offered by Wide World Airlines is $0.25 \bullet 120 = 30$.

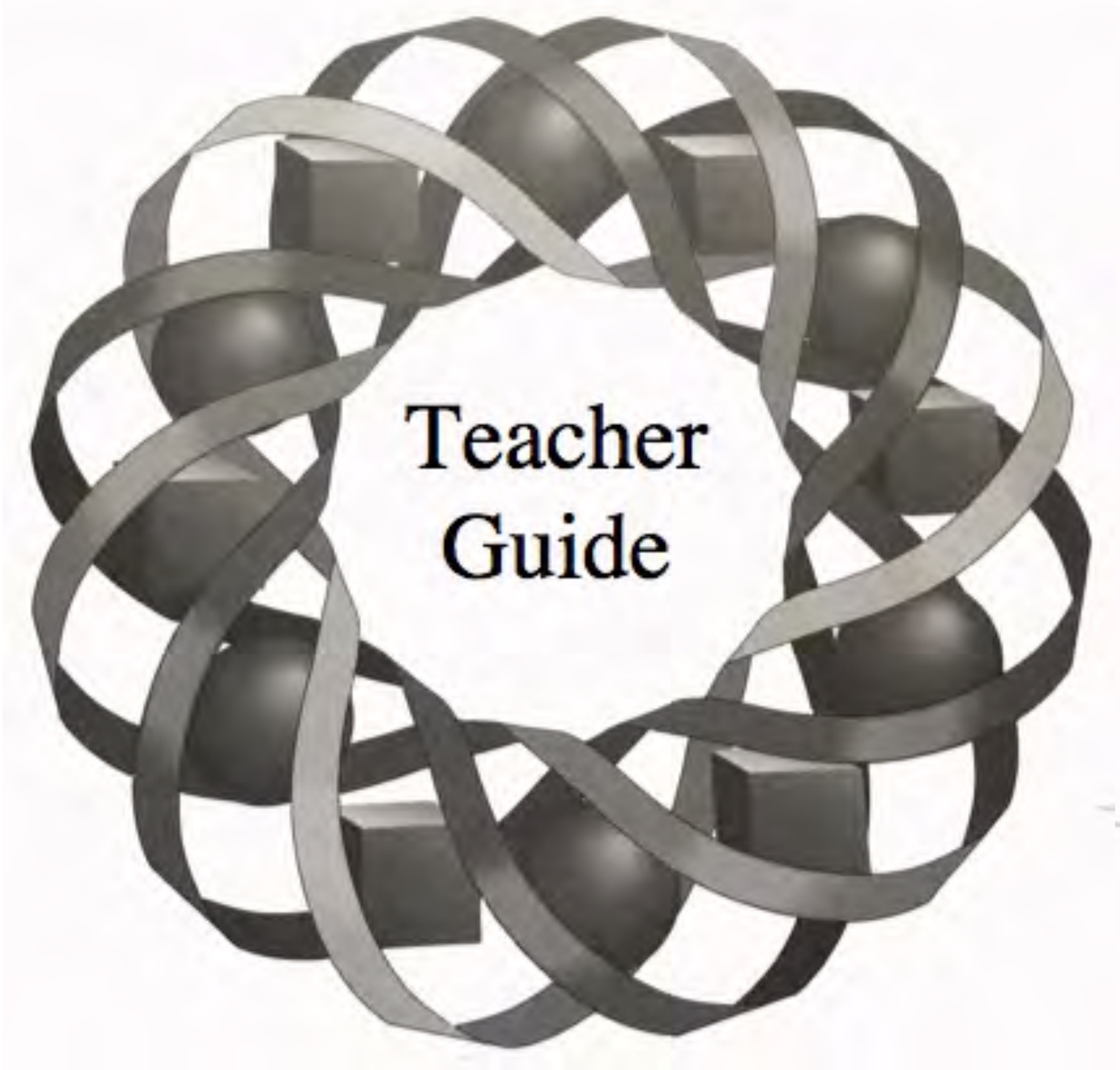
4.3 **a.** The decrease in value is \$15.

b. The percentage of the original value is $15/60 = 0.25 = 25\%$.

4.4 **a.** The increase in value is \$2500.

b. The percentage of last year's value is $2500/24,000 \approx 10.4\%$.

Yesterday's Food Is Walking and Talking Today



You are what you eat, give or take a few calories. But how those calories get used depends largely on what you do. In this module, you examine how some daily activities—like walking, talking, or doing the backstroke—affect your dietary needs.

John Carter • Janet Higgins • Paul Swenson • Steve Yockim

Teacher Edition

Yesterday's Food is Walking and Talking Today

Overview

This module uses calorie intake and expenditure to extend students' understanding of linear relationships. By examining slope, y-intercept, and point of intersection, students gain experience analyzing graphs and solving equations.

Objectives

In this module, students will:

- use a graphing utility to display data
- analyze scatterplots and line graphs
- examine ratio as a measure of slope
- examine slope as a rate
- examine the slopes of parallel lines
- write a linear equation given the slope and the y-intercept
- write a linear equation given two data points
- identify the domain and range of a linear relation
- graphically solve systems of linear equations
- solve linear equations for y in terms of x.

Prerequisites

For this module, students should know:

- metric conversions
- how to express a rate
- how to plot and interpret points on a coordinate plane.

Time Line

Activity	Intro.	1	2	3	Summary Assessment	Total
Days	1	3	3	2	1	10

Materials Required

Materials	Activity				
	Intro.	**1**	**2**	**3**	**Summary Assessment**
large can	X				
small can	X				
crucible (or tuna can)	X				
metal rod	X				
thermometer (˚C)	X				
100–500 mL water	X				
matches or lighter	X				
paper clips	X				
roasted mixed nuts	X				
balance	X				
safety goggles	X				

Technology

Software	Activity				
	Intro.	**1**	**2**	**3**	**Summary Assessment**
graphing utility		X	X	X	
spreadsheet	X	X	X		
symbolic manipulator		X	X	X	
temperature probe	X				
science interface device	X				

Teacher Note

In all activities, students use a graphing utility and spreadsheet to graph scatterplots and linear equations. You may wish to allow students to use a symbolic manipulator to transform linear equations from point-slope form to slope-intercept form.

In the exploration in the introduction, you may wish to use a science interface device and temperature probe instead of a thermometer.

Yesterday's Food is Walking and Talking Today

Introduction
(page 59)

Students burn several different kinds of nuts, measure the amount of heat produced, and convert their data into calories and kilocalories.

Materials List

- safety goggles (one pair per student)
- large can with both ends removed and vent holes added (one per group)
- small can with open top (one per group)
- 100–500 mL water at room temperature (per group)
- crucible (one per group; a tuna can or jar lid may be substituted)
- thermometer (one per group)
- balance (one per group)
- metal rod (one per group)
- matches or lighter (one per group)
- length of wire or paper clip (one per group)
- roasted mixed nuts (enough for each group to burn several nuts)

Technology

- science interface device (optional)
- temperature probe (optional)
- spreadsheet (optional)

Teacher Note

You may wish to conduct this exploration as a demonstration. A small quantity of vegetable oil (such as olive oil) may be substituted for the nuts.

Exploration
(page 59)

Students must wear eye protection throughout the experiment.

a. The experimental setup illustrated in Figure **1** is just one of many possible configurations. You may wish to consult a science teacher for suggestions on equipment. A length of wire may be carefully forced through each nut or a paper clip bent to support the nut as it burns.

b. See sample data given in Parts **e–f** below.

c. Beginning with water at room temperature and positioning the nut no more than 5 cm below the can of water yields better experimental results. Although it is important to measure the volume of water carefully, the quantity may vary from 100 mL to 500 mL, depending on the size of the can.

d. In the setup illustrated in Figure **1**, the nut can be ignited by moving the large and small cans, then replacing them after the nut has begun burning.

e–f. Sample data:

Table 1: Data for calorie experiment

Type of Nut	Initial Temperature (˚C)	Maximum Temperature (˚C)	Initial Mass (g)	Final Mass(g)	Volume of Water (mL)
brazil nut	23.0	25.0	24.66	24.62	100
peanut	24.0	30.0	20.58	20.49	100
almond	23.0	24.0	21.49	21.45	100
cashew	22.0	24.5	21.39	21.30	100

g–h. Sample data:

Table 2: Kilocalories in different kinds of nuts

Type of Nut	Change in Temperature (˚C)	Change in Mass (g)	Volume of Water (mL)	Calories per Gram	Kcal per Gram
brazil nut	2.0	0.04	100	5000	5.0
peanut	6.0	0.09	100	6667	6.7
almond	1.0	0.04	100	2500	2.5
cashew	2.5	0.09	100	2778	2.8

Teacher Note

The calorie content of food is typically determined using a bomb calorimeter. This device consists of a metal sphere in which food is placed along with pure oxygen. By submerging the calorimeter in water and igniting the contents electronically, the heat energy released can be measured precisely. Results are then adjusted according to the body's ability to metabolize each type of food.

Discussion

(page 61)

a. Experimental results may vary considerably. Most nuts contain close to 6 kcal/g.

b. One package of dry roasted peanuts reports 170 Calories in a 28-g serving, or approximately 6.1 kcal/g.

c. While an incompletely burned nut will give off less heat, it would also lose less mass. The results should not be affected since the smaller loss of mass is accounted for when calculating kilocalories.

d. Using the experimental setup illustrated in Figure **1**, some heat is lost both while lighting the nut and during its burning. Experimental error may also be introduced in measuring the volume and temperature of the water and the mass of the nuts.

e. Sample response: The quantity of water has no bearing on the results since the calculations are adjusted accordingly.

(page 62)

Activity 1

Students calculate rates of kilocalorie usage for a variety of physical activities and use these rates to write equations of the form $y = mx$. Slope is defined as a rate and as the ratio of the change in vertical distance to the change in horizontal distance in the graph of a line. **Note:** Throughout this module, graphs of lines are used to model real-world relationships. Portions of a graph that lie outside the reasonable domain for the problem setting are represented as dashed lines.

Materials List

- none

Technology

- graphing utility
- spreadsheet

Teacher Note

You may wish to use Parts **d** and **e** of the discussion as an opportunity to demonstrate the use of dimensional analysis. This process can be especially helpful when performing a multi-step conversion. For example, to convert volume measured in barrels (bbl) to cubic meters, students may "cancel" units as follows:

$$100\ \cancel{\text{bbl}} \cdot \frac{42\ \cancel{\text{gal}}}{1\ \cancel{\text{bbl}}} \cdot \frac{3.8\ \cancel{\text{L}}}{1\ \cancel{\text{gal}}} \cdot \frac{100\ \cancel{\text{mL}}}{1\ \cancel{\text{L}}} \cdot \frac{1\ \cancel{\text{cm}^3}}{1\ \cancel{\text{mL}}} \cdot \frac{1\ \text{m}^3}{1 \cdot 10^6\ \cancel{\text{cm}^3}} = 15.96\ \text{m}^3$$

Discussion 1

(page 62)

a. As indicated in Table **3**, strenuous physical activities such as jumping rope, aerobic dancing, and swimming burn calories at a high rate.

b. As indicated in Table **3**, activities that require little physical exertion, such as lying at ease and sitting quietly, burn calories at a low rate.

c. The human body uses energy to maintain a constant temperature near 37˚ C (98.6˚ F), circulate blood, and perform other basic functions. The body ceases to burn calories only after death.

d. Sample response: Since you are looking for an answer in kilocalories, you must cancel minutes and kilograms by multiplying the table entry for lying at ease

$$0.022\ \frac{\text{kcal}}{\text{min} \bullet \text{kg}}$$

by 30 min and 60 kg.

e.

1. Sample response: Using the appropriate entry from Table **3**, the energy usage for a 60-kg person playing basketball for *x* minutes can be represented by the following expression:

$$60\ \text{kg}\left(0.138\ \frac{\text{kcal}}{\text{min} \bullet \text{kg}}\right)x\ \text{min}$$

When this expression is multiplied, the units of kilograms and minutes cancel, leaving $8.28x$ kcal.

2. To solve for x, divide both sides of the equation by 8.28:

$$500 = 8.28x$$
$$60.39 \approx x$$

Exploration

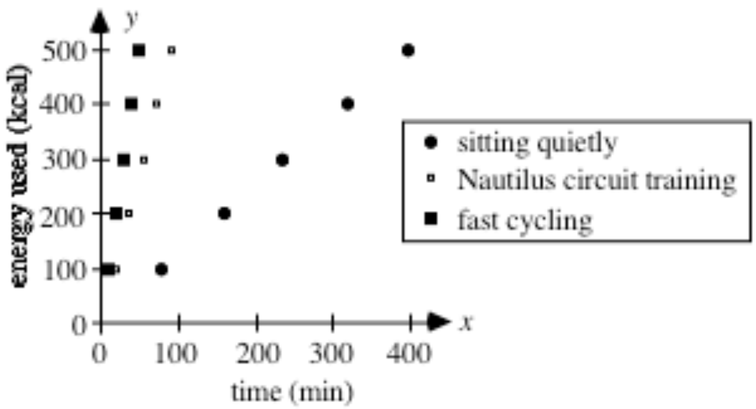
(page 63)

a–c. Sample response:

Table 4: Time required to burn kilocalories for three activities

Energy Required	**High**	**Moderate**	**Low**
Activity	fast cycling	Nautilus circuit training	sitting quietly
Minutes to burn 100 kcal	9.9	18.1	79.4
Minutes to burn 200 kcal	19.7	36.2	158.7
Minutes to burn 300 kcal	29.6	54.3	238.1
Minutes to burn 400 kcal	39.4	72.5	317.5
Minutes to burn 500 kcal	49.3	90.6	396.8

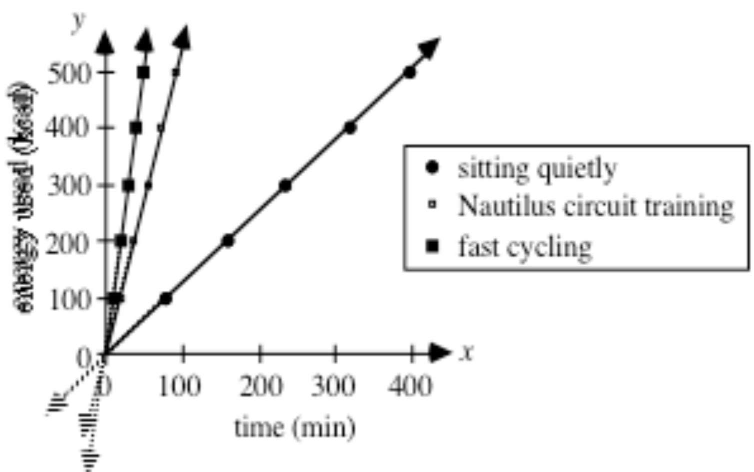
d. The following scatterplot use the sample data given above.



e. The table below lists equations for the sample activities given in Part **a**

Activity	Equation
fast cycling	$y = 10.14x$
Nautilus circuit training	$y = 5.52x$
sitting quietly	$y = 1.26x$

f. Sample graph:



Discussion 2

(page 65)

a. The graphs all contain the origin.

b. High-energy activities have graphs that are closest to vertical.

c. Low-energy activities have graphs that are closest to horizontal.

d. **1.** An 80-kg person uses 6 kcal of energy while playing cards for 3 min.

2. In this situation, the rate is energy usage in kilocalories per minute.

e. **1.** For the three sample equations given in Part **e** of the exploration, the slopes are 10.14, 5.52, and 1.26, respectively.

2. The slope of each equation can be calculated using any two values from a given column in Table **4**. For fast cycling, two points are 9.9 min for 100 kcal and 19.7 min for 200 kcal. The change in y (energy used) is $200 - 100 = 100$. The change in x (time) is $19.7 - 9.9 = 9.8$. Therefore, the slope is:

$$\frac{100 \text{ kcal}}{9.8 \text{ min}} \approx 10.2 \text{ kcal/min}$$

3. The slopes of the equations represent the rate of energy usage in kilocalories per minute.

f. **1.** Each domain is the set of all real numbers.

2. Each range is the set of all real numbers.

g. **1.** Sample response: Since it is not possible to play cards for a negative number of minutes, only non-negative real numbers make sense.

2. The corresponding range is the set of non-negative real numbers. **Note:** In Figure **2**, the unused part of the graph is indicated by a dotted line.

Assignment

(page 66)

1.1 **a.** $(57 \text{ kg})(23 \text{ min})\left(0.178 \dfrac{\text{kcal}}{\text{min} \bullet \text{kg}}\right) \approx 230 \text{ kcal}$

b. $(61 \text{ kg})(120 \text{ min})\left(0.132 \dfrac{\text{kcal}}{\text{min} \bullet \text{kg}}\right) \approx 970 \text{ kcal}$

c. $y = (58 \text{ kg})(x \text{ min})\left(0.085 \dfrac{\text{kcal}}{\text{min} \bullet \text{kg}}\right)$ or $y \approx 4.9x$

d. Both the domain and the range are the set of non-negative real numbers.

1.2 **a.** $\dfrac{300 \text{ kcal}}{(58 \text{ kg})\left(0.195 \dfrac{\text{kcal}}{\text{min} \bullet \text{kg}}\right)} \approx 27 \text{ min}$

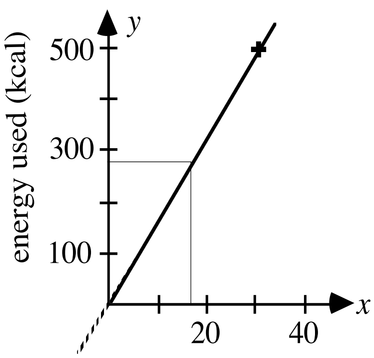
b. $\dfrac{320 \text{ kcal}}{(72 \text{ kg})\left(0.135 \dfrac{\text{kcal}}{\text{min} \bullet \text{kg}}\right)} \approx 33 \text{ min}$

c. $y = (60 \text{ kg})(x \text{ min})\left(0.027 \dfrac{\text{kcal}}{\text{min} \bullet \text{kg}}\right)$ or $y = 1.6x$; $x = \dfrac{y}{1.6}$

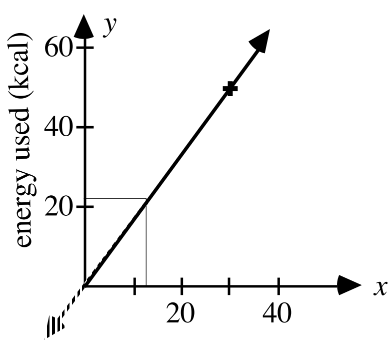
1.3 In Parts **a** and **b**, students may check their responses using proportional reasoning. For example,

$$\frac{500}{30} = \frac{x}{17}$$

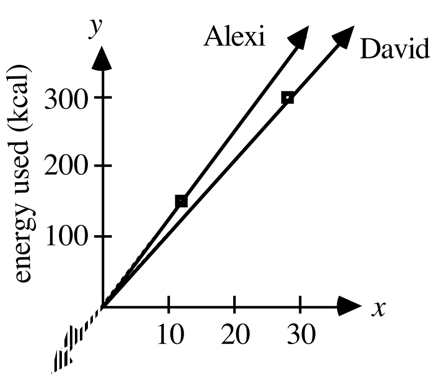
a. Alexi uses about 280 kcal in 17 min of running. Sample graph:



b. David uses about 20 kcal in 12 min of typing. Sample graph:



c. Sample response: The graph of Alexi's energy usage has a greater slope than the graph of David's energy usage. Since Alexi burns calories at a faster rate, he will burn more calories in 1 hr.

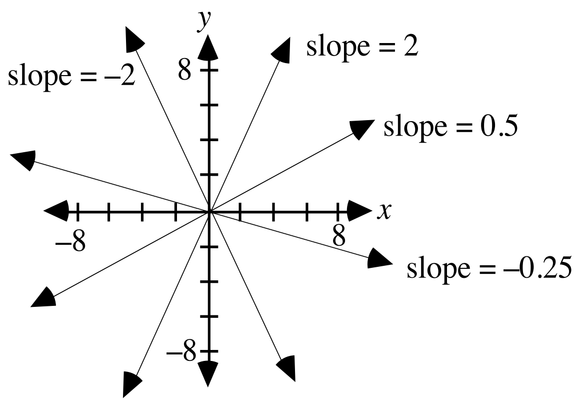


d. The range of each relation is the number of kilocalories since the range is represented by the y values and the y values on the graph represent the number of kilocalories used.

1.4 **a.** A completed table is shown below

Change in Vertical Distance	Change in Horizontal Distance	Slope
8	4	2
4	8	0.5
–8	4	–2
3	–12	–0.25

b. Sample graph:



1.5 **a.** $\dfrac{3-7}{12-3}=-\dfrac{4}{9}$

b. $\dfrac{-4-2}{6-5}=-6$

c. $\dfrac{4-(-8)}{10-12}=-6$

d. $\dfrac{2-(-2)}{-3-(-7)}=1$

e. $\dfrac{-\frac{3}{5}-\frac{2}{5}}{-\frac{5}{7}-\frac{9}{7}}=\dfrac{1}{2}$

1.6 **a.** In the exploration, the rate of energy use for a 60-kg person cycling fast is relatively large and positive (10.14 kcal/min). A speed of 100 km/hr is another example.

b. In the exploration, the rate of energy use for a 60-kg person sitting quietly is relatively small and positive (1.26 kcal/min). A speed of 0.01 km/hr is another example.

c. In Activity **2**, students investigate negative rates by considering the kilocalories held in reserve during exercise. For example, the rate of energy loss for a 60-kg person cycling fast is –10.14 kcal/min.

1.7 **a.** The coordinates are $A(2,2)$, $B(4,4)$, and $C(8,8)$.

b. **1.** The vertical change is 2, the horizontal change is 2, and the slope is 1.

2. The vertical change is 4, the horizontal change is 4, and the slope is 1.

3. The vertical change is 6, the horizontal change is 6, and the slope is 1.

c. No. Slope is the ratio

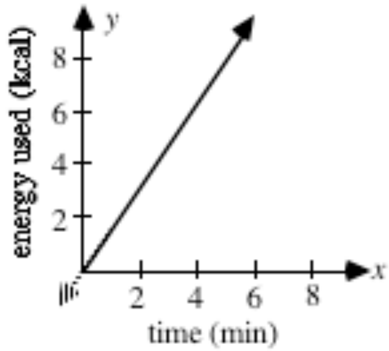
$$\frac{\text{change in vertical distance}}{\text{change in horizontal distance}}$$

This ratio remains constant for any two points on the line.

***1.8** The following sample responses use painting as the activity.

a. $y = (50 \text{ kg})(x \text{ min})\left(0.034 \dfrac{\text{kcal}}{\text{min} \bullet \text{kg}}\right)$ or $y = 1.7x$

b. The following sample graph shows the energy used over time while a 50-kg person is painting.



c. In this setting, both the domain and the range are the set of non-negative real numbers. Values in the domain represent the number of minutes spent painting, while values in the range represent the number of kilocalories used.

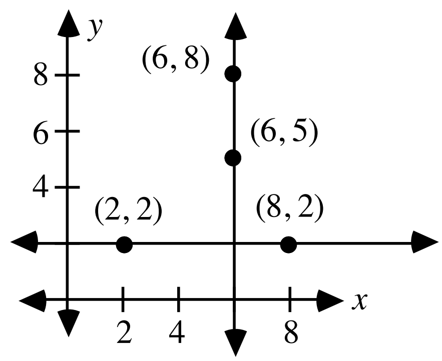
d. The slope of the line in Part **b** is about 1.7. In this situation, it represents the rate (in kilocalories per minute) at which a 50-kg person uses energy while painting.

e. **1.** $y = 1.7(30) = 51$ kcal

2. $y = 1.7(100) = 170$ kcal

f. Using the sample responses given above, 30 min of painting results in an energy usage of 51 kcal, while 100 min of painting results in an energy usage of 170 kcal. The change in y (kilocalories) is 119, the change in x (time) is 70 min, and the ratio or slope is about 1.7.

***1.9** **a–d.** Sample graph:



The slope of the horizontal line is:

$$\frac{2-2}{8-2}=\frac{0}{6}=0$$

The slope of the vertical line is undefined:

$$\frac{8-5}{6-6}=\frac{3}{0}$$

e. Answers will vary.

1. The equation of the sample horizontal line is $y=2$.

2. The domain is the set of all real numbers and the range is 2.

f. Answers will vary.

1. The equation of the sample vertical line is $x=6$.

2. The domain is 6 and the range is the set of all real numbers.

* * * * *

1.10 While jumping rope, Sam burns kilocalories at the rate below:

$$\left(0.162\ \frac{\text{kcal}}{\text{min} \bullet \text{kg}}\right)(75\ \text{kg}) \approx 12\ \text{kcal/min}$$

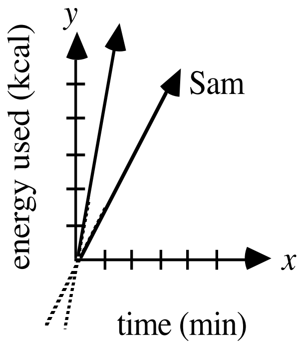
At this rate, the time to burn 865 kcal can be found as follows:

$$\frac{865\ \text{kcal}}{12\ \frac{\text{kcal}}{\text{min}}} \approx 72\ \text{min}$$

1.11 **a.** $y = 227 \bullet 0.022 \bullet x$ or $y = 5x$

b. $y = 75 \bullet 0.022 \bullet x$ or $y = 1.7x$

c. Sample graph:



d. The sumo wrestler's graph is closer to vertical. This means he uses energy at a faster rate than Sam.

1.12 **a.** $\dfrac{1220 \text{ m} - 1000 \text{ m}}{4 \text{ hr}} = 55 \text{ m/hr}$

b. $\dfrac{655{,}000 - 534{,}000}{1990 - 1980} = \dfrac{121{,}000 \text{ people}}{10 \text{ yr}} = 12{,}100 \text{ people/yr}$

c. $\dfrac{\$32 - \$8}{1990 - 1950} = \dfrac{\$24}{40 \text{ yr}} = \$0.60/\text{yr}$

d. $\dfrac{1500 - 2400}{3 - 1} = \dfrac{-900 \text{ m}}{2 \text{ sec}} = -450 \text{ m/sec}$

e. Sample response: The domain is the times from 1 sec to 3 sec. The range is the altitudes from 2400 m to 1500 m.

* * * * * * * * * *

(page 71)

Activity 2

This activity introduces students to the slope-intercept form of linear equations.

Materials List

- none

Technology

- spreadsheet
- graphing utility
- symbolic manipulator (optional)

Teacher Note

Table **5** gives only a sample listing of breakfast foods. You may wish to obtain complete lists from the original sources.

Exploration

(page 71)

a. The kilocalories needed for a 62-kg person to play racquetball for 1 hr can be found as follows:

$$\left(0.178\ \frac{\text{kcal}}{\text{min} \bullet \text{kg}}\right)(60\text{ min})(62\text{ kg}) \approx 662\text{ kcal}$$

An adequate breakfast might include a glass of 2% milk (112 kcal), an omelet (290 kcal), a glass of orange juice (120 kcal), and two pieces of wheat toast (140 kcal).

b–g. While playing racquetball, a 62-kg person burns approximately 55 kcal every 5 min. Sample table:

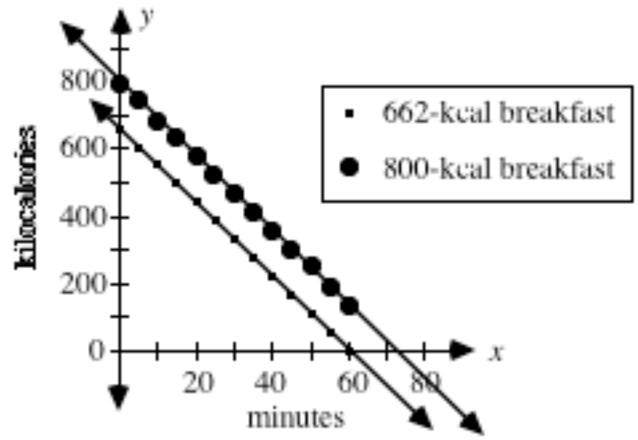
Time (min)	Energy Remaining from 662-kcal Meal	Energy Remaining from 800-kcal Meal
0	662	800
5	607	745
10	552	690
⋮	⋮	⋮
55	57	195
60	2	140

Both graphs have the same slope. Using the points $(5, 607)$ and $(10, 552)$, for example, the slope is:

$$\frac{607 - 552}{5 - 10} = -11$$

The graphs intersect the *y*-axis at 662 and 800, respectively.

The equation that models the energy remaining after a 662-kcal breakfast is $y = -11x + 662$. The equation that models the energy remaining after a 800-kcal breakfast is $y = -11x + 800$. The following graph shows both equations along with the respective scatterplots.



Discussion (page 72)

a. Sample response: The graph of the equation goes through the points on the scatterplot.

b. In this situation, *y* represents the kilocalories remaining from breakfast as the person plays racquetball, while *x* represents the minutes that the person plays racquetball.

c. The two identifying characteristics of a line are slope and *y*-intercept.

d. The slopes of the equations are equal; therefore, the graphs of the equations are parallel.

e. When a linear equation is written in slope-intercept form, the slope is the coefficient of *x*, while the *y*-intercept is the constant. In the equation $y = 7x + 6$, for example, the slope is 7 and the *y*-intercept is 6.

Teacher Note

Students may find symbolic manipulators helpful in the assignment.

Assignment (page 73)

***2.1** **a.** The kilocalories needed for a 50-kg person to dance aerobically for 1 hr can be found as follows:

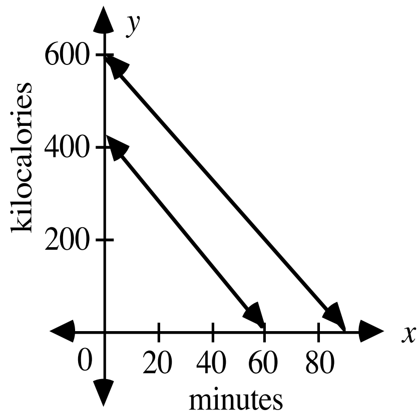
$$\left(0.135 \ \frac{\text{kcal}}{\text{min} \bullet \text{kg}}\right)(60 \ \text{min})(50 \ \text{kg}) = 405 \ \text{kcal}$$

One possible breakfast includes wheat toast (70 kcal) and an egg with muffin (340 kcal) for a total of 410 kcal.

b. The corresponding equation for the sample breakfast in Part **a** is $y = -6.75x + 410$, where *y* represents the kilocalories remaining from the meal and *x* represents time in minutes.

c. $y = -6.75x + 600$

d. Sample graph:



2.2 **a.** 0

b. $y = 3x + 4$

c. $y = -2x + 5$

d. $y = \frac{7}{3}x + \frac{2}{5}$

e. $y = \frac{2}{5}x - 3$

2.3 **a.** $y = 2x - 3$

b. $y = 3x + 7$

c. $y = -4x - 7$

d. $y = -\frac{3}{4}x + \frac{7}{4}$

e. $y = \frac{2}{3}x - 2$

2.4 **a.** $y = -10x$

b. $y = \frac{4}{7}x$

c. $y = -4x$

2.5 **a.** $\frac{4-2}{6-2} = \frac{1}{2}$

b. $\frac{y-2}{x-2}$

c. $\frac{y-2}{x-2} = \frac{1}{2}$ or $y - 2 = \frac{1}{2}(x-2)$

2.6 **a.** $\dfrac{y_2 - y_1}{x_2 - x_1}$

b. $\dfrac{y - y_1}{x - x_1}$

c. Sample response: These two expressions are equal because they both represent the slope of the same line.

d. $\dfrac{y - y_1}{x - x_1} = \dfrac{y_2 - y_1}{x_2 - x_1}$ or $y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

2.7 **a.** **1.** $y - 8 = -3(x - 5)$

2. $y - 10 = -\dfrac{1}{2}(x - 2)$ or $y - 5 = -\dfrac{1}{2}(x - 12)$

3. $y - 0 = \dfrac{2}{3}(x - 0)$ or $y = \dfrac{2}{3}x$

4. $y - 4 = \dfrac{1}{4}(x - (-6))$ or $y - 5 = \dfrac{1}{4}(x - (-2))$

b. **1.** $y = -3x + 23$

2. $y = -\dfrac{1}{2}x + 11$

3. $y = \dfrac{2}{3}x$

4. $y = \dfrac{1}{4}x + \dfrac{11}{2}$

* * * * *

2.8 **a.** linear

b. nonlinear

c. nonlinear

d. nonlinear

e. linear

f. nonlinear

g. linear

2.9 **a.** **1.** $m \approx 0.16$

2. $m = 0.145$

b. **1.** $y - 8.2 = 0.16(x - 6.4)$ or $y - 7.5 = 0.16(x - 1.9)$

2. $y - 0.120 = 0.145(x - 9.00)$ or $y - (-0.460) = 0.145(x - 5.00)$

c. **1.** $y = 0.16x + 7.2$

2. $y = 0.145x - 1.19$

* * * * * * * * * *

(page 77)

Activity 3

Students find the intersection of two equations and use the distributive property to transform equations in point-slope form to slope-intercept form.

Materials List

- none

Technology

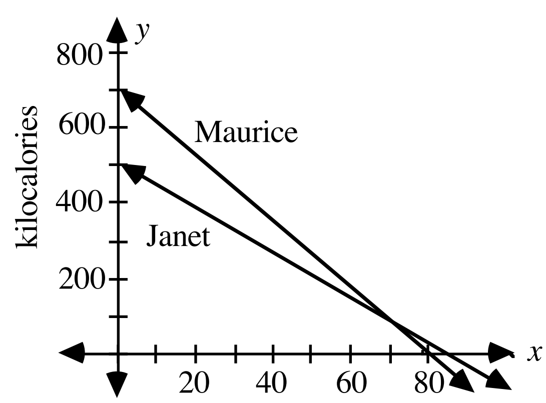
- graphing utility
- symbolic manipulator (optional)

Exploration

(page 77)

a. The slope of each equation represents the number of kilocalories burned per minute while cycling. The y-intercept represents the number of kilocalories in each person's breakfast.

b. Sample graph:



c. The point common to both lines is $(70, 80)$. Students should be able to estimate these coordinates from the graph or by using the intersection capabilities of a graphing utility.

d. Sample response: After 70 min of cycling, both Maurice and Janet have 80 kcal remaining from their breakfasts.

Discussion
(page 78)

a. The y-intercept of each equation represents the number of kilocalories in each person's breakfast. The slopes indicate the rates at which each person uses calories while cycling.

b. The equations would have to have the same slope.

c. The x-coordinate of the point of intersection indicates when Maurice and Janet had the same number of kilocalories remaining from breakfast.

d. Sample response: When an equation is written in slope-intercept form, it is easy to determine both the slope of the line and point where the line intersects the y-axis.

e. Each equation can be written in slope-intercept form using the distributive property.

1. $y = 5x - 20$

2. $y = -3x - 6$

3. $y = -7x + 35$

f. The following steps can be used to change an equation in point-slope form to an equation in slope-intercept form. Note that the y-intercept is represented by $-mx_1 + y_1$.

$$y - y_1 = m(x - x_1)$$
$$y - y_1 = mx - mx_1$$
$$y = mx - mx_1 + y_1$$

Assignment
(page 79)

3.1

a. $y = 2x - 3$

b. $y = 2x + 9$

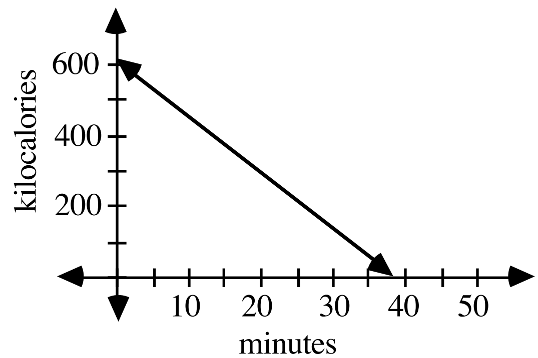
c. $y = 16x - 13$

d. $y = 5x + 10$

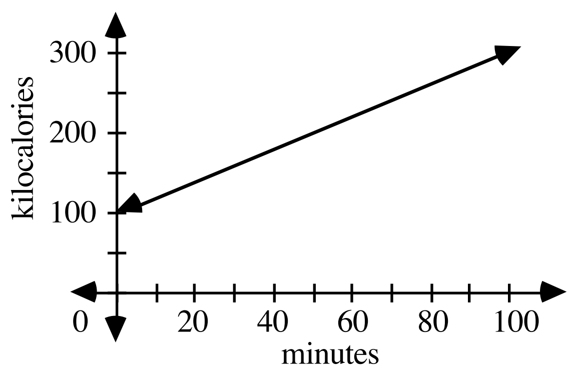
e. $y = 3x + 2$

f. $y = 6x - 18$

3.2 **a.** The equation in slope-intercept form is $y = -16x + 620$. The slope represents Ricardo's rate of energy usage in kilocalories per minute.



b. The equation in slope-intercept form is $y = 2x + 100$. The *y*-intercept represents the number of kilocalories Perry had used prior to beginning the decathlon.



c. Sample response: In the form given, the equation yields little useful information. If rewritten in slope-intercept form

$$y = \frac{5}{3}x + 30$$

then you can tell that Kelly used 30 kcal of energy before beginning to read and is currently burning them at a rate of 5/3 kcal/min.

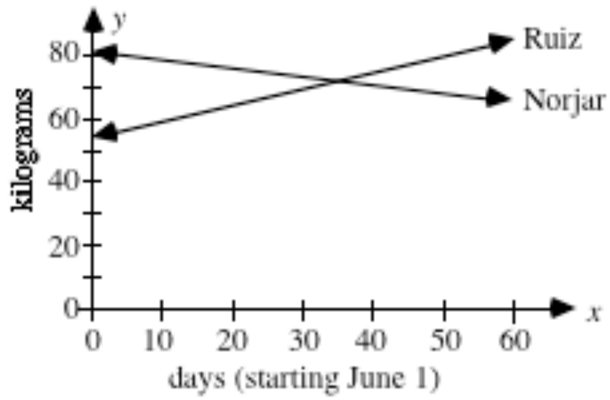
***3.3** **a.** Using ordered pairs, this information can be expressed as $(12, 78)$ and $(16, 77)$. The slope of the line through these points is

$$\frac{77 - 78}{16 - 12} = -\frac{1}{4}$$

b. $y = -\frac{1}{4}x + 81$

c. $y = \frac{1}{2}x + 55$

d. Sample graph:



e. The point common to both lines shows when the masses of the two wrestlers are equal.

f. If students represent dates as shown in the graph in Part **d**, then the coordinates are approximately $(35, 72)$.

g. Since there are 30 days in June, their masses are equal on July 5.

h. Their masses should be about 72 kg.

3.4 **a.** $\frac{8-14}{4-6} = 3$

b. $y - 8 = 3(x - 4)$

c. $y - 14 = 3(x - 6)$

d. The equations are equivalent. Both can be written as $y = 3x - 4$.

3.5 The equation for Kimberly's change in mass, where *y* represents mass in kilograms and *x* represents time in days, is:

$$y = \frac{3}{10}x + 60$$

The equation for Manuel's change in mass is:

$$y = -\frac{1}{5}x + 75$$

By finding the intersection of the graphs of these equations, students should determine that the launch can proceed on day 30.

3.6 An acceptable alternate for Kimberly or Manuel must have a mass of 69 kg on day 30.

a. The table below shows the equation for each potential alternate.

Britte	$y = 0x + 69$
Kwasi	$y = -\frac{7}{30}x + 76$
Sergei	$y = -\frac{9}{10}x + 96$
Yukawa	$y = \frac{13}{40}x + \frac{237}{4}$

b. Since she does not have to gain or lose mass, Britte is the likely choice.

***3.7** **a.** $y = 15x + 60$

b. The coordinates of the point of intersection are approximately $(3.8, 117)$. This represents the time when Rolf and Tanya have used the same number of kilocalories. After 3.8 min, each person has used 117 kcal.

3.8 **a.** Students should find the points of intersection graphically.

1. $(-2, -10)$
2. $(-12, -21)$
3. $\left(\frac{16}{3}, \frac{14}{3}\right)$
4. $(5, 4)$

b. To verify each solution, students should substitute the coordinates of the point of intersection into both equations.

* * * * *

3.9 **a.** $y = -x - 1$

b. $y = -1$

c. $y = \frac{1}{2}x$

3.10 **a.** $y = 150x$

b. The slope of the line is 150. This indicates that Denali climbs at a constant rate of 150 m/hr.

c. Denali will reach the top of the cliff in 2 hr.

3.11 **a.** $(7,4)$

b. $\left(\frac{4}{3},\frac{4}{3}\right)$

c. $(5,-2)$

* * * * * * * * * *

Teacher Note

To complete the summary assessment, students will need access to the information given in Table **3**.

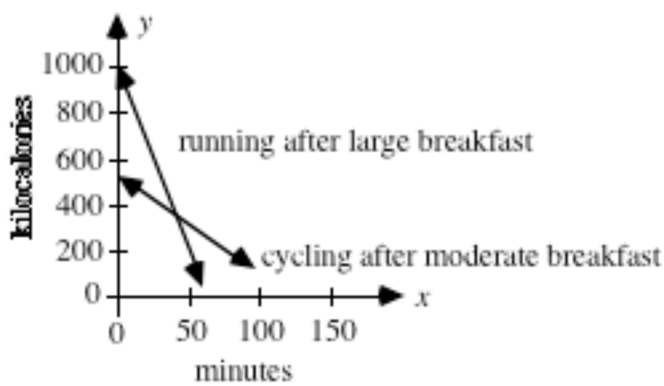
Answers to Summary Assessment (page 83)

1. Rick burns 16.6 kcal/min while running and 4.2 kcal/min while cycling.

a. The equation for kilocalories remaining while running after a large breakfast is $y = -16.6x + 1022$.

b. The equation for kilocalories remaining while cycling after a moderate breakfast is $y = -4.2x + 522$.

Sample graph:



2. **a.** The approximate coordinates of the intersection are (40,350).

b. Sample response: After approximately 40 min of running, there will be about 350 kilocalories remaining from Rick's large breakfast. The same number of kilocalories will remain from the moderate breakfast after he cycles for 40 min.

3. While writing, Rick burns kilocalories at 1.9 kcal/min. Since he has 350 kcal left from breakfast, it will take

$$\frac{350 \text{ kcal}}{1.9 \text{ kcal/min}} \approx 184 \text{ min}$$

to use the remaining kilocalories.

4. **a.** Rick burns kilocalories at a faster rate while running. Since the line which is closer to vertical has a larger slope, it represents the faster rate of kilocalorie usage while running. The line with the smaller slope represents the energy used while cycling.

b. The slope of each line can be determined by estimating the ratio of vertical change to horizontal change. The line that represents energy used while running has a vertical change of approximately 32 kcal for a horizontal change of 2 min. The slope is approximately $32/2$ or 16 kcal/min. The line that represents energy use while cycling has a vertical change of approximately 8 kcal for a horizontal change of 2 min. The slope is approximately 4 kcal/min.

c. The domain and range for both activities are the non-negative real numbers.

5. **a.** The slope of the line is:

$$\frac{270-590}{60-20}=\frac{-320}{40}=-8$$

Two possible equations in point-slope form are $y-590=-8(x-20)$ and $y-270=-8(x-60)$.

b. In slope-intercept form, the equation of the line is $y=-8x+750$. This shows that Rick had a 750-kcal breakfast.

c. Since the slope of the line is –8, Rick is using 8 kcal/min while exercising. Since Rick weighs 66 kg, the rate per kilogram of body mass can be found as follows

$$\frac{8\ \text{kcal}/\text{min}}{66\ \text{kg}}\approx 0.12\,\frac{\text{kcal}}{\text{min}\bullet\text{kg}}$$

One activity from Table **3** that uses kilocalories close to this rate is circuit training on a Universal gym, which burns

$$0.116\,\frac{\text{kcal}}{\text{min}\bullet\text{kg}}$$

Module Assessment

1. Determine the number of kilocalories used in each of the following situations:

a. an 82-kg person walking for 25 min

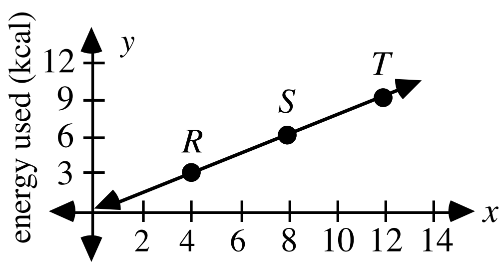
b. a 72-kg person running at 5 min/km for *x* min.

2. Find the mass of a person who burns 115 kcal while playing the piano for 45 min.

3. Find the time required for a person with a mass of *x* kg to burn *y* kilocalories at a rate of

$$m \frac{\text{kcal}}{\text{min} \bullet \text{kg}}$$

4. Use the following graph to complete Parts **a–e** below.



a. List the coordinates of points *R*, *S*, and *T*.

b. Find the vertical change, the horizontal change, and the slope between points *R* and *T*.

c. Find the *y*-intercept.

d. Write the equation of the line that passes through all three points.

e. Describe what the line represents.

5. Steve and Chia-Heng are divers for the National Oceanic and Atmospheric Administration (NOAA). For an experiment on oxygen consumption rates, they must have approximately the same mass. To meet this goal, Chia-Heng has increased her daily consumption of calories, while Steve has reduced his.

Their masses in kilograms for the previous 21 days are shown in the table below. If the mass of each diver continues to change at the same rate, when can the experiment begin?

Diver	Day 0	Day 7	Day 14	Day 21
Steve	70.15 kg	no data	67.65 kg	66.40 kg
Chia-Heng	58.50 kg	60.25 kg	no data	63.75 kg

6. Rewrite each of the following equations in slope-intercept form.

a. $y + 11 = -7x$

b. $y + 7 = 4(x - 3)$

c. $y - 7 = 0.6(x + 20)$

7. Write the equation of the line through each of the following pairs of points in slope-intercept form.

a. $(3, 4)$ and $(5, 12)$

b. $(-5, 9)$ and $(7, 3)$

Answers to Module Assessment

1. **a.** $(82\ \text{kg})\left(0.080\ \dfrac{\text{kcal}}{\text{min} \bullet \text{kg}}\right)(25\ \text{min}) = 164\ \text{kcal}$

b. $(72\ \text{kg})\left(0.208\ \dfrac{\text{kcal}}{\text{min} \bullet \text{kg}}\right)(x\ \text{min}) \approx 15x\ \text{kcal}$

2. To determine that the person has a mass of 64 kg, students should solve the following equation for *x*:

$$(x\ \text{kg})\left(0.040\ \frac{\text{kcal}}{\text{min} \bullet \text{kg}}\right)(45\ \text{min}) = 115\ \text{kcal}$$

3. In the following expression, *t* represents time in minutes:

$$t\ \text{min} = \frac{y\ \text{kcal}}{(x\ \text{kg})\left(m\ \dfrac{\text{kcal}}{\text{min} \bullet \text{kg}}\right)}$$

4. **a**. $R(4,3)$, $S(8,6)$, and $T(12,9)$

b. The vertical change is 6, the horizontal change is 8, and the slope is $3/4$.

c. The *y*-intercept is 0.

d. $y = \dfrac{3}{4}x$

e. The line describes the number of kilocalories burned over time.

5. Sample response: Steve's mass is described by the equation $y \approx -0.1786x + 70.15$, where *x* represents time in days. Chia-Heng's mass is described by the equation $y = 0.25x + 58.5$. The coordinates of the point of intersection of these two lines are approximately (27,65). This means that the experiment can begin on day 27.

6. **a.** $y = -7x - 11$

b. $y = 4x - 19$

c. $y = 0.6x + 19$

7. **a**. Using the point-slope form, then solving for *y*.

$$y - 12 = \frac{12 - 4}{5 - 3}(x - 5)$$
$$y = 4x - 8$$

b.

$$y - 3 = \frac{3 - 9}{7 - (-5)}(x - 7)$$
$$y = -\frac{1}{2}x + \frac{13}{2}$$
$$y = -0.5x + 6.5$$

Selected References

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Williams, J. "A Day in the Life of Jack LaLanne." *Family Health* (September 1979): 48–50.

Flashbacks

Activity 1

1.1 What does the prefix *kilo*- mean?

1.2 What is the relationship between a liter and a milliliter?

1.3 Describe three situations that involve rates, including the appropriate units in each case.

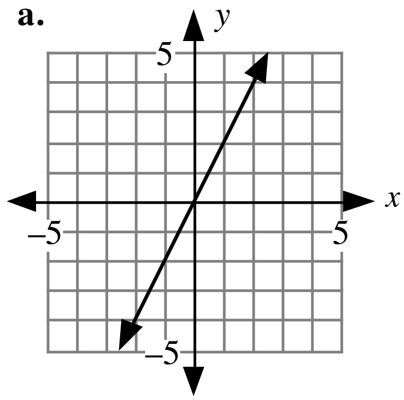
Activity 2

2.1 Draw two parallel lines on an *xy*-coordinate system.

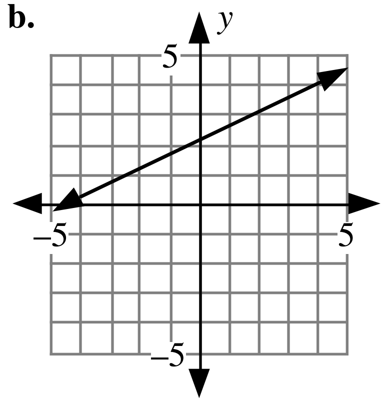
2.2 Draw two perpendicular lines on an *xy*-coordinate system.

2.3 Determine the slope of each line in Parts **a–d** below.

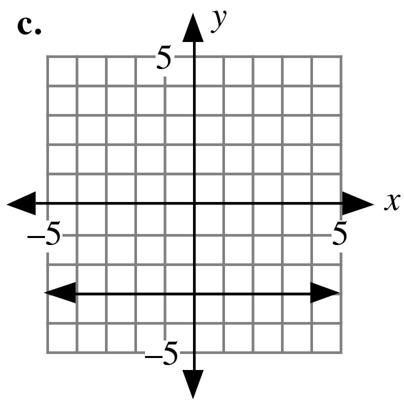
a.



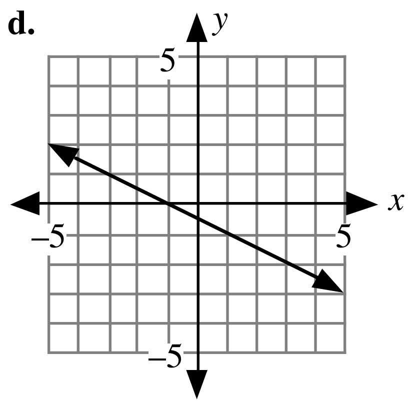
b.



c.



d.



Activity 3

3.1 Multiply each of the following:

a. $(-4)(-6)$

b. $(-2x)(5)$

c. $(-3)(-8x)$

3.2 Find the slope of the line that passes through each of the following pairs of points:

a. $(-2,-6)$ and $(-8,0)$

b. $(4,1)$ and $(-8,0)$

c. $(-6,-2)$ and $(-6,2)$

3.3 Solve each of the equations below for *y*:

a. $\frac{y}{2} = x + 5$

b. $-3y = x - 10$

Answers to Flashbacks

Activity 1

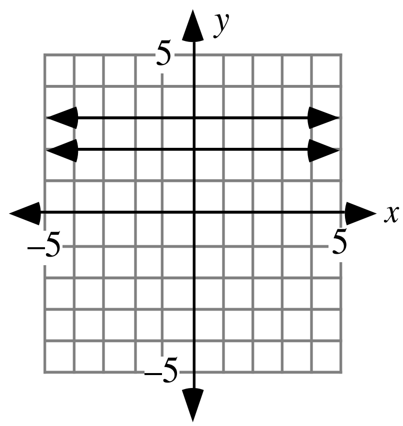
1.1 The prefix *kilo*- stands for thousand.

1.2 There are 1000 mL in 1 L of water.

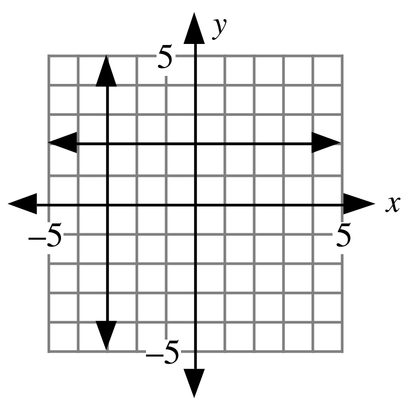
1.3 Three familiar situations that involve rates are driving a car at 60 km/hr, typing at 80 words/min, and running at 5.6 min/km.

Activity 2

2.1 Sample response:



2.2 Sample response:



2.3 **a.** 2

b. 1/2

c. 0

d. −1/2

Activity 3

3.1 **a.** $(-4)(-6) = 24$

b. $(-2x)(5) = -10x$

c. $(-3)(-8x) = 24x$

3.2 **a.** $\frac{0-(-6)}{-8-(-2)} = -1$

b. $\frac{0-1}{-8-4} = \frac{1}{12}$

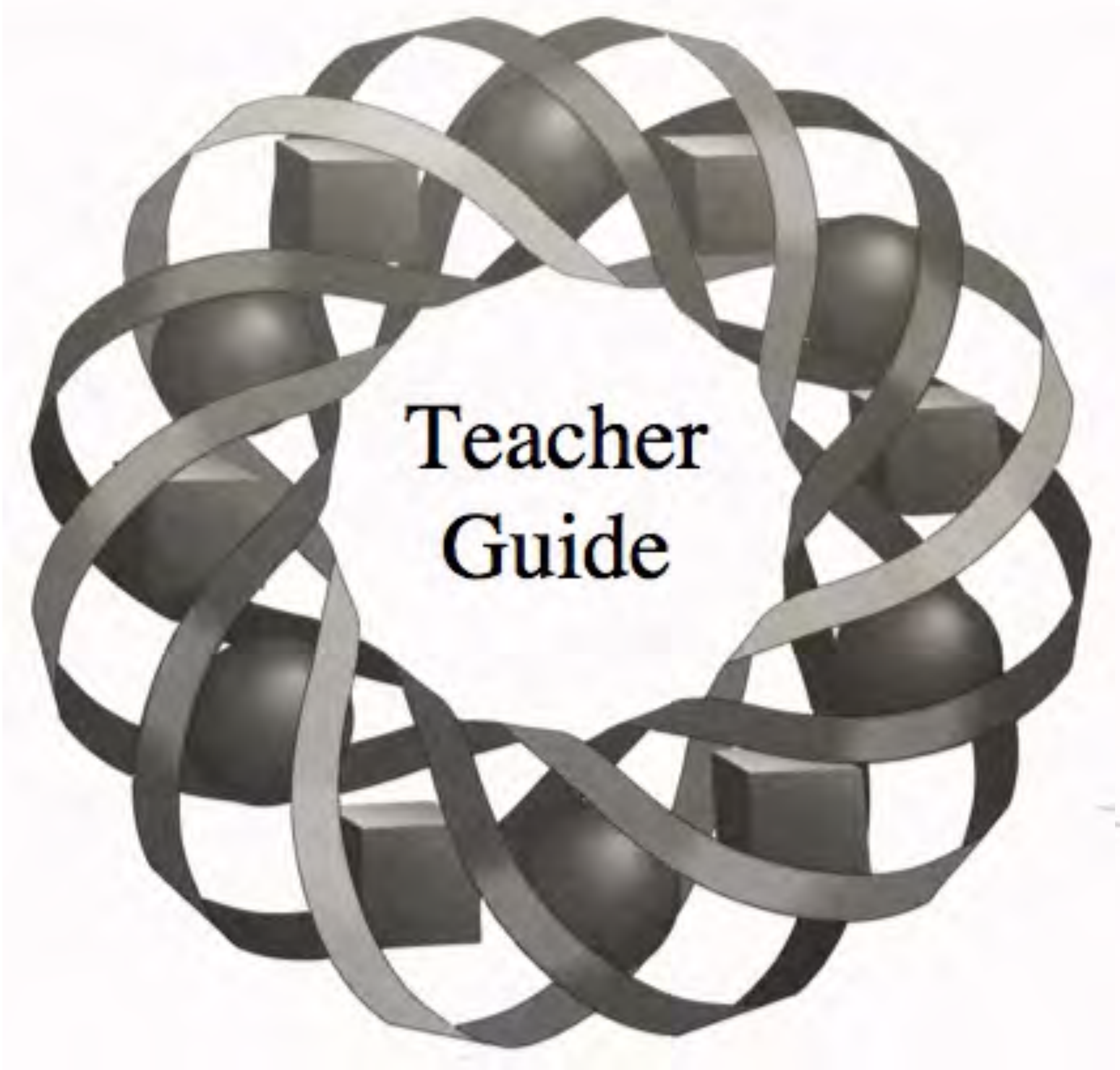
c. The slope is undefined.

$$\frac{2-(-2)}{-6-(-6)} = \frac{4}{0}$$

3.3 **a.** $y = 2x + 10$

b. $y = -\frac{1}{3}x + \frac{10}{3}$

Yesterday's Food Is Walking and Talking Today



You are what you eat, give or take a few calories. But how those calories get used depends largely on what you do. In this module, you examine how some daily activities—like walking, talking, or doing the backstroke—affect your dietary needs.

John Carter • Janet Higgins • Paul Swenson • Steve Yockim

Teacher Edition

A New Look at Boxing

Overview

In this module, students explore three-dimensional solids constructed from two-dimensional templates. Students work with percentages and ratios, as well as areas and tessellations.

Objectives

In this module, students will:

- determine nets for three-dimensional solids
- use nets to find the surface area of solids
- tessellate polygons
- find the area of regular polygons
- calculate the waste created by a template and a shape that encloses it.

Prerequisites

For this module, students should know:

- the definitions of a parallelogram, rectangle, square, triangle, and trapezoid
- the definition of a regular polygon
- how to find the area of a square, rectangle, triangle, and trapezoid
- how to make a scale drawing
- how to use a geometry utility
- how to use a spreadsheet
- how to find percent increase and decrease
- how to use metric measurements.

Time Line

Activity	1	2	3	Summary Assessment	Total
Days	2	3	3	2	10

Materials Required

Materials	Activity			
	1	**2**	**3**	**Summary Assessment**
scissors	X	X		X
rulers	X		X	X
tape or glue sticks	X			X
protractors			X	
pentagon template		X		
polygon template		X		
grid paper template	X	X		X
box template			X	
rectangular boxes (such as cereal boxes)	X			
construction paper or light cardboard		X		X

Teacher Note

Students should bring rectangular boxes to class on the first day of the module. Cereal or cracker boxes work well.

Blackline masters for the templates appear at the end of the teacher edition FOR THIS MODULE.

Technology

Software	Activity			
	1	**2**	**3**	**Summary Assessment**
geometry utility			X	X
spreadsheet		X	X	

A New Look at Boxing

Introduction

(page 91)

Students review the definition of a prism and discuss several examples.

Discussion

(page 91)

a. A cereal box can be considered a prism because it is formed by two congruent polygons in parallel planes—the top and bottom of the box. The front, back, and sides of the cereal box are parallelograms which can be formed by joining the corresponding vertices of the top and bottom.

Since the front and back are two congruent polygons in parallel planes, they could also be considered as the bases of the prism. The same reasoning can be used to designate the two sides of this box as bases.

b. The two triangular faces are the bases of the prism.

c. Many familiar objects are prisms, including compact disc boxes, briefcases, refrigerators, and some television sets.

d. A tennis-ball container is a cylinder, not a prism. While the top and bottom of the can are congruent and lie in parallel planes, they are not polygons. Since the bases are circles instead of polygons, there are no vertices which can be connected to form parallelograms.

(page 92)

Activity 1

Students use cereal boxes to explore templates, nets, and surface area. They draw templates and nets, explore shapes that enclose templates, and calculate percentage waste.

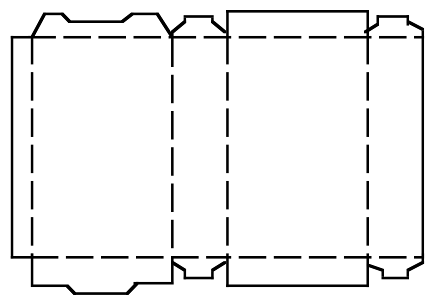
Materials List

- scissors (one pair per student)
- rulers (one per student)
- tape or glue sticks (one per group)
- grid paper (several sheets per group)
- cereal box or other rectangular box (one per group)

Exploration (page 92)

a. Encourage students to discover their own methods of determining the surface area of the box (without tearing apart the box). Sample response: The dimensions of the box are 30 cm ∞ 19 cm ∞ 8 cm. Its surface area is approximately $1900\ \text{cm}^2$.

b. The following sample sketch shows a template for a typical cereal box. Students may or may not draw the tabs.



Scale: 1 cm = 10 cm

c–d. The area of the net should be close to the estimate of the surface area made in Part **a**.

e. Answers will vary depending on the template.

Discussion (page 92)

a. The folded paper patterns should resemble the original boxes.

b. The paper patterns should be similar to the templates (or nets, if students did not draw tabs).

c. The area of the template is greater than the area of the net.

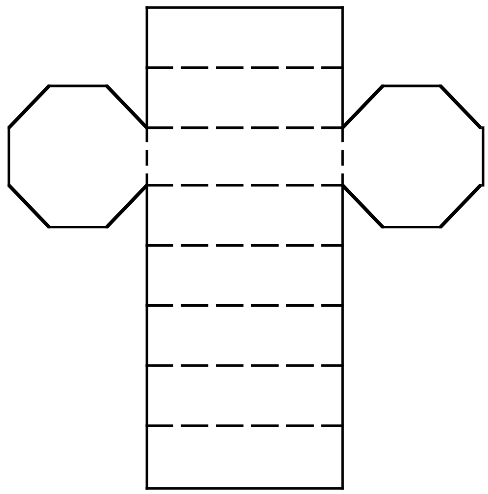
d. Sample response: Most manufacturers try to minimize waste and its associated costs.

e. The area of the net is the surface area of the box, but the area of the template is greater than the surface area of the box.

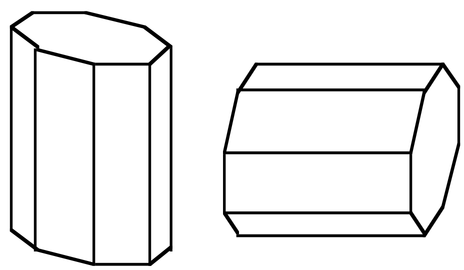
f. Sample response: All containers are not prisms. A can of tomato paste, for example, is a cylinder.

Assignment (page 93)

1.1 **a.** The diagram below shows the corresponding net.



b. Sample sketches:



*__1.2__ Answers will vary. The following sample responses were calculated using a box 30 cm ∞ 19 cm ∞ 8 cm.

a. The dimensions of the smallest rectangle are 60 cm ∞ 38 cm. Its area is approximately $2280\ \text{cm}^2$.

b. The area of the template is approximately $2166\ \text{cm}^2$.

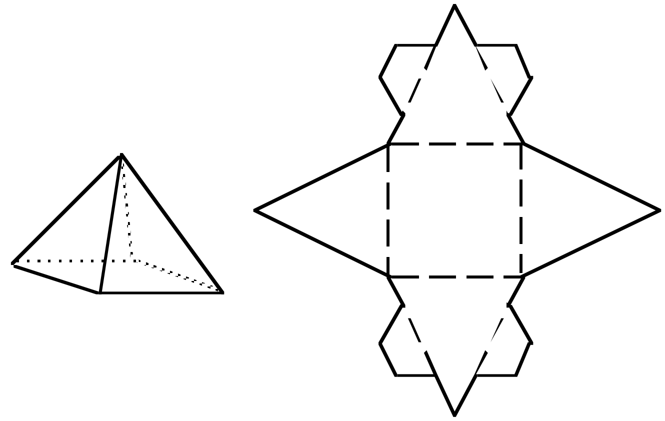
c. Since the area of the rectangle minus the area of the template equals the amount wasted, $2280\ \text{cm}^2 - 2166\ \text{cm}^2 = 114\ \text{cm}^2$. The percentage wasted can be calculated as follows:

$$\frac{114\ \text{cm}^2}{2280\ \text{cm}^2} \bullet 100 \approx 5\%$$

d. Assuming that the dimensions of a roll of cardboard are multiples of the dimensions of the smallest rectangle that encloses the template, the cost of the waste can be calculated as follows:

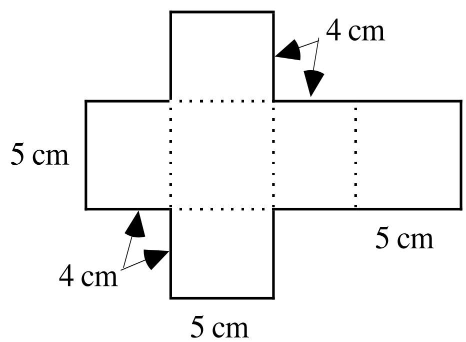
$$\frac{114\ \text{cm}^2}{1\ \text{box}} \bullet \frac{1\ \text{m}^2}{(100\ \text{cm})^2} \bullet \frac{\$0.14}{1\ \text{m}^2} \bullet 500{,}000\ \text{boxes} \approx \$800.00$$

1.3a–b. Sample sketch:

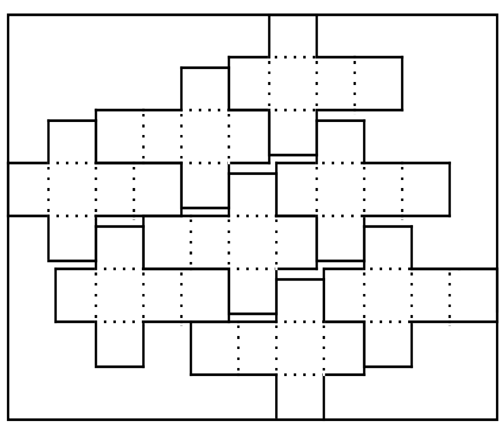


c. Sample response: This container is not a prism because it does not have two congruent faces in parallel planes.

1.4 **a.** The dimensions of the corresponding box for the sample net shown below are 5 cm ∞ 5 cm ∞ 4 cm.



b. Sample response: Positioning several nets as shown in the following diagram could reduce the amount of wasted cardboard.



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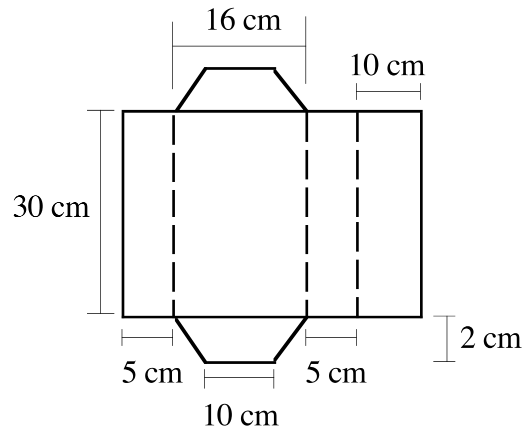
1.5 Answers may vary. Several possible methods are described below.

- Lay a grid over the octagon, and count the grid squares within the octagonal region. Add half the boundary squares to this total.
- Cut the octagon into triangles from one vertex, then determine the sum of the areas of the triangles.
- Cut the octagon into triangles from the center, then determine the sum of the areas of the triangles.
- Cut the octagon into triangles and rectangles using a tic-tac-toe pattern, then determine the sum of the areas of those polygons.

1.6 **a.** Sample drawing:



b. The surface area of the block is 1132 cm^2.

* * * * * * * * * *

(page 95)

Activity 2

In this activity, students investigate tessellations and determine which regular polygons tessellate the plane. Students also examine figures which are not regular that tessellate the plane.

Materials List

- polygon template (six copies per student)
- pentagon template (one copy per student)
- grid paper template (one copy per student)
- scissors (one pair per student)
- construction paper or light cardboard (optional)

Technology

- spreadsheet
- geometry utility (optional)

Teacher Note

In Part **a** of the exploration, six copies of the polygon template are necessary to supply each student (or group) with an appropriate number of polygons: 18 triangles, 15 squares, 4 pentagons, 10 hexagons, 8 octagons, and 6 dodecagons.

As an alternative, you may wish to photocopy the template onto heavyweight paper (such as 90# stock). Each student may then use one copy of each polygon to trace shapes on paper and explore tessellations. A blackline master of the template appears at the end of the teacher edition for this module.

Exploration

(page 96)

a. Regular triangles, squares, and hexagons will tile a plane. Students may record their drawings by tracing around templates. They may realize that the sum of the angle measures around any vertex of a tiling is 360˚.

b.
1. The sum of the measures of the exterior angles of the triangle is 360˚.
2. The measure of each exterior angle of the triangle 120˚.
3. The measure of an interior angle of an equilateral triangle is 60˚.
4. The number of equilateral triangles that could fit at one vertex is 6.

c. Sample spreadsheet:

No. of Sides in Polygon	**Measure of Exterior Angle (x)**	**Measure of Interior Angle (m)**	**No. of Polygons that "Fit" at One Vertex ($360˚/m$)**
3	120˚	60˚	6
4	90˚	90˚	4
5	72˚	108˚	≈ 3.33
6	60˚	120˚	3
7	≈ 51.43˚	≈ 128.57˚	2.8
8	45˚	135˚	≈ 2.67
9	40˚	140˚	≈ 2.57
10	36˚	144˚	2.5
11	≈ 32.73˚	≈ 147.27˚	≈ 2.44
12	30˚	150˚	2.4

Discussion

(page 97)

a. Sample response: The right-hand column of the table shows how many of the polygons could fit at one vertex. If this value is a natural number, this means that the polygons could fit with no gaps or overlaps.

b. Triangles, squares, and hexagons are the only regular polygons that tessellate a plane. The sum of the angle measures around any vertex of a tiling of these regular polygons is 360°.

c. The notion of "walking around" the exterior angles may to be extended to any regular polygon.

d. The sum of the measures of the exterior angles of any regular polygon is 360°.

e. The measure of an exterior angle of a polygon with n sides is $(360/n)^\circ$.

f. The measure of an interior angle of a polygon with n sides is $180^\circ - (360/n)^\circ$.

g. Sample response: Set the measure of the interior angle equal to $180^\circ - (360/n)^\circ$ and solve the equation for n, the number of sides.

Teacher Note

To complete Problem **2.6**, each student will need one sheet of grid paper. To complete Problem **2.7**, each student will need one copy of the pentagon template. Blackline masters appear at the end of the teacher edition for this module.

Assignment

(page 97)

2.1 **a.** $180^\circ - (360/24)^\circ = 165^\circ$

b. $(24)165^\circ = 3960^\circ$

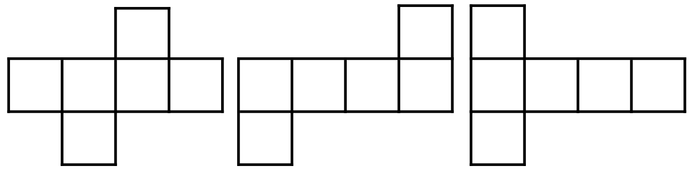
2.2 $180^\circ - (360/102)^\circ = 176\frac{8}{17}^\circ \approx 176.47$

2.3 At least three regular polygons must fit around a point to form a tessellation. For any regular polygon with more than six sides, the sum of the measures of three interior angles will always be more than 360°. Therefore, there are no regular polygons with more than six sides that tessellate the plane.

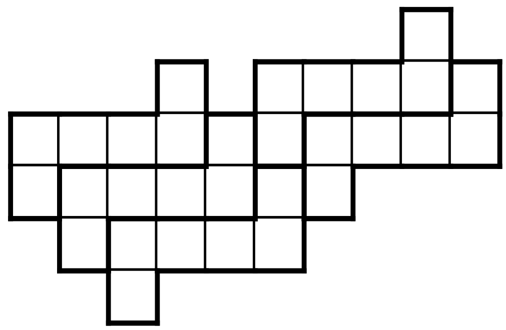
***2.4** Answers will vary. Sample response: One tiling is made of triangles placed on a flat surface. Each triangle has three sides of equal length. The pattern is created by placing these identical triangles side by side so that there are no gaps or overlaps. Each triangle is immediately adjacent to three other triangles and a total of six triangles meet at any one vertex.

2.5 $\frac{400 \text{ cm}}{35.5 \text{ cm}} \approx 11$; $\frac{6000 \text{ m}}{0.224 \text{ m}} \approx 26,785$; $11 \bullet 26,785 = 294,635$

2.6 **a.** Sample response:



b. Sample response:



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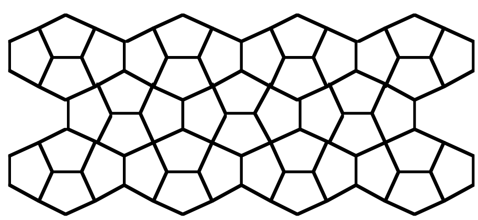
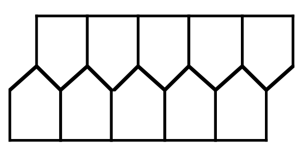
2.7 **a.** The diagram below shows one way in which this pentagon can tessellate the plane. This is often referred to as a "Cairo tessellation" because such tiles were used on the streets of Cairo, Egypt.



b. Sample design:



2.8 Yes. The sum of the measures of the interior angles of any quadrilateral is 360˚. By placing the four different vertices of a quadrilateral at a vertex of the tiling, the sum of the measures at that vertex is 360˚. This guarantees the quadrilateral can be used to tile the plane.

2.9 **a.** The interior angles measure 157.5°, while the exterior angles measure 22.5°.

b. The sum of the measures of the interior angles is 2520°, while the sum of the measures of the exterior angles is 360°.

c. The polygon contains 36 sides. This can be found by solving the following equation for n.

$$180 - \frac{360}{n} = 170$$

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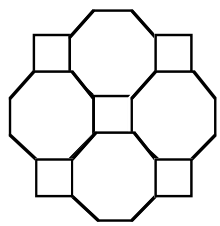
Research Project

(page 99)

The research project allows students to continue their explorations of tiling patterns to include combinations of polygons. For example, the tiling shown below is formed by a combination of squares and octagons.



Note: See Grünbaum and Shephard's *Tiling and Patterns* for a complete discussion of the tilings of regular polygons.

(page 99)

Activity 3

In this activity, students use a geometry utility to construct regular polygons and complete a table of the apothems, side lengths, and areas. They then use a spreadsheet to help discover a formula for the area of a regular polygon.

Materials List

- ruler (one per student)
- protractor (one per student)
- box template (one per student)

Technology

- geometry utility
- spreadsheet

Teacher Note

You may wish to demonstrate the construction of a regular pentagon by following the steps described in Part **a** of the exploration. Students should recall how to find the central angle of a polygon from the module "Reflect on This." The measure of each central angle should be 72˚.

Exploration

(page 99)

a. Students should recognize how constructing a circle helps them draw the regular polygon.

b–e. Sample table:

Polygon	No. of Triangles	Apothem (a)	Length of Side (s)	Area of Polygon
pentagon	5	5.0 cm	7.2 cm	90.0 cm^2
heptagon	7	4.0 cm	3.8 cm	53.2 cm^2
decagon	10	6.0 cm	3.9 cm	117 cm^2
n-gon	n	a	s	$\frac{1}{2}a \bullet s \bullet n$

f. Sample response: The area determined by the geometry utility is about the same as the area calculated using triangles.

g. Students repeat Parts **a–f**, using 7 points to create a regular heptagon and 10 points to create a regular decagon.

h. Sample response: The formula for the area of a regular n-gon is $\text{Area} = 0.5 \bullet a \bullet s \bullet n$, where a represents the length of the apothem, s represents the length of a side, and n represents the number of triangles.

Discussion

(page 101)

a. Sample response: As a polygon's number of sides increases, it begins to look more like a circle.

b. Sample response: As the measure of the central angle decreases, the polygon begins to look more like a circle.

c. The two areas should be approximately equal.

d. Sample response: Yes, this equation is equivalent to the formula because the number of sides is equal to the number of triangles.

Teacher Note

To complete Problem **3.4**, each student will need one copy of the box template. A blackline master appears at the end of the teacher edition for this module.

Assignment

(page 101)

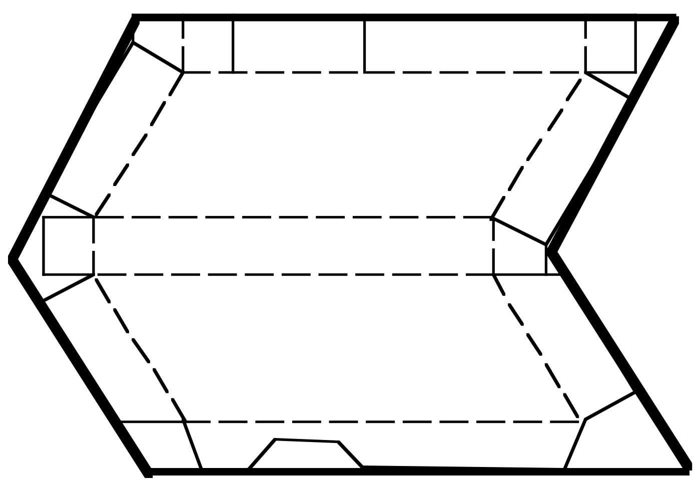
3.1 The side length of the hexagon and the perpendicular distance from the center point to a side (the apothem) are the only measurements needed.

3.2 Students should recognize that the perimeter of a regular polygon equals the number of sides multiplied by the side length. This should lead them to the following equation, where *a* represents the apothem and *P* represents the perimeter:

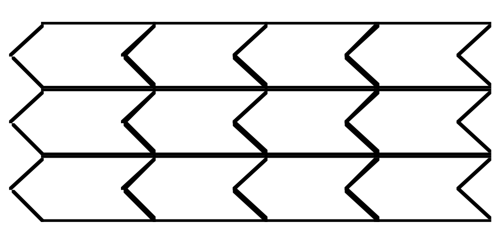
$$A = \frac{1}{2}aP$$

3.3 The area of the net is approximately 820 cm^2. This box is similar to a container in which cupcake papers are sold.

***3.4** **a.** The area of the template is approximately 46 cm^2. The area of the shape that encloses the template is approximately 49 cm^2.



b. Sample tessellation:



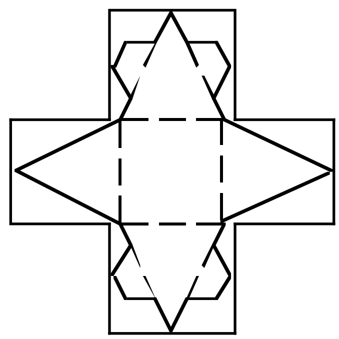
c. The percentage of cardboard wasted can be calculated as follows:

$$\frac{49\text{ cm}^2 - 46\text{ cm}^2}{49\text{ cm}^2} \approx 0.06 = 6\%$$

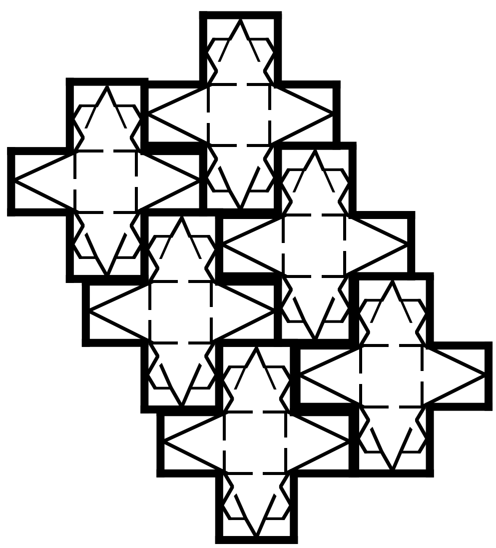
* * * * *

3.5 Answers will vary, the following sample response uses a container shaped like a pyramid.

a. Sample response:



b. Sample tessellation:



c. The area of the sample template shown in Part **a** is 7.1 cm^2. The area of the shape that encloses it is 11.1 cm^2.

d. The percentage of cardboard wasted can be calculated as follows:

$$\frac{11.1\text{ cm}^2 - 7.1\text{ cm}^2}{11.1\text{ cm}^2} \approx 0.36 = 36\%$$

3.6 The length of the apothem of the hexagon is approximately 3.9 cm. The area of each hexagon is $0.5 \bullet 3.9 \text{ cm} \bullet 6 \bullet 4.5 \text{ cm} \approx 53 \text{ cm}^2$.

The length of the apothem of the pentagon is approximately 3.1 cm. The area of each pentagon is $0.5 \bullet 3.1 \text{ cm} \bullet 5 \bullet 4.5 \text{ cm} \approx 35 \text{ cm}^2$.

The total surface area of the soccer ball is approximately $12 \bullet 35 \text{ cm}^2 + 20 \bullet 53 \text{ cm}^2 = 1480 \text{ cm}^2$.

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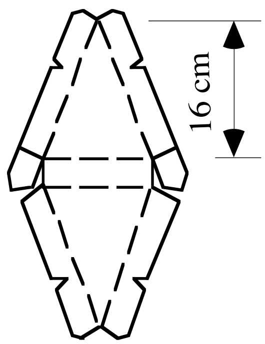
Answers to Summary Assessment (page 103)

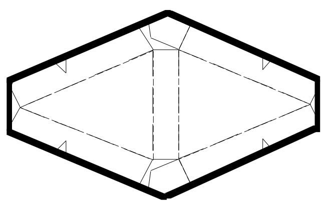
Answers will vary, depending on the design of the template. Students should select a written or verbal format that includes all of the following information.

1. Sample template:

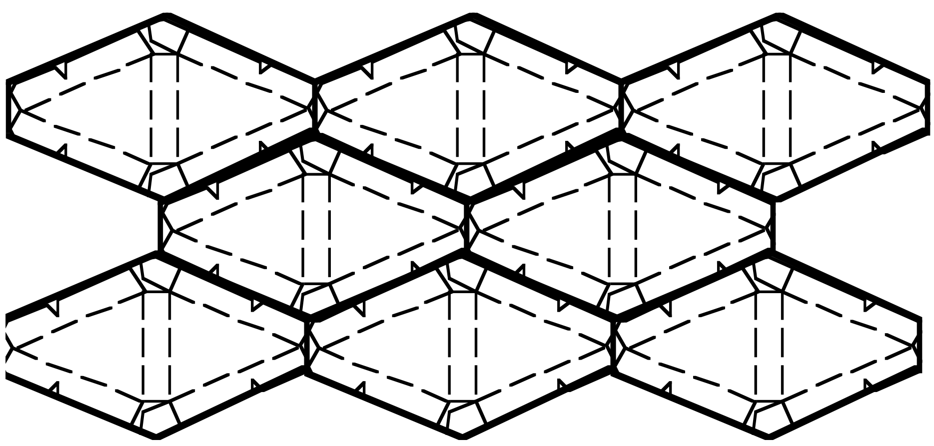


2. The area of the sample template above is approximately 454 cm^2.

3. The shape that encloses the sample template is shown below.



4. This shape can tessellate the plane as follows:



5. The area of the shape is 468 cm^2. The cost of the cardboard required to make one template is:

$$\frac{468\ \text{cm}^2}{1\ \text{box}} \bullet \frac{1\ \text{m}^2}{10{,}000\ \text{cm}^2} \bullet \frac{\$0.22}{1\ \text{m}^2} \approx \frac{\$0.01}{1\ \text{box}}$$

6. The percentage of cardboard wasted can be calculated as follows:

$$\frac{(468 - 454)\ \text{cm}^2}{468\ \text{cm}^2} \approx 0.03 = 3\%$$

7. Logos will vary.

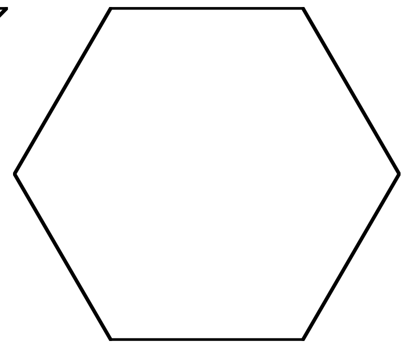
8. Students should provide models of their containers.

Module Assessment

1. Determine the area of each polygon shown below in square centimeters. Show all your calculations.

a.

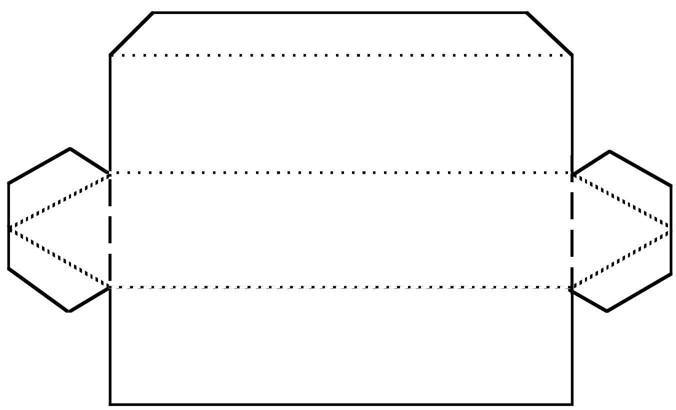
b.



2. Describe how a plane can be tessellated with a shape or shapes other than a quadrilateral. Use a drawing to support your response.

3. Explain why a regular polygon will not tessellate if the measure of each of its interior angles is 108°.

4. Use the template shown below to complete Parts **a–c**.



a. Calculate the surface area of the three-dimensional object formed by the template.

b. Draw a shape that will enclose the template and tessellate the plane.

c. Calculate the percentage of cardboard wasted in making one template.

Answers to Module Assessment

1. **a.** Using the formula $A = bh$, the area is $4.5 \bullet 1.9 = 8.6\ \text{cm}^2$.

b. Using the formula

$$A = \frac{1}{2}aP$$

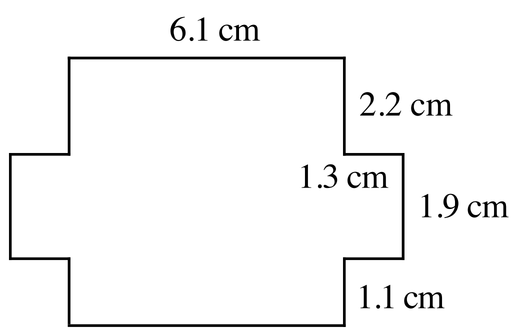
the area is $(0.5)(2.2)(15.0) = 16.5\ \text{cm}^2$. Students may use other methods to arrive at approximately the same answer.

2. Answers will vary. Many tessellations are possible.

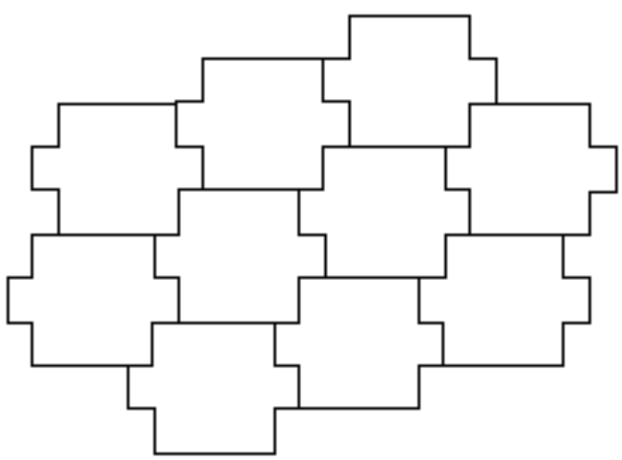
3. Sample response: The regular polygon will not tessellate because $360^\circ/108^\circ$ is not a whole number.

4. **a.** The area of the net is approximately $29.9\ \text{cm}^2$.

b. Answers will vary. Some students may use the smallest rectangle that encloses the template. Others may use a tessellating shape like the one shown below. The area of this shape is approximately $36.7\ \text{cm}^2$.



Sample tessellation:



c. The area of the template equals the areas of the 2 triangles, 4 small trapezoids, 1 large trapezoid, and 3 rectangles:

$$A = 2.0 + 2.5 + 2.8 + 27.9 = 35.2 \text{ cm}^2$$

The area of the sample shape described in Part **b** is approximately 36.7 cm^2. The percentage of cardboard wasted can be calculated as follows:

$$\frac{36.7 - 35.2}{36.7} \approx 0.04 = 4\%$$

The area of the smallest rectangle that encloses the template is approximately 45.2 cm^2. The percentage of cardboard wasted is:

$$\frac{45.2 - 35.2}{45.2} \approx 0.22 = 22\%$$

Selected References

Britton, J., and D. Seymour. *Introduction to Tessellations*. Palo Alto, CA: Dale Seymour Publications, 1989.

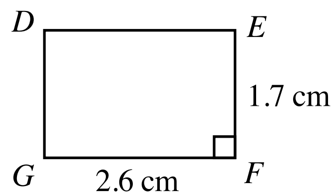
Consortium for Mathematics and Its Applications (COMAP). *For All Practical Purposes*. New York: W. H. Freeman and Co., 1991.

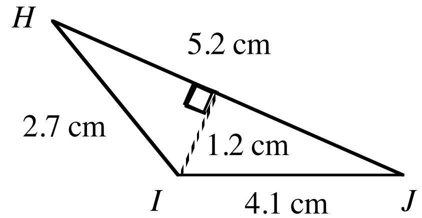
Grünbaum, B., and C. G. Shephard. *Tilings and Patterns*. New York: W. H. Freeman and Co., 1987.

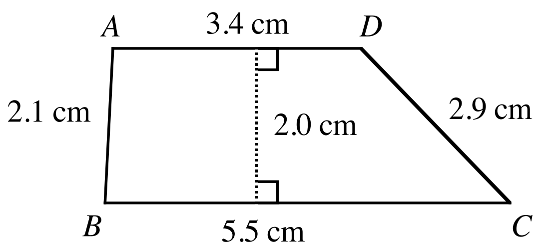
Flashbacks

Activity 1

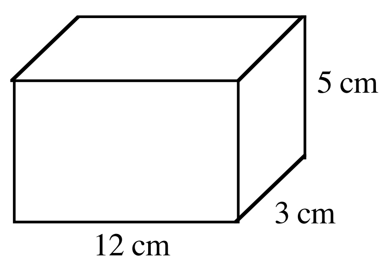
1.1 Find the area of each of the following polygons to the nearest tenth of a square centimeter (0.1 cm^2).







1.2 **a.** Find the surface area of the following box.



b. What percentage of the total surface area is represented by the area of the bottom of the box?

1.3 **a.** How many centimeters are there in one meter (1 m)?

b. How many square centimeters are there in a square meter (1 m^2)?

1.4 A contractor wishes to cover a floor with square tiles. The floor is 10 m by 20 m. Each square tile is 10 cm by 10 cm.

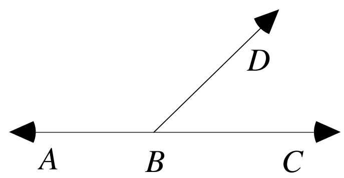
a. How many tiles will she need to cover the floor?

b. If each tile costs $0.25, how much will she spend on materials?

Activity 2

2.1 The dimensions of a cardboard rectangle are 18 cm by 24 cm. How many rectangles with dimensions 10 cm by 2 cm can be cut from this larger rectangle?

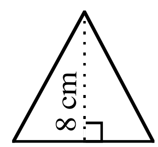
2.2 Find the sum of the measures of the adjacent angles *DBA* and *DBC* in the diagram below.



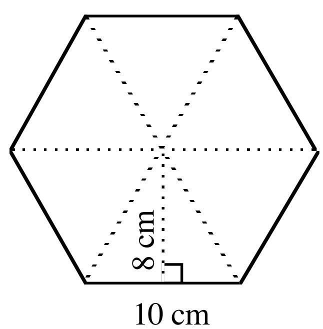
2.3 What characteristics are common to all regular polygons?

Activity 3

3.1 Find the area of the triangle below in square centimeters.



3.2 Find the area of the hexagon below in square centimeters.



Answers to Flashbacks

Activity 1

1.1 The area of quadrilateral *DEFG* is 4.4 cm^2. The area of triangle *HIJ* is 3.1 cm^2. The area of trapezoid *ABCD* is 8.9 cm^2.

1.2 **a.** The surface area is 222 cm^2.

b. The area of the bottom is 36 cm^2. This represents $36/222 \approx 16\%$ of the total.

1.3 **a.** There are 100 cm in 1 m.

b. There are $10{,}000 \text{ cm}^2$ in 1 m^2.

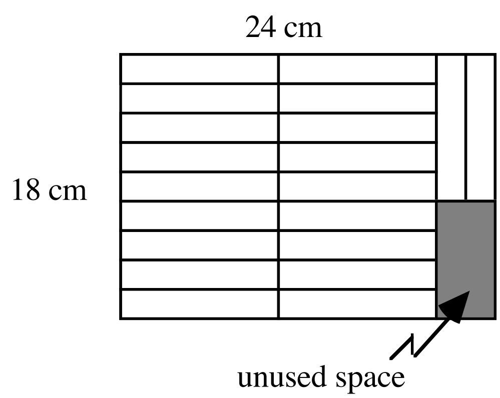
1.4 **a.** The number of tiles required can be calculated as follows:

$$\frac{200 \text{ m}^2}{1 \text{ floor}} \bullet \frac{10{,}000 \text{ cm}^2}{1 \text{ m}^2} \bullet \frac{1 \text{ tile}}{100 \text{ cm}^2} = \frac{20{,}000 \text{ tiles}}{\text{floor}}$$

b. $20{,}000 \bullet \$0.25 = \$5000.00$

Activity 2

2.1 As shown in the diagram below, the maximum number of smaller rectangles is 20.



2.2 The sum of the measures of the adjacent angles is 180°.

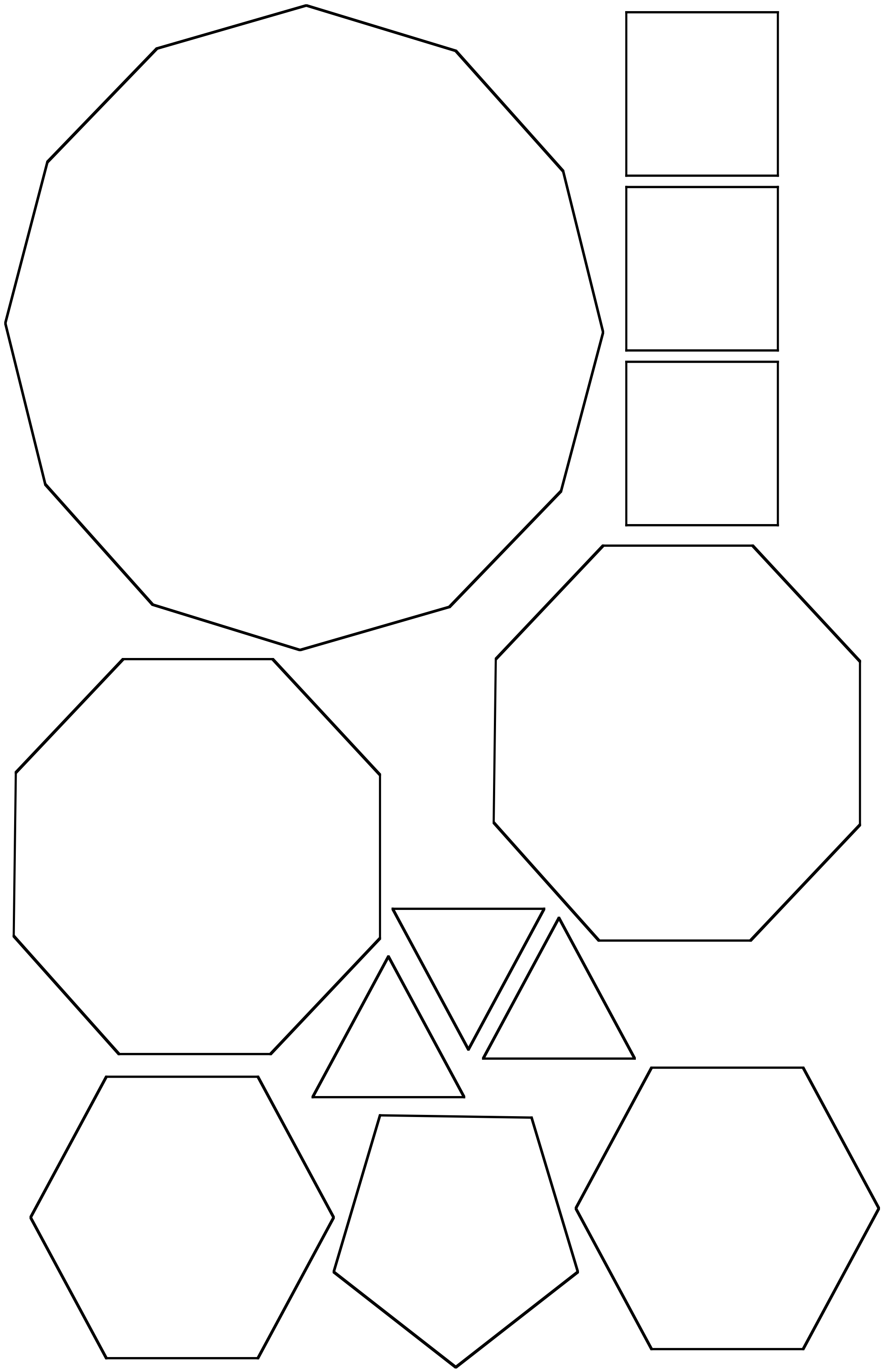
2.3 Regular polygons are equilateral and equiangular.

Activity 3

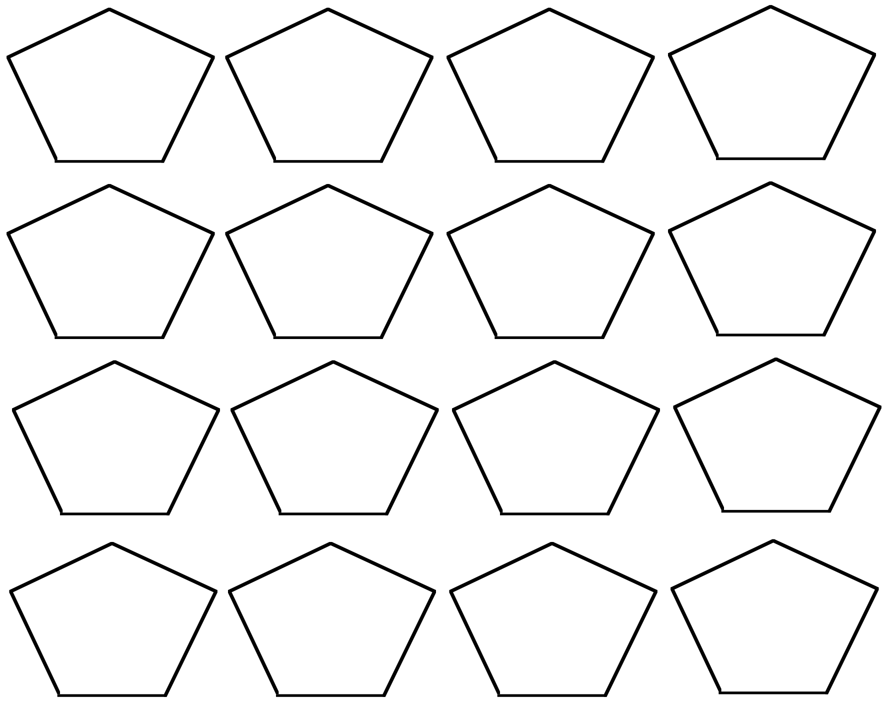
3.1 The area of the triangle is 40 cm^2.

3.2 The area of the hexagon is $6 \bullet 40 \text{ cm}^2 = 240 \text{ cm}^2$.

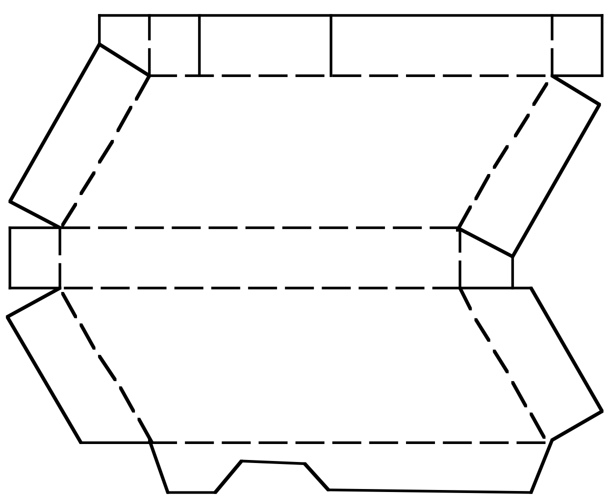
Polygon Template



Pentagon Template for Problem 2.7

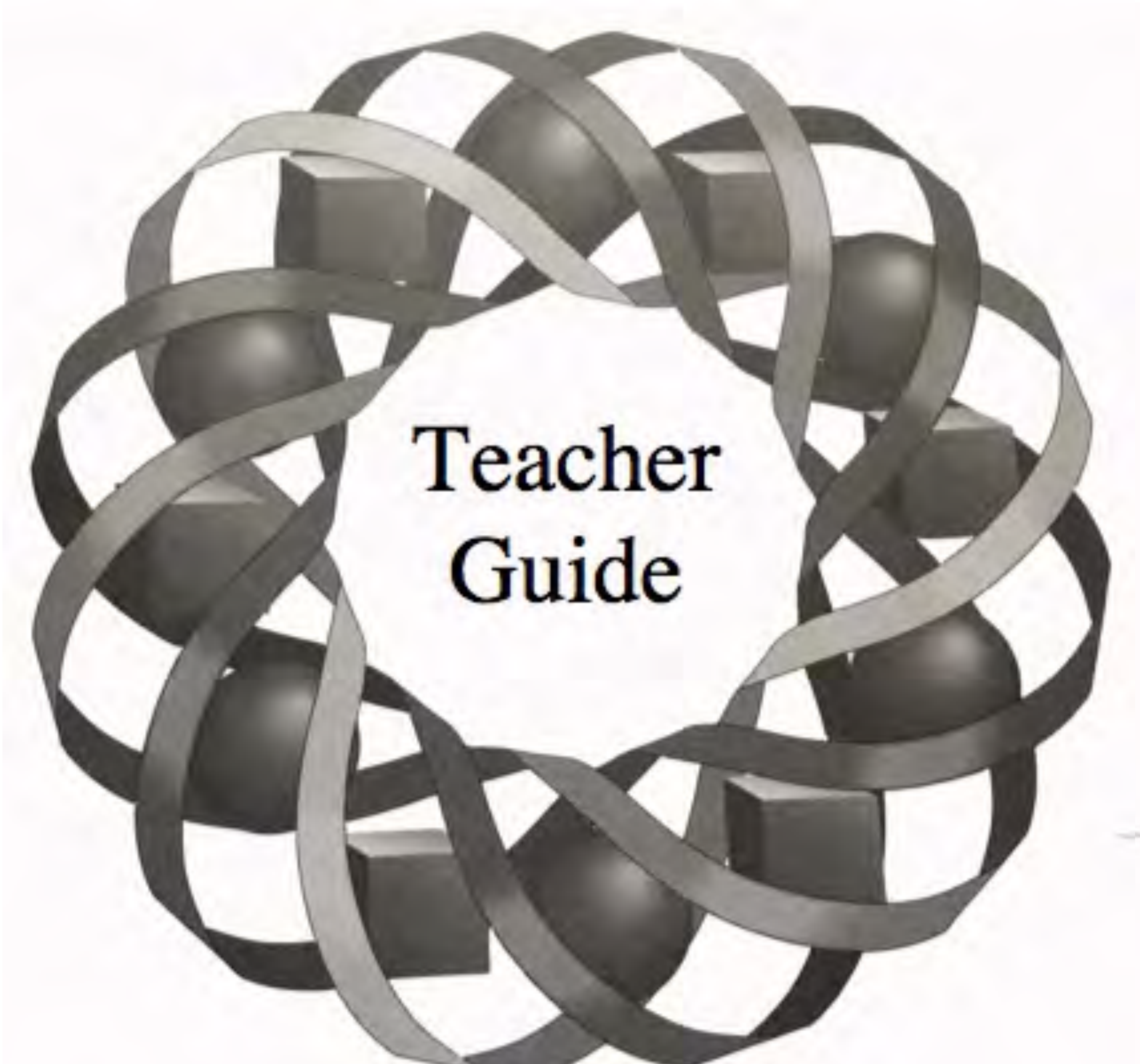


Box Template for Problem 3.4



1-cm Grid Paper

What Will We Do When the Well Runs Dry?



The availability of fresh, clean water affects us personally, locally, and globally. In this module, you'll use volume, rates of change, and linear models to assess individual water use.

Kyle Boyce • Karen Longhart • Mike Lundin • Karen Umbaugh

Teacher Edition

What Will We Do When the Well Runs Dry?

Overview

Students examine personal water use and continue their investigation of linear equations. The primary focuses of the module are volume, rate of change, slope, linear modeling, and residuals.

Objectives

In this module, students will:

- determine the volumes of triangular, rectangular, and trapezoidal prisms
- estimate and calculate the volumes of three-dimensional solids
- investigate the relationships among cubic centimeters, cubic decimeters, and liters
- construct and interpret graphs
- develop and use linear models
- determine rates of change using slope
- convert rates to different units
- examine residuals and use them to evaluate models.

Prerequisites

For this module, students should know:

- the definition of a polygon
- how to calculate the area of rectangles, triangles, and trapezoids
- how to draw the net of a three-dimensional solid
- how to find the slope of a line
- how to write linear equations given two data points
- how to write an equation in the form $y = mx + b$
- how to use graphing utilities and spreadsheets
- the definition of absolute value
- how to report calculations using measured quantities in significant digits
- how to write numbers in scientific notation.

Time Line

Activity	1	2	3	Summary Assessment	Total
Days	3	3	2	2	10

Materials Required

Materials	Activity		
	1	**2**	**3**
metric rulers	X	X	
cardboard	X	X	
scissors	X	X	
tape	X	X	
1-L containers	X	X	
centimeter graph paper	X	X	X
rice	X	X	
stopwatches or timers		X	
large cans or buckets	X	X	
straightedges		X	X

Teacher Note

In Activities **1** and **2**, students will require sheets of light, strong cardboard (such as tagboard) to construct models. Manila file folders also work well.

Plastic soda bottles with the necks removed may be substituted for the 1-L containers. Buckets should hold at least 4 L.

A blackline master for 1-cm grid paper appears at the end of the teacher edition FOR THIS MODULE.

Technology

Software	Activity		
	1	**2**	**3**
graphing utility			X
spreadsheet (optional)			X

What Will We Do
When the Well Runs Dry?

Introduction

(page 109)

The introduction is designed to relate national and global water resource issues to potential problems in the local area.

Discussion

(page 109)

a. Some students may argue that water may become even more expensive if population growth exceeds our ability to produce drinkable water or if worldwide standards of living place higher demands on water.

Other students may respond that the price of water may remain stable if cost-effective ways are developed to conserve existing water supplies or desalinate sea water.

b. Answers will vary, depending on the region and on different perceptions of the term *shortage*. If a local shortage does exist, encourage students to discuss possible causes for this situation.

c.

1. Some possible causes are drought, waste, water pollution, deforestation, and the paving of water recharge areas.
2. Answers will vary. Some causes may be reversed, while others may be controlled or lessened.

d.

1. Students will determine their average daily use in the research project in Activity **3**. In 1990, the average U.S. citizen used approximately 420 L of water per day.
2. Answers will vary. Some students may suggest monitoring a household water meter, then dividing household use by the number of family members.

(page 109)

Activity 1

In this activity, students estimate and calculate the volume of prisms and other three-dimensional solids.

Materials List

- metric rulers (one per group)
- cardboard (approximately 50 cm ∞ 30 cm per group)
- scissors (one pair per group)
- tape (one roll per group)
- 1-L containers (one per group)
- rice (at least 1 kg per group)
- centimeter graph paper (at least two sheets per group)
- large cans or buckets (one per group)

Teacher Note

You may wish to point out the lateral faces, bases, edges, and heights of various prisms in your classroom. In this module, students examine only right prisms; therefore, the heights are the same as the lengths of the edges. In an oblique prism, the height is not the same as the length of an edge.

Exploration 1

(page 110)

Using cubic centimeters and cubic decimeters, students develop a sense of the volume equivalent to a liter.

a–b. Students construct cubes with an edge length of 10 cm.

c. **1.** 1000 cm^3

2. 1 dm^3

d. $1000 \text{ cm}^3 = 1 \text{ dm}^3$

e. Estimates will vary. The volume of the cube is 1 L.

f. Students check their estimates from Part **e** by pouring rice into their cubes.

g. **1.** $1 \text{ L} = 1000 \text{ cm}^3$

2. $1 \text{ L} = 1 \text{ dm}^3$

Discussion 1

(page 111)

a. **1.** The cube created in the exploration is a prism because it is a three-dimensional figure formed by two parallel and congruent polygons (squares) and the parallelograms (squares) formed by connecting the corresponding vertices of these polygons.

2. The bases and the lateral faces of a cube are indistinguishable since all six are congruent squares. Any two parallel faces may be designated as the bases; the remaining four faces may then be identified as lateral faces.

b. 1. Every prism has two bases.

2. Every prism with an *n*-gon as its bases has *n* lateral faces. For example, since the bases of a cube have 4 sides, it has 4 lateral faces.

3. Every prism with an *n*-gon as its bases has *n* lateral edges. For example, since the bases of a cube have 4 sides, it has 4 lateral edges.

4. Every prism with an *n*-gon as its bases has a total of $3n$ edges. For example, since the bases of a cube have 4 sides, it has a total of 12 edges.

c. 1. $1 \text{ dm}^3 = 1000 \text{ cm}^3$

2. $1 \text{ L} = 1000 \text{ cm}^3$

3. $1 \text{ L} = 1 \text{ dm}^3$

Exploration 2 (page 111)

In this exploration, students investigate a method for finding the volume of objects that are not prisms. **Note:** In Part **c**, each group should construct just one three-dimensional model.

a. Students may draw circles, ellipses, or other figures consisting of curves.

b. If students use 1-cm grid paper, the area of each square is 1 cm^2. Therefore, the sum of Steps **2** and **3** approximates the area of the figure in square centimeters.

c. Students use cardboard to construct three-dimensional solids 10 cm high.

d. 1. To calculate the volume of the solid in cubic centimeters, students should multiply the estimated area from Part **b** by 10 cm.

2. To calculate the volume in liters, students should divide the volume in cubic centimeters (found in Step **1** above) by 1000.

e. Students use rice to verify their estimates from Part **d**.

Discussion 2 (page 112)

a. Answers may vary. Sample response: The area can be found by estimating the average length and average width of the figure, then finding the product of the two.

b. **1.** Sample response: We multiplied the estimated area by the height of 10 cm to find the volume in cubic centimeters, then divided this value by 1000 to find the equivalent volume in liters.

2. Answers will vary, depending on the accuracy of the estimates of base area.

c. **1.** Sample response: The volume can be calculated by multiplying the area of the base by the height of the prism. To calculate the area of the base, use the standard formula for the area of the polygon.

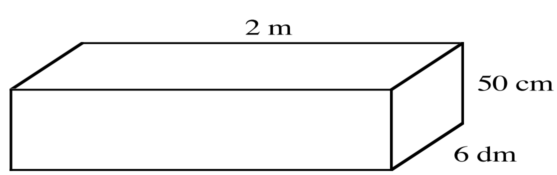
2. Sample response: Regardless of the shape of the base, the volume can be calculated by multiplying the area of the base by the height of the prism. To determine the area of the base, divide the polygon into triangles, rectangles, or trapezoids, find the areas of these using the standard area formulas, then add them.

Assignment

(page 113)

1.1 The volume of Thaddeus's tub can be calculated as follows: $(5\text{ dm})(3\text{ dm})(15\text{ dm}) = 225\text{ dm}^3 = 225\text{ L}$. Since the volume of José's tub is 250 L, it holds more.

1.2 **a.** Sample drawing:



b. $(200\text{ cm})(60\text{ cm})(50\text{ cm}) = 600{,}000\text{ cm}^3$

***1.3** **a.** Considering the water trough as a rectangular prism, the volume is: $(120\text{ cm})(13\text{ cm})(10\text{ cm}) = 15{,}600\text{ cm}^3 = 15.6\text{ L}$

b. Considering the toilet tank as a trapezoidal prism, the volume is: $(0.5)(42 + 40)(25)(15) = 15{,}375\text{ cm}^3 = 15.375\text{ L}$.

***1.4** **a.** To estimate the volume, students should first determine the approximate surface area of the lake, then multiply by the average depth of 15 m. Some students may suggest tracing the lake onto centimeter graph paper (since the scale is 1 cm to 1 km), then counting the number of squares. Others may estimate the average length and width of the lake, then find the product of these two.

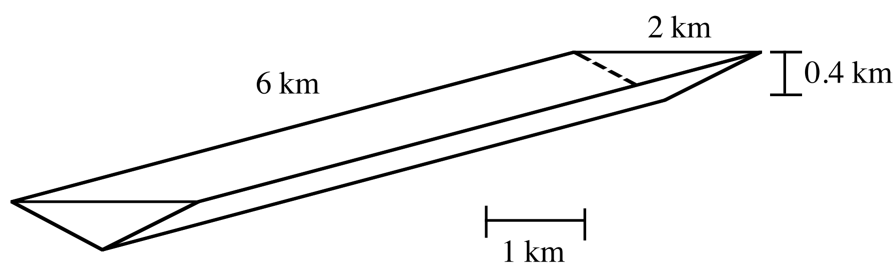
Using the latter method, the volume of the lake is approximately $(6500 \text{ m})(5700 \text{ m})(15 \text{ m}) \approx 5.6 \bullet 10^8 \text{ m}^3 = 5.6 \bullet 10^{11} \text{ L}$.

b. Answers will vary, depending on the estimate of the lake's volume. The volume of water used by the Canadian town each day is $(50,000)(380) = 1.9 \bullet 10^7 \text{ L}$. Using the estimate from Part **a**, the time that the water will last can be calculated as follows:

$$\frac{5.6 \bullet 10^{11}}{1.9 \bullet 10^7} \approx 29,000 \text{ days} \approx 79 \text{ yr}$$

* * * * *

1.5 **a.** Sample drawing:



b. The shape of the reservoir is a triangular prism. Using decimeters, the area of the base is $(0.5)(4000)(20,000) = 4 \bullet 10^7 \text{ dm}^2$.

Using the formula for the volume of a prism, $(4 \bullet 10^7 \text{ dm}^2)(6 \bullet 10^4 \text{ dm}) = 2.4 \bullet 10^{12} \text{ dm}^3$ or $2.4 \bullet 10^{12}$ L.

c. The volume of water used by the city each day is $(6,000,000)(420) = 2.52 \bullet 10^9 \text{ L}$. Therefore, the time that the water will last can be calculated as follows:

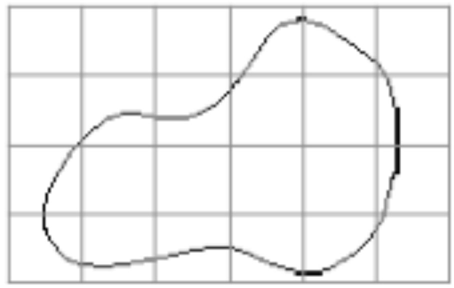
$$\frac{2.4 \bullet 10^{12}}{2.52 \bullet 10^9} \approx 950 \text{ days} \approx 2.6 \ yr$$

1.6 Because $1000 \text{ cm}^3 = 1 \text{ L}$, $1 \text{ cm}^3 = 0.001 \text{ L}$. Since 1 mL also represents 0.001 L, $1 \text{ cm}^3 = 1 \text{ mL}$.

***1.7** Considering the pool as a trapezoidal prism, its volume is: $(0.5)(1.5 \text{ m} + 4.5 \text{ m})(10 \text{ m})(5 \text{ m}) = 150 \text{ m}^3$ or 150,000 L.

1.8 Considering the driveway as a rectangular prism, its volume is $(20 \text{ ft})(15 \text{ ft})(0.5 \text{ ft}) = 150 \text{ ft}^3$. Since $27 \text{ ft}^3 = 1 \text{ yd}^3$, $150 \text{ ft}^3 \approx 5.6 \text{ yd}^3$.

***1.9** Answers may vary. Sample response: By tracing the outline of the pond on a 1-cm square grid, the surface area is estimated to be approximately 12 m^2. The volume of the pond is roughly $(12\text{ m}^2)(4\text{ m}) = 48\text{ m}^3 = 48{,}000\text{ L}$.



* * * * * * * * * *

(page 116)

Activity 2

In this activity, students explore rate of flow and make connections between rates and the slopes of linear models.

Materials List

- 1-L containers (one per group)
- metric rulers (one per group)
- scissors (one pair per group)
- cardboard (three sheets, approximately 50 cm ∞ 30 cm, per group)
- tape (one roll per group)
- rice (approximately 2 kg per group)
- stopwatches or timers (one per group)
- buckets (one per group)
- centimeter graph paper (several sheets per student)
- straightedges (one per student)

Teacher Note

In this activity, rice is used to simulate water. Any dry material that flows freely and does not pose a hazard to electronic equipment may be substituted.

In the exploration, students model the flow of rice through a funnel with a linear function. In reality, the change in pressure due to the decreasing depth of the rice results in data that is not truly linear. (Water flow depending on pressure is a classic differential equations problem.) However, given the depth of the funnels used, a linear model is acceptable for this activity.

If water is available in your classroom (and the dangers of spills can be minimized), you may wish to modify the exploration accordingly. For example, students may measure the time required for a plastic jug of water to empty through openings of three different sizes—0.5 cm, 1.0 cm, and 2.0 cm—by drilling holes of the appropriate diameters in three different lids.

Exploration (page 116)

This exploration requires students to work in groups of at least two: one student to hold the funnel and another to start the timer. You may wish to demonstrate Parts **a** and **b** before students begin work.

a–d. Sample data:

Funnel Opening (cm)	Time for Trial 1	Time for Trial 2	Time for Trial 3
2	27.0	27.0	24.0
3	6.0	6.0	6.0
4	3.0	3.0	3.0

e. Answers will vary. Using the sample data in Table **1**, the average time for the 2-cm funnel is 26.0 sec, the average for the 3-cm funnel is 6.0 sec, and the average for the 4-cm funnel is 3.0 sec.

f. Answers will vary. The following sample responses show the rate of flow for the 2-cm funnel.

1. $2.0 \text{ L}/26.0 \text{ sec} \approx 0.08 \text{ L/sec}$

2. Since there are 60 sec in 1 min,

$$\left(\frac{2.0 \text{ L}}{26.0 \text{ sec}}\right)\left(\frac{60 \text{ sec}}{1 \text{ min}}\right) \approx 4.6 \text{ L/min}$$

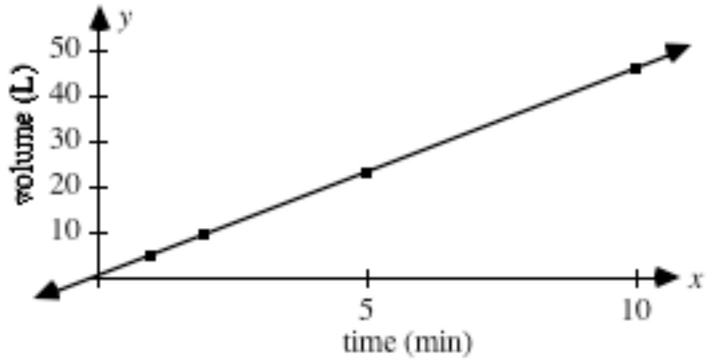
3. Since there are 60 min in 1 hr,

$$\left(\frac{2.0 \text{ L}}{26.0 \text{ sec}}\right)\left(\frac{60 \text{ sec}}{1 \text{ min}}\right)\left(\frac{60 \text{ min}}{1 \text{ hr}}\right) \approx 280 \text{ L/hr}$$

g. Answers will vary. The predictions in the following table were made using the sample data given above.

Time (min)	Volume (L)		
	2-cm Funnel	3-cm Funnel	4-cm Funnel
1	4.6	20	40
2	9.2	40	80
5	23	100	200
10	46	200	400

h. **1–2.** Sample scatterplot:



3. The slope of the line on the sample graph above can be calculated as follows:

$$\frac{46-0}{10-0}=\frac{23}{5}=4.6$$

i. Students should use the graph of the line to make their predictions. The sample responses below correspond to the graph shown in Part **h**.

1. 32 L

2. 55 L

Discussion

(page 117)

a. Some possible causes for the variations in experimental data include errors in measurement, differences in the shape of the funnel opening, and differences in timing.

b. Sample response: As the size of the funnel opening increases, so does the rate of flow.

c. Answers will vary. As the sample data shows fairly consistent results, this suggests that the average rate is a reliable measurement.

d. Sample response: The rate of flow and the slope are the same. As the rate increases, the slope of the line increases.

e. The volume of water may be calculated as follows:

$$\left(\frac{0.50\text{ mL}}{1\text{ sec}}\right)\left(\frac{60\text{ sec}}{1\text{ min}}\right)\left(\frac{60\text{ min}}{1\text{ hr}}\right)\left(\frac{24\text{ hr}}{1\text{ day}}\right)\approx 43{,}000\text{ mL/day}=43\text{ L/day}$$

Assignment (page 118)

2.1 **a.** **1.** The y-intercept is 0.

2. Sample response: It represents the volume of rice that has passed through the funnel after 0 min.

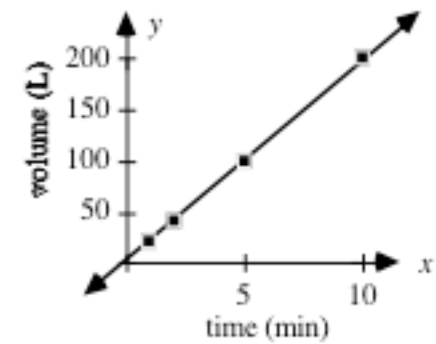
b. One equation for the line is $y = 4.6x + 0$.

c. Using the sample equation given in Part **b**, the predicted values are $(4.6)(7) = 32.2\text{ L}$ and $(4.6)(12) = 55.2\text{ L}$.

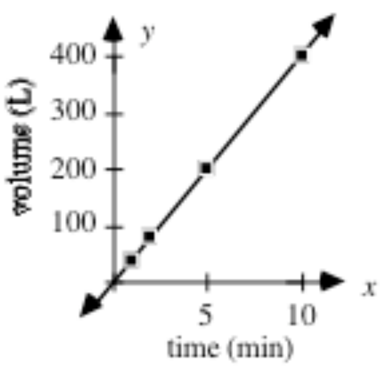
d. The predicted values found using the graph of the line should be very close to those found using its equation.

***2.2** The following responses use the sample data given in the exploration.

a. An equation for the line in the sample graph below is $y = 20x + 0$.



b. An equation for the line in the sample graph below is $y = 40x + 0$.



***2.3** Sample response: Since the volume of rice that has passed through each funnel after 0 min is 0, all three lines have y-intercepts of 0. The slope of the line for the 4-cm funnel is much steeper than the others. This is because the rate of flow through the 4-cm funnel is faster than the rates for either of the other two funnels. The bigger the opening, the faster the flow rate and the steeper the line.

2.4 **a.** The rate of flow through the 0.6-cm opening is:

$$\frac{2.0 \text{ L}}{78.0 \text{ sec}} \approx 0.03 \text{ L/sec} \approx 1.5 \text{ L/min} \approx 92 \text{ L/hr}$$

The rate of flow through the 1.3-cm opening is:

$$\frac{2.0 \text{ L}}{18.0 \text{ sec}} \approx 0.11 \text{ L/sec} \approx 6.7 \text{ L/min} \approx 400 \text{ L/hr}$$

The rate of flow through the 2.5-cm opening is:

$$\frac{2.0 \text{ L}}{5.0 \text{ sec}} = 0.40 \text{ L/sec} = 24 \text{ L/min} = 1440 \text{ L/hr}$$

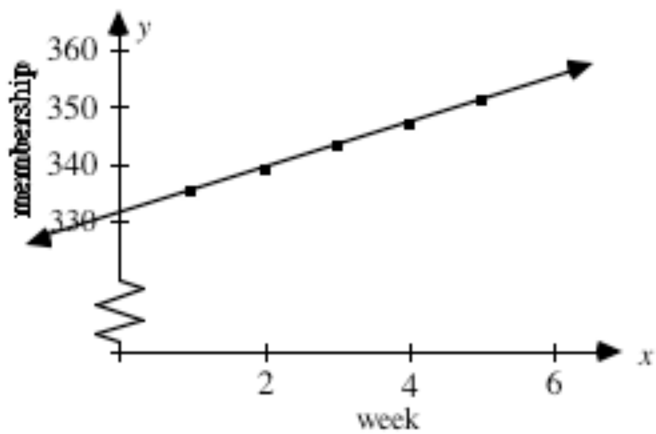
b. The equation for the amount of water flowing through the 0.6-cm opening is $y = 1.5x$.

The equation for the amount of water flowing through the 1.3-cm opening is $y = 6.7x$.

The equation for the amount of water flowing through the 2.5-cm opening is $y = 24x$.

* * * * *

2.5 **a.** Sample graph:



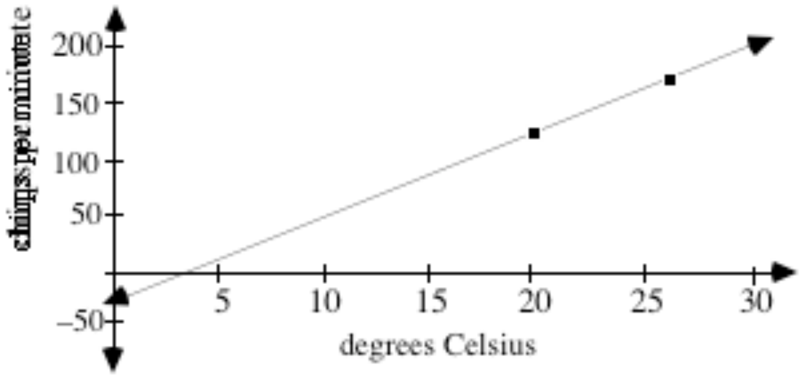
b. **1.** The slope of the line is positive.

2. The positive slope indicates that the membership is increasing.

c. **1.** Answers may vary. For the sample line given in Part **a**, the *y*-intercept is 331.

2. The *y*-intercept represents the number of students in the All School Club when the membership drive began.

***2.6** **a–b.** Sample graph:



c. For the sample graph given in Part **a** above, the *y*-intercept is approximately –22.

d. For the sample graph, the slope is approximately 7.3.

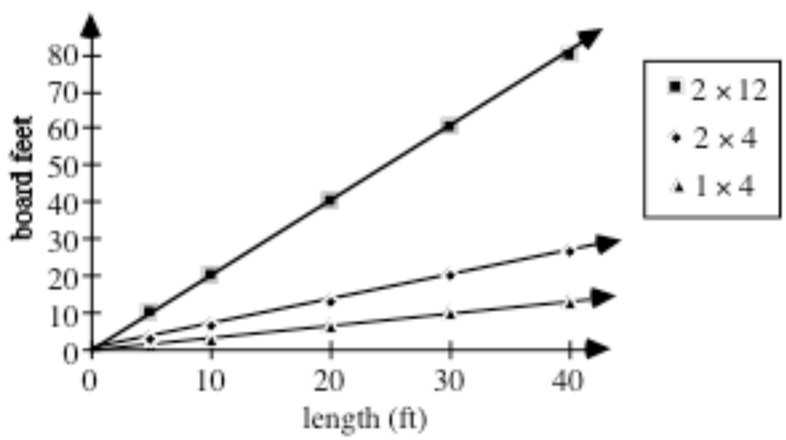
e. An equation for the line given in Part **a** is $y = 7.3x - 22$.

f. The predicted temperature is approximately 24° C.

Teacher Note

The board foot is a unit of volume. One board foot represents the volume of wood in a board with dimensions $1 \text{ in.} \times 12 \text{ in.} \times 12 \text{ in.}$, or 144 in.^3. For finished lumber, the actual dimensions are slightly smaller than the nominal dimensions.

2.7 **a.** Sample graph:



b. Sample response: It is a linear relationship. As the length of a board increases, the number of board feet it contains also increases. This is shown by the positive association of the scatterplots.

c. See sample graph in Part **a** above.

d. The equation for 2×12 lumber is $y = 2x$, where *y* represents number of board feet and *x* represents length in feet. The equation for 2×4 lumber is $y = (2/3)x$. The equation for 1×4 lumber is $y = (1/3)x$.

e. Answers will vary. Sample response: A 2×12 that measures 0.5 ft long is equal to 1 board foot. This can be found by tracing the graph of the equation for a 2×12 until the number of board feet equals 1. This occurs at a length of 0.5 ft.

It is also possible to substitute 1 for y in the equation $y = 2x$ and solve for x. This also results in a board 0.5 ft long.

* * * * * * * * * *

(page 120)

Activity 3

In this activity, students use residuals to determine how closely a line models a set of data points.

Teacher Note

In this module, students are introduced only to the sum of the absolute value of the residuals as a method for evaluating fit. In later modules, students investigate the principle of least squares and other curve-fitting techniques.

Materials List

- straightedges (one per group)
- centimeter graph paper (several sheets per group)

Technology

- graphing utility
- spreadsheet (optional)

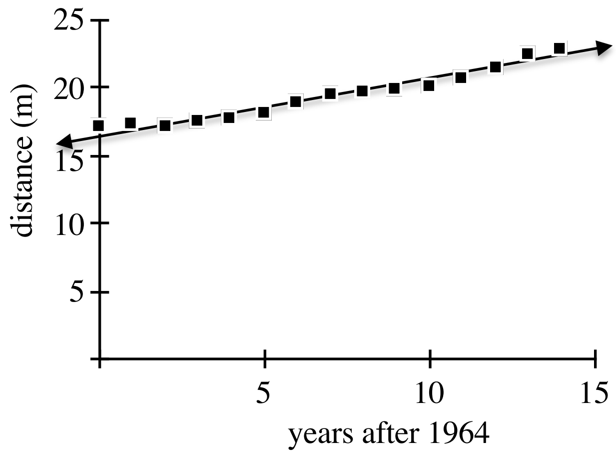
Exploration

(page 121)

In this exploration, students find a linear model for the distances to the high-water levels of the High Plains Aquifer and use residuals to compare the data set to the model.

a. See completed spreadsheet in Part **d** below.

b. Sample graph:



c. Students draw a line that closely models the data, such as the one shown in Part **b** above. The equation of this line is $y = 0.4x + 17$.

d–f. The predicted values in the following table were made using the equation $y = 0.4x + 17$.

Year after 1964	Actual Distance	Predicted Distance	Residual	Absolute Value of Residual
0	17.1	17.0	–0.1	0.1
1	17.2	17.4	0.2	0.2
2	17.1	17.8	0.7	0.7
3	17.4	18.2	0.8	0.8
4	17.7	18.6	0.9	0.9
5	18.1	19.0	0.9	0.9
6	18.9	19.4	0.5	0.5
7	19.4	19.8	0.4	0.4
8	19.7	20.2	0.5	0.5
9	19.8	20.6	0.8	0.8
10	20.1	21.0	0.9	0.9
11	20.6	21.4	0.8	0.8
12	21.3	21.8	0.5	0.5
13	22.3	22.2	–0.1	0.1
14	22.7	22.6	–0.1	0.1
		Sum	7.6	8.2

g. Using the sample responses given in Parts **d–f**, the average distance from each data point to the model is $8.2/15 \approx 0.55$.

Discussion

(page 124)

a. Sample response: The line does not fit the data exactly because it does not go through all the data points. An exact fit would have all residuals of 0.

b. Sample response: If the average distance from each data point to the model is small when compared to the spread of the data, then the model could be considered a reasonable one.

c. Sample response: This appears to be a good model because the sum of the absolute values of the residuals is only 8.2. Since there are 15 data points, this is only about 0.5 m per data point, which is small compared to the distances in the data.

d. Sample response: No. There could be a point 100 units from the line on one side and another point 100 units on the other side. The two corresponding residuals would be 100 and –100. Their sum would be 0, but the line would not fit the data exactly. **Note:** In general, the horizontal line $y = a$, where a is the mean of the y-values in the data, results in a sum of the residuals of 0.

e. Sample response: The slope of the line is positive, indicating that the distance from the surface to the water is increasing with time. This means that the level of the aquifer is dropping.

f. Table **3** contains data for 15 years. Although this may seem like a long time to most students, it may not be long enough to describe a long-term trend in water levels. Natural climatic variations and drought cycles often occur over periods of many years.

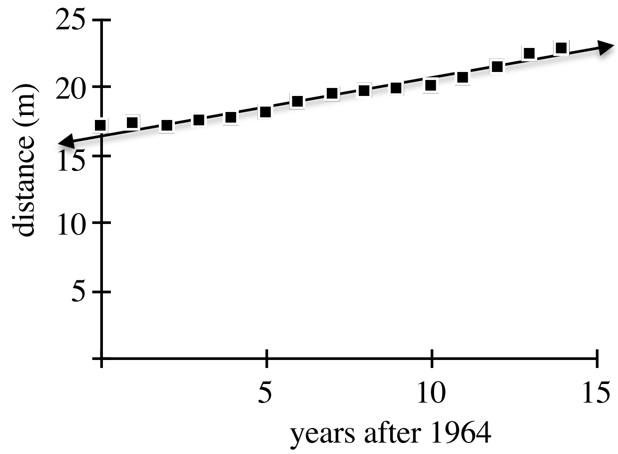
g. Students may suggest modifying irrigation techniques, planting drought-tolerant crops, reducing personal water use, or controlling population growth.

h. Answers will vary by region. You may wish to consult the local department of natural resources or water quality bureau for more information.

Assignment

(page 124)

3.1 **a–b.** Sample graph:



c. Answers will vary. An equation for the sample line in Part **a** is $y = 0.7x + 17$. The slope was calculated using the points (1,18.1) and (13,26.5) as follows:

$$m = \frac{26.5 - 18.1}{13 - 1} = \frac{8.3}{12} \approx 0.7$$

The y-intercept of 17 was approximated from the graph.

d–e. The predicted values in the following table were made using the equation $y = 0.7x + 17$.

Year after 1964	Actual Distance	Predicted Distance	Residual	Absolute Value of Residual
0	18.1	17.0	–1.1	1.1
1	18.1	17.7	–0.4	0.4
2	18.2	18.4	0.2	0.2
3	18.2	19.1	0.9	0.9
4	19.4	19.8	0.4	0.4
5	19.8	20.5	0.7	0.7
6	20.7	21.2	0.5	0.5
7	21.3	21.9	0.6	0.6
8	21.3	22.6	1.3	1.3
9	22.3	23.3	1.0	1.0
10	22.9	24.0	1.1	1.1
11	23.8	24.7	0.9	0.9
12	26.0	25.4	–0.6	0.6
13	26.5	26.1	–0.4	0.4
14	27.6	26.8	–0.8	0.8
			Sum	10.9

f. Answers will vary. The sample line is a good model because the sum of the absolute values of the residuals is only 10.9. Since there are 15 data points, this is only about 0.7 m per data point. The value of 0.7 is small compared to any value in the data, so the error in the model is small.

***3.2** The equation $y = 30x + 16$ is the better model since it produces the smaller sum of the absolute values of the residuals, as shown in the following tables.

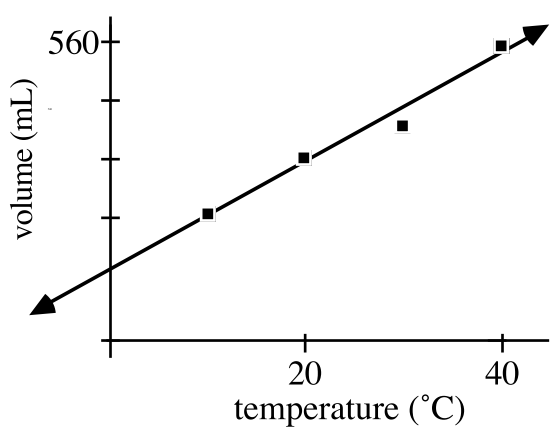
Residuals for $y = 30x + 16$

Time (min)	Mass (g)	Predicted Value (g)	Absolute Value of Residual
5	160	166	6
10	350	316	34
15	470	466	4
20	570	616	46
25	790	766	24
		Sum	114

Residuals for $y = 33x + 15$

Time (min)	Mass (g)	Predicted Value (g)	Absolute Value of Residual
5	160	180	20
10	350	345	5
15	470	510	40
20	570	675	105
25	790	840	50
		Sum	220

***3.3** **a–b.** Sample graph:



c. Answers will vary. An equation for the sample line shown in Part **a** is $y = 1.93x + 481$.

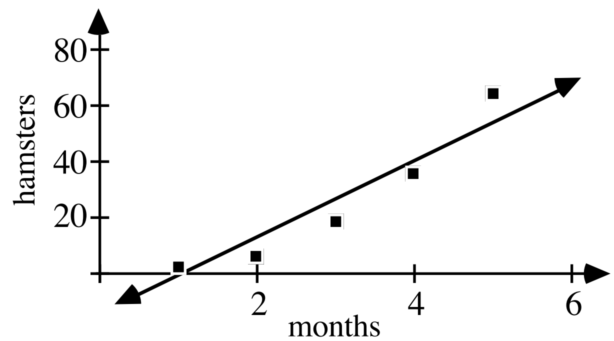
d. **1.** Using the equation given in Part **c**, the sum of the absolute values of the residuals is 8.8.

2. The relatively small sum indicates that this linear model provides a good fit for the data.

e. Using the equation given in Part **c**, the predicted volume of the balloon is 674 mL.

* * * * *

3.4 **a–b.** Sample graph:

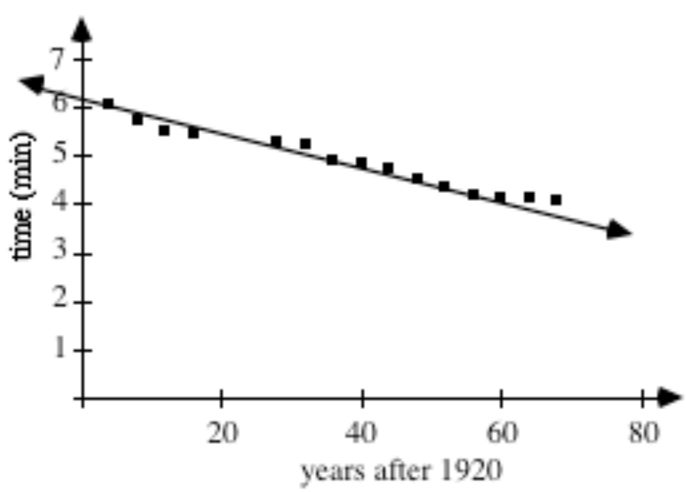


c. Answers will vary. An equation for the sample line shown in Part **a** is $y = 11.7x - 9.7$.

d. Using the equation given in Part **c**, the sum of the absolute values of the residuals is 32.4.

e. Sample response: No. The scatterplot appears to be curved, not linear. The large sum of the absolute values of the residuals also indicates that the model may not be a good fit.

3.5 **a.** Sample graph:



b. An equation of the sample line shown in Part **a** is $y = -0.03x + 6.1$

.

c. Sample response: The sum of the absolute value of the residuals is about 1.8. The model appears to fit the data well since the average distance from the line to each data point is only 0.12 min, while the spread in the data is 1.98 min.

d. The following sample predictions were made using the equation from Part **b**.

1. $-0.03(80) + 6.1 = 3.7$ min

2. $-0.03(160) + 6.1 = 1.3$ min

f. Sample response: While the winning time predicted for the year 2000 may be possible, the time predicted for the year 2080 does not seem realistic. Judging from the scatterplot, the change in recent winning times seems to be decreasing from one Olympics to the next. Using the line, the predicted winning time will eventually be 0.

3.6 **a.** In the following sample responses, the predicted values were calculated using the equation $y = 0.4x + 17$.

1. $\left|\frac{17 - 17.1}{17.1}\right| \approx 0.5\%$

2. $\left|\frac{19.4 - 18.9}{18.9}\right| \approx 2.6\%$

3. $\left|\frac{21.4 - 20.6}{20.6}\right| \approx 3.9\%$

b. Students should find the predicted values for each year using the equation $y = 0.7x + 17$.

1. $\left|\frac{17.7 - 18.1}{18.1}\right| \approx 2.2\%$

2. $\left|\frac{21.2 - 20.7}{20.7}\right| \approx 2.4\%$

3. $\left|\frac{24.7 - 23.8}{23.8}\right| \approx 3.8\%$

c. Some students may argue that since percent error varies from point to point, it is not a good measure of fit for an entire set of data. Others may claim that it is a good measure of fit if all the percent errors are small or the largest percent error for the data is small.

* * * * * * * * * *

Research Project

(page 128)

The information in the sample table below was compiled by an actual SIMMS class. In this example, student water usage was close to the national average.

Use	Rate of Use	No. of Uses or Time Used	Daily Volume
washing machine	7 L/min	3 min	21 L
dishwasher	8 L/wash	1 wash	8 L
bathroom sink	8 L/min	2 min	16 L
kitchen sink	8 L/min	20 min	160 L
toilet	15 L/flush	5 flushes	75 L
shower or bath	16 L/min	8 min	128 L
		Total	408 L

Note: In areas where residential water use is metered, students can determine their daily consumption by reading the meter. By examining household water bills, they can also determine the daily cost of this water.

Answers to Summary Assessment (page 129)

1. **a.** The farmers do not have enough water for their plan. The approximate area of the reservoir is $240,000\ \text{m}^2$, Since the average depth is 8.0 m, its volume is $(240,000\ \text{m}^2)(8.0\ \text{m}) = 1.9 \bullet 10^6\ \text{m}^3$ or $1.9 \bullet 10^9$ L.

Since the farmers have a total of 8 sprinklers, the daily volume of water used can be calculated as follows:

$$(8)\left(\frac{4000\ \text{L}}{\text{min}}\right)\left(\frac{60\ \text{min}}{\text{hr}}\right)\left(\frac{16\ \text{hr}}{\text{day}}\right) \approx 3.1 \bullet 10^7\ \text{L/day}$$

Therefore, the total volume of water needed for 90 days is $(90)(3.1 \bullet 10^7) = 2.8 \bullet 10^9$ L. This exceeds the volume of the reservoir.

b. The reservoir can supply the farmers with enough water for approximately 61 days. This can be calculated as follows:

$$\frac{1.9 \bullet 10^9\ \text{L}}{3.1 \bullet 10^7\ \text{L/day}} \approx 61\ \text{days}$$

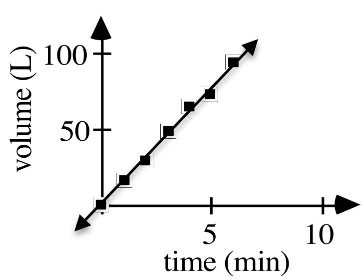
c. The volume of water available per day can be calculated as follows:

$$\frac{1.9 \bullet 10^9\ \text{L}}{90\ \text{days}} \approx 2.1 \bullet 10^7\ \text{L/day}$$

The maximum number of hours per day is therefore:

$$\frac{2.1 \bullet 10^7\ \text{L}}{8\left(\frac{4000\ \text{L}}{\text{min}}\right)\left(\frac{60\ \text{min}}{\text{hr}}\right)} \approx 10\ \text{hr}$$

2. **a–b.** Sample graph:



c. Answers may vary. Using the data points (6,94) and (0,0), one possible equation is $y = 15.7x$.

d. **1.** 15.7 L/min

2. 0.26 L/sec

3. 942 L/hr

e. The rate of flow in liters per minute is the slope of the line.

f. The following responses were calculated using the sample equation given in Part **c**.

1. $(15.7)(15) \approx 236$ L

2. $(120)(15.7) \approx 1880$ L

g. The predicted values in the following table were calculated using the equation $y = 15.7x$.

Time (min)	Actual Volume (L)	Predicted Volume (L)	Absolute Value of the Residual
0	0	0	0
1	15	15.7	0.7
2	29	31.4	2.4
3	48	47.1	0.9
4	65	62.8	2.2
5	73	78.5	5.5
6	94	94.2	0.2
		Sum	11.9

3. Sample response: The sum of the absolute value of the residuals is a measure of how well the line fits the data. Since the sum seems relatively small, this model appears to be a good fit.

h. The following table shows the predicted values calculated using the equation $y = 14x + 2$. Based on the sum of the absolute values of the residuals, the equation $y = 15.7x$ is the better model.

Time (min)	Actual Volume (L)	Predicted Volume (L)	Absolute Value of the Residual
0	0	2	2
1	15	16	1
2	29	30	1
3	48	44	4
4	65	58	7
5	73	72	1
6	94	86	8
		Sum	24

Module Assessment

1. By carefully analyzing his water consumption, Terry determined that he used an average of 275 L of water per day during the past year. His New Year's resolution is to reduce his water consumption by 15%.

a. What is the total volume of water Terry plans to save in one year?

b. Determine the dimensions of two different types of containers that will hold this volume of water.

c. The table below shows the volume of water Terry used each day from January 1 to January 15.

Day	Water Used (L)	Day	Water Used (L)	Day	Water Used (L)
1	230	6	230	11	230
2	235	7	235	12	250
3	240	8	235	13	230
4	225	9	240	14	230
5	225	10	230	15	235

1. Create a scatterplot of the total amount of water Terry had saved by the end of each day in the table.

2. Draw a line that closely approximates the scatterplot.

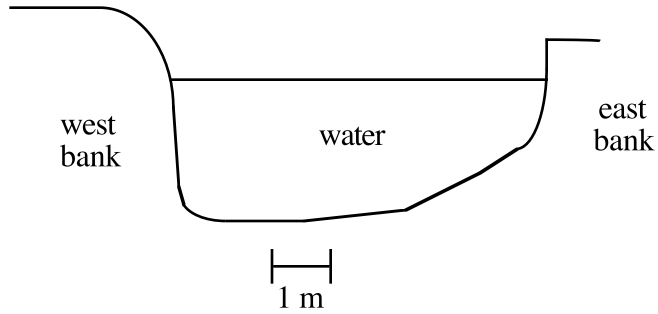
3. What does the slope of the line indicate?

d. 1. Determine an equation for the line in Part **c** and explain what it describes about Terry's water usage.

2. Use residuals to describe how well the line fits the scatterplot.

e. If Terry continues to save water at the same rate as from January 1 to January 15, will he reach his goal?

2. The diagram below shows a cross-section of a stream.



a. Find the approximate volume of water (in liters) in a 100-m length of the stream.

b. Isabel is fishing from the east bank of the stream. If the water moves at a rate of 5 km/hr, what volume of water flows past her in 10 min?

3. The following table shows the number of new households that connected to a town water system each year from 1980 to 1988.

Year	Number of Households
1980	28
1981	36
1982	35
1983	36
1984	37
1985	39
1986	43
1987	46
1988	45

a. Make a scatterplot of the data. Let *y* represent the number of households and *x* represent the number of years after 1980.

b. **1.** On the scatterplot, draw a line that closely models the data.

2. Write an equation of the line in the form $y = mx + b$.

3. Use residuals to describe how well your line models the data.

c. Use your model to predict the number of new households that will connect to the water system in the year 2000.

Answers to Module Assessment

1. **a**. The total amount of water Terry plans to save can be calculated as follows:

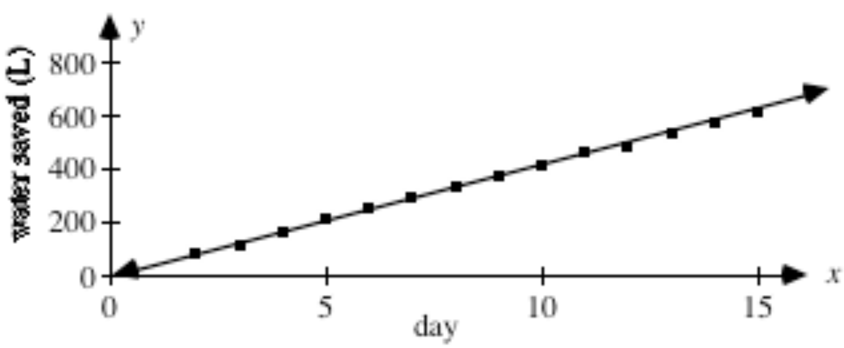
$$(0.15)\left(\frac{275\text{ L}}{\text{day}}\right)\left(\frac{365\text{ days}}{\text{year}}\right) \approx 15{,}000\ \text{ L/year}$$

b. Answers will vary. Students should convert liters to another metric unit, such as cubic meters, then determine appropriate dimensions. Since $15{,}000\text{ L} = 15\text{ m}^3$, some possible dimensions are
5 m ∞ 3 m ∞ 1 m, 5 m ∞ 2 m ∞ 1.5 m, and 10 m ∞ 3 m ∞ 0.5 m.

c. The table below shows the total amount of water Terry saved for each day from January 1 to January 15.

Day	Total Saved (L)	Day	Total Saved (L)	Day	Total Saved (L)
1	45	6	250	11	455
2	80	7	290	12	480
3	115	8	330	13	525
4	165	9	365	14	570
5	215	10	410	15	610

1–2. Sample graph:



3. The slope of the line indicates the rate at which Terry is saving water in liters per day.

d. **1.** An equation for the sample line given in Part **c** is $y = 41x$. This equation describes the total volume of water Terry has saved, where *y* represents the volume in liters and *x* represents the number of days after December 31.

2. This model appears to fit the data well. The average distance from the model to each data point is approximately 4.7, which is small compared to the spread in the data.

Day	Volume Saved (L)	Predicted Volume (L)	Absolute Value of Residual
1	45	41	4
2	80	82	2
3	115	123	8
4	165	164	1
5	215	205	10
6	250	246	4
7	290	287	3
8	330	328	2
9	365	369	4
10	410	410	0
11	455	451	4
12	480	492	12
13	525	533	8
14	570	574	4
15	610	615	5
		Sum	71

e. Using the equation $y = 41x$, the number of days required to save 15,000 L can be calculated as follows:

$$15,000 = 41x$$
$$x \approx 365 \text{ days}$$

If he continues at the same rate, Terry should come very close to reaching his goal.

2. **a.** The approximate area of the cross-section is $(1.6\ \text{m})(5.0\ \text{m}) = 8.0\ \text{m}^2$. The volume is $(8.0\ \text{m}^2)(100\ \text{m}) = 800\ \text{m}^3 = 800{,}000\ \text{L}$.

b. The speed of the water in kilometers per minute can be calculated as follows:

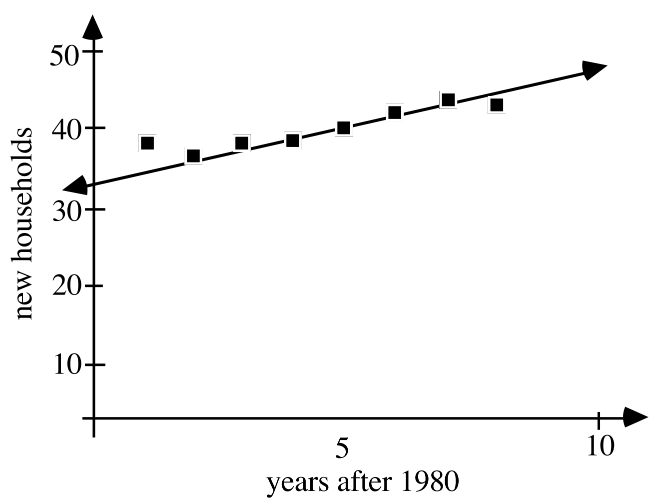
$$\left(\frac{5\ \text{km}}{\text{hr}}\right)\left(\frac{1\ \text{hr}}{60\ \text{min}}\right) \approx \frac{0.083\ \text{km}}{\text{min}}$$

Since the area of the cross-section is approximately $8.0\ \text{m}^2$, the volume of water that passes by in 1 min can be calculated as shown:

$$(8.0\ \text{m}^2)\left(\frac{100\ \text{dm}^2}{1\ \text{m}^2}\right)\left(\frac{0.083\ \text{km}}{1\ \text{min}}\right)\left(\frac{10{,}000\ \text{dm}}{1\ \text{km}}\right) \approx \frac{660{,}000\ \text{dm}^3}{\text{min}}$$

Since $1\ \text{dm}^3 = 1\ \text{L}$, the volume of water that flows past her in 10 min is $(10)(664{,}000) = 6.6 \bullet 10^6\ \text{L}$.

3. **a.** Sample graph:



b. **1.** Lines drawn may vary. One possible line is shown on the scatterplot in Part **a.**

2. An equation for the sample line is $y = 2x + 31$.

3. This model appears to fit the data well. The average distance from the model to each data point is approximately 1.6, which is small compared to the spread in the data.

Years after 1980	**Number of Households**	**Predicted households**	**Absolute Value of Residuals**
0	28	31	3
1	36	33	3
2	35	35	0
3	36	37	1
4	37	39	2
5	39	41	2
6	43	43	0
7	46	45	1
8	45	47	2
		Sum	14

c. Answers will vary. Using the equation $y = 2x + 31$, the predicted number of new households is 71.

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Flashbacks

Activity 1

1.1 Calculate each of the following to the correct number of significant digits:

a. (100)(2.75)

b. $34.5 + 16 + 12.32 + 2$

c. $25.5 \div 5.0$

1.2 Write each of the following numbers in scientific notation:

a. 1,600,000,000

b. 2,750,000,000,000

1.3 Complete each of the following equations:

a. 12 dm = ____ cm

b. 12 cm = ____ dm

c. 23 m = ____ dm

d. 425 dm = ____ m

1.4 Complete each of the following equations:

a. $8\ m^2$ = ____ cm^2

b. $6.2\ cm^2 = dm^2$

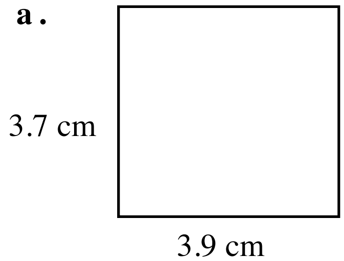
c. $90\ dm^2 = m^2$

d. $215\ dm^3$ = ____ cm^3

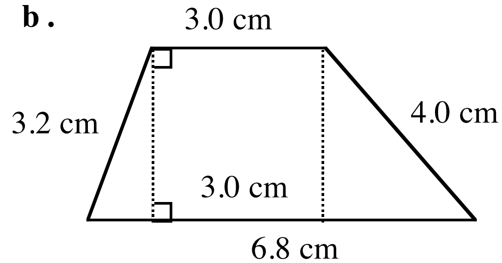
e. $3426\ m^3$ = ____ dm^3

1.5 Find the area (in square centimeters) of each of the following figures:

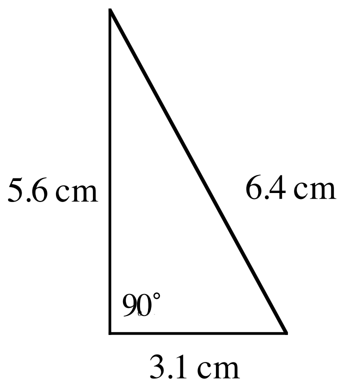
a.



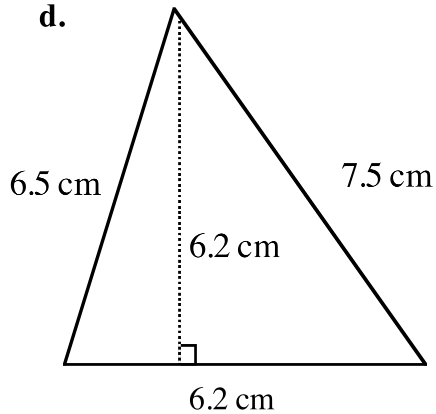
b.



c.



d.



Activity 2

2.1 If a water pipe leaks at the rate of 300 mL/min, what is the rate of water loss in liters per day?

2.2 Find the slope of the line that contains each of the following pairs of points:

a. (1,5) and (4,11)

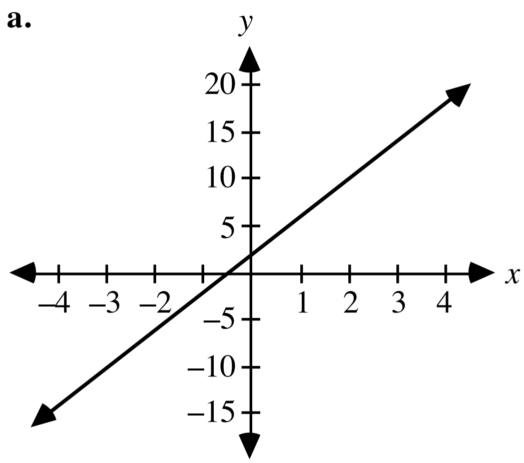
b. (0,–4) and (2,–7)

c. (4,6) and (8,6)

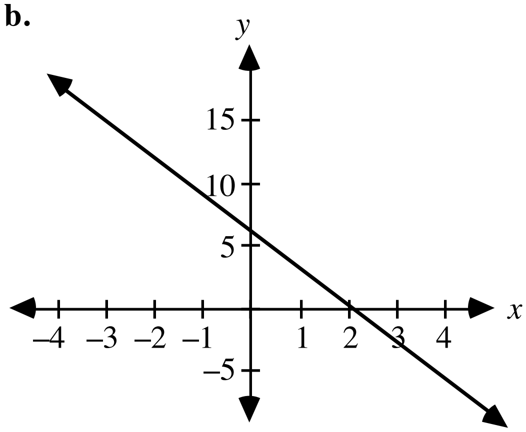
d. (6,4) and (6,8)

2.3 Find the slope and y-intercept of each of the following lines:

a.



b.



2.4 Write an equation for each line in Flashback **2.2** in the form $y = mx + b$, where m is the slope and b is the y-intercept.

Activity 3

3.1 Determine the absolute value of each of the following:

a. 23

b. –3.2

c. 0

3.2 Evaluate each of the expressions below:

a. $|-5|$

b. $|9.6|$

3.3 Write each of the following as a percentage:

a. 12 out of 25

b. 3/11

c. $\frac{9}{16}$

d. 7:20

e. $131 \div 75$

Answers to Flashbacks

Activity 1

1.1 **a.** $(100)(2.75) \approx 300$

b. $34.5 + 16 + 12.32 + 2 \approx 60$

c. $25.5 \div 5.0 = 5.1$

1.2 **a.** $1.6 \bullet 10^9$

b. $2.75 \bullet 10^{12}$

1.3 **a.** 12 dm = 120 cm

b. 12 cm = 1.2 dm

c. 23 m = 230 dm

d. 425 dm = 42.5 m

1.4 **a.** $8 \text{ m}^2 = 80{,}000 \text{ cm}^2$

b. $6.2 \text{ cm}^2 = 0.062 \text{ dm}^2$

c. $90 \text{ dm}^2 = 0.9 \text{ m}^2$

d. $215 \text{ dm}^3 = 215{,}000 \text{ cm}^3$

e. $3426 \text{ m}^3 = 3{,}426{,}000 \text{ dm}^3$

1.5 The following responses are not given in significant digits.

a. 14.43 cm^2

b. 14.7 cm^2

c. 8.68 cm^2

d. 19.22 cm^2

Activity 2

2.1 Sample response:

$$\frac{300\text{ mL}}{1\text{ min}} \bullet \frac{60\text{ min}}{1\text{ hr}} \bullet \frac{24\text{ hr}}{1\text{ day}} \bullet \frac{1\text{ L}}{1000\text{ mL}} = 432\ \text{L/day}$$

2.2 **a.** 2

b. –1.5

c. 0

d. undefined

2.3 **a.** The slope is approximately 4; the *y*-intercept is approximately 2.

b. The slope is approximately –3; the *y*-intercept is approximately 6.

2.4 **a.** $y = 4x + 2$

b. $y = -3x + 6$

Activity 3

3.1 **a.** 23.

b. 3.2.

c. 0

3.2 **a.** 5

b. 9.6

3.3 **a.** 48%

b. approximately 27%

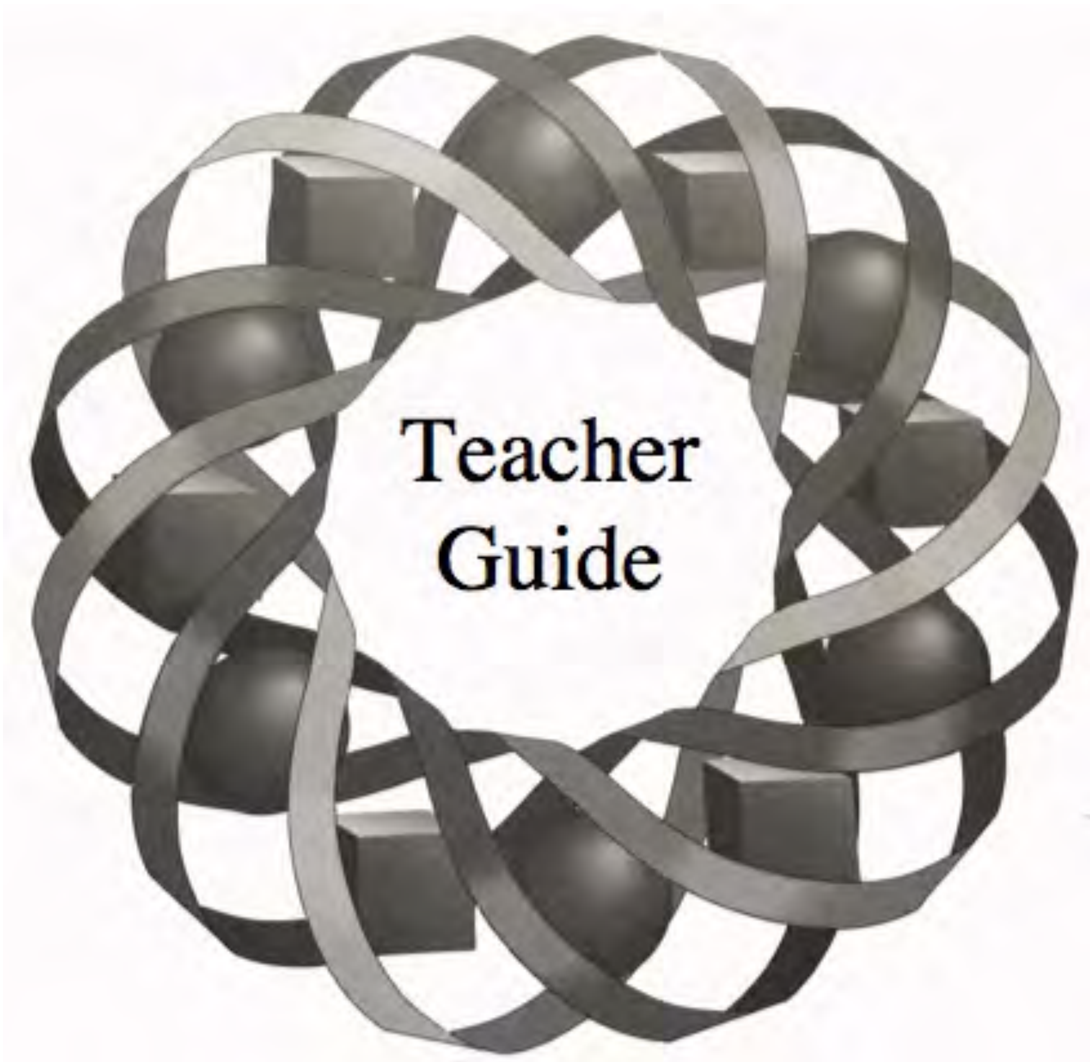
c. approximately 56%

d. 35%

e. approximately 175%

1-cm Grid Paper

Skeeters Are Overrunning the World



How does the size of a population change over time? In this module, you use a simple model to shed light on some complicated issues.

Shirley Bagwell • Gary Bauer • Patricia Bean • Mike Trudnowski

Teacher Edition

Skeeters Are Overrunning the World

Overview

In this module, students look at how the size of a population increases or decreases over time. Students use population models to simulate different growth rates and study the effects of those rates on population size. These models and the mathematics behind them are used to predict future population changes.

Note: A population is a collection of individuals of the same species in a given area. A few examples of populations are grizzly bears in the Greater Yellowstone ecosystem, humpback whales in the Pacific Ocean, or humans on Earth. Scientists study a variety of population characteristics, including changes in age distribution, ratios of males to females, and total size.

Objectives

In this module, students will:

- develop and use a mathematical model for population growth
- determine the growth rate of a population
- graph and interpret an exponential function in the form $y = a \bullet b^x$.

Prerequisites

For this module, students should know:

- exponential notation
- how to use a spreadsheet
- how to graph data on an *xy*-coordinate system
- how to calculate percent change
- how to evaluate an equation.

Time Line

Activity	Intro.	1	2	3	4	Summary Assessment	Total
Days	1	3	2	2	2	1	11

Materials Required

Materials	Activity					
	Intro.	**1**	**2**	**3**	**4**	**Summary Assessment**
graph paper	X	X				X
multicolored candies or disks with distinctive mark on one side	X	X				
boxes with lids	X	X				
colored pencils		X				

Technology

Software	Activity					
	Intro.	**1**	**2**	**3**	**4**	**Summary Assessment**
spreadsheet		X	X	X	X	X
graphing utility					X	

Teacher Note

The activities in this module are based on the following five-step modeling process:

- creating a simplified model of a given situation
- translating the model into mathematics
- using mathematics to solve a problem
- relating the results of the mathematical manipulations to the simplified model and the given situation
- revising the model as necessary and repeating the process.

Skeeters Are Overrunning the World

Introduction

(page 135)

Students discuss populations and population growth. They engage in data collection and organization, then look for patterns and make predictions. (For additional information on human population growth, see Joel Cohen's "How Many People Can the Earth Hold?")

Materials List

- multicolored candies or disks with a mark on one side (approximately 500 per group)
- container with a large, flat bottom and a lid (one per group)
- graph paper (several sheets per group)

Teacher Note

The multicolored candies or disks are referred to as Skeeters in this module. In the following exploration, students require only a single color of Skeeters. In Activity **1**, students will need five different colors. Round, flattened candies such as M&Ms™ or Skittles™ work well. You may also use beans that have been marked on one side, or any other objects with two distinct sides that will not stack up when shaken.

Shoe boxes or pizza boxes work well for containers.

Discussion 1

(page 135)

a. Students should observe that population numbers are increasing with time. Students may also notice that the pattern is nonlinear.

b. Answers will vary. Sample response: If the population continues to rise as in the past 40 years, there will be about 14 billion people in 2075.

c. Students may identify a variety of causes for the change in the rate of growth. Improved nutrition and medical care may increase the rate of survival. On the other hand, the population may be limited by the availability of land, food, water, and energy; or the death rate may increase due to diseases such as AIDS, famine, and pollution.

Exploration

(page 136)

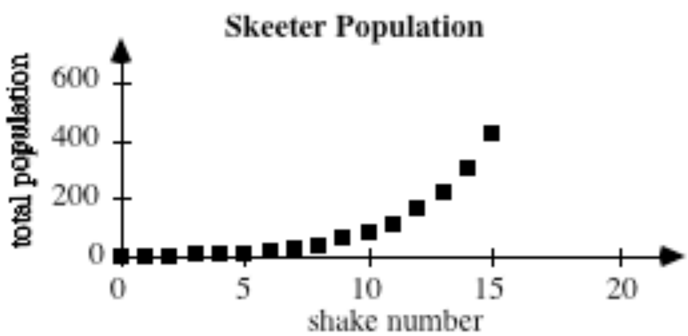
In this exploration, Skeeters provide a physical representation of a real-world population. The simulation produces data to help model population growth.

a–b. Students should make an initial prediction before beginning the simulation. Sample response: The number of Skeeters will increase with each shake.

The following table shows sample data for 15 shakes.

Shake	Population	Shake	Population
0	2	8	40
1	3	9	61
2	4	10	87
3	6	11	114
4	7	12	164
5	12	13	225
6	17	14	308
7	26	15	424

c. **1.** Sample scatterplot:



2. Students should notice a nonlinear, curved pattern.

d. **1.** Sample response: The total population after shake 20 will be approximately 1300.

2. Answers will vary. One possible strategy involves estimating the volume of a single Skeeter, then multiplying this value by the predicted population.

3. Answers may vary substantially. Using the table in Part **a-b** above, the total population after shake 15 should be about 424. The total population after shake 16 should be about 636.and after shake 17 about 954. Therefore, it would take approximately 17 shakes.

Discussion 2

(page 137)

a. Students should notice that their scatterplots have the same general, nonlinear shape.

b. Sample response: The population increases after each shake. Each subsequent shake adds a greater number of Skeeters than the previous shake.

c.
1. Students should observe that the population size remains relatively unchanged for small shake numbers, but increases rapidly as the shake number (or time) increases.
2. Students should note that these graphs do not seem to be linear, as they get steeper as the number of shakes increases. Some may describe the shape of this pattern as like the letter *J*, the side of a mixing bowl, or a skateboard ramp.

d.
1. This pattern of growth may sometimes be observed in populations of bacteria or—for a relatively short period of time—in other animals and plants that rapidly colonize a new habitat.
2. Students may point out several limitations in using this model to simulate natural populations. For example, Skeeters reproduce asexually, whereas most animals and plants reproduce sexually. The simulation also does not account for predator-prey relationships, deaths, or environmental changes. In other words, the simulation is too simplified to model most populations very well.

(page 137)

Activity 1

Students use equations of the form $y = b^x$, where *y* represents total population and *x* represents the shake number, to model Skeeters data. In this equation, *b* represents the factor by which the population is changing.

Materials List

- multicolored candies or disks (approximately 500 per group)
- container with a large, flat bottom and a lid (one per group)
- graph paper (several sheets per group)
- colored pencils to match Skeeter colors (optional; one set per group)

Technology

- spreadsheet

Teacher Note

The colors of the Skeeters described in the exploration may be changed to fit the colors of your manipulatives.

Students may choose to use technology for recording and graphing their data. If students make paper-and-pencil graphs in Part **g** of the exploration, they may want colored pencils to indicate the different populations.

Students should save their tables and graphs for use later in this activity as well as in Activities **2** and **3**.

Exploration

(page 138)

Students may run out of yellow or green Skeeters before completing 10 shakes. However, the doubling and tripling patterns should be clear before this occurs.

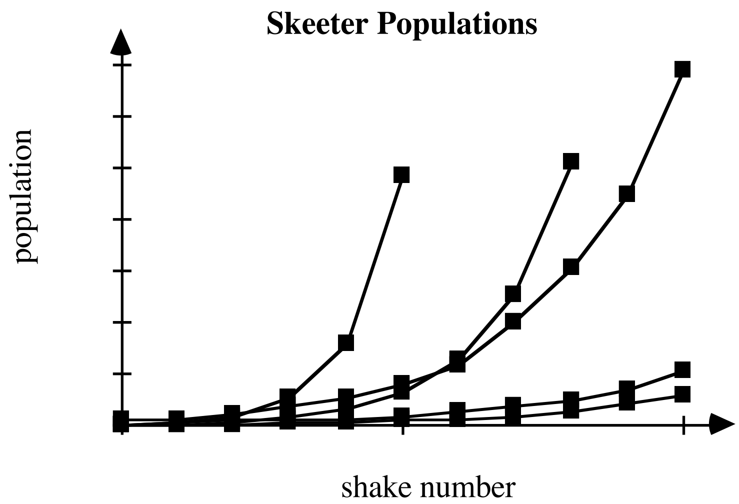
a. **1.** After three shakes, the green Skeeter population will be 27.

2. After 10 shakes, the green Skeeter population will be the largest.

b–f. Sample data:

Shake	Green	Yellow	Orange	Red	Purple
0	1	1	1	2	5
1	3	2	1	4	7
2	9	4	2	6	11
3	27	8	3	6	20
4	81	16	4	7	28
5	243	32	5	10	41
6	729	64	7	13	60
7	2187	128	10	20	102
8	6561	256	14	26	153
9	19,683	512	22	36	226
10	59,049	1024	31	53	346

g. Sample graph:



Discussion

(page 139)

a. Students should notice that the population doubles from shake to shake or that the new population is always equal to the previous population multiplied by 2.

b. **1.** Sample response: total number $= 2^{\text{shake number}}$

2. This relationship can be expressed as $y = 2^x$, where *y* represents the number of Skeeters at the end of the shake and *x* represents the shake number.

c. The equation from Part **b** can be used to describe the population of Yellow Skeeters after any shake. To help verify that the equation works, students may evaluate it for the values of *x* from 0 to 10. The corresponding values for *y* should match the population numbers obtained in the exploration.

d. Sample response: The relationship between shake number and population is a function for every color since, in each set of data, the shake number is different in each ordered pair.

e. The values for shake number represent the domain; the values for population represent the range.

f. The growth rate is constant for the yellow and green Skeeter populations. For the other colors, the growth rate changes slightly from shake to shake.

Assignment

(page 140)

1.1 **a.**

Shake Number	Total Population	Expanded Notation	Exponential Notation
0	1	$1 \bullet 1$	$1 \bullet 2^0$
1	2	$1 \bullet 2$	$1 \bullet 2^1$
2	4	$1 \bullet 2 \bullet 2$	$1 \bullet 2^2$
3	8	$1 \bullet 2 \bullet 2 \bullet 2$	$1 \bullet 2^3$
⋮	⋮	⋮	⋮
n	2^n		$1 \bullet 2^n$

b. Sample response: $T = 1 \bullet 2^n$, where *T* represents the total population and *n* represents the shake number.

c. Sample response: Yes. Each shake number is the first term in an ordered pair and occurs only once in the set of all ordered pairs.

d. **1.** Sample response: Substitute the shake number for *n* in the equation $T = 1 \bullet 2^n$.

2. Students may need some instruction on the use of the exponential function on their calculators: $2^{20} = 1,048,576$.

e. **1.** 2

2. Students may recognize this number as the base of the exponent in the equation from Part **b**.

f. **1.** 100% per shake

2. Students may recognize (1 + the growth rate) as the base of the exponent in the equation from Part **b**.

1.2 **a**. Sample table:

Shake Number	Total Population	Expanded Notation	Exponential Notation
0	1	$1 \bullet 1$	$1 \bullet 3^0$
1	3	$1 \bullet 3$	$1 \bullet 3^1$
2	9	$1 \bullet 3 \bullet 3$	$1 \bullet 3^2$
3	27	$1 \bullet 3 \bullet 3 \bullet 3$	$1 \bullet 3^3$
⋮	⋮	⋮	⋮
n	3^n		$1 \bullet 3^n$

Sample equation: $T = 1 \bullet 3^n$, where *T* represents the total population and *n* represents the shake number.

b. Since $2^{24} = 16,777,216$, one possible response is 15 ($3^{15} = 14,348,907$).

c. **1.** 3

2. Students may recognize this as the base of the exponent in the equation from Part **a**.

d. **1.** 200% per shake

2. The base of the exponent is (1 + the growth rate) expressed as a decimal.

Teacher Note

You may want to point out that which variable is dependent and which is independent may be a matter of convenience.

***1.3** **a**. Sample response: In both equations, the shake number is the exponent. The bases, however, are different.

b. Since the population total depends on the shake number, it is the dependent variable.

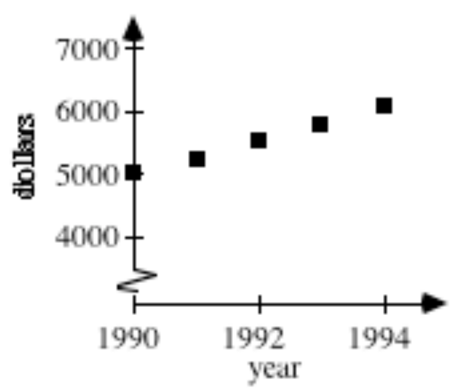
c. The equation for Problem **1.1b** is $y = 2^x$; the equation for Problem **1.2a** is $y = 3^x$.

1.4 **a.** After 10 shakes, the population is $5^{10} = 9,765,625$.

b. The population after each shake is 5 times the population before the shake.

* * * * *

1.5 **a.** Sample scatterplot:



b. In this setting, the annual growth rate is the annual interest rate. It can be calculated as follows:

$$\frac{5250.00 - 5000.000}{5000} = 0.05 = 5\%$$

c. In 1995, the balance is $(\$6077.53 \bullet 0.05) + \$6077.53 \approx \$6381.41$.

d. Students should use technology to determine an estimate. After the interest is deposited in the 15th year, the account balance will be $10,394.64. This is the first time that the balance is at least double the original deposit.

1.6 **a.** Sample table:

Shake	Population
0	1
1	4
2	16
3	64
4	256
5	1024
6	4096

b. $y = 1 \bullet 4^x$

c. $y = 1 \bullet 4^{10} = 1{,}048{,}576$

c. The constant growth rate for this population is 300% per shake.

* * * * * * * * * *

(page 143)

Activity 2

Students determine a method for calculating the growth rates of the orange, red, and purple Skeeter populations. At this point in the module, it is unlikely that they will know a way to describe growth other than a simple doubling or tripling from shake to shake. In the assignment, students examine a growth rate of 50% (or 0.5) for the orange Skeeter population.

Materials List

- none

Technology

- spreadsheet

Discussion 1

(page 143)

a. Sample response: You would expect 50% of the population, or 5 Skeeters, to land with a mark showing.

b. If the coin is fair, the probabilities are the same: 50%.

c. The expected growth rate is 50% or 0.5 per shake.

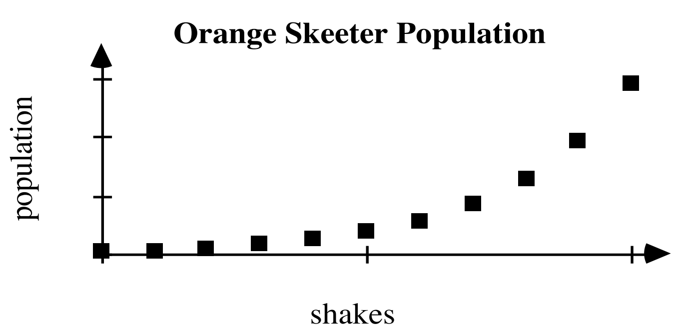
Exploration

(page 143)

a. Sample spreadsheet:

Shake	Expected Population	Actual Population	Actual Growth Rate (from Previous Shake)
0	1.0	1	
1	1.5	1	0
2	2.3	2	100%
3	3.4	3	50%
4	5.1	4	33%
5	7.6	5	25%
6	11.4	7	40%
7	17.1	10	43%
8	25.6	14	40%
9	38.4	22	58%
10	57.7	31	41%

b. Sample graph:



c. **1.** $y = 1 \bullet 1.5^x$

2. The graph of the equation contains all of the expected population values. See Part **b** for the graph of the line.

Discussion 2

(page 144)

a. Sample response: The graph of the actual data is close to the graph of the expected data. However, the expected data contains fractional values. This cannot occur with actual Skeeters.

b. Answers will vary, depending on individual sets of data. The equation should model the data reasonably well.

Assignment

(page 144)

***2.1**

a. Sample response: At the end of the first shake, you would expect to add $0.5(20) = 10$ Skeeters. The expected total population would be $20 + 10 = 30$.

b. Sample response: At the end of the second shake, you would expect to add $0.5(30) = 15$ Skeeters. The expected total population would be $30 + 15 = 45$.

c. $0.5p$

d. 50% or 0.5 per shake

e. Students should understand that the total population equals the present population plus 0.5 times the present population (or 1.5 times the present population). This can be expressed as $T = p + 0.5p$ or $T = 1.5p$.

f. Sample response: The number you would expect to add is 0.5 times the expected total population or $0.5(p + 0.5p)$ or $0.5(1.5p)$.

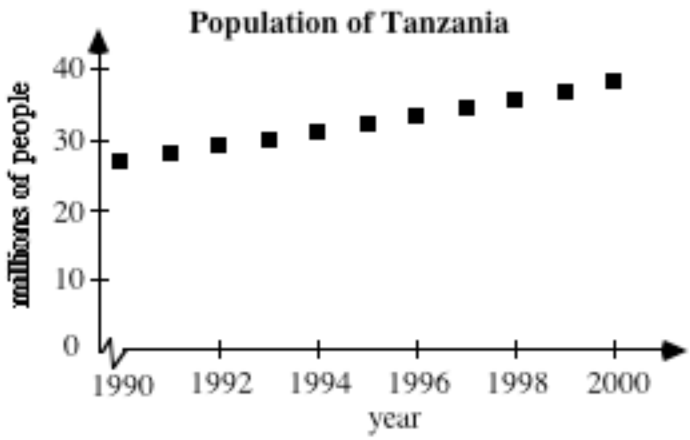
g. The expected total population after a shake equals the population before the shake plus 0.5 times the population before the shake.

2.2 Sample response: Yes. As long as each Skeeter has the opportunity to be tossed, the probability of a mark appearing is $1/2$.

2.3 **a.** Sample response:

Year	Population
1990	27,000,000
1991	27,945,000
1992	28,923,075
1993	29,935,383
1994	30,983,121
1995	32,067,530
1996	33,189,894
1997	34,351,540
1998	35,553,844
1999	36,798,229
2000	38,086,167

b. Sample graph:



c. Sample response: If the actual growth rate remains constant at about 3.5% per year, the expected values will give a reasonable approximation of the actual population. If the growth rate changes dramatically, the expected values may be very different.

* * * * *

2.4 Answers will vary. Sample response: Chauncy will receive more money over the 5-year period if he chooses the $7 raise. If he chooses the 40% increase, then he will receive more money only in the last 2 years. This can be seen in the table below.

Year	$7 Raise	40% Increase
0	$10.00	$10.00
1	$17.00	$14.00
2	$24.00	$19.60
3	$31.00	$27.44
4	$38.00	$38.42
5	$45.00	$53.78
Total (52 weeks/year)	$8,580.00	$8488.48

2.5 **a.** The annual growth rate can be calculated as follows:

$$\frac{3905-4338}{4338} \approx -0.10 = -10\%$$

b. Students may approach this problem by representing each year's tax as $p - 0.1p$, where p represents the previous year's tax. If the growth rate remains the same, Sue's taxes should be $2561.55.

c. Sample response: The growth rate for taxes is usually not constant. It depends on a number of things including inflation and legislation.

* * * * * * * * * *

(page 145)

Activity 3

In this activity, students examine how initial population affects population growth.

Materials List

- none

Technology

- spreadsheet

Teacher Note

To complete this activity, students will need their data from the orange, red, and purple Skeeter populations from the exploration in Activity **1**. Displaying the data both as a table and as a graph provides students with an opportunity to observe patterns involving growth rate and initial population.

Discussion 1

(page 145)

a. Although the growth rates are the same for all three populations, the total populations after 10 shakes should be quite different. Students should recognize that initial population size affects total population. Encourage them to use tables and graphs to justify their observations.

b. Students may recognize that higher initial populations lead to very large population differences at low shake numbers.

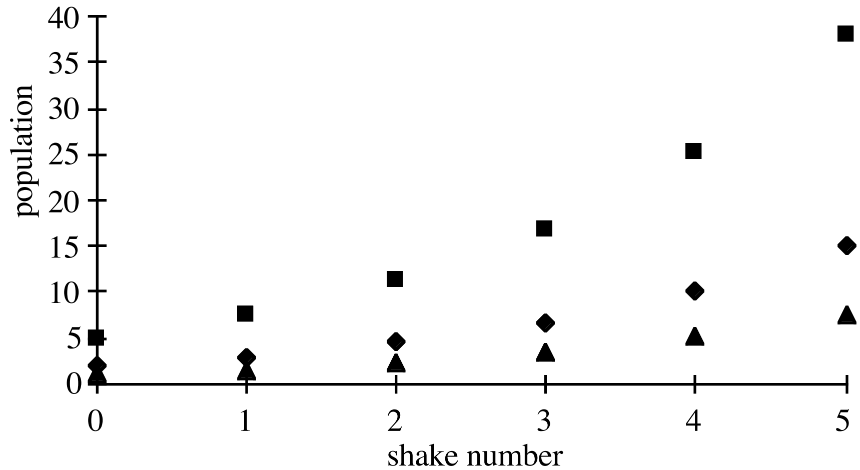
Exploration (page 145)

a–b. **Note:** Students may need instructions on using spreadsheet formula commands. You may wish to ask them to suggest the mathematical operations needed to create each cell from the preceding cell.

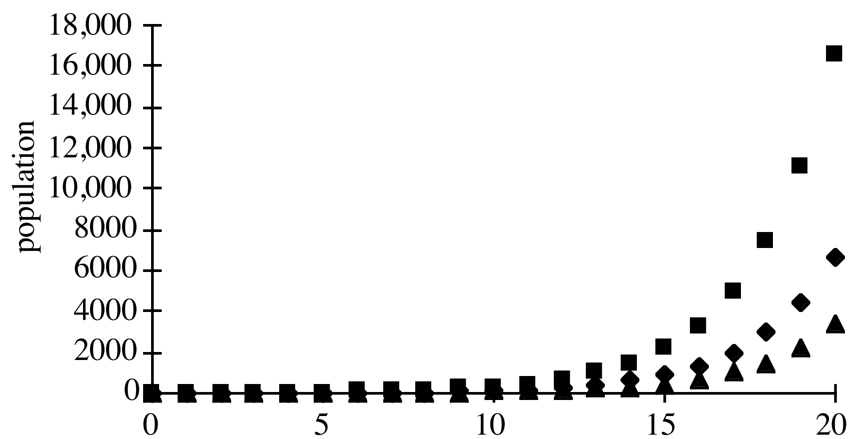
In the following sample spreadsheet, all numbers were rounded to the nearest 0.01.

Shake	Orange	Red	Purple
0	1.00	2.00	5.00
1	1.50	3.00	7.50
2	2.25	4.50	11.25
3	3.38	6.75	16.88
4	5.06	10.13	25.31
5	7.59	15.19	37.97
6	11.39	22.78	56.95
7	17.09	34.17	85.43
8	25.63	51.26	128.145
9	38.44	76.89	192.22
10	57.67	115.33	288.33
11	86.50	173.00	432.49
12	129.75	259.49	648.73
13	194.62	389.24	973.10
14	291.93	583.86	1459.65
15	437.89	875.79	2189.47
16	656.84	1313.68	3284.20
17	985.26	1970.52	4926.31
18	1477.90	2955.78	7389.46
19	2216.84	4433.68	11084.19
20	3325.26	6650.52	16626.29

c. Sample graph:



d. Sample graph:



Discussion 2

(page 146)

a. The actual growth rates for the three populations are not constant. However, they tend to be close to 0.5 from shake to shake.

b. Students should notice that the expected data generated by technology gives decimal fractions of Skeeters. The data from the exploration in Activity **1** consists of discrete, whole-number values. The general patterns in corresponding sets of data should be reasonably close.

c.

1. Sample response: All three graphs rise to the right, showing a positive relationship between total population and shake number. Some graphs rise "faster" than others and each one intersects the *y*-axis at a different point.

2. The initial size of each population is the *y*-intercept.

d. Sample response: To calculate the population after the first shake, add the value of the previous cell to 50% of that value. From there, fill down the column.

e. The total population equals the initial population plus the increase in population. This can be expressed as $T = p + pr$. Using the distributive property, this expression can be rewritten as $T = p(1 + r)\cdot$

f. **1.** $\left[p(1 + r)\right](1 + r) = p(1 + r)^2$

2. In general, students should recognize the following relationship:

$$T = p(1 + r)^n$$

where T is the total population, p is initial population, r is growth rate, and n is the shake number.

Assignment

(page 147)

3.1 Since the actual populations consist only of whole numbers, the relationship found in Part **f** of Discussion **2** will not fit the data exactly. It should, however, describe the general trend in both populations.

3.2 **a**. The variable y would represent the total population; a would represent the initial population; b would represent $1 + r$; x would represent the shake number.

b. Changing the initial population changes the value of a in the equation and changes the y-intercept in its graph.

c. Students should graph $y = 4(1 + 1.5)^x$ .This problem gives students the opportunity to enter an exponential equation in a graphing utility. They may need help selecting the appropriate intervals for the domain and range.

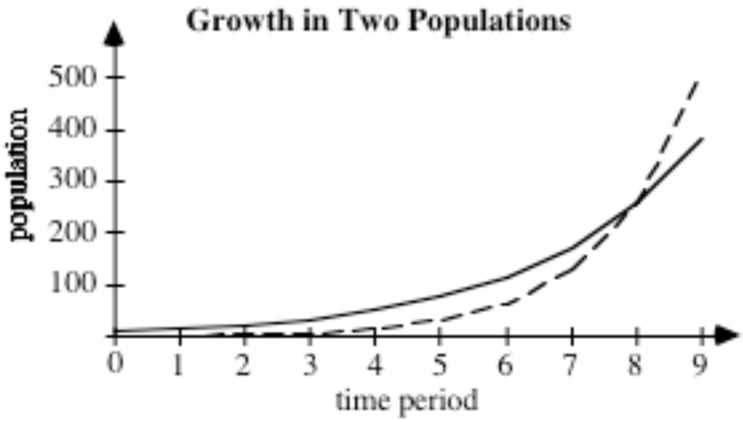
3.3 Sample response: The population increases by 50% each time the box is shaken. Fifty percent of a small number is small, but 50% of a large number is large.

***3.4** Students should substitute the appropriate values into the general form of an exponential equation: $2,734,375 = 7b^8$. Since $2,734,375/7 = 390,625$, $b^8 = 390,625$. Through guess-and-check, $5^8 = 390,625$. Students also may use a symbolic manipulator to solve the equation $2,734,375 = 7b^8$. Since $b = 5$, $r = 5 - 1 = 4 = 400\%$ per shake.

3.5 **a.** The domain is all non-negative integers. The range is a subset of the positive integers. Some integers are not part of the range because there are no integer values for x that satisfy $3 \bullet 2^x = y$. For example, there is no integer value for x that satisfies $3 \bullet 2^x = 5$.

b. This is a function because every x-value is paired with a y-value and every x-value occurs in only one ordered pair.

3.6 **a.** Students may need to experiment to find appropriate intervals for the domain and range. Since $y_1 = 10 \bullet 1.5^8 = 256.3$ and $y_2 = 1 \bullet 2^8 = 256$, both equations are approximately equal to 256 when $x = 8$. Sample graph:



b. When $x = 0$, $y_1 = 10 \bullet 1.5^0 = 10$ and $y_2 = 1 \bullet 2^0 = 1$. These values represent the initial population sizes. **Note:** Students may need to review the mathematical meaning of b^0 when *b* is not 0.

***3.7** Sample equation: $y = 2 \bullet (1 + 2.5)^x = 2 \bullet 3.5^x$. **Note:** The population totals in the table have been rounded to the nearest whole number.

* * * * *

3.8 **a.** $y = 4000(1.04)^x$

b. When $x = 0$, $y = \$4000$ (the initial deposit).

c. Students should use technology or trial-and-error to find an approximate value for *x* in the equation $8000 = 4000(1.04)^x$. Since $x \approx 17.67$, the value of the account will double in the year 2008.

3.9 To estimate the growth rate, students should use technology to solve the equation $53.9 \bullet 10^6 = 50(b)^{98}$ for *b*. The value of *b* is approximately 1.15 and since $b = 1 + r$, the value of r is approximately 0.15. Therefore, the painting's value increased by approximately 15% over 98 years.

* * * * * * * * * *

Research Project (page 149)

Responses will vary. As part of the research project, students should include a story about the population, a description of its growth rate, and an explanation of the consequences of the chosen growth rate over time.

(page 149)

Activity 4

In this activity, students examine the change in the U.S. population from 1790 to 1990. Students also examine the effects of a zero growth rate on a population.

Materials List

- none

Technology

- spreadsheet
- graphing utility

Teacher Note

We use $y = ab^x$ to describe population growth. A special case of this model is $T = pe^{rt}$, where p represents the initial population, r represents the growth rate, t represents time, and T represents the total population at time t.

Exploration

(page 149)

The following exploration is recommended for small groups. Students may use a spreadsheet to calculate the growth rates between census counts. **Note:** Each student will need a copy of the calculated growth rates for the assignment.

a. Students may see a variety of patterns in this data. For example, they may note that growth slowed during times of war, during the Great Depression of the 1930s, and during the latter half of this century.

b. Sample spreadsheet:

Census Date	Population	Growth Rate	Census Date	Population	Growth Rate
1790	3,929,214		1900	75,994,575	0.21
1800	5,308,483	0.35	1910	91,972,266	0.21
1810	7,239,881	0.36	1920	105,710,620	0.15
1820	9,638,453	0.33	1930	122,775,046	0.16
1830	12,866,020	0.33	1940	131,669,275	0.07
1840	17,069,453	0.33	1950	150,697,361	0.14
1850	23,191,876	0.36	1960	179,323,175	0.19
1860	31,443,321	0.36	1970	203,302,031	0.13
1870	39,818,449	0.27	1980	226,545,805	0.11
1880	50,155,783	0.26	1990	248,709,873	0.10
1890	62,947,714	0.26			

Discussion (page 150)

a. Students may mention a range of possible explanations, including historical events such as the Civil War, World War II, and the Great Depression, as well as social trends like the change from a rural to an urban society, advancements in medical science, or the decline in the number of children per family.

b. Students may choose the mean, median, or mode as a representative growth rate—or they may choose something entirely different. Students should be able to justify their choices using the data provided.

c. The death rate is 0.05.

d. $y = 65 \bullet (1 - 0.05)^x = 65 \bullet 0.95^x$

e. The population decreases.

Assignment (page 150)

4.1 This problem is designed to help students develop an understanding of the consequences of unchecked population growth.

a. The period from 1790 to 1990 represents 20 ten-year intervals. Using a growth rate of 35% per decade, $3{,}929{,}214 \bullet 1.35^{20} \approx 1{,}588{,}000{,}000$. This prediction is almost 6.4 times greater than the actual population in 1990 (248,709,863).

b. Sample response: It is possible to make a rough estimate of the population for the year 2040 by using technology to create a scatterplot of the data, extending the trend of the last five points in the scatterplot, then using the trace function to estimate the population size.

Given the appropriate initial population and growth rate, an exponential equation provides a quick way to predict population sizes.

c. Using a growth rate of 10% per decade, the predicted population is $248{,}709{,}873 \bullet 1.10^5 \approx 400{,}550{,}000$.

4.2 **a.** Sample response: –5% per decade.

b. Since the period from 1990 to 2040 represents 5 ten-year intervals, $248{,}709{,}873 \bullet (1 - 0.05)^5 \approx 192{,}446{,}959$.

c. Sample response: Circumstances that could cause a population to decrease include famine, natural disasters, diseases, or increased emphasis on birth control.

4.3 **a.** The predicted population in 1940 can be calculated as follows:

$$122,775,046 \bullet 1.16^1 \approx 142,419,000$$

Likewise, the predicted population in 1950 is:

$$122,775,046 \bullet 1.16^2 \approx 165,206,000$$

b. The predicted populations are somewhat larger than the actual populations.

***4.4** The following table shows the predicted population for each city.

New York	$14,628,000 \bullet 1.00^{49} \approx 14,628,000$
London	$9,168,000 \bullet 0.99^{49} \approx 5,603,000$
Lagos	$8,487,000 \bullet 1.048^{49} \approx 84,422,000$

Sample response: In 50 years, the population of Lagos will be about 6 times larger than that of New York due to the difference in growth rates. The predicted decrease in population size for London shows the effect of a negative growth rate.

4.5 As seen in Problem **4.4**, the size of a population with a growth rate of 0 does not change over time.

* * * * *

4.6 **a.** The growth rate is approximately 165% per decade:

$$\frac{42,968,000,000 - 16,185,000,000}{16,185,000,000} \approx 1.65$$

b. Since there are 5 ten-year periods from 1940 to 1990, the predicted debt is $42,968,000,000 \bullet (1+1.65)^5$ or about $5.6 trillion.

c. **1.** The growth rate is approximately 256%:

$$\frac{3.233 - 0.9077}{0.9077} \approx 2.56$$

2. Since there are 5 ten-year periods from 1990 to 2090, the predicted debt is $3.233(1 + 2.56)^5$ or about $1849 trillion.

4.7 **a.** The growth rate is approximately 1.2% per year. At this rate, the student population will be about 1017 in 5 years. Therefore, the addition to the high school will be required before this time.

b. In 20 years, the student population will be approximately 1216 and overcrowding will again be a problem.

* * * * * * * * * *

Answers to Summary Assessment (page 152)

1. The table below shows a sample equation for each nation.

Country	Growth Equation
Canada	$y = 27,700,000 \bullet 1.010^x$
China	$y = 1,177,600,000 \bullet 1.014^x$
Hungary	$y = 10,324,000 \bullet 0.997^x$
India	$y = 903,159,000 \bullet 1.021^x$

2. Using the sample equation given for Canada, the year in which the 1993 population will have doubled can be found by substituting as shown below, then solving for x.

$$2(27,700,000) = 27,700,00(1.010)^x$$
$$2 = 1(1.010)^x$$

Since $x \approx 67$ years, the corresponding year is $1993 + 67 = 2060$.

Because the growth rate for Hungary's population is negative, its population is decreasing and should not be expected to double in the future. (If this negative rate remains constant, Hungary's population would reach one-half of its 1993 level in about 231 years.)

3. **a.** Students may use a spreadsheet or graphing utility to approach this problem, or use a symbolic manipulator to determine what value of x makes the following equation true:

$$1,177,600,000(1.014)^x = 903,159,000(1.021)^x$$

Since $x \approx 39$ years, India's population is predicted to surpass that of China in 2032.

b. India's predicted population can be found as follows:

$$y = 903,159,000(1.021)^{39}$$
$$\approx 2,031,272,000$$

This results in a population density of about $683.5 \text{ people/km}^2$.

4. India is predicted to have the greatest population density—over 2400 people/km^2. The table below shows the predicted population and density for each nation.

Nation	**2093 Population**	**Land Area (km^2)**	**Density (people/km^2)**
Canada	74,923,000	8,968,000	8.4
China	4,729,261,000	9,327,000	507.1
Hungary	7,645,000	92,000	83.1
India	7,216,696,000	2,972,000	2428.2

Module Assessment

1. Write an equation of the form $y = a \bullet b^x$ to describe the pattern in the following data.

Time Period	Total Population
0	3
1	6
2	12
3	24
4	48
5	96

2. **a.** Describe the pattern that exists in the following numbers: $1.5, 3.45, 7.935, 18.2505$.

b. Determine the next number in the pattern.

3. The Whitetail Wildlife Refuge is home to a population of 30 deer. This population is expected to grow each year by a factor of 1.3.

a. Make a table that shows the expected population in each of the next 10 years.

b. Create a scatterplot of the data and describe the shape of the graph.

c. Write a mathematical equation that describes the growth of the population.

d. Predict what the deer population will be in 20 years.

4. The pocket gophers in Prairie County have a population density of 215 per km^2. This population is expected to grow at a rate of 4% every 5 years. Use this expected growth rate to predict what the population density will be in 20 years.

Answers to Module Assessment

1. $y = 3 \bullet 2^x$

2.

a. Multiply each term by 2.3 to obtain the following term.

b. The next term is 41.97615.

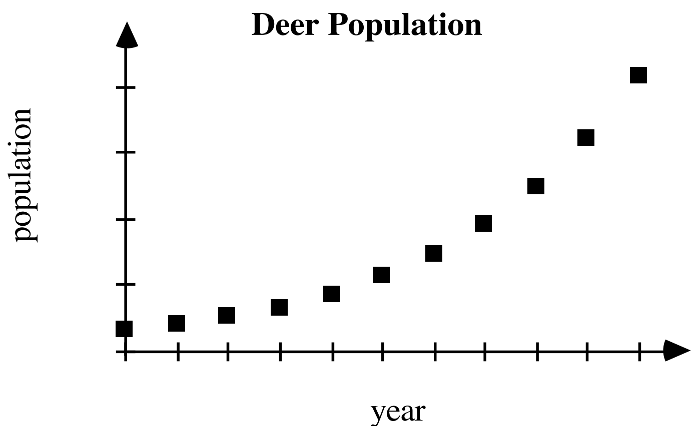
3.

a. The values in the sample table on the left were obtained using continuous values for p. The values in the table on the right were obtained by multiplying the preceding population by 1.3, then rounding to the nearest integer.

Using Continuous Values	
Year	**Population**
0	30
1	39
2	51
3	66
4	86
5	111
6	145
7	188
8	245
9	318
10	414

Using Discrete Values	
Year	**Population**
0	30
1	39
2	51
3	66
4	86
5	112
6	146
7	190
8	247
9	321
10	417

b. The graph shows the J-shape of an exponential equation with a y-intercept of 30. Sample scatterplot:



c. Sample equation: $P = 30 \bullet 1.3^y$, where P represents the total population and y represents time in years.

d. $P = 30 \bullet 1.3^{20} \approx 5701$ deer

4. Since there are 4 five-year periods in 20 years, the predicted population density can be calculated as follows: $d = 215 \bullet 1.04^4 \approx 252$ gophers/km^2.

Selected References

Cohen, J. E. "How Many People Can the Earth Hold?" *Discover* 13 (November 1992): 114–119.

U.S. Bureau of the Census. *Statistical Abstract of the United States: 1993*. Washington, DC: U.S. Government Printing Office, 1993.

Flashbacks

Activity 1

1.1 By what factor are the following numbers increasing?

a. $8, 16, 32, 64, \ldots$

b. $4, 12, 36, 108, \ldots$

1.2 For each of the following pairs of numbers, determine the percent increase from the first value to the second value.

a. $15, 30$

b. $8, 12$

1.3 **a.** Express $3 \bullet 3 \bullet 3 \bullet 3$ using exponential notation.

b. Write 4^3 as a product.

c. Explain the meaning of x^5.

d. Explain the meaning of 5^x.

1.4 **a.** Describe the pattern that exists in the following numbers:

$$1, \frac{5}{2}, \frac{25}{4}$$

b. Determine the next number in the pattern.

Activity 2

2.1 The following table shows the population of an ant colony during a 4-month period.

Month	Population
May	5000
June	5500
July	6050
August	6655

a. Create a scatterplot of the data.

b. What is the monthly growth rate in the population?

c. Predict the population of the ant colony in September and October.

Activity 3

3.1 Evaluate each of the following expressions:

a. 2^3

b. $(-6)^2$

c. $-(6^2)$

3.2 **a.** Create a graph of the equation $y = 2^x$.

b. Using the graph from Part **a**, determine the value of y for each of the following values of x:

1. $x = 3$

2. $x = 8$

c. Using the graph from Part **a**, determine the value of x for each of the following values of y:

1. $y = 1$

2. $y = 16$

3.3 Determine the approximate value of x in each of the following equations.

a. $310 = 5(1 + x)$

b. $48 = 3 \bullet 4^x$

Activity 4

4.1 Consider the data in the following table.

x	y
1	3
2	9
3	27
⋮	⋮

a. Write an equation that best models the numbers in the table where x is the independent variable and y is the dependent variable.

b. Create a graph of the equation.

c. Determine the value of y for each of the following values of x.

1. 4

2. 6

3. 25

4.2 Convert each of the following percentages to decimal form.

a. 0.05%

b. 0.7%

c. 1.3%

Answers to Flashbacks

Activity 1

1.1 **a.** 2

b. 3

1.2 **a.** 100%

b. 50%

1.3 **a.** 3^4

b. $4 \bullet 4 \bullet 4$

c. x used as a factor 5 times

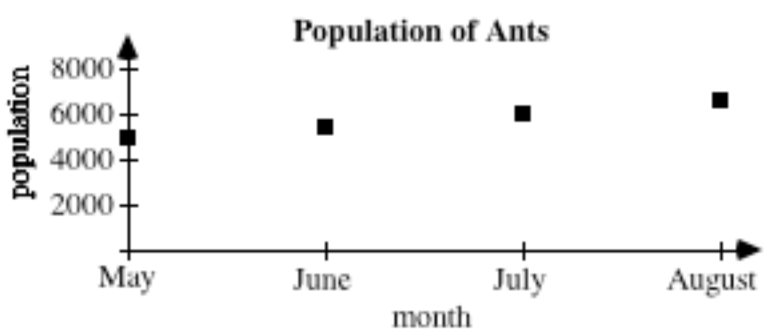
d. 5 used as a factor x times

1.4 **a.** Sample response: Each number in the sequence is the previous number multiplied by $5/2$.

b. $\frac{125}{8}$

Activity 2

2.1 **a.** Sample scatterplot:



b. 10% per month

c. The predicted population for September is 7321; the predicted population for October is 8053.

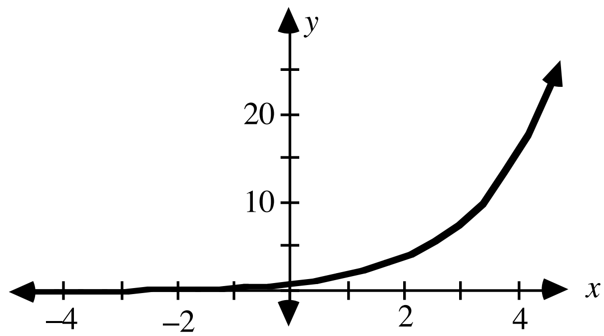
Activity 3

3.1 **a.** 8

b. 36

c. –36

3.2 **a.** Sample graph:



b. **1.** $y = 8$

2. $y = 256$

c. **1.** $x = 0$

2. $x = 4$

3.3 Students may use technology or guess-and-check to estimate solutions.

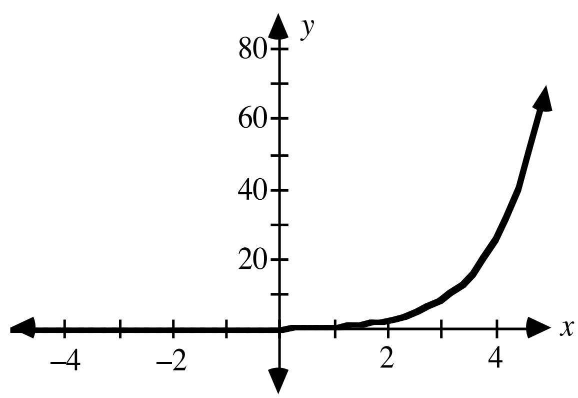
a. $x = 61$

b. $x = 2$

Activity 4

4.1 **a.** $y = 3 \bullet (1+2)^x = 3 \bullet 3^x$

b. Sample graph:



c. **1.** $y = 243$

2. $y = 2187$

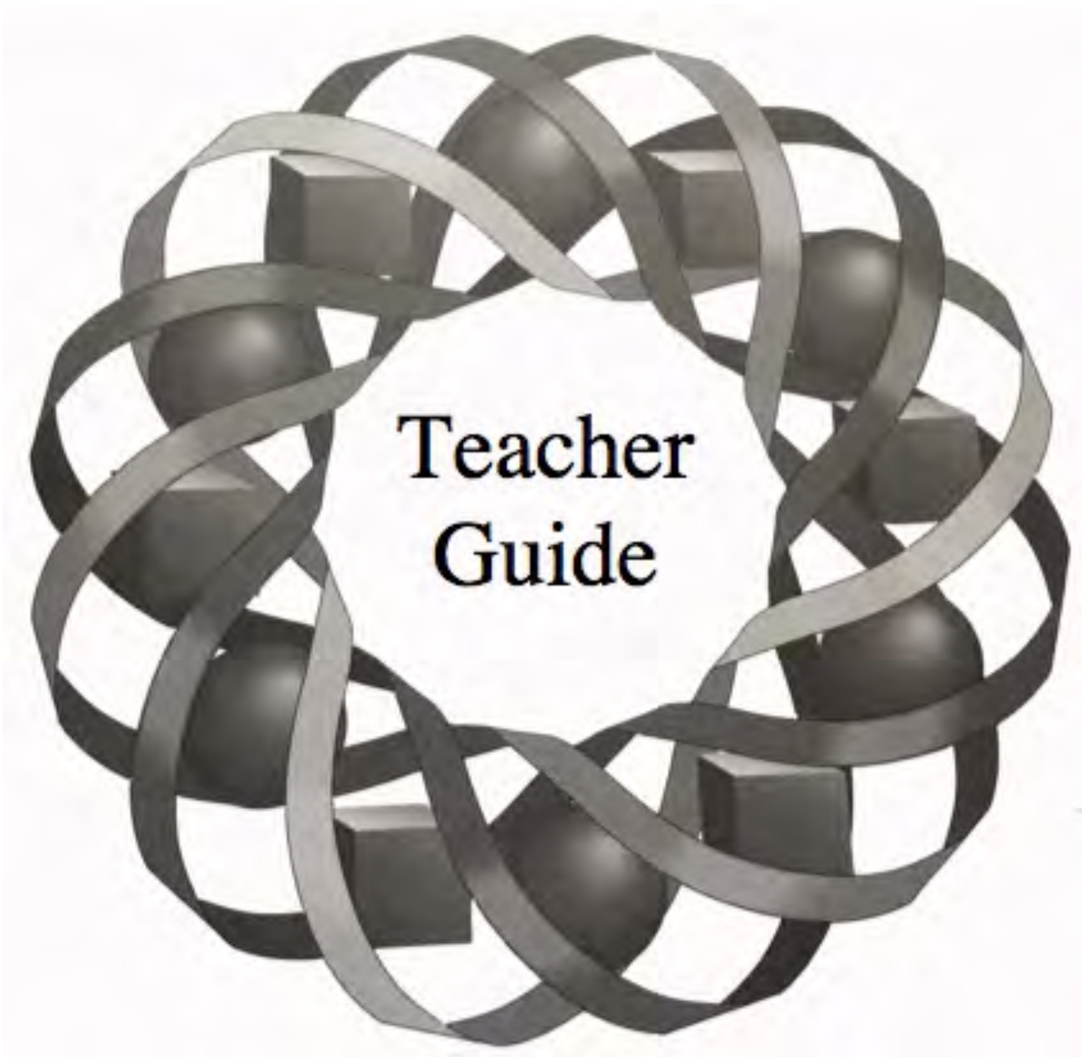
3. $y = 2,541,865,828,000$

4.2 **a.** 0.0005

b. 0.007

c. 0.013

Oil: Black Gold



Oil spills can have disastrous effects on the environment. When planning their response, clean-up crews must estimate both the volume of the spill and the area it covers. In this module, you investigate direct and inverse relationships in the context of oil spills.

Gary Bauer • Kyle Boyce • Mike Lundin • Karen Umbaugh

Teacher Edition

Oil: Black Gold

Overview

This module is designed to provide experience in studying natural events by using simulations. Students simulate oil spills on land and water and develop mathematical models of these phenomena. Through these models, they investigate relationships between volume and surface area.

Objectives

In this module students will:

- determine the area of irregularly-shaped figures
- determine the volume of cylinders and prisms
- develop and graph direct and inverse proportions
- develop mathematical models of real-world events
- use mathematical models to make prediction about data sets.

Prerequisites

For this module, students should know:

- how to calculate the area of circles, triangles, and rectangles
- how to determine the equation of a line
- how to report calculations using measured quantities in significant digits
- how to write numbers in scientific notation.

Time Line

Activity	Intro.	1	2	3	Summary Assessment	Total
Days	1	2	2	2	1	8

Materials Required

Materials	Activity				
	Intro.	1	2	3	Summary Assessment
water	X		X	X	
used motor oil	X		X		
medicine droppers	X		X		
paper towels	X		X		
shallow containers at least 15 cm in diameter	X				
liquid detergent	X				
centimeter grid paper		X			
metric rulers		X	X	X	
soft-drink cans		X			
wax paper			X		
toilet tissue			X		
tape			X		
cylindrical containers				X	
200-mL beaker				X	

Teacher Note

You may wish to discuss the proper disposal of oil-contaminated water and oil-soaked paper with colleagues in the science department at your school.

Technology

Software	Activity				
	Intro.	1	2	3	Summary Assessment
spreadsheet			X	X	
graphing utility			X		

Oil: Black Gold

Introduction (page 157)

Students investigate how water and oil interact. They experiment, make observations, test hypotheses, and communicate information to their classmates.

Materials List

- water
- used motor oil or clean oil with some red food coloring in it (several drops per group)
- shallow container at least 15 cm in diameter (one per group)
- medicine droppers (one per group)
- paper towels (for clean up)
- liquid detergent (optional; several drops per group).

Discussion 1 (page 157)

a. Answers will vary. One gallon contains about 3.8 L, and 1000 L is equivalent to 1 m^3. If the dimensions of the room were 6 m ∞ 9 m ∞ 2.5 m, for example, this amount of oil would fill the room 1100 times.

b. Sample response: Both plants and animals that live in the water may be killed or injured. Birds whose feathers become fouled with oil often die of hypothermia. When oil reaches the coastline, it can also harm plant and animal life on shore.

c. When oil spills occur on water, workers typically attempt to contain it with floating booms or rings. When practical, a skimmer is used to draw off a layer of oil and water. The oil is then separated from this mixture. In most cases, less than 20% of spilled oil is recovered, usually much less. A substantial fraction of the oil (30%) or more evaporates.

In some cases, the oil may be burned off. Dispersants can also be used to break the slick into smaller particles. These particles tend to sink and biodegrade. The more recently developed process of bioremediation involves the use of microorganisms to “eat” the oil.

On land, oil penetrates the soil and does not disperse as widely as it does on water. The cleanup generally consists of removing the contaminated earth, then using a variety of methods to separate the oil from the earth.

Teacher Note

As an extension to this exploration, you may wish to allow students to add a few drops of liquid detergent to the oil after completing Part **d**. The two liquids repel each other dramatically. This extension allows students to simulate one phase in the clean-up of an oil spill.

Exploration

(page 157)

a. To clean accidental spills, students should have paper towels nearby.

b. Sample response: After adding one drop of oil to the water, the water and oil do not mix. The oil disperses quickly on the water's surface into a somewhat circular shape.

c. Sample response: When a second drop of oil is placed directly on top of the first one, the oil doesn't disperse nearly as fast as the first drop did. The circular oil slick becomes larger.

d. Sample response: When several drops of oil are added at once to the existing oil slick, the area of the slick gets even larger and it takes longer for the oil to disperse.

Discussion 2

(page 157)

a. They are the same. The shape changes but the volume does not.

b. Students will probably note that the surface of the oil slick is roughly circular or elliptical. The actual shape of the slick is a lens, thicker near the center and thinner near the edges. **Note:** In the remainder of this module, students use cylinders to model the shapes of oil slicks.

c. Answers will vary. Milgram, et al., describe "thin" layers of oil as about 0.05 cm deep and "thick" layers as about 0.4 cm deep. Highly refined oil on a calm body of water can spread to a very thin film, about $2.5 \bullet 10^{-3}$ cm thick.

d. Sample response: Since the oil slick is somewhat circular, estimate its radius and use this value to calculate the area.

e. The physical properties of the oil itself can greatly influence the spread of a slick. For example, heavy crude spreads more slowly than gasoline. Some environmental factors that affect the spread of a slick are wind, salinity, temperature, wave action, and shorelines.

(page 158)

Activity 1

In this activity, students review the concepts of area and volume and estimate areas of irregular shapes.

Materials List

- metric rulers (one per student)
- centimeter grid paper (one sheet per student)
- soft-drink can (one per student)

Teacher Note

Since many of the calculations in this module use measured quantities, you may wish to review the appropriate use of significant digits. In Part **b** of the exploration students are instructed to find the mean radius and use it to approximate the area of an irregular shape. You may wish to have students explore squaring the radii and using the mean of the radii squared to approximate the area.

Exploration

(page 158)

a. **1.** In this case, each square on a centimeter grid represents 100 m^2. The area of slick A is approximately 1300 m^2. The area of slick B is approximately 1400 m^2.

2. The volume of oil in slick A is $1300 \text{ m}^2 \bullet 0.001 \text{ m} = 1.3 \text{ m}^3$. The volume of oil in slick B is $1400 \text{ m}^2 \bullet 0.001 \text{ m} = 1.4 \text{ m}^3$.

b. **1–3.** Sample response: Slick A has a mean radius of about 2.0 cm. Slick B has a mean radius of about 1.9 cm.

4. Using the sample responses given above, the area of slick A is about $\pi(20 \text{ m})^2 \approx 1300 \text{ m}^2$. The area of slick B is approximately $\pi(19 \text{ m})^2 \approx 1100 \text{ m}^2$.

5. Using the sample responses given above, the volume of oil in slick A is $1300 \text{ m}^2 \bullet 0.001 \text{ m} = 1.3 \text{ m}^3$. The volume of oil in slick B is $1100 \text{ m}^2 \bullet 0.001 \text{ m} = 1.1 \text{ m}^3$.

Discussion

(page 159)

a. Sample response: There are about as many partially covered squares that are more than half covered as there are squares that are less than half covered. Counting all partially covered squares then dividing by 2 gives a reasonable estimate for the area of the partially covered squares.

b. Sample response: To get a more accurate estimate of the area, you could use a grid with smaller squares.

c. Response teams can estimate the area of the surface covered by the slick using aerial mapping techniques or mathematical models involving volume, time, and surface tension.

d.
1. In the sample responses given in the exploration, the two estimates for slick A are the same, while the two estimates for slick B differ by 300 m^2.
2. Sample response: The larger estimate should be used to make sure that clean-up crews are prepared to encounter at least that much oil.

Teacher Note

In order to complete Problem **1.9**, each student or group will need an ordinary soft-drink can.

Assignment

(page 160)

1.1

a.
1. approximately $98,300 \text{ cm}^3$
2. approximately $19,000 \text{ cm}^3$

b.
1. 98.3 L
2. 19 L

1.2 The dimensions of sheet of letter-size paper are 21.5 cm by 27.9 cm. The volume of one sheet can be found as follows:

$$\frac{21.5 \text{ cm} \bullet 27.9 \text{ cm} \bullet 5.2 \text{ cm}}{500} \approx 6.2 \text{ cm}^3$$

1.3 $\pi \bullet (1.94/2)^2 \bullet 12.19 \approx 36.0 \text{ m}^3$

1.4

a. The oil slick could be described as a right circular cylinder.

b. The area covered by the slick is $\pi \bullet (405 \text{ m})^2 \approx 515,000 \text{ m}^2$. Since $V = B \bullet h$, the volume can be calculated as follows:

$$515,000 \bullet 2.5 \bullet 10^{-5} \text{ m} \approx 13 \text{ m}^3$$

Since $1 \text{ m}^3 = 1000 \text{ L}$, the volume of the spill is approximately 13,000 L.

1.5 **a.** Since there are 42 gal in 1bbl and 3.8 L in 1 gal:

$$1 \text{ bbl} \bullet \left(\frac{42 \text{ gal}}{1 \text{ bbl}}\right) \bullet \left(\frac{3.8 \text{ L}}{1 \text{ gal}}\right) \approx 160 \text{ L}$$

Students may then use their response to Problem **1.4** to write the following proportion:

$$\frac{160 \text{ L}}{B \text{ m}^2} = \frac{13,000 \text{ L}}{515,000 \text{ m}^2}$$

By solving for B, the area is approximately 6300 m^2.

b. Students should realize that the spill will probably not reach the estimated size because some of the oil will evaporate over time, and that some will disperse as wave action breaks the slick into smaller droplets.

***1.6** **a.** Sample response: There are 29 full squares and 25 partially covered squares. After dividing the number of partially covered squares by 2 and adding this to the number of full squares, the total is 41.5 squares. Each square on the grid represents an area of 1 km^2. This gives an approximate area of 41.5 km^2. **Note:** Students may also use the mean radius to estimate area.

b. Using a cylinder to model the shape of the slick:

$$V = \left(41.5 \text{ km}^2 \bullet \frac{1 \bullet 10^{10} \text{ cm}^2}{1 \text{ km}^2}\right)(0.05 \text{ cm}) = 2 \bullet 10^{11} \text{ cm}^3$$

This can be converted to barrels as follows:

$$\left(2 \bullet 10^{11} \text{ cm}^3\right)\left(\frac{1 \text{ mL}}{1 \text{ cm}^3}\right)\left(\frac{1 \text{ L}}{1000 \text{ mL}}\right)\left(\frac{1 \text{ gal}}{3.8 \text{ L}}\right)\left(\frac{1 \text{ bbl}}{42 \text{ gal}}\right) \approx 1 \bullet 10^6 \text{ bbl}$$

1.7 **a.** Students may convert gallons to cubic kilometers as follows:

$$\left(1.9 \bullet 10^7 \text{ gal}\right)\left(\frac{3.8 \text{ L}}{1 \text{ gal}}\right)\left(\frac{1000 \text{ mL}}{1 \text{ L}}\right)\left(\frac{1 \text{ cm}^3}{1 \text{ mL}}\right)\left(\frac{1 \text{ km}^3}{1 \bullet 10^{15} \text{ cm}^3}\right) \approx 7.2 \bullet 10^{-5} \text{ km}^3$$

Since $V = B \bullet h$ the thickness of the spill can be found as follows:

$$\frac{7.2 \bullet 10^{-5} \text{ km}^3}{260 \text{ km}^2} = 2.8 \bullet 10^{-7} \text{ km}$$

This is equivalent to 0.028 cm.

b. As mentioned in Problem **1.4**, highly refined oil can spread to a thickness of approximately $2.5 \bullet 10^{-3}$ cm under ideal conditions. This spill is about 10 times thicker.

* * * * *

1.8 Sample response: The dimensions of the field are 840 m by 1030 m. The area of the land, including the pond, is 865,200 m^2. The pond has a mean radius of approximately 135 m. The area of the pond is approximately 56,300 m^2. The area of the field without the pond is 808,900 m^2. This equals about 200 acres; therefore the farmer appears to be in compliance.

1.9
a. Sample response: The diameter is 6.6 cm and the height is 11.7 cm.

b. $V = \pi \bullet r^2 \bullet h = \pi \bullet (3.3 \text{ cm})^2 \bullet 11.7 \text{ cm} \approx 400 \text{ cm}^3$

c. Sample response: Since $1 \text{ cm}^3 = 1 \text{ mL}$, the volume of the can is about 400 mL. The volume printed on the can is 355 mL. The difference occurs because the printed volume measures the liquid in the can. The can is not filled exactly to the top and is not truly cylindrical (it has indentations in the top and bottom).

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(page 162)

Activity 2

In this activity, students simulate oil spills on land. The ratio of the number of drops of oil to the resulting surface area of the spill is used to introduce direct proportions.

Materials List

- used motor oil
- paper towels
- medicine dropper (one per group)
- toilet tissue (eight sheets per group)
- metric rulers (one per group)
- wax paper (eight sheets per group)
- tape

Technology

- graphing utility
- spreadsheet

Teachers Note

Plans for the disposal of oil-soaked paper should be made before beginning the exploration. Used motor oil is recommended because its spread is easy to observe. Cooking oil or mineral oil will also work but are more difficult to observe.

The experiment requires a smooth, level, nonabsorbent surface. Placing each sheet of toilet paper on a sheet of wax paper improves the experimental results and simplifies clean-up. Sheets of wax paper should be taped down to prevent movement.

Exploration

(page 163)

This exploration works best in small groups. Students should find a direct proportion between the number of drops of oil and the area of the spill. (Since each drop represents a fixed volume, the values found for the constant of proportionality depend on the thickness of the paper and the time allowed for the oil to spread. The following sample responses are based on an observation time of 20 min.)

a–c. To obtain accurate data, students must keep each sheet level and motionless until the oil is absorbed. To ensure that enough time is allowed for 8 drops of oil to spread (about 15 min), students should start with sheet 8.

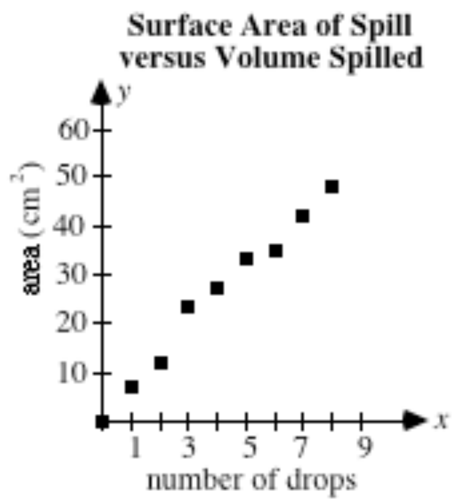
d. Sample response: More oil dropped on the paper produces larger shapes. Most of the time the oil spreads in a circular or elliptical fashion. **Note:** Uneven or tilted work surfaces, wrinkles in the paper, patterns in the paper, and the orientation of fibers within the paper may contribute to the formation of irregular shapes.

e. Students should use the method developed in Activity **1** to find the mean radius.

f. **1.** Sample data:

Drops	**Radius (cm)**	**Area (cm^2)**
0	0	0
1	1.5	7.0
2	2.0	13
3	2.7	23
4	3.2	32
5	3.5	38
6	3.5	38
7	3.7	43
8	4.0	50

2. Sample scatterplot:



3. Answers will vary. Students might use a string or piece of dry spaghetti attached to the graph at the origin to try and hit the most points. For the sample data, either $y = 6x$ or $y = 7x$ provide reasonable models.

g. The following sample predictions were made using the equation $y = 7.0x$, where y represents the area in square centimeters and x represents the number of drops.

1. approximately 0.3 cm^2
2. approximately 4 cm^2
3. approximately 18 cm^2
4. approximately 180 cm^2
5. approximately 180,000 cm^2

h. The following table shows the ratios, to the nearest whole number, for the sample data given in Part **f**.

Drops	Area (cm^2)	Ratio
0	0	
1	7	7
2	13	7
3	23	8
4	32	8
5	38	8
6	38	6
7	43	6
8	50	6

Discussion

(page 164)

a. Students report their observations. You may wish to remind students that, in this exploration, the volume of each spill after the oil hits the paper is modeled by a cylinder whose height is the thickness of the paper. The volume of this cylinder is a combination of oil and the volume of paper fibers.

b. Sample response: The shapes were not exactly circles and it was hard to determine the outer edges of the spill. Reading the ruler as it was suspended above the spill was also difficult.

c.
1. The accuracy of the area is restricted by the number of significant digits in the measurements.
2. Sample response: Since measuring even small spills is difficult, the reported areas should be considered as estimates only.

d.
1. Sample response: The predicted area was approximately $180{,}000\ \text{cm}^2$, or about $18\ \text{m}^2$.
2. Sample response: Probably not. By the time 1 L of oil could spread on such a large sheet of paper, some of it would have evaporated.

e. Sample response: Yes, it is reasonable because when no oil is dropped, no spill is created.

f.
1. In the sample data given in Part **f** of the exploration, all the ratios are either 6, 7, or 8.
2. The mean of the ratios provides a reasonable representative value.
3. The variable y represents the area covered by the spill in square centimeters; x represents the volume of oil in drops. The ratio of the area to the volume is m.
4. Students should observe that the values of m in both equations are close.
5. Sample response: No. In the experiment, a small amount of oil was spread very thin on a flat surface. Oil would soak deeper into soil and cover less surface area. You could adjust the model by collecting data for an experiment on soil with larger amounts of oil.

g.
1. As the values of x increase, the values of y increase.
2. As the values of x increase, the values of y decrease.

h. Sample response: Yes. A scatterplot of area versus volume appears to form a linear pattern with the origin included in the graph.

i. Because the line contains the origin, the slope of the line is:

$$m = \frac{q-0}{p-0} = \frac{q}{p}$$

Assignment

(page 166)

2.1 **a.** In this data set, x and y are directly proportional. The ratio of x to y is constant $(1/3)$.

b. In this data set, x and y are not directly proportional. The ratio of x to y is not constant.

c. In this data set, x and y are not directly proportional. Although the graph contains the origin, the ratio of x to y is not constant.

2.2 **a.** This graph does not represent a direct proportion. Although it does seem to show a set of data with a linear pattern, the graph does not pass through the origin.

b. Since this graph does not show data with a constant ratio between x and y, it cannot represent a direct proportion.

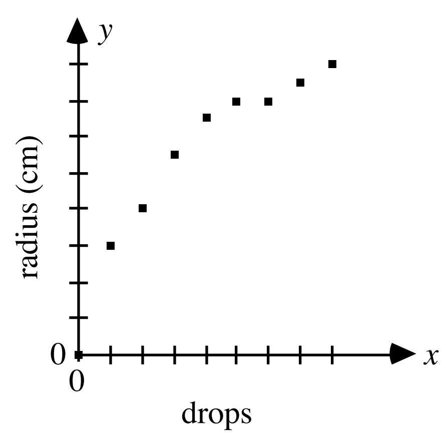
c. This graph represents a direct proportion. The data appears to show a constant ratio between x and y and includes the origin. It could be described by an equation of the form $y = mx$.

d. This graph does not represent a direct proportion. Although it does seem to show a set of data with a linear pattern, the graph does not show data with a constant ratio between x and y and does not pass through the origin.

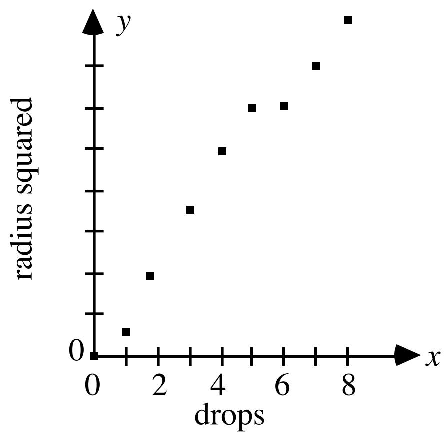
2.3 **a.** Students should use a graphing utility to complete this problem.

b. Sample response: The graphs of direct proportions are all straight lines with different slopes. They all pass through the origin and have slopes that are the constants of proportionality.

2.4 **a.** The following graph uses the sample data given in Part **f** of the exploration. It does not show a direct proportion. (The radius increases in proportion to the square root of the volume.)



b. The following graph uses the sample data given in Part **f** of the exploration. This appears to be a direct proportion.



***2.5** Sample response: Yes. The equation $V = Bh$ can be rewritten as $V = hB$. This equation is in the form of $y = mx$, a direct proportion in which the constant of proportionality is *h*, the thickness of the paper.

* * * * *

2.6 **a.** $C = \pi \bullet d$

b. **1.** This is a direct proportion. The relationship can be described by the equation $C = 2\pi \bullet r$, where the constant of proportionality is $2\pi \approx 6.28$.

2. This is not a direct proportion. The relationship can be described by the equation $A = \pi \bullet r^2$, which is not of the form $y = mx$. Its graph is not a line.

2.7 Sample response: This is not a direct proportion because the equation is not of the form $y = mx$ and the graph does not contain the origin.

2.8 **a.** 0.13 cm

b. One equation for this relationship is $r = 0.13t$, where *r* represents the radius in centimeters and *t* represents time in seconds.

c. The hailstone would have to remain in high clouds more than 19 sec.

$$r = 0.13t$$
$$2.5 = 0.13t$$
$$2.5/0.13 = t$$
$$19 \approx t$$

2.9 **a.** $\$18,699 - \$9948 = \$8751$

b. Assuming that the relationship is a direct proportion, the yearly increases are equal.

1. $875.10

2. One equation for this relationship is $I = 875.1t$, where I represents the increase and t represents the number of years after 1980.

c. 1. $875.10 • 14 + $9948 = $22,199.40

2. One equation for this relationship is $P = 875.1t + 9948$, where P represents the per capita income and t represents the number of years after 1980.

3. Sample response: This is not a direct proportion because the equation is not of the form $y = mx$ and the graph does not contain the origin.

d. Sample response: The model may not be accurate because there are many factors in the economy that cause increases in income to vary from year to year.

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(page 168)

Activity 3

In this activity, students investigate the relationship between the volume of cylinders and prisms and the area of the base. The concept of inverse proportion is also developed in this context.

Materials List

- containers in the shape of right cylinders (at least eight different sizes)
- metric rulers (one per group)
- 200-mL beaker (one per group)
- water (200 mL per group)

Technology

- graphing utility
- spreadsheet

Teacher Note

In order to generate a sufficient amount of data, students require at least eight containers with different base areas, including one with a relatively large diameter and one with a relatively small diameter. To show students that the relationship is the same regardless of the shape of the base, you may wish to include some prisms in your collection of containers.

Exploration

(page 169)

This exploration is designed for work in groups. To save time, each group is instructed to measure one container, then combine their data with the rest of the class. As an alternative to this procedure, you may wish to prepare eight stations and allow students to take turns measuring each container.

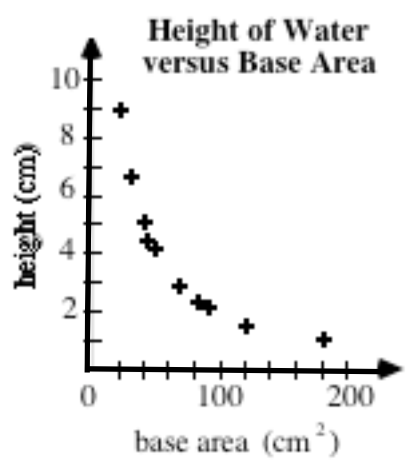
a. By measuring the diameter of each cylinder to the nearest 0.1 cm, students can include two significant digits in the area estimates.

b. Students should also measure the heights to two significant digits.

c–d. Sample data:

Base Area (cm^2)	**Height of Water (cm)**	**Volume** (cm^3)
21	9.0	190
30	6.7	200
39	5.1	200
42	4.5	190
49	4.2	210
67	2.9	190
83	2.4	200
90	2.2	200
120	1.6	190
180	1.1	200

e. The following graph was created using the sample data from in Part **d**.



Discussion

(page 169)

a. **1.** Sample response: Each entry in the column should be close to 200 cm^3. **Note:** Students should recall that $1 \text{ cm}^3 = 1 \text{ mL}$.

2. Experimental values will be influenced by rounding errors and the accuracy of measuring tools. In most cases, the volume should be reasonably close to 200 cm^3.

b. Sample response: The graph shows a curved pattern. As the area of the base increases, the height approaches 0; as the area of the base approaches 0, the height becomes very large.

c. Given equal volumes of water, the height of the water depends only on the area of the base.

d. **1.** The height decreases as the area of the base increases. The height increases as the area of the base decreases.

2. Since the ratio of the two quantities is not constant, they are not directly proportional.

e. Models can be used to describe and make predictions. The sequence of containers models an oil spill in the sense that as oil spreads, the height of the spill decreases and the surface area of the spill increases. As with most models, however, there are limitations to its precision.

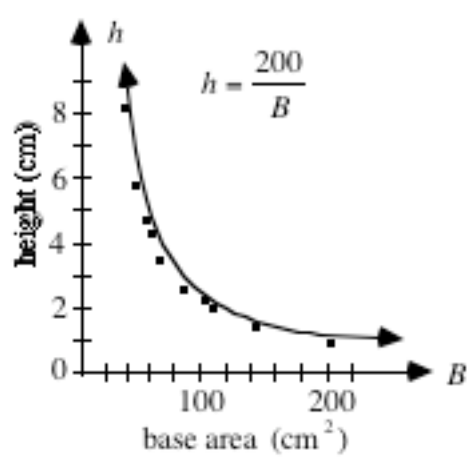
Assignment

(page 170)

3.1 **a.** $200 = Bh$

b. $h = 200/B$

c. Sample graph:



d. Students should realize that the equation describes the exact relationship between base area and the height of the water. The scatterplot gives only an approximate relationship, since it is based on measurements. These measurements introduce error in the model.

e. Since h represents height, its value must be a positive number. **Note:** In real-world interpretations of graphs, negative domain values are frequently excluded because they have no meaning.

3.2 **a.** $B = 200/h$

b. **1.** $B = 200/(2.5 \bullet 10^{-3}) = 80,000 \text{ cm}^2 = 8.0 \text{ m}^2$

2. $d = 2\sqrt{8.0/\pi} \approx 3.2 \text{ m}$

***3.3** **a.** $100 = Bh$ or $B = 100/h$

b. Students may make the conversion as follows:

$$100 \text{ bbl} \bullet \frac{160 \text{ L}}{1 \text{ bbl}} \bullet \frac{1000 \text{ cm}^3}{1 \text{ L}} = 1.6 \bullet 10^7 \text{ cm}^3$$

c. $B = (1.6 \bullet 10^7)/h$

d. Students should substitute as follows:

$$B = \frac{1.6 \bullet 10^7 \text{cm}^3}{2.5 \bullet 10^{-3} \text{cm}} = 6.4 \bullet 10^9 \text{ cm}^2 = 0.64 \text{ km}^2$$

e. $d = 2\sqrt{0.64/\pi} \approx 0.90 \text{ km}$

3.4 **a.** $y = 20/x$

b. Sample response:

x	y
2	10
4	5
5	4
20	1
40	0.5

c. As x increases, y decreases. As x decreases, y increases. This is consistent with the results from the exploration.

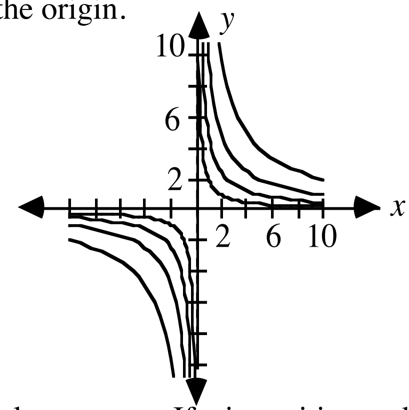
3.5 **a.** **1.** $y = 20/x$, $k = 20$

2. $y = 10/x$, $k = 10$

3. $y = 5/x$, $k = 5$

4. $y = 0.5/x$, $k = 0.5$

b. Sample response: Each graph is similar in shape and none pass through the origin.



c. **1.** Sample response: If x is positive and close to 0, the y-values assume very large positive values. If x is negative and close to 0, the y-values assume very small negative values.

2. The y-values are positive and approach 0.

3. The y-values are negative and approach 0

d. Sample response: As the constant of proportionality decreases, the graph shifts toward the origin; as the constant increases the graph shifts away from the origin.

e. The constant of proportionality represents the volume of water.

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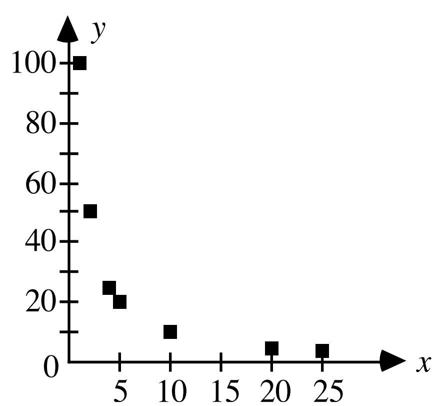
3.6 **a.** Sample response: The graph of an inverse proportion is curved and does not contain the origin. The graph of a direct proportion is a line that passes through the origin.

b. Sample response: In an inverse proportion, the product of the two variables is constant. In a direct proportion, the quotient is constant.

3.7 **a.** Sample graph:



b. The y-values become small.

c. The y-values become large.

d. Sample response: This data represents an inverse proportion. The graph fits the pattern and the two variables multiplied together produce a constant.

e. $xy = 100$, $k = 100$

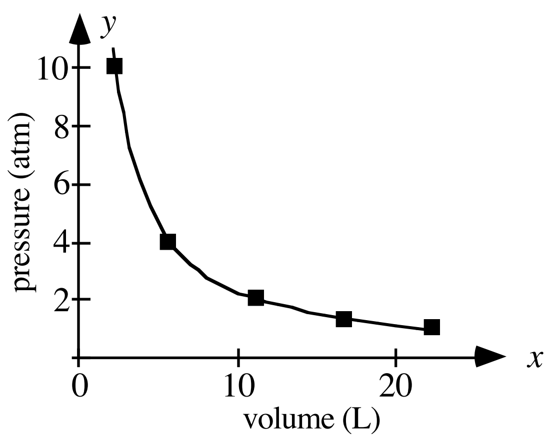
f. Sample response: I must travel 100 km. The chart of values represents my speed in kilometers per hour and the time in hours it takes to travel that distance.

3.8 **a.** See sample graph in Part **c** below.

b. Sample response: Pressure and volume are inversely proportional. When multiplied together, they produce a constant.

c. **1.** $y = 22.4/x$, $k = 22.4$

2. Sample graph:



d. Students should substitute as follows:

$$\begin{aligned} y &= 22.4/x \\ &= 22.4/20.5 \\ &\approx 1.09 \text{ atm} \end{aligned}$$

3.9 **a.** Sample response: The data is inversely proportional because as the wavelength increases, the frequency decreases.

b. $y = (3.0 \bullet 10^8)/x$

c. The constant of proportionality is $3.0 \bullet 10^8$. **Note:** This is the speed of light in meters per second. The equation given in Part **b** is often written as $c = \lambda \bullet \nu$, where c represents the speed of light.

d. Students should substitute as follows:

$$\begin{aligned} y &= \frac{3.0 \bullet 10^8}{4.0 \bullet 10^{-7}} \\ &= 7.5 \bullet 10^{14} \text{ Hz} \end{aligned}$$

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Answers to Summary Assessment (page 175)

1. In order to estimate the volume of oil spilled, students must first estimate the area covered by the slick. The mean radius of the slick in the diagram is about 2.8 cm. Using the given scale, this corresponds to an actual mean radius of 560 km. The area of the oil slick is approximately 9900 km^2.

Using a cylinder to model the shape of the spill,

$$\begin{aligned} V &= Bh \\ &= 9900 \text{ km}^2 (2.5 \bullet 10^{-8} \text{ km}) \\ &\approx 2.5 \bullet 10^{-4} \text{ km}^3 \end{aligned}$$

Students may convert this value to gallons as follows:

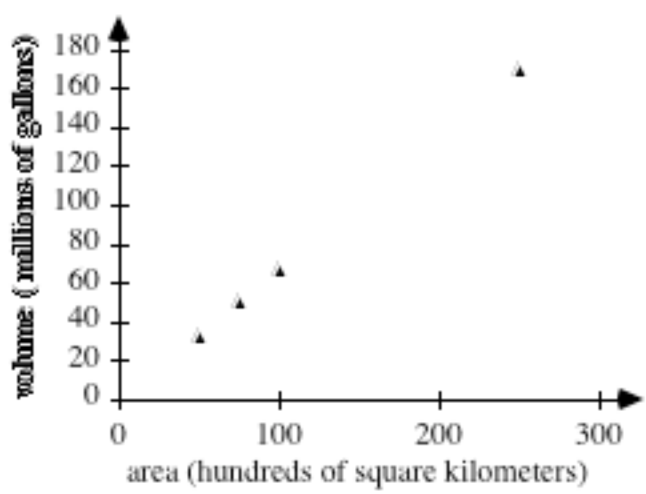
$$\left(2.5 \bullet 10^{-4} \text{ km}^3\right)\left(\frac{1 \bullet 10^{15} \text{ cm}^3}{1 \text{ km}^3}\right)\left(\frac{1 \text{ mL}}{1 \text{ cm}^3}\right)\left(\frac{1 \text{ L}}{1000 \text{ mL}}\right)\left(\frac{1 \text{ gal}}{3.8 \text{ L}}\right) = 6.6 \bullet 10^7 \text{ gal}$$

Note: This is close to the volume of oil spilled in the 1978 grounding of the *Amoco Cadiz* off the coast of France.

2. **a.** Sample table:

Area of Slick (km^2)	Volume of Spill (gal)
5000	33,000,000
7500	50,000,000
10,000	67,000,000
25,000	170,000,000

b. The volume of the slick is directly proportional to the area. Sample graph:



c. One equation that fits the data is $y = 6700x$, where y represents volume in gallons and x represents area in square kilometers.

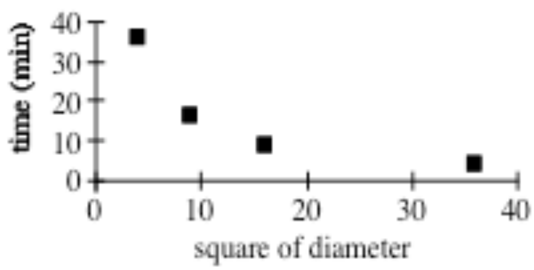
d. Using the equation from Part **c**:

$$\begin{aligned} y &= 6700x \\ &= 6700(71{,}000\ \text{km}^2) \\ &\approx 4.8 \bullet 10^8\ \text{gal} \end{aligned}$$

3. **a.** A completed table is shown below.

Diameter (cm)	Square of Diameter	Time (min)
2	4	36
3	9	16
4	16	9
6	36	4

b. Sample graph:



c. One possible equation is $x \bullet y = 144$, where y represents the time in minutes and x represents the square of the diameter of the pipe.

d. Students should substitute as follows:

$$\begin{aligned} 10^2 \bullet y &= 144 \\ 100 \bullet y &= 144 \\ y &= 1.44\ \text{min} \end{aligned}$$

e. Students should substitute as follows:

$$\begin{aligned} x \bullet 30 &= 144 \\ x &= 144/30 \\ x &= 4.8 \end{aligned}$$

Since x represents the square of the diameter, $d = \sqrt{4.8} \approx 2.2\ \text{cm}$.
Note: Time is inversely proportional to the area of the base, which is itself directly proportional to the square of the diameter.

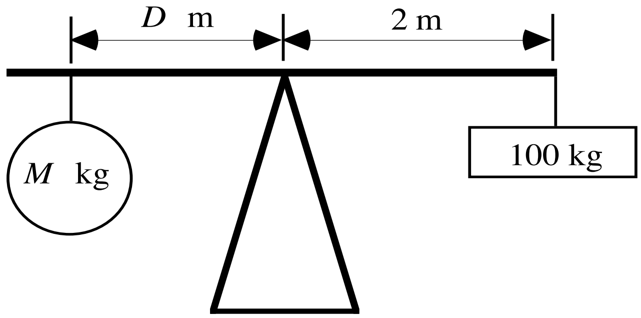
Module Assessment

1. When you see lightning strike far off, you do not hear the thunder immediately. The sound created by the lightning takes time to travel to your position.

a. Given that sound travels at a constant speed, describe the relationship between the distance from a lightning strike and the time required to hear the thunder.

b. When lightning strikes at a distance of 6.4 km, the thunder can be heard 20 sec later. Write an equation that describes this situation and identify the constant of proportionality.

2. The following diagram shows two objects hung from a beam supported in the center by a fulcrum. The product of the mass and its distance from the fulcrum on one side equals the product of the mass and its distance from the fulcrum on the opposite side.



a. If D is less than 2, what can you conclude about M?

b. To keep the beam balanced, describe how M would have to change in each of the following situations:

1. as D decreases

2. as D increases

c. Write an equation that models the relationship between M and D.

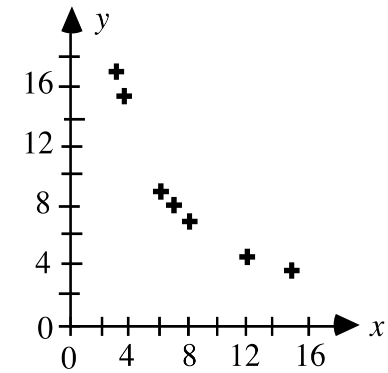
3. Describe each of the following situations as involving an inverse proportion or a direct proportion.

a. When paid on an hourly basis, your gross wages vary according to the number of hours worked.

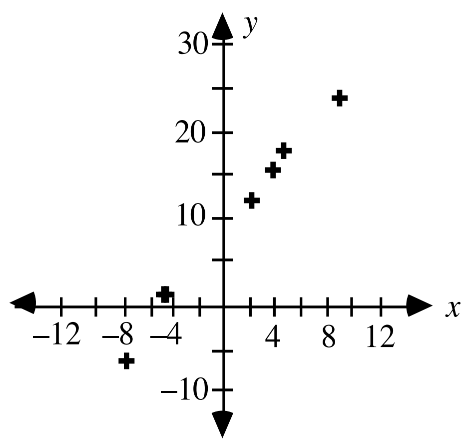
b. When pushing a piano down the hall, the force required to move the piano at a constant speed varies with its mass.

c. When mowing lawns, the time required to finish one lawn varies with the number of people working.

4. Does the scatterplot below appear to represent a direct proportion, an inverse proportion, or neither? Explain your response. If the scatterplot appears to represent a direct or inverse proportion, suggest a possible equation for the relationship.



5. Does the scatterplot below appear to represent a direct proportion, an inverse proportion, or neither? Explain your response. If the scatterplot appears to represent a direct or inverse proportion, suggest a possible equation for the relationship.



Answers to Module Assessment

1. **a.** Sample response: The relationship between the distance from the lightning and the time required to hear it is a direct proportion.

b. This situation can be described by an equation of the form $y = mx$, where y represents the distance in kilometers and x represents time in sec. By substituting as follows:

$$6.4 \text{ km} = m(20 \text{ sec})$$
$$m = 0.32 \text{ km/sec} = 320 \text{ m/sec}$$

The constant of proportionality is the approximate speed of sound in air.

2. **a.** Sample response: The value of M must be more than 100 kg.

b. Sample response: As D decreases, M would have to increase in order to keep the beam balanced. As D increases, M will have to decrease in order to keep the beam balanced.

c. $MD = k$ or $MD = 200$

3. **a.** In this situation, the gross wages are directly proportional to the hours worked.

b. In this situation, the force required is directly proportional to the mass of the piano.

c. In this situation, the time required is inversely proportional to the number of workers.

4. The shape of the graph suggests an inverse proportion. From the graph, three of the points appear to be close to (3,17), (6,9), and (8,7). The three corresponding products are 51, 54, and 56. Since the average of these products is approximately 54, one possible equation would be $xy = 54$.

5. The shape of the graph appears to be linear, ruling out an inverse proportion. However, the graph does not pass through the origin. It is neither a direct proportion nor an inverse proportion.

Selected References

Milgram, J. H., and R. G. Donnelly, R. J. Van Houten, and J. M. Camperman. *Effects of Oil Slick Properties on the Dispersion of Floating Oil into the Sea*. U. S. Department of Transportation Report No. CG-D-64-78. Springfield, VA: National Technical Information Service, 1978.

Stanglin, D. "Toxic Wasteland." *U.S. News and World Report* 112 (13 April 1992): 40–46.

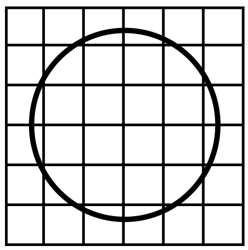
U.S. Congress, Office of Technology Assessment. *Coping with an Oiled Sea: An Analysis of Oil Spill Response Technologies*. OTA-BP-O-63. Washington, DC: U.S. Government Printing Office, March 1990.

Flashbacks

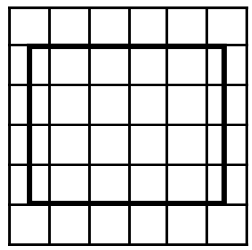
Activity 1

1.1 Estimate the area of each figure below to the nearest square centimeter. **Note:** Each square on a grid represents 1 cm^2.

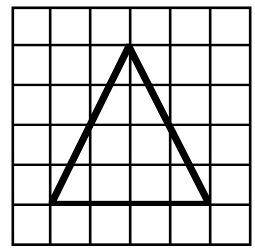
a.



b.



c.



1.2 Round 236.321 to the nearest:

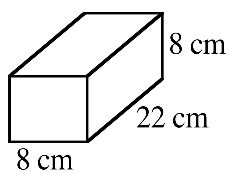
a. hundred

b. hundredth

c. ten

d. tenth.

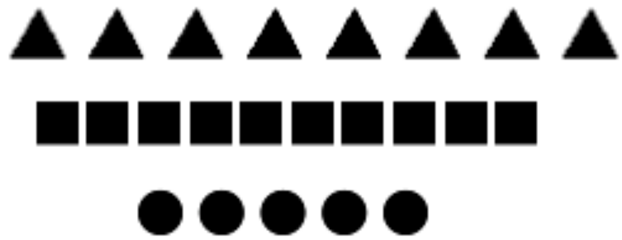
1.3 **a.** Find the volume of the rectangular prism shown below.



b. Given that 1 dm^3 = 1 L , determine the volume of the prism in liters.

Activity 2

2.1 Use the figure below to complete Parts **a–c**.



a. What is the ratio of the number of triangles to the number of circles?

b. What is the ratio of the number of circles to the number of squares?

c. What is the ratio of the number of squares to the number of triangles?

2.2 **a.** Graph the equation $y = 3x - 6$.

b. What does the number 3 represent in the equation?

c. What is the y-intercept of the graph?

2.3 Write an equation that could be used to model each of the following:

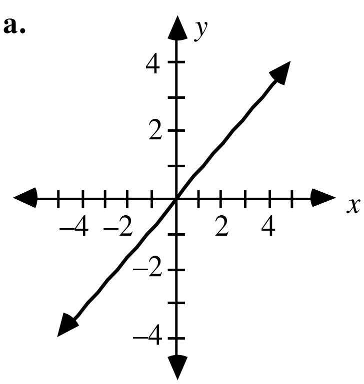
a. a positive association

b. a negative association

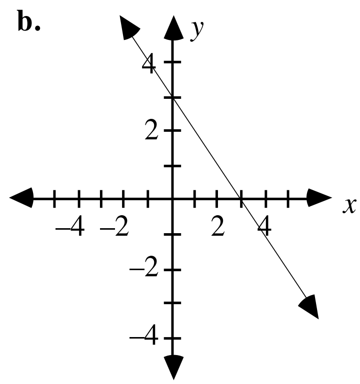
Activity 3

3.1 For each of the following graphs, describe the change in the y-values as the x-values increase and as the x-values decrease.

a.



b.



3.2 **a.** Write an equation of the form $y = mx + b$ that describes a relationship in which the y-values increase as the x-values decrease.

b. Write an equation of the form $y = mx + b$ that describes a relationship in which the y-values increase as the x-values increase.

Answers to Flashbacks

Activity 1

1.1 **a.** approximately 18 cm^2

b. approximately 20 cm^2

c. approximately 8 cm^2

1.2 **a.** 200

b. 236.32

c. 240

d. 236.3

1.3 **a.** 1408 cm^3

b. 1.408 L

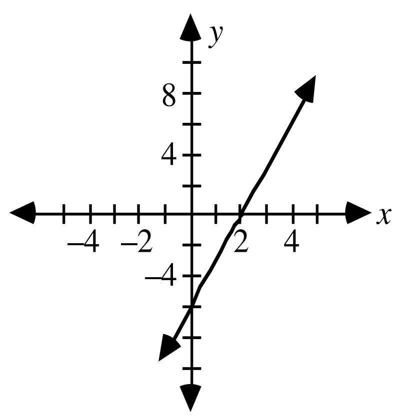
Activity 2

2.1 **a.** 8/5

b. 1/2

c. 5/4

2.2 **a.** Sample graph:



b. The number 3 is the slope of the line.

c. The y-intercept is –6.

2.3 **a.** Sample response: $y = 3x$.

b. Sample response: $y = -3x$.

Activity 3

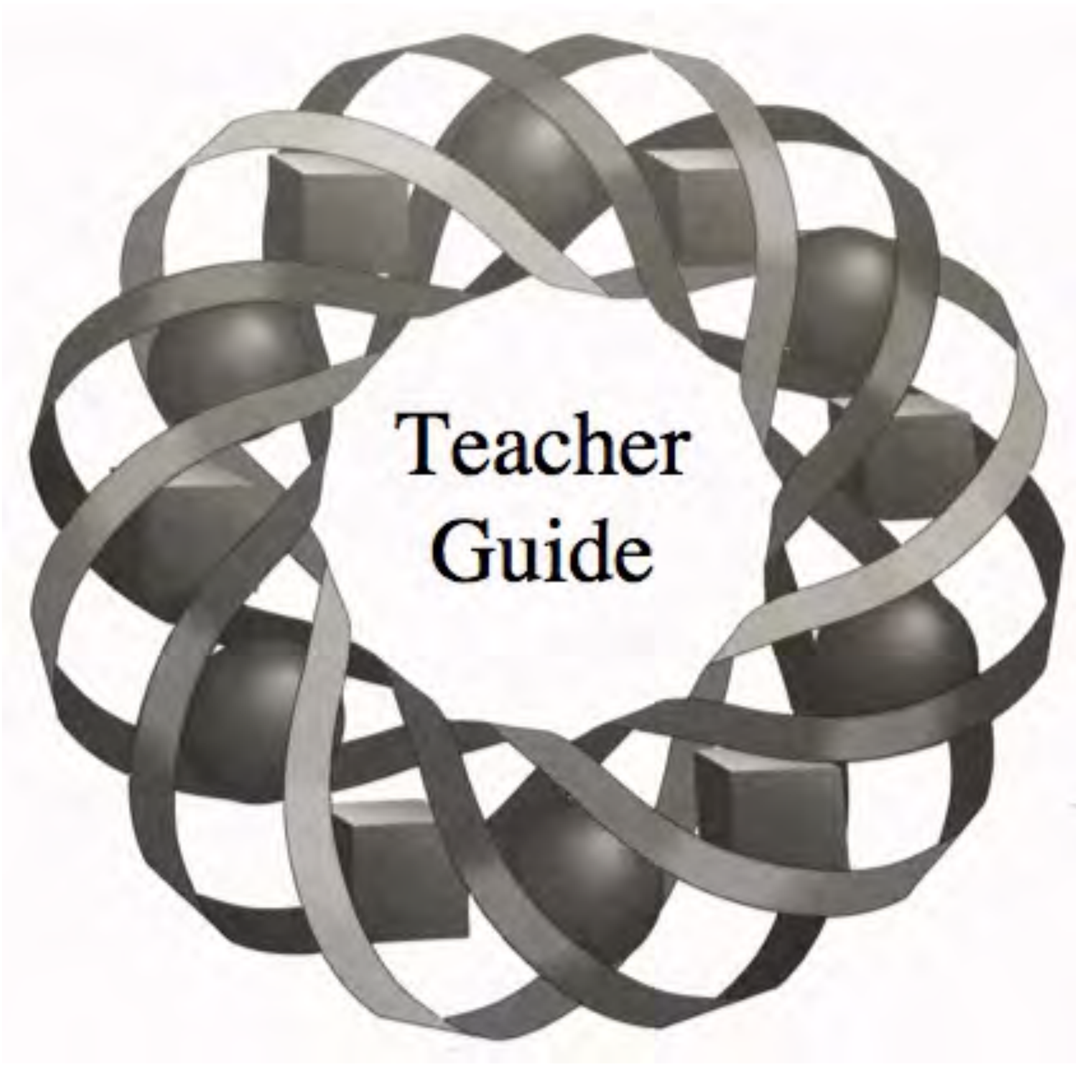
3.1 **a.** The y-values increase as the x-values increase and decrease as the x-values decrease.

b. The y-values decrease as the x-values increase and increase as the x-values decrease.

3.2 **a.** Sample response: $y = -3x + 5$.

b. Sample response: $y = 3x - 2$.

I’m Not So Sure Anymore



Are you betting on the lottery to change your life? The odds of hitting that big jackpot are not as good as you might think. In this module, you’ll explore the mathematical probability of waking up a winner.

Ted Dreith • Anne Merrifield • Dean Preble • Deanna Turley

Teacher Edition

I'm Not So Sure Anymore

Overview

In this module, students simulate various simple lotteries and determine the probabilities of winning. These probabilities are then used to find expected values.

Objectives

In this module, students will:

- use a variety of methods for simulation
- determine experimental probability
- calculate theoretical probability
- find that the sum of the probabilities for all the outcomes of an experiment is 1
- identify and extend data patterns
- calculate expected value.

Prerequisites

For this module, students should know:

- how to use a spreadsheet
- the definition of a simulation.

Time Line

Activity	Intro.	1	2	3	Summary Assessment	Total
Days	1	2	2	2	2	9

Materials Required

Materials	Activity				
	Intro.	1	2	3	Summary Assessment
numbered objects	X				
container	X				
paper clips		X			
spinner template		X			
playing cards		X	X		
sample space template					X

Technology

Software	Activity				
	Intro.	1	2	3	Summary Assessment
random number generator	X	X		X	
spreadsheet			X	X	

I'm Not So Sure Anymore

Introduction

(page 181)

Students use experimental probability to analyze a lottery game.

Teacher Note

To introduce the module, you may wish to show students some actual lottery tickets.

Materials List

- six objects, each labeled with one of the digits from 1 to 6 (per group)
- container (one per group)

Exploration

(page 181)

a. Sample response: Officials could mark 6 balls with a different number from 1 to 6, mix them in a container, then pick 2 balls out of the 6.

b. Predictions will vary. After calculating the experimental probabilities for each event in Part **h**, students make another set of predictions.

c. Apple Lottery rules state that players must pick two different numbers. Students may find the exploration less confusing if they keep the same lottery ticket throughout the exploration; however, changing the choice of numbers will not affect their results.

d. Each group requires six objects, each labeled with one of the digits from 1 to 6, and a container.

e–f. Students may want to organize their data in a table. Sample data:

Color of Apple	Number of Wins
red	11
yellow	2
green	12

g. The following experimental probabilities were calculated using the sample data given in Part **f**:

1. $11/25 = 44\%$

2. $2/25 = 8\%$

3. $12/25 = 48\%$

4. $25/25 = 100\% = 1$

h. You may wish to organize a table for the entire class. The following experimental probabilities were calculated using the sample data in the table below.

Color of Apple	Number of Wins
red	43
yellow	8
green	49

1. 43/100 = 43%
2. 8/100 = 8%
3. 49/100 = 49%
4. 100/100 = 1

i.
1. Using the sample data given in Part **h**, some appropriate predictions are 430 red apples, 80 yellow apples, and 490 green apples.
2. The predictions based on experimental results may contradict the predictions made in Part **b**.

Discussion

(page 184)

a. Students should recognize that the combined data is likely to give more reliable experimental results. Sample response: The probabilities found in Part **h** should give a better estimate. Flipping a coin 4 times may result in 4 heads, but it seems very unlikely to flip a coin 400 times and see only heads.

b. Sample response: The probability is 100% or 1 because you win an apple every time you play the game.

c.
1. Students may suggest rolling dice, drawing cards, using a random number table, or using a calculator or computer.
2. Answers will vary, depending on the responses given above. For example, rolling a pair of dice may result in 2 sixes.
3. The rules of the Apple Lottery require two different numbers.
4. Sample response: When using two dice, disregard a roll that results in a pair. Some may argue that this is not a good simulation. The only way to find out is to try it and see if you seem to get consistent empirical results.

d. Sample response: Technology can allow you to generate many trials quickly. Having the results of more trials can produce a better estimate of the true chances of winning.

(page 184)

Activity 1

Students use technology to simulate a large number of trials, then calculate experimental probabilities from their data.

Material List

- paper clips (one per student)
- spinner template (one copy per student; a blackline master appears at the end of the teacher edition FOR THIS MODULE)
- playing cards (optional)

Technology

- random number generator

Teacher Note

Many forms of technology have a built-in random number generator. Students should refer to the manual for specific instructions. Typically, this feature generates a number in the range (0,1).

Students may suggest using the first digit after the decimal point (ignoring 0, 7, 8, and 9) to obtain a random selection from 1 through 6.

As an alternative, students may multiply the random number by 6, resulting in a number in the range (0,6). It then may be possible to identify only the integer portion of the random number, resulting in a random integer in the range [0,5]. Adding 1 results in a random integer in the range [1,6].

Exploration

(page 184)

a. Students select two numbers according to Apple Lottery rules. They may find data collection easier if they keep the same numbers throughout the exploration.

b. The following sample table corresponds to the selection 3, 4:

Pair of Numbers	Number of Matching Digits	Color of Apple Won
1, 2	0	R
1, 3	1	G
1, 4	1	G
1, 5	0	R
1, 6	0	R
2, 3	1	G
2, 4	1	G
2, 5	0	R
2, 6	0	R
3, 4	2	Y
3, 5	1	G
3, 6	1	G
4, 5	1	G
4, 6	1	G
5, 6	0	R

c–e. Students may want to organize their data in a table. Sample data:

Color of Apple	Number of Wins
red	45
yellow	6
green	49

f. The following experimental probabilities were calculated using the sample data given in Part **e**:

1. $45/100 = 45\%$
2. $6/100 = 6\%$
3. $49/100 = 49\%$

g. The sample space for the Apple Lottery contains 15 possible outcomes.

1. $6/15 = 40\%$
2. $1/15 \approx 6.7\%$
3. $8/15 \approx 53.3\%$
4. $15/15 = 100\% = 1$

Discussion

(page 186)

a. Answers may vary. The sample data given in Part **e** results in experimental probabilities that are reasonably close to the theoretical probabilities.

b–c. The combined data should give experimental probabilities that support the theoretical probabilities.

d. The theoretical probabilities of winning each type of apple are not affected by the pair of numbers selected.

Teacher Note

Each student will require a paper clip and a copy of the spinner template to complete Problem **1.3**. A blackline master appears at the end of the teacher edition for this module.

Students may wish to use playing cards to help determine the sample space in Problem **1.5**.

Assignment

(page 187)

1.1 Changing tickets does not change the theoretical probabilities because the number of outcomes in the sample space and the number of times each event occurs remain the same. No matter what pair of numbers is chosen, 6 cells in Table **1** correspond to a red apple, 1 cell to a yellow apple, and 8 cells to a green apple.

1.2 Students should reason that 40% of the apples will be red, about 53% green, and about 7% yellow. By multiplying the theoretical probability for each apple by 1000, the predicted results are 400 red, 533 green, and 67 yellow.

1.3 **a.** A spin represents one free throw attempt.

b–d. The experimental results should be close to the theoretical probability of 80%.

***1.4** **a.** The sample space is the five letter grades: {A, B, C, D, F}.

b. The sample space is {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}.

1.5 **a.** There are 16 outcomes in the sample space. In the following table, A represents an ace, K a king, Q a queen, J a jack, S a spade, and D a diamond.

AD, AS	AD, KS	AD, QS	AD, JS
KD, AS	KD, KS	KD, QS	KD, JS
QD, AS	QD, KS	QD, QS	QD, JS
JD, AS	JD, KS	JD, QS	JD, JS

b. Since there are 4 winning combinations, the theoretical probability is $4/16 = 25\%$.

c. Answers will vary. Sample response: This game does not seem fair because Alicia has less than a 50% chance of winning the game.

d. Sample response: No. If all eight cards were shuffled together, two diamonds or two spades could be drawn, which means Camie has a lower chance of winning.

* * * * *

1.6 **a.** The experimental probability of getting three heads is $29/250 = 11.6\%$.

b. The theoretical probability of getting three heads is $1/8 = 12.5\%$.

c. Students should observe that the experimental probability supports the theoretical probability.

***1.7** **a.** The sample space is listed in the table below. The types of apples won correspond with the numbers 3, 4.

Pair of Numbers	Number of Matching Digits	Color of Apple Won
1, 2	0	R
1, 3	1	G
1, 4	1	G
1, 5	0	R
2, 3	1	G
2, 4	1	G
2, 5	0	R
3, 4	2	Y
3, 5	1	G
4, 5	1	G

b. The theoretical probability of winning a red apple is $3/10 = 30\%$, a yellow apple $1/10 = 10\%$, and a green apple $6/10 = 60\%$.

c. **1.** Use technology to randomly generate two numbers from 1 to 5 and repeat this 100 times.

2. Sample data:

Color of Apple	Number of Wins
red	32
yellow	14
green	54

3. Using the sample data, the experimental probability of winning a red apple is $32/100 = 32\%$, a yellow apple $14/100 = 14\%$, and a green apple $54/100 = 54\%$.

d. Sample response: Yes, the experimental probability supports the theoretical probability. The values are very close.

e. Sample response: Theoretically, you would have a better chance of winning a yellow apple in the Apple Lottery in which players choose numbers from 1 to 5. In this version, the probability is 10%, while it was less than 7% in the original version.

* * * * * * * * * *

Research Project

(page 189)

a. The sample space for the Match Lottery contains 36 outcomes, as shown in the following 6×6 matrix:

		Second Pick					
		1	**2**	**3**	**4**	**5**	**6**
	1						
	2						
First	**3**						
Pick	**4**						
	5						
	6						

In Activity **1**, the theoretical probabilities for the Apple Lottery are calculated using a sample space of 15 outcomes. In that sample space, each possible outcome represents two ordered pairs and each has an equally likely chance of occurring. For example, the ordered pairs (1,2) and (2,1) are considered as the same outcome. However, it is also possible to consider the ordered pairs (1,2) and (2,1) as different outcomes, resulting in a sample space of 30 equally likely outcomes. This does not affect the theoretical probabilities for each event.

Since the Match Lottery allows pairs, this adds six more outcomes to the sample space: (1,1), (2,2), (3,3), (4,4), (5,5), and (6,6). If a sample space of 36 outcomes is not used, an outcome with two different numbers is more likely to occur than an outcome with two identical numbers. Therefore, ordered pairs such as (1,2) and (2,1) must be considered as different outcomes.

b. Since the Match Lottery does not require the two numbers chosen to be different, duplicate numbers are no longer a concern in simulating the game. If students use six numbered objects to simulate the lottery (as in the introduction), the first object drawn must be replaced and mixed with the others before the second object is drawn.

c. The table below shows the results of 100 trials using a ticket in which the numbers 1 and 2 were selected.

Color of Apple	Number of Wins
red	47
yellow	5
green	48

d. The following table shows the results of 100 trials using a ticket in which the numbers 1 and 1 were selected.

Color of Apple	Number of Wins
red	69
yellow	5
green	26

e. Using the sample data for a ticket in which two different numbers were selected, the experimental probability of winning a red apple is 47%, a yellow apple 5%, and a green apple 48%.

Using the sample data for a ticket in which two identical numbers were selected, the experimental probability of winning a red apple is 69%, a yellow apple 5%, and a green apple 26%.

f. Using a Match Lottery ticket in which the numbers 1 and 2 were selected, the apples won in any particular drawing can be indicated as follows (where Y, G, and R represent yellow, green, and red, respectively):

		Second Pick					
		1	**2**	**3**	**4**	**5**	**6**
	1	G	Y	G	G	G	G
	2	Y	G	G	G	G	G
First	**3**	G	G	R	R	R	R
Pick	**4**	G	G	R	R	R	R
	5	G	G	R	R	R	R
	6	G	G	R	R	R	R

When two different numbers are chosen, the theoretical probability of winning a red apple is $16/36 \approx 44\%$, a yellow apple $2/36 \approx 6\%$, and a green apple $18/36 = 50\%$.

Using a Match Lottery ticket in which the numbers 1 and 1 were selected, the apples won in any particular drawing can be indicated as follows:

		Second Pick					
		1	**2**	**3**	**4**	**5**	**6**
	1	Y	G	G	G	G	G
	2	G	R	R	R	R	R
First	**3**	G	R	R	R	R	R
Pick	**4**	G	R	R	R	R	R
	5	G	R	R	R	R	R
	6	G	R	R	R	R	R

When two identical numbers are chosen, the theoretical probability of winning a red apple is $25/36 \approx 69\%$, a yellow apple $1/36 \approx 3\%$, and a green apple $10/36 \approx 28\%$.

g. The sample results given in Parts **c** and **d** are reasonably close to the theoretical probabilities.

h. Based on the theoretical probabilities, a ticket with two different numbers is more likely to win a yellow apple than a ticket with two identical numbers.

(page 190)

Activity 2

In this activity, students use a spreadsheet to investigate the number of possible outcomes in large sample spaces.

Materials List

- playing cards (optional)

Technology

- spreadsheet

Teacher Note

Playing cards may serve as useful manipulatives in the following exploration.

Exploration

(page 190)

a. In Parts **b** and **c**, students should discover a simple pattern that will help them fill in the spreadsheet.

b. **1.** There are 4 singles in the sample space:

1	2	3	4

2. There are 6 doubles in the sample space:

1, 2	1, 3	1, 4	2, 3	2, 4	3, 4

3. There are 4 triples in the sample space:

1, 2, 3	1, 2, 4	1, 3, 4	2, 3, 4

4. There is 1 quadruple in the sample space:

1, 2, 3, 4

c. The value in any cell of the spreadsheet can be determined by adding the value of the cell directly above it with the value of the cell up one row and to the left one column. For example, the number of triples in the sample space for the set $\{1, 2, 3, \ldots, 50\}$ is 19,600 because $18,424 + 1176 = 19,600$. **Note:** A complete table appears at the end of this teacher edition.

Numbers for the Lottery	No. of Singles	No. of Pairs	No. of Triples	No. of Quadruples	No. of Quintuples
$\{1\}$	1	0	0	0	0
$\{1, 2\}$	2	1	0	0	0
$\{1, 2, 3\}$	3	3	1	0	0
$\{1, 2, 3, 4\}$	4	6	4	1	0
$\{1, 2, 3, 4, 5\}$	5	10	10	5	1
$\{1, 2, 3, \ldots, 6\}$	6	15	20	15	6
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\{1, 2, 3, \ldots, 49\}$	49	1176	18,424	211,876	1,906,884
$\{1, 2, 3, \ldots, 50\}$	50	1225	19,600	230,300	2,118,760

Discussion (page 191)

a. Answers may vary. See response to Part **c** of the exploration.

b. Sample response: It is best to work through the first few cells in each column. This helps to see the patterns to generate formulas for the spreadsheet.

c. Each cell represents the size of the sample space for a lottery that draws *n*-tuples from a set of available numbers.

d. Sample response: By looking at the intersection of the column titled "number of triples" and the row titled $\{1, 2, 3, \ldots, 20\}$, you can discover that there are 1140 outcomes in the sample space. If all three numbers must match, then there is only one way to win. The probability is $1/1140 \approx 0.09\%$.

Assignment (page 191)

2.1 The number of quadruples is 91,390. This can be found in the intersection of the column titled "number of quadruples" and the row titled $\{1, 2, 3, \ldots, 40\}$.

***2.2** **a.** The probability is $1/10,626 \approx 0.00009 \approx 0.009\%$.

b. The probability is $10/10,626 \approx 0.0009 \approx 0.09\%$.

2.3 **a.** The sample space contains 8 outcomes, as shown below.

(1,1)	(1,2)	(1,3)	(1,4)
(2,1)	(2,2)	(2,3)	(2,4)

b. Since 4 possible white balls can be drawn for each of the 2 black balls, the size of the sample space is $2 \bullet 4 = 8$. (This problem shows a brief, informal use of the fundamental counting principle, which will be developed in later modules.)

***2.4** **a.** The size of the sample space is 324,632.

b. The size of the sample space is 35.

c. Using the method from Problem **2.3**, the size of the sample space is $324,632 \bullet 35 = 11,362,120$.

d. For any one ticket, the theoretical probability of matching the five white balls and one black ball is:

$$\frac{1}{11,362,120} \approx 0.000009\%$$

2.5 Answers will vary. If the class contains 30 students, then there are 142,506 different groups of 5.

* * * * *

***2.6** Sample response: I disagree with Josh. By extending the spreadsheet from the exploration, there are 38,760 possible ways to draw six numbers from the set $\{1, 2, 3, \ldots, 20\}$. However, there are 43,758 possible ways to draw eight numbers from the set $\{1, 2, 3, \ldots 18\}$.

2.7 There are 45 ways to pick 2 gloves from a set of 10. Since, there are only 5 different pairs, the probability of picking a matching pair is $5/45 \approx 11\%$.

2.8
a. The size of the sample space is 1,221,759.

b. The size of the sample space is 45.

c. The size of the sample space is $45 \bullet 1{,}221{,}759 = 54{,}979{,}155$.

d. For any one ticket, the theoretical probability of matching all six numbers is:

$$\frac{1}{54{,}979{,}155} \approx 0.000002\%$$

2.9 Since the size of the sample space is 2970 and the number of quadruples that can be selected from the set $\{1, 2, 3, \ldots, 12\}$ is 495, the number of elements in the second set is $2970/495 = 6$.

* * * * * * * * * *

(page 193)

Activity 3

Students calculate the expected value for various games and determine if a game is fair.

Materials List

- none

Technology

- spreadsheet

Exploration

(page 193)

Students play the New Apple Lottery 20 times and record their results.

a–d. Sample data:

Event	Prize	No. of Wins	Total Winnings
two matches	\$3.00	2	\$6.00
one match	\$1.00	8	\$8.00
no matches	\$0.00	10	\$0.00
	Sum	20	\$14.00

e. Using the sample data, the mean amount is $\$14/20 = \$0.70$.

f–g. The sum of the expected winnings is the same as the mean amount won per game. Sample table:

Event	Prize	Experimental Probability	Expected Winnings
two matches	\$3.00	$1/10$	\$0.30
one match	\$1.00	$2/5$	\$0.40
no matches	\$0.00	$1/2$	\$0.0
	Sum	1	\$0.70

Discussion

(page 194)

a. Since 20 plays is a relatively small number of trials, the mean amount may vary considerably.

b. As shown below, the expressions for mean amount won per game and total expected winnings are equivalent:

$$\frac{2(\$3.00) + 8(\$1.00) + 10(\$0.00)}{20} = \frac{1}{10}(\$3.00) + \frac{2}{5}(\$1.00) + \frac{1}{2}(\$0.00)$$

c. Answers will vary. Sample response: Since each game costs \$1.00 to play and the mean amount won is \$0.70, you can expect a mean loss of \$0.30 per game.

d. Sample response: To make the coin game a fair one, the cost of playing must equal the expected value. If a \$2.00 prize was given for a match, then the expected value would be \$1.00. Another possibility would be a \$1.50 prize for a match and a \$0.50 prize for no match.

e. Answers will vary. Using the sample data given in the exploration, the New Apple Lottery does not appear to be a fair game because the cost of playing (\$1.00) does not equal the expected value (\$0.70).

f. Sample response: Most lotteries are not fair games because they are designed to make a profit for their organizers.

Assignment

(page 195)

***3.1** **a.** In the table below, dollar amounts are rounded to the nearest cent.

Event	Value	Theoretical Probability	Product
two matches	\$3.00	1/15	\$0.20
one match	\$1.00	8/15	\$0.53
no matches	\$0.00	6/15	\$0.00
	Sum	1	\$0.73

b. Since the cost of playing (\$1.00) does not equal the expected value (\$0.73), this is not a fair game.

c. The following table shows one possible payoff scheme.

Event	Value	Theoretical Probability	Product
two matches	\$3.00	1/15	\$0.20
one match	\$1.50	8/15	\$0.80
no matches	\$0.00	6/15	\$0.00
	Sum	1	\$1.00

d. There are many other payoff schemes that make this a fair game (for example, a prize of \$6.60 for matching both numbers, \$1.05 for matching one number, and \$0.00 for matching no numbers).

***3.2** **a.** The expected value can be calculated as follows:

$$\$20,000\left(\frac{1}{435,897}\right) + \$200\left(\frac{160}{435,897}\right) + \$5\left(\frac{4960}{435,897}\right) \approx \$0.18$$

b. Buying more tickets will not make this a fair game. For example, if a player buys 5 tickets, the cost to play would be \$5.00. Since each time the game is played, the expected value is approximately \$0.18, five plays would have an expected value of approximately $\$0.18 \bullet 5 = \$0.90$. The cost of playing is still not equal to the expected value.

3.3 **a.** **1.** 1/20

2. 1/20

b. The expected value (in cents) can be calculated as follows:

$$1 \bullet \frac{1}{20} + 2 + \frac{1}{20} + 3 \bullet \frac{1}{20} + \cdots + 20\frac{1}{20} = 10.5$$

Since the expected value does not equal the cost to play, this is not a fair game.

3.4 **a.** $30 \bullet 80\% = 24$ shots

b. $500 \bullet 80\% = 400$ shots

c. $80\% \bullet n$ or $0.8n$ shots

3.5 **a.** Since there are 750 tickets, the probability that any one ticket will win is $1/750$.

b. The expected value can be calculated as follows:

$$\$300 \bullet \frac{1}{750} = \$0.40$$

c. Since the expected value of $0.40 is not equal to the cost of a raffle ticket, this is not a fair game.

3.6 **a.** The expected value can be calculated as follows:

$$\$30 \bullet \frac{1}{38} \approx \$0.79$$

b. Since the expected value of the game is approximately $0.79, the cost to play should also be $0.79.

* * * * *

3.7 The probability of winning with one ticket is $1/400$. The expected value is:

$$\$100 \bullet \frac{1}{400} = \$0.25$$

3.8 **a.** From the spreadsheet created in Activity **2**, there are 4845 ways in which four numbers may be chosen from 20. For each one of the 4845 ways to choose four red numbers, there are 20 possible blue numbers. This results in a sample space of $4845 \bullet 20 = 96,900$. Since only one choice will match, the probability is $1/96,900$.

b. Using the definition of expected value:

$$\$x\left(\frac{1}{96,900}\right) + \$200\left(\frac{16}{24,225}\right) + \$5\left(\frac{12}{1615}\right) = \$0.70$$

Therefore, $x = \$51,430$.

c. The expected earnings for the lottery equal the difference between the cost to play and the expected value for the player. For 100,000 tickets, the expected earnings are $100,000(\$1.00 - \$0.70) = \$30,000$.

* * * * * * * * * *

Teacher Note

You may wish to distribute copies of the sample space template (a completed version of Table **2**) for use in this assessment. A blackline master appears at the end of the teacher edition for this module.

Answers to Summary Assessment (page 198)

Sample response: The Sample Lottery costs $1.00 for each ticket. Players pick three different numbers from 1 to 10. The prizes for 3, 2, 1, and 0 matches are $10, $5, $1, and $0, respectively.

From the spreadsheet made in Activity **2**, there are 120 different ways that 3 numbers can be chosen from 10. Therefore, the probability for matching all three different numbers is $1/120 \approx 0.83\%$. Using the spreadsheet again, the probability for matching two numbers is $1/45 \approx 2.22\%$, while the probability for matching one number is $1/10 = 10\%$.

The probability for no matches is what's leftover:

$$100\% - 0.83\% - 2.22\% - 10\% \approx 87\%$$

The table below show the theoretical probabilities for each prize and the expected value for the game.

Matches	Prize	Probability	Winnings
three	$10.00	0.0083	$0.08
two	$5.00	0.0222	$0.11
one	$1.00	0.1	$0.10
none	$0.00	0.8695	$0.00
	Total	1.0	$0.29

Since the expected value per play is $0.29, the expected revenue for the lottery is $0.71 per play. If 1 million people play the game, the expected revenue is $710,000.

Since 10% of lottery profits go to its creator, I expect to make 10% of $0.71 or a little more than 7 cents per play. This will amount to $71,000 for 1 million plays.

The following table shows the results of 1000 trials simulated using technology. (**Note:** Students are not required to run 1000 trials.)

Matches	Prize	No. of Wins	Payoff
three	$10.00	5	$50.00
two	$5.00	26	$130.00
one	$1.00	119	$119.00
none	$0.00	850	$0.00
	Total	1000	$299.00

The total winnings of $299.00 are close to the expected value of $1000(\$0.29) = \$290$. The following table shows that the experimental probabilities are reasonably close to the theoretical probabilities.

Matches	**Theoretical Probability**	**Experimental Probability**
three	0.83%	0.5%
two	2.22%	2.6%
one	10.00%	11.9%
none	86.95%	85%

Module Assessment

1. **a.** Complete 20 trials of the experiment described in Steps **1–3** below.

   1. Randomly draw three cards from the set {ace, king, queen, jack, 10}.

   2. Record the results.

   3. Replace the cards and shuffle the set.

   b. Based on the data you collected in Part **a**, what is the experimental probability of drawing the set {ace, king, queen}?

   c. **1.** How many different outcomes are there in the sample space for this experiment?

   2. List the sample space.

   d. What is the theoretical probability of drawing the set {ace, king, queen}?

   e. If you completed 100 trials of this experiment, how many times would you expect to see the set {ace, king, queen}?

2. The city manager has asked you, the local expert on lotteries, for your opinion of a new game devised by the city council. The back of the ticket for the new lottery appears below.

   Analyze the game and make a complete report to the council. Your report should identify the game's expected value and the amount of money the city should expect to gain or lose if the game costs $5.00 to play. You should also include any suggestions you have for improving the game.

   1. Choose 5 numbers from the front of the lottery card.
   2. At 6:00 P.M. Monday, the lottery randomly selects 5 numbers.
   3. Prizes will be awarded as shown below.

Matches	**Prize**	**Probability**
five	$1,600,000	1/1,533,939
four	$80,000	1/7304
three	$4,000	1/178
two	$200	1/13
one	$10	1/3

Answers to Module Assessment

1. **a.** Student data will vary.

b. Sample response: Since I drew the set {ace, king, queen} 3 times in 20 trials, the experimental probability is $3/20 = 15\%$.

c. The following table shows the 10 outcomes in the sample space.

{ace, king, queen}	{ace, king, jack}
{ace, king, 10}	{ace, queen, jack}
{ace, queen, 10}	{ace, jack, 10}
{king, queen, jack}	{king, queen, 10}
{king, jack, 10}	{queen, jack, 10}

d. The theoretical probability is $1/10 = 10\%$.

e. 10

2. The table below shows the expected value for the game.

Matches	**Prize**	**Probability**	**Product**
five	$1,600,000	1/1,533,939	$1.04
four	$80,000	1/7304	$10.95
three	$4,000	1/178	$22.47
two	$200	1/13	$15.38
one	$10	1/3	$3.33
		Total	$53.17

Since the expected value is $53.17 and the game cost only $5.00 to play, the city can expect to lose $48.17 per play. Students should recognize that this is a great game for players but a terrible one for the city. In order to make this game more profitable for the city, students should suggest changes in the payoff scheme.

Selected References

Packal, E. *The Mathematics of Games and Gambling*. Washington, DC: Mathematical Association of America, 1981.

Paulos, J. A. *Innumeracy: Mathematical Illiteracy and its Consequences*. New York: Vintage Books, 1988.

Paulos, J. A. *Beyond Numeracy: Ruminations of a Numbers Man*. New York: Alfred A. Knopf, 1991.

Flashbacks

Activity 1

1.1 Complete the following table.

Percentage	**Decimal**	**Fraction**
		1/2
12%		
	0.8	
		3/2
	1.0	
0.5%		
		3/50

Activity 2

2.1 A bag contains 24 black marbles and 18 red marbles. If one marble is randomly selected from the bag,

a. what is the probability that the marble is red?

b. what is the probability that the marble is black?

2.2 **a.** When tossing a fair coin, determine the theoretical probability of obtaining each of the following:

1. tails

2. heads.

b. In 20 trials of a coin toss, 12 of the trials came up tails. Using this data, determine the experimental probability of obtaining each of the following:

1. tails

2. heads.

2.3 List the sample space for a lottery game that involves picking two different numbers from 1 to 4.

Activity 3

3.1 **a.** What number is 68% of 2960?

b. Five percent of what number is 1240?

c. Fifteen is 200% of what number?

d. What number is $21/35$ of 320?

e. Six-sevenths of what number is 660?

3.2 In a lottery game, numbers are drawn from the set $\{1, 2, 3, 4, 5\}$. Determine the probability that a single ticket will match all the numbers in each of the following situations:

a. when two different numbers are drawn

b. when three different numbers are drawn

c. when four different numbers are drawn.

Answers to Flashbacks

Activity 1

1.1

Percentage	Decimal	Fraction
50%	0.5	1/2
12%	0.12	3/25
80%	0.8	4/5
150%	1.5	3/2
100%	1.0	1
0.5%	0.005	1/200
6%	0.06	3/50

Activity 2

2.1 **a.** $18/42 \approx 43\%$

b. $24/42 \approx 57\%$

2.2 **a.** **1.** $1/2 = 50\%$

2. $1/2 = 50\%$

b. **1.** $12/20 = 60\%$

2. $8/20 = 40\%$

2.3 One way to list the sample space, in which (1,2) and (2,1) are considered identical outcomes, is shown below.

1, 2	1, 3	1, 4	2, 3	2, 4	3, 4

Activity 3

3.1 **a.** $x = 0.68 \bullet 2960 = 2012.8$

b.

$$0.05x = 1240$$
$$x = 24,800$$

c.

$$15 = 2.00x$$
$$x = 7.5$$

d.

$$x = 320\left(\frac{21}{35}\right)$$
$$= 192$$

e.

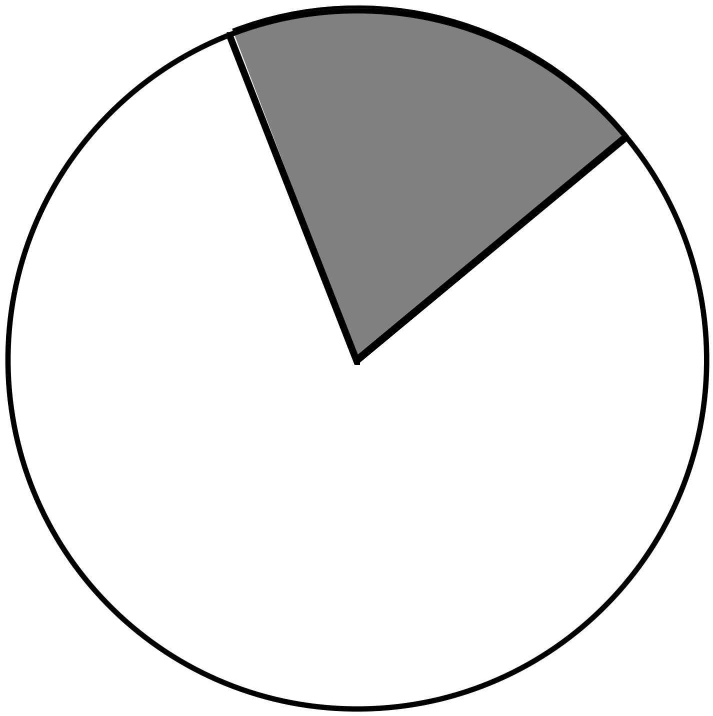
$$\frac{6}{7}x = 660$$
$$x = 770$$

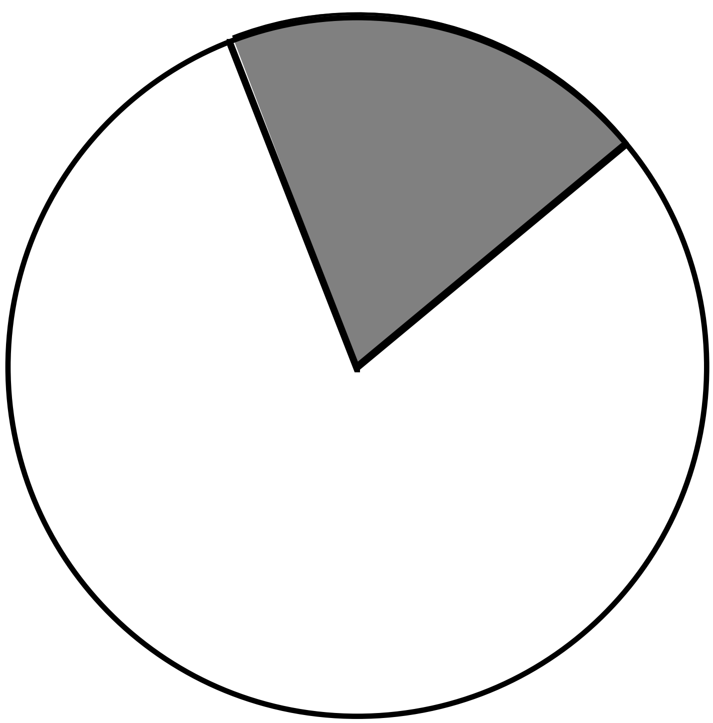
3.2 **a.** 1/10

b. 1/10

c. 1/5

Spinner Template

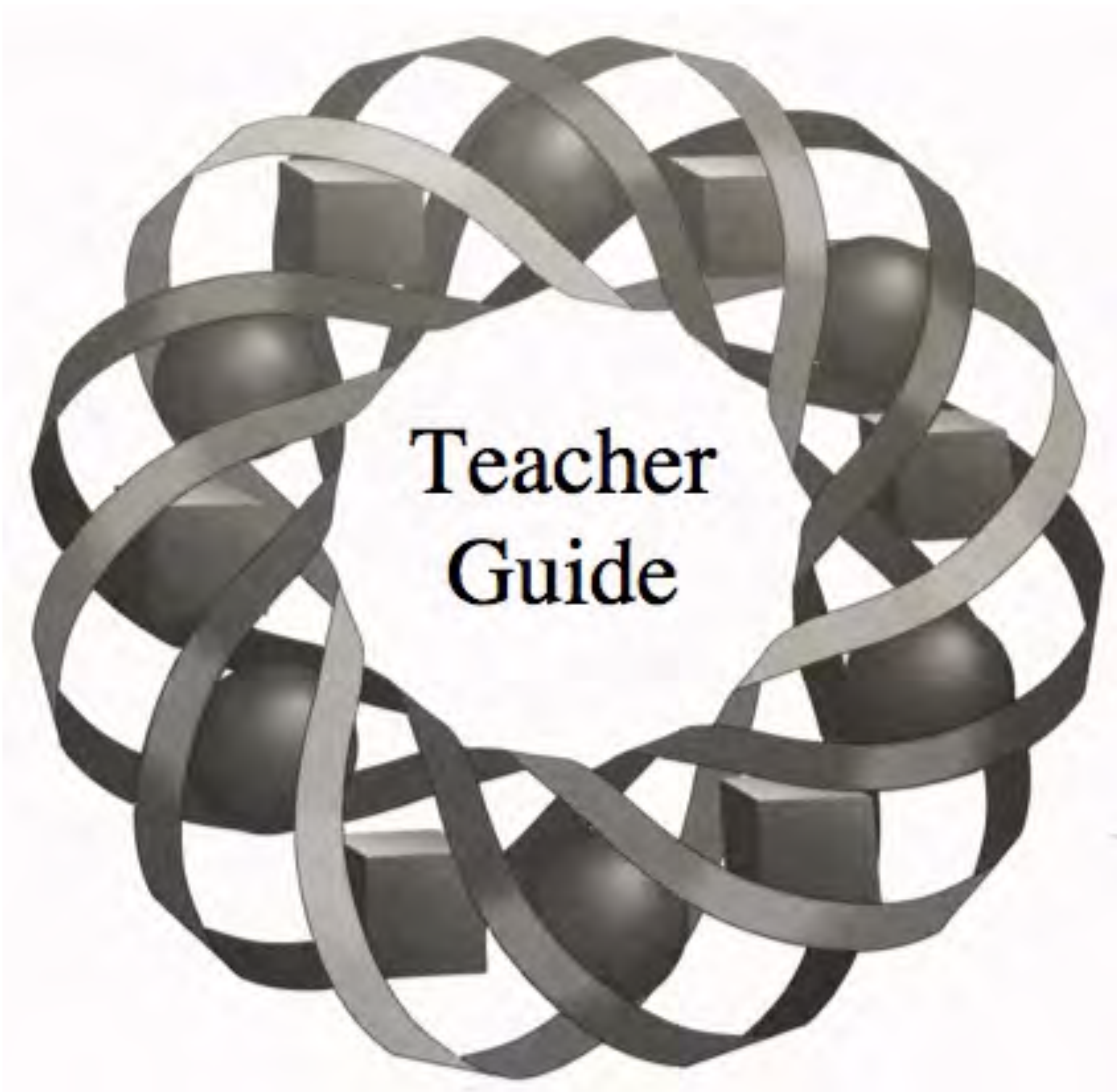




Sample Space Template

Numbers for the Lottery	No. of Singles	No. of Pairs	No. of Triples	No. of Quadruples	No. of Quintuples	No. of Sextuples
{1}	1	0	0	0	0	0
{1,2}	2	1	0	0	0	0
{1,2,3}	3	3	1	0	0	0
{1,2,3,4}	4	6	4	1	0	0
{1,2,3,4,5}	5	10	10	5	1	0
{1,2,3,…,6}	6	15	20	15	6	1
{1,2,3,…,7}	7	21	35	35	21	7
{1,2,3,…,8}	8	28	56	70	56	28
{1,2,3,…,9}	9	36	84	126	126	84
{1,2,3,…,10}	10	45	120	210	252	210
{1,2,3,…,11}	11	55	165	330	462	462
{1,2,3,…,12}	12	66	220	495	792	924
{1,2,3,…,13}	13	78	286	715	1287	1716
{1,2,3,…,14}	14	91	364	1001	2002	3003
{1,2,3,…,15}	15	105	455	1365	3003	5005
{1,2,3,…,16}	16	120	560	1820	4368	8008
{1,2,3,…,17}	17	136	680	2380	6188	12376
{1,2,3,…,18}	18	153	816	3060	8568	18564
{1,2,3,…,19}	19	171	969	3876	11628	27132
{1,2,3,…,20}	20	190	1140	4845	15504	38760
{1,2,3,…,21}	21	210	1330	5985	20349	54264
{1,2,3,…,22}	22	231	1540	7315	26334	74613
{1,2,3,…,23}	23	253	1771	8855	33649	100947
{1,2,3,…,24}	24	276	2024	10626	42504	134596
{1,2,3,…,25}	25	300	2300	12650	53130	177100
{1,2,3,…,26}	26	325	2600	14950	65780	230230
{1,2,3,…,27}	27	351	2925	17550	80730	296010
{1,2,3,…,28}	28	378	3276	20475	98280	376740
{1,2,3,…,29}	29	406	3654	23751	118755	475020
{1,2,3,…,30}	30	435	4060	27405	142506	593775
{1,2,3,…,31}	31	465	4495	31465	169911	736281
{1,2,3,…,32}	32	496	4960	35960	201376	906192
{1,2,3,…,33}	33	528	5456	40920	237336	1107568
{1,2,3,…,34}	34	561	5984	46376	278256	1344904
{1,2,3,…,35}	35	595	6545	52360	324632	1623160
{1,2,3,…,36}	36	630	7140	58905	376992	1947792
{1,2,3,…,37}	37	666	7770	66045	435897	2324784
{1,2,3,…,38}	38	703	8436	73815	501942	2760681
{1,2,3,…,39}	39	741	9139	82251	575757	3262623
{1,2,3,…,40}	40	780	9880	91390	658008	3838380
{1,2,3,…,41}	41	820	10660	101270	749398	4496388
{1,2,3,…,42}	42	861	11480	111930	850668	5245786
{1,2,3,…,43}	43	903	12341	123410	962598	6096454
{1,2,3,…,44}	44	946	13244	135751	1086008	7059052
{1,2,3,…,45}	45	990	14190	148995	1221759	8145060
{1,2,3,…,46}	46	1035	15180	163185	1370754	9366819
{1,2,3,…,47}	47	1081	16215	178365	1533939	10737573
{1,2,3,…,48}	48	1128	17296	194580	1712304	12271512
{1,2,3,…,49}	49	1176	18424	211876	1906884	13983816
{1,2,3,…,50}	50	1225	19600	230300	2118760	15890700

Are You Just A Small Giant?



How do you measure up against the world's tallest person? In this module, you'll examine some human—and inhuman—proportions and investigate the concept of similarity.

Randy Carspecken • Bonnie Eichenberger • Sandy Johnson • Terry Souhrada

Teacher Edition

Are You Just a Small Giant?

Overview

In this module, students investigate the relationship between physical dimensions and biological growth. Students examine linear measures and scale factors, determine the relationship of scale factors to area and volume in similar objects, and investigate the impact of area and volume on growth.

Objectives

In this module, students will:

- model growth using similarity
- identify and use relationships among scale factor, length, area, and volume of similar objects
- explore how area changes as objects change size proportionally
- examine how volume changes as objects change size proportionally
- use relationships among mass, density, weight, and pressure to describe proportional size changes
- examine how the values of *a* and *b* affect graphs of power equations of the form $y = ax^b$.

Prerequisites

For this module, students should know:

- how to write, graph, and solve linear equations
- how to use ratios and proportions.

Time Line

Activity	**1**	**2**	**3**	**4**	**5**	**6**	**Summary Assessment**	**Total**
Days	2	1	2	2	2	3	1	13

Materials Required

Materials	Activity						
	1	**2**	**3**	**4**	**5**	**6**	**Summary Assessment**
rulers	X		X	X	X	X	
metersticks	X		X	X	X	X	
centimeter graph paper		X		X	X	X	
Wadlow template	X						
unit cubes			X				
paper						X	
tape						X	

Teacher Note

A blackline master of the Wadlow template appears at the end of the teacher edition for this module.

Technology

Software	Activity						
	1	**2**	**3**	**4**	**5**	**6**	**Summary Assessment**
graphing utility			X	X		X	X
geometry utility		X					
spreadsheet			X	X		X	X

Teacher Note

In Activity **5**, students investigate pressure as force per unit area. In Activity **6**, they use this notion to explore limits on biological growth. These may be treated as optional activities.

Teacher Edition

Are You Just a Small Giant?

Introduction

(page 203)

Similarity and proportionality are used throughout this module. You may wish to ask students to describe some examples of similar objects.

Teacher Note

The discussion of Wadlow's dimensions continues in Activity **1**. Figure **2** shows a visual comparison of Wadlow and his father. You may want to mark Wadlow's height and arm span on a wall for student comparisons. As students discuss their similarity to Wadlow, you may wish to point out that his height and arm span are nearly the same. This ratio is close to 1 for most people.

Discussion

(page 204)

a. Sample response: Yes, the enlarged image is similar to the original image because the ratios of corresponding lengths are proportional and they have the same shape.

b. A photographic image of a person is not similar to the actual person. Although the ratios of corresponding lengths will be proportional, a two-dimensional object cannot have the same shape as a three-dimensional one. A statue, on the other hand, is similar to its subject.

c.

1. Sample response: The scale factor tells how much the dimensions in the drawing have been reduced or enlarged from the dimensions in the actual object.
2. Scale drawings of two-dimensional objects produce similar figures because the drawing has the same shape as the original object and the ratios of corresponding lengths are proportional.

   Two-dimensional scale drawings of three-dimensional objects do not produce similar figures because they do not have the same shape. Architectural models, however, are similar to the actual buildings.

d. $y = \frac{a}{b}x$

e. Yes, the equation in Part **d** represents a direct proportion in which a/b is the constant of proportionality. **Note:** The ratio a/b is also the scale factor.

(page 204)

Activity 1

Students continue their exploration of proportions and similarity.

Materials List

- metersticks (one per group)
- rulers (one per group)
- Wadlow template (a blackline master appears at the end of the teacher edition for this module; one per student)

Exploration

(page 205)

Students investigate similarity by measuring the height of two scale drawings of Robert Wadlow: one shown in Figure **2**, the other on a copy of the Wadlow template.

a. **1.** In Figure **2**, Wadlow's image is 9.9 cm tall. In the template, Wadlow's image is 19.9 cm tall.

2. The ratio of these two measurements is $9.9/19.9 \approx 0.5$.

b. **1.** In Figure **2**, the image of Wadlow's father is 6.6 cm tall. In the template, the image of Wadlow's father is 13.2 cm tall.

2. The ratio of the two measurements is $6.6/13.2 \approx 0.5$.

c. **1.** Yes, the results in Parts **a** and **b** suggest that the images in the two drawings are proportional.

2. Since the two drawings have the same shape and are in proportion, the drawings are similar.

d. Sample response:

$$\frac{\text{width of father's hat in Figure } \mathbf{2}}{\text{width of father's hat in template}} = \frac{1 \text{ cm}}{2 \text{ cm}} = 0.5$$

e. **1.** Students should write the following ratio:

$$\frac{\text{actual height}}{\text{height in Figure } \mathbf{2}} = \frac{272 \text{ cm}}{9.9 \text{ cm}} \approx 27.5$$

2–3. Students should write and solve the proportion below:

$$\frac{272 \text{ cm}}{9.9 \text{ cm}} = \frac{\text{father's height}}{6.6 \text{ cm}}$$

$$\frac{272 \text{ cm}(6.6 \text{ cm})}{9.9 \text{ cm}} = \text{father's height}$$

$$181 \text{ cm} \approx \text{father's height}$$

f. **1.** Students should write the following ratio:

$$\frac{\text{actual height}}{\text{height in template}} = \frac{272 \text{ cm}}{19.9 \text{ cm}} \approx 13.7$$

2–3. Students should write and solve the proportion below:

$$\frac{272 \text{ cm}}{19.9 \text{ cm}} = \frac{\text{father's height}}{13.2 \text{ cm}}$$

$$\frac{272 \text{ cm}(13.2 \text{ cm})}{19.9 \text{ cm}} = \text{father's height}$$

$$180 \text{ cm} \approx \text{father's height}$$

g. The two heights are approximately the same.

h. **1.** Sample response: My height is 160 cm and my shoe length is 24.6 cm.

2. Sample response: The ratio of my height to Wadlow's height is:

$$\frac{160 \text{ cm}}{272 \text{ cm}} \approx 0.59$$

The ratio of my shoe length to Wadlow's shoe length is:

$$\frac{24.6 \text{ cm}}{47 \text{ cm}} \approx 0.52$$

We are not similar because the ratios are not equal.

i. **1.** Sample response: My height is 160 cm and my shoe length is 24.6 cm. My friend's shoe length is 27.1 cm.

$$\frac{27.1 \text{ cm}}{24.6 \text{ cm}} = \frac{\text{friend's height}}{160 \text{ cm}}$$

$$\frac{27.1 \text{ cm}(160 \text{ cm})}{24.6 \text{ cm}} = \text{friend's height}$$

$$176 \text{ cm} \approx \text{friend's height}$$

2. Sample response: My friend's actual height is 180 cm. He is a little taller than I predicted using the ratios.

3. Sample response: The results appear to show that my friend and I are not similar. However, since the difference between the predicted height and the actual height is small, this could be explained by an error in measurement.

Discussion

(page 206)

a. Sample response: Given their shoe lengths, it is possible to predict the heights of other people using my measurements. However, these predictions may not be exact because I am not similar to everyone.

b. Corresponding angles of similar polygons are congruent.

c. Sample response: The values of b and d can't be 0 because you can't divide by 0.

d. Sample response: When a photocopier enlarges or reduces an image, you select a percentage. This is the scale factor. The resulting copy is similar to the original and corresponding lengths are proportional.

Assignment

(page 207)

1.1 **a.** Students may write and solve the following proportion:

$$\frac{\text{height}}{272\text{ cm}} = \frac{40\text{ cm}}{47\text{ cm}}$$
$$\text{height} \approx 231\text{ cm}$$

b. Students may write and solve the following proportion:

$$\frac{\text{shoe length}}{47\text{ cm}} = \frac{165\text{ cm}}{272\text{ cm}}$$
$$\text{shoe length} \approx 29\text{ cm}$$

1.2 **a.** **1.** brother: (12, 72)

2. mother: (24, 144)

3. sister: (18, 108)

4. father: (36, 216)

b. A completed table is shown below.

Family Member	father	mother	brother	sister
father	1:1	3:2	3:1	2:1
mother	2:3	1:1	2:1	4:3
brother	1:3	1:2	1:1	2:3
sister	1:2	3:4	3:2	1:1

1.3 Sample response: My thumb length is 5.5 cm and my height is 160 cm. If we are similar, the person's height can be estimated as follows:

$$\frac{\text{height}}{160\text{ cm}} = \frac{6.5\text{ cm}}{5.5\text{ cm}}$$
$$\text{height} \approx 189\text{ cm}$$

1.4 Students should measure their own heights and head circumferences then compare ratios. Sample response: I am not similar to this baby. My ratio of head circumference to height is:

$$\frac{59\text{ cm}}{160\text{ cm}} \approx 0.37$$

The baby's ratio is:

$$\frac{33\text{ cm}}{46\text{ cm}} \approx 0.72$$

***1.5** **a.** Sample response: Assuming the image in the photograph is proportional to the real shoe print, the length of the print can be found as shown below.

$$\frac{\text{real penny}}{\text{photo penny}} = \frac{\text{real footprint}}{\text{photo footprint}}$$
$$\frac{1.9\text{ cm}}{0.8\text{ cm}} = \frac{\text{real footprint}}{12\text{ cm}}$$
$$29\text{ cm} \approx \text{real footprint}$$

b. If the suspect is similar to Robert Wadlow, then the suspect's height can be found as follows:

$$\frac{\text{height of suspect}}{272\text{ cm}} = \frac{29\text{ cm}}{47\text{ cm}}$$
$$\text{height of suspect} \approx 168\text{ cm}$$

c. A detective might also consider other aspects of the print, such as the style of the shoe or the wear on the tread, to make predictions about the suspect's lifestyle or employment. Footprint photography and analysis are common topics of study for investigators.

* * * * *

1.6 Sample response: My wrist circumference is 0.16 m and my height is 1.6 m. Assuming I am similar to the basket ball player:

$$\frac{\text{wrist circumference}}{0.16\text{ m}} = \frac{2.5\text{ m}}{1.6\text{ m}}$$
$$\text{wrist circumference} \approx 0.25\text{ m}$$

1.7 **a.** Sample response: Yes, he will have time to finish the project. Using the following proportion, the typing will take 87.5 min.

$$\frac{2000\text{ words}}{3500\text{ words}} = \frac{50\text{ min}}{x\text{ min}}$$

b. Using the proportion below, the amount of time needed is 10.5 min.

$$\frac{3\text{ min}}{1000\text{ words}} = \frac{x\text{ min}}{3500\text{ words}}$$

c. Sample response: Yes, he will need more time. The total time required is $87.5 + 10.5 = 98$ min.

1.8 If speed and distance from the center are inversely proportional, then:

$$\text{speed} = \frac{k}{\text{distance}}$$

Substituting into this relationship,

$$\begin{aligned} k &= (900 \text{ cm})7.8 \text{ cm/sec} \\ &= 7020 \text{ cm}^2\text{/sec} \end{aligned}$$

If the distance from the center is 10 cm, the speed is:

$$\begin{aligned} \text{speed} &= \frac{7020 \text{ cm}^2\text{/sec}}{10 \text{ cm}} \\ &= 702 \text{ cm/sec} \end{aligned}$$

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(page 209)

Activity 2

Students investigate the relationship between scale factor and area for similar objects.

Materials List

- centimeter graph paper (one sheet per student)

Technology

- geometry utility

Exploration

(page 209)

In this exploration, students discover that, for two similar figures, the ratio of the areas is the square of the scale factor.

a–b. Students may estimate areas by counting squares on the graph paper. Some may observe that the relationship between the areas can be obtained by squaring the scale factor.

c. A completed table is shown below.

Square	Side Length (cm)	Area (cm^2)
A	1	1
B	2	4
C	3	9
D	4	16
E	5	25
F	6	36
G	1.5	2.25
H	2.5	6.25
I	1.2	1.44
J	z	z^2
K	y	y^2

d. A completed table is shown below.

Squares	Ratio of Side Lengths (Scale Factor)	Ratio of Areas
A to B	$1/2$	$1/4$
E to A	$5/1$	$25/1$
B to D	$1/2$	$1/4$
E to C	$5/3$	$25/9$
I to A	$6/5$	$36/25$
G to A	$3/2$	$9/4$
J to F	$z/1$	$z^2/1$
J to K	z/y	z^2/y^2

e. The ratio of the areas is the square of the scale factor.

f. Answers will vary. Students consider this question in general in Part **h** of the discussion.

g. **1–3.** Students should show that the ratio of corresponding sides (scale factor) is 2 and that the corresponding angles have equal measures.

4. The ratio of the areas is $4/1$. This is the square of the scale factor.

Discussion

(page 211)

a. Sample response: The four angles of a square are right angles and the fours sides are of equal length. Therefore, for any two squares, the ratio of corresponding sides will be the same and the corresponding angles will have equal measures.

b. The area increases by a factor of 4.

c. The area increases by a factor of 9.

d. The side length is the square root of the area: $s = \sqrt{a}$.

e. The scale factor can be found by taking the square root of the ratio of the areas:

$$\sqrt{\frac{49}{1}} = \frac{\sqrt{7^2}}{\sqrt{1^2}} = \frac{7}{1} = 7$$

f. The square root of the ratio of the areas can be written as:

$$\sqrt{\frac{z^2}{y^2}} = \frac{\sqrt{z^2}}{\sqrt{y^2}} = \frac{z}{y}$$

g. Solve for the area of square B in the porportion:

$$\frac{\text{area of A}}{\text{area of B}} = \left(\frac{\text{side length of A}}{\text{side length of B}}\right)^2$$

h. The relationship between scale factor and the ratio of areas is true for any pair of similar figures. (For similar polygons, an argument can be constructed based on similar triangles.)

Assignment

(page 213)

2.1 Each length was multiplied by 15. This is the scale factor found by taking the square root of the ratio of the areas, $225/1$.

2.2 The area of the larger circle is 10^2 or 100 times greater.

2.3 The area of the larger rectangle is $3^2 \bullet 80 = 720 \text{ m}^2$ ·

2.4 **a.** The scale factor for the two pizzas is $41/30$.

b. The ratio of the areas is:

$$\left(\frac{41}{30}\right)^2 \approx 1.87$$

c. If price were proportional to area, then

$$\left(\frac{41}{30}\right)^2 = \frac{x}{11.30}$$

The 41-cm pizza should cost $21.11.

d. At $18.95, the 41-cm pizza is a better buy because it costs less per square centimeter.

***2.5** The ratio of foot areas is:

$$\frac{\text{Nelson's foot area}}{\text{pro's foot area}} = \frac{325}{468}$$

The scale factor (ratio of corresponding lengths) is therefore:

$$\frac{\sqrt{325}}{\sqrt{468}} \approx 0.83$$

Since the basketball player is 215 cm tall, Nelson's height is $215 \bullet 0.83 \approx 179 \text{ cm}$.

2.6 **a.** Since the diagonal is a length measurement, the ratio of the diagonals is the scale factor: $63/33 \approx 1.9/1$. The ratio of the areas is the square of the scale factor: $(1.9)^2 \approx 3.6$.

b. If the ratio of the areas is 2, the scale factor is $\sqrt{2}$. The diagonal of the larger screen measures $\sqrt{2}(33) \approx 46.7 \text{ cm}$.

2.7 **a.** $A = 0.5 \bullet 4 \bullet 3 = 6 \text{ cm}^2$

b. Sample response: These two right triangles are similar. Since the scale factor is 6, the ratio of the areas is $6^2/1$. The area of the larger triangle is $6 \text{ cm}^2(36) = 216 \text{ cm}^2$.

c. $A = 0.5 \bullet 24 \bullet 18 = 216 \text{ cm}^2$

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2.8 **a.** The larger room is 6.56 m long and 3.84 m wide.

b. Answers may vary. Sample response: No. Since the ratios of corresponding lengths are not equal, the rectangles are not similar. (Some students may conjecture that the ratio will be $1.4^2 = 1.96$ because 1.4 is the mean of 1.2 and 1.6.)

c. The area of the smaller room is 13.12 m^2; the area of the larger room is 25.19 m^2.

d. Answers will vary. The ratio of areas is 1.92.

2.9 The $50 section of turf and a football field are similar rectangles. The scale factor is 5; therefore, the ratio of areas is $25/1$. To keep the price proportional to the area, the company should charge $1250 to clean a football field.

2.10 **a.** In the following table, the perimeters and areas for the four rectangles are listed from smallest to largest. For any pair, the ratio of the areas is the square of the ratio of the perimeters.

Rectangle	Perimeter	Area
A	16	15
B	48	135
C	64	240
D	80	375

b. Sample response: All of the rectangles are similar to each other. For any pair, the ratios of corresponding sides are equal and all angles are congruent.

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(page 215)

Activity 3

In this activity, students explore the relationship between scale factor and volume for similar objects.

Materials List

- unit cubes (27 per group)
- rulers (one per group)

Technology

- graphing utility
- spreadsheet

Exploration

(page 215)

By building models with unit cubes, students investigate the relationship between scale factor and volume.

a. A completed table appears below.

Cube	Edge Length	Volume
A	1	1
B	2	8
C	3	27
N	n	n^3

b. **1.** A completed table appears below.

Cubes	Ratio of Edge Lengths (scale factor)	Ratio of Volumes
B to A	2/1	8/1
B to C	2/3	8/27
C to A	3/1	27/1
	a/b	a^3/b^3

2. Students should observe that the ratio of volumes is the cube of the scale factor.

Discussion

(page 216)

a. **1.** The ratio of the volumes is 125/8.

2. The scale factor is 5/2.

b. The ratio of the volumes is equivalent to the cube of the scale factor (ratio of the edge lengths).

c. Since $4 = \sqrt[3]{64}$, the scale factor is 4.

d. The length of each edge is $\sqrt[3]{d}$.

e. The relationship is true for all similar three-dimensional objects. For example, doubling the measurements of a room increases its volume by a factor of 8. Tripling the radius of a sphere increases its volume by a factor of 27.

Teacher Note

In Problems **3.2**, **3.4**, and **3.6**, students extend the concept of similarity to eggs and fish. In living things, mass may not be strictly proportional to volume. To make their estimates, students should assume that densities are nearly equal among similar animals. **Note:** The biological literature cites fish as examples of living organisms that are truly close to similar.

Assignment

(page 217)

3.1 Since the ratio of the volumes is 1/343, the scale factor is:

$$\sqrt[3]{\frac{1}{343}} = \frac{1}{7}$$

3.2 Since the scale factor is 2/1, the ratio of their volumes is:

$$\left(\frac{2}{1}\right)^3 = \frac{8}{1}$$

3.3 **a.** Since density is the ratio of mass to volume, the density is:

$$\frac{261 \text{ g}}{33.5 \text{ cm}^3} \approx 7.8 \text{ g/cm}^3$$

b. Since the density of a pure substance is a constant, the density of the larger sphere is the same: 7.8 g/cm^3.

c. $3/2 = 1.5$

d. Since volume varies as the cube of the scale factor, the ratio is:

$$\left(\frac{3}{2}\right)^3 = \frac{27}{8}$$

e. **1.** The ratio of the volumes is the same as the ratio of the mass.

2. The ratio of the volumes is the cube of the scale factor.

***3.4** From mouth to tail, the image of the smaller fish measures about 40 mm. The image of the larger fish measures about 48 mm. The scale factor between the fish is $48/40 = 1.2$. Since the smaller fish is 25 cm long, the length of the larger fish is $25(1.2) = 30 \text{ cm}$.

The volume of the larger fish is greater by the cube of the scale factor: $1.2^3 \approx 1.7$. Assuming that mass is proportional to volume, its mass should also be greater by the cube of the scale factor. Since the smaller fish has a mass of 0.75 kg, the mass of the larger fish is: $0.75(1.7) \approx 1.3 \text{ kg}$. (This is not far from doubling the mass, which may surprise some students.)

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3.5 Since the scale factor is $8/9$, the ratio of the volumes is $(8/9)^3 \approx 0.7$. Therefore, an 8-inch pie should require $6(0.7) = 4.2$ cups of apples.

3.6 If one egg has three times the circumference of another egg, then the scale factor is $3/1$. Assuming that mass is proportional to volume, the ratio of the masses is the cube of the scale factor: $27/1$.

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Activity 4

Students collect data on shoe length and shoe-print area and model the data using equations of the form $y = ax^b$.

Materials List

- metersticks (one per group)
- rulers (one per group)
- centimeter graph paper (one sheet per student)

Technology

- graphing utility
- spreadsheet

Exploration

(page 218)

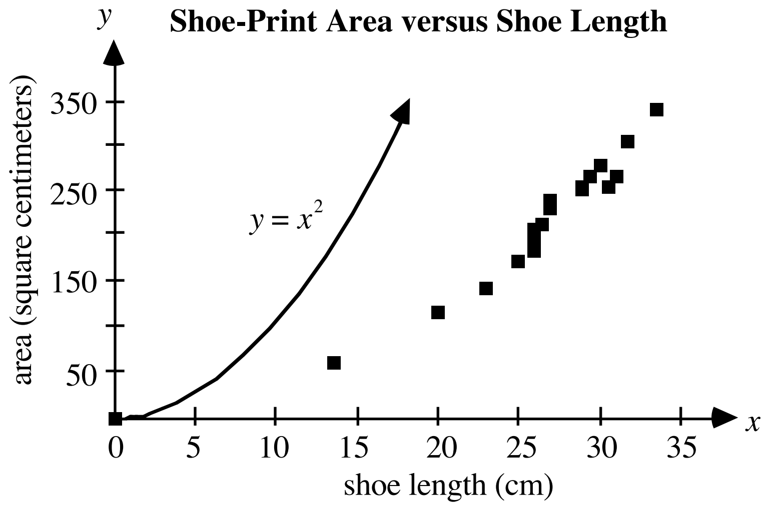
a–b. Students can estimate area by counting squares. The child's shoe print is approximately 13.5 cm long, with an area of approximately 57 cm^2.

c. **1.** Sample data:

Length (cm)	Area (cm^2)	Length (cm)	Area (cm^2)
0.0	0.0	27.0	237.0
13.5	57.0	29.0	248.0
20.0	115.0	29.0	251.0
23.0	140.0	29.0	252.0
25.0	170.0	29.5	262.0
26.0	182.0	30.0	275.0
26.0	190.0	30.5	251.0
26.0	203.0	31.0	262.0
26.5	210.0	31.8	300.0
27.0	228.0	33.5	336.0

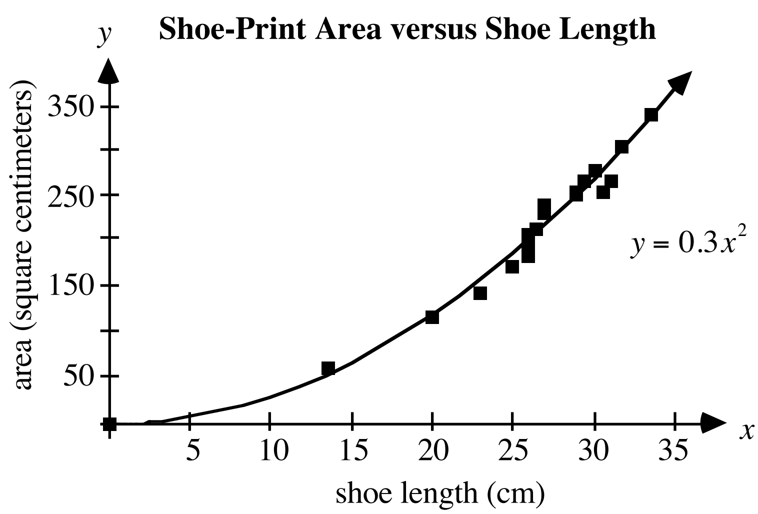
2. See sample graph in Part **d1** below.

d. **1.** Sample graph:



2. Sample response: The two graphs have the same general shape. As the length increases, the area increases. Although both graphs are nonlinear, the graph of $y = x^2$ is steeper than the scatterplot.

e. Students find an equation that models the data by experimenting with different values of a. For the sample data, the equation $y = 0.3x^2$ fits very well. Sample graph:



f. The values in the following table were generated using the sample data given in Part **c** and the equation $y = 0.3x^2$.

Length (*x*)	**Area (*y*)**	**Predicted Area**	**Absolute Value of Residual**
0.0	0.0	0.0	0.0
13.5	57.0	54.7	2.3
20.0	115.0	120.0	5.0
23.0	140.0	158.7	18.7
25.0	170.0	187.5	17.5
26.0	182.0	202.8	20.8
26.0	190.0	202.8	12.8
26.0	203.0	202.8	0.2
26.5	210.0	210.7	0.7
27.0	228.0	218.7	9.3
27.0	237.0	218.7	18.3
29.0	248.0	252.3	4.3
29.0	251.0	252.3	1.3
29.0	252.0	252.3	0.3
29.5	262.0	261.1	0.9
30.0	275.0	270.0	5.0
30.5	251.0	279.1	28.1
31.0	262.0	288.3	26.3
31.8	300.0	303.4	3.4
33.5	336.0	336.7	0.7
		Sum	175.8

g. For the sample data, the equation $y = 0.3x^2$ produces the smallest sum of the absolute values of the residuals for *a* rounded to the nearest tenth. The equation $y = 0.298x^2$ produces a slightly smaller sum (172).

h. This graph should be very similar to the sample graph shown in Part **e**. The segment drawn in Step **3** provides a graphic representation of the residual. Students will evaluate how well their models fit their own data points in Part **b** of the discussion.

Discussion

(page 219)

a. Sample response: As shoe length increases, the area of the shoe print increases by the square of the length.

b. Students should have obtained approximately the same equations.

c. Answers will vary. Students should recognize that the size of the residual is one measure of the accuracy of a model.

d. Sample response: The value of a affects the steepness of the curve. **Note:** At this point in the module, students have used only positive values for a. They continue their investigation of power equations in Problem **4.1**.

e. The ratio of the areas is 9 to 1.

Assignment

(page 220)

4.1 **a–c.** Students use technology to explore graphs of $y = ax^b$.

d. Sample response: The value of a affects the steepness of the curve. The sign of a determines which quadrants contain the graph. The value of b changes the shape of the curve. If b is even, the curve resembles a parabola. If b is odd, the curve resembles the graph of $y = x^3$. The size of b also influences the steepness of the curve.

4.2 Answers will vary. Some possible equations are listed below:

a. $y = x^2$, $y = x^4$

b. $y = -x^3$, $y = -x^5$

c. $y = x^3$, $y = x^5$

d. $y = -x^2$, $y = -x^4$

***4.3** Substituting Wadlow's shoe length from Table **1** (47 cm) into the equation $y = 0.3x^2$, the area of Wadlow's shoe print is approximately 641 cm^2. **Note:** It may help students to rewrite their equations using more meaningful variables, such as $A = 0.3s^2$. You may wish to discuss why the generic variables x and y are used in tools such as graphing calculators.

4.4 Sample response: My height is 160 cm and my shoe length is 24.6 cm. If Neva is similar to me, her shoe length can be found as follows:

$$\frac{\text{Neva's shoe}}{24.6 \text{ cm}} = \frac{170 \text{ cm}}{160 \text{ cm}}$$

$$\text{Neva's shoe} \approx 26.1 \text{ cm}$$

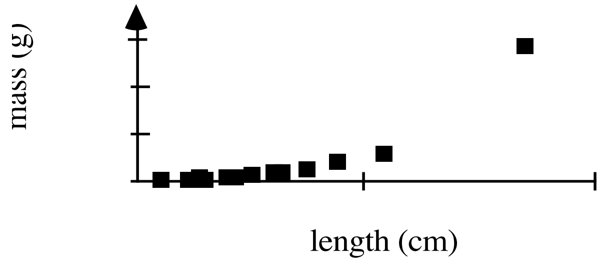
Using the equation from the exploration, the area of her shoe print is:

$$\begin{aligned} y &= 0.3x^2 \\ &= 0.3(26.1 \text{ cm})^2 \\ &\approx 204 \text{ cm}^2 \end{aligned}$$

Note: Even if the student and Neva are not similar, the same relationship betwen height and shoe-print area may hold.

4.5 **a.** Sample graph:

Mass vs. Length of Bird Eggs



b. One equation that models the data is $y = 0.28x^3$.

c. Sample response: The condor egg appears to be a little below the upward-curving trend of the other data points. The data point for the condor is (11,270). Substituting 11 into the power equation $y = 0.28x^3$ yields a predicted mass of 373 g, a difference of 103 g.

d. Substituting the length of the bald eagle's egg into the sample equation yields a mass of 108.9 g.

e. Sample response: The graph of the data for mass and length of bird eggs can be modeled by the equation $y = 0.28x^3$. This equation states that the mass of an egg is proportional to the cube of its length. In other words, as the length is increased by a scale factor of x, the mass of an egg increases by x^3.

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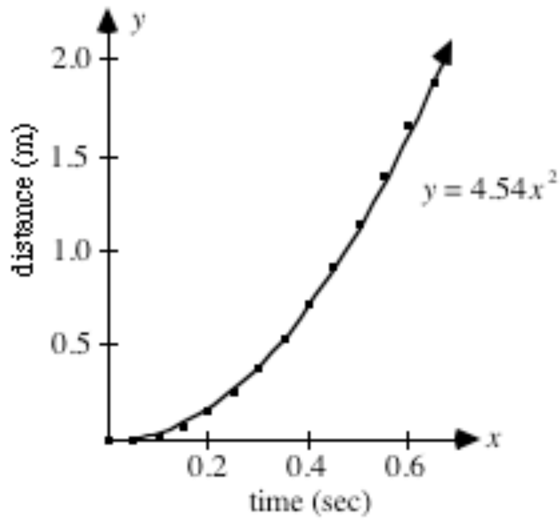
4.6 **a.** See sample graph in Part **c**.

b. One equation that models the data is $y = 4.54x^2$. The sum of the absolute values of the residuals is 0.267.

Time (x)	**Distance (y)**	**Predicted Distance**	**Absolute Value of the Residual**
0.00	0.000	0.000	0.000
0.05	0.000	0.011	0.011
0.10	0.017	0.045	0.028
0.15	0.072	0.102	0.030
0.20	0.152	0.182	0.030
0.25	0.256	0.284	0.028
0.30	0.387	0.409	0.022
0.35	0.539	0.556	0.017
0.40	0.717	0.726	0.009
0.45	0.920	0.919	0.001
0.50	1.144	1.135	0.009
0.55	1.393	1.373	0.020
0.60	1.668	1.634	0.034
0.65	1.889	1.918	0.029
		Sum	0.267

c. Sample graph:



d. Using the sample equation given in Part **b**, the object would fall about 18.2 m in 2 sec.

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Activity 5

Students investigate pressure as force per unit area. **Note:** Although this activity may be treated as optional, it must be completed if you wish to proceed to Activity **6**.

Materials List

- metersticks (one per group)
- rulers (one per group)
- centimeter graph paper (one sheet per student)

Exploration

(page 222)

Students discover that shoes which add area to the footprint lower the force per unit area on the ground (and vice versa). **Note:** This can be demonstrated visually by stepping into a box of soft sand.

a.

1. Students can estimate the area of a footprint by counting squares. Sample response: The area of my footprint is 400 cm^2.

2. This area should be larger than the area in Step **1**. Sample response: The area of my shoe print is 420 cm^2.

3. This area should be considerably less than the area in Step **1**. Sample response: The area of my print while wearing a small-heeled shoe is 240 cm^2.

b. Sample response: The areas vary considerably. The area of the largest shoe print is 175% of the area of the smallest shoe print.

c. Sample response: My weight in newtons can found by multiplying my mass in kilograms by 9.8 m/sec^2.

$$135 \text{ lbs} \bullet \frac{1 \text{ kg}}{2.2 \text{ lbs}} \bullet \frac{9.8 \text{ m}}{\text{sec}^2} = \frac{600 \text{ kg} \bullet \text{ m}}{\text{sec}^2} \approx 600 \text{ N}$$

d. The following pressures were calculated using the sample responses given in Parts **a** and **c**.

1. without shoes:

$$\frac{600 \text{ N}}{2(400 \text{ cm}^2)} = 0.75 \text{ N/cm}^2$$

2. shoes with flat soles:

$$\frac{600 \text{ N}}{2(420 \text{ cm}^2)} \approx 0.71 \text{ N/cm}^2$$

3. shoes with small heels:

$$\frac{600 \text{ N}}{2(240 \text{ cm}^2)} = 1.25 \text{ N/cm}^2$$

e. Sample response: The three pressures vary by a considerable amount. The largest pressure exerted is approximately 176% of the least pressure exerted.

Discussion

(page 223)

a. Sample response: Hiking boots have large soles which spread your weight over a larger area. This makes them comfortable for long hikes. Shoes worn by rock climbers have small soles to increase the weight per unit area and improve the gripping power of the shoes.

Track shoes for racing also have small soles. Besides reducing the mass, this increases the weight per unit area and (along with spikes) improves traction. **Note:** In Problem **5.2**, students are asked to describe how snowshoes make it easier to walk on snow.

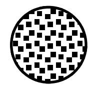
b. Sample response: As the area of the shoe print decreases, the force per unit area increases. High heels could damage floors made of wood or other substances.

c. Sample response: Four casters are sufficient to support the weight of a box spring and mattress. The weight of a waterbed must be distributed over a larger area.

Assignment

(page 223)

5.1 **a.** Sample response:



print of Arlis' heel

b. Assuming half of her weight rests on the heel:

$$\frac{250 \text{ N}}{0.75 \text{ cm}^2} \approx 333 \text{ N/cm}^2$$

***5.2** Snowshoes distribute the wearer's weight over a greater surface area, decreasing the force per unit area. Estimating the area of the snowshoe is difficult due to its shape and the webbing. However, multiplying its length by its width: 2625 $75 \text{ cm} \bullet 35 \text{ cm} = 2625 \text{ cm}^2$. Even if the actual area is as little as one-third of 2625 or 875 cm^2, it is still over three times the area of a typical adult shoe print.

5.3 From Problem **4.3**, the area of one of Wadlow's shoe prints is 641 cm^2. From Table **1**, his weight is 1950 N. The pressure on the ground can be calculated as follows:

$$\frac{1950 \text{ N}}{2 \bullet 641 \text{ cm}^2} \approx 1.52 \text{ N/cm}^2$$

Students should compare this value with the pressures found in Part **d** of the exploration.

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5.4 **a.** The volume of the block is 180 cm^3. Using the given density, the block has a mass of approximately 2040 g or 2.04 kg. Multiplying this mass by the acceleration due to gravity (9.8 m/sec^2) gives a force of approximately 20 N. Since the area of the base is 45 cm^2, the pressure is:

$$\frac{20 \text{ N}}{45 \text{ cm}^2} \approx 0.44 \text{ N/cm}^2$$

b. Since the area of the cube's base is 6.25 cm^2, the pressure is:

$$\frac{20 \text{ N}}{6.25 \text{ cm}^2} = 3.2 \text{ N/cm}^2$$

c. Sample response: When the lead block is balanced on the cube, the weight is distributed over a smaller area. It is the same idea as distributing my weight over high-heeled shoes.

5.5 The area of the base is approximately 3848 cm^2. Multiplying this area by the pressure gives a force of 1924 N. Dividing this force by the acceleration due to gravity gives a mass of approximately 196 kg.

5.6 Students should select one of the pressures determined in Part **d** of the exploration. They should then divide this pressure by 3. Sample response: While standing upright without shoes, I exert a pressure of $0.75\ \text{N/cm}^2$ on Earth. Since the gravity on Mars is one-third the gravity on Earth, my weight on Mars would be one-third my weight on Earth. Therefore, the pressure I would exert on the Martian surface is $0.25\ \text{N/cm}^2$.

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Activity 6

Students make models of bones, compare the graphs of linear and second-degree curves, and investigate one limit to human growth. **Note:** This may be treated as an optional activity.

Materials List

- tape
- metersticks (one per group)
- rulers (one per group)
- paper (two sheets per group)
- centimeter graph paper (one sheet per student)

Technology

- graphing utility
- spreadsheet

Exploration

(page 225)

Students investigate the relationship between area and volume.

a. The two models are obviously not similar. The ratio of the diameters is:

$$\frac{\text{femur diameter of horse}}{\text{femur diameter of human}} = \frac{4.5}{3} = 1.5$$

If the horse and human femurs were similar and 1.5 was the scale factor, the length of the horse femur would have to be $1.5 \bullet 50.5\ \text{cm} = 75.75\ \text{cm}$ (almost twice its actual length). The actual ratio of the lengths is:

$$\frac{\text{femur length of horse}}{\text{femur length of human}} = \frac{39.5}{50.5} = 0.78$$

The circumference of the horse femur is 14.1 cm; the circumference of the human femur is 9.4 cm.

b. **1.** Sample response: The area of a cross section of the human femur is about 6 cm^2. The area of a cross section of the horse femur is about 18 cm^2.

2–3. Using the formula for the area of a circle, the human and horse femurs have cross-sectional areas of approximately 7 cm^2 and 16 m^2, respectively.

4. The horse femur has approximately 2.3 times the cross-sectional area of the human femur. Therefore, it is about twice as strong as a human femur. (Since a horse also has twice as many legs, the total cross-sectional area of leg bones in an individual horse is 4.6 times that of a human.)

c. **1–2.** Since the height is doubled, the scale factor is 2. Therefore, the diameter of the giant's femur is $2(3 \text{ cm}) = 6 \text{ cm}$; the length is $2(50.5 \text{ cm}) = 101 \text{ cm}$; and the circumference is $2(9.4 \text{ cm}) = 18.8 \text{ cm}$. The cross-sectional area of the bone increases by the square of the scale factor: $2^2(7 \text{ cm}) = 28 \text{ cm}^2$.

3. For the 600-N person, the pressure is:

$$\frac{600 \text{ N}}{2 \bullet 7 \text{ cm}^2} = 42.9 \text{ N/cm}^2$$

Volume increases as the cube of the scale factor. Because mass is directly proportional to volume and weight is directly proportional to mass, weight also increases as the cube of the scale factor. The giant's weight would therefore be $2^3(600 \text{ N}) = 4800 \text{ N}$. Since the cross-sectional area has increased only as the square of the scale factor, the giant must support more weight per square centimeter of bone. For the giant, the pressure is:

$$\frac{4800 \text{ N}}{2 \bullet 28 \text{ cm}^2} \approx 85.7 \text{ N/cm}^2$$

4. Students should summarize their measurements and calculations and demonstrate their understanding of the relationship between mass and area. Sample response: The weight of the giant increased as the cube of the scale factor but the bone cross-sectional area increased only as the square of the scale factor. Since the scale factor was 2, the giant's femurs bear twice the weight for each unit of cross-sectional area.

5. Sample response: As the tension on the bone structure increases, bones are more likely to break. A giant might be less mobile than a smaller person and more prone to fractures.

d. Sample response: Yes, when a human gets too tall, the bones of the femur would break.

Teacher Note

In Part **e** of the exploration, the formula used to calculate pressure in the right-hand column of the spreadsheet must account for two femurs per person. The table below shows some possible formulas:

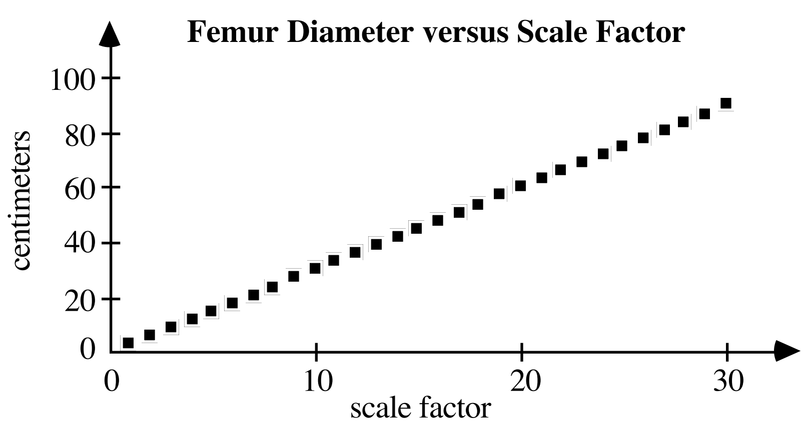
	A	B	C	D	E	F
1	**Scale Factor**	**Body Height (cm)**	**Femur Diameter (cm)**	**Cross-Sectional Area of Femur (cm^2)**	**Body Weight (N)**	**Pressure on Femur (N / cm^2)**
2	1	180	3	=(C2/2)^2*PI()	600	=E2/(D2*2)
3	=A2+1	=B2+180	=C2+3	=(C3/2)^2*PI()	=A3^3*600	=E3/(D3*2)
4	=A3+1	=B3+180	=C3+3	=(C4/2)^2*PI()	=A4^3*600	=E4/(D4*2)

e. Sample spreadsheet: Area is rounded to the nearest whole number while pressure is rounded to the nearest tenth.

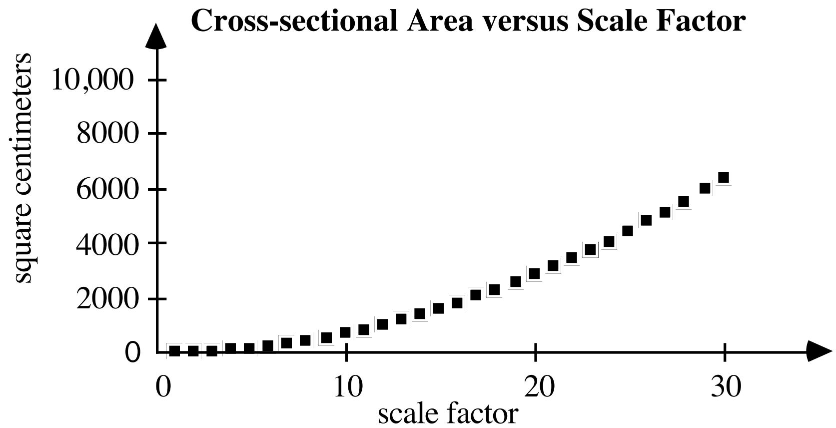
Scale Factor	**Body Height (cm)**	**Femur Diameter (cm)**	**Cross-Sectional Area of Femur (cm^2)**	**Body Weight (N)**	**Pressure on Femur (N/cm^2)**
1	180	3	7	600	42.9
2	360	6	28	4800	85.7
3	540	9	64	16,200	126.6
⋮	⋮	⋮	⋮	⋮	⋮
28	5040	84	5542	13,171,200	1188.3
29	5220	87	5945	14,633,400	1230.7
30	5400	90	6362	16,200,000	1273.2

f. Answers will vary. As shown in the sample spreadsheet above, the pressure on the femur exceeds 1200 N/cm^2 when the scale factor is 29. This giant would be 57.6 m tall and weigh 14,633,400 N. Students may argue that other factors would limit a person's growth long before this size could be reached.

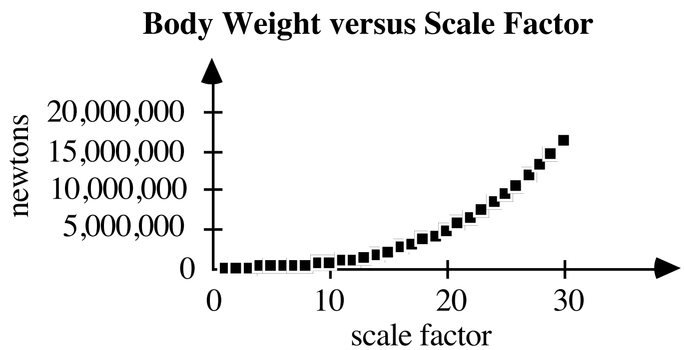
g. **1.** Sample graph:



2. Sample graph:

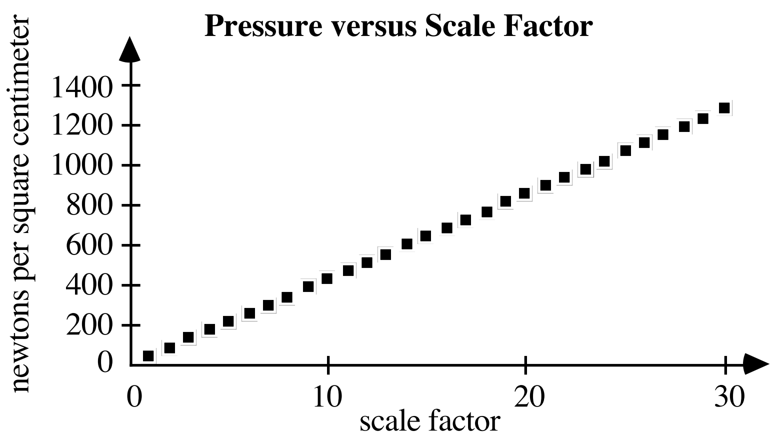


3. Sample graph:



h. **1.** Answers will vary. Since cross-sectional area increases as the square of the scale factor and weight increases as the cube of the scale factor, then pressure (or weight / area) increases linearly as the scale factor.

2. Sample graph:



Discussion (page 227)

a. Sample response: The femurs of humans and horses are not similar since corresponding lengths do not have the same ratios. Horse femurs, although shorter in length, have a larger diameter and are much stronger than human femurs.

b. If the strength of bone were the only limitation on human growth (and assuming that human bones can withstand a pressure 1200 N/cm^2 before crushing), then a human could be much larger than Wadlow. However, the limitations on growth are much more complicated than this model suggests. You may wish to ask students to consider, for example, the power of the pump required to circulate blood from head to toe in a person 52 m tall.

c. The three graphs show linear, quadratic, and cubic increases. Students should use the knowledge that area increases as the square of the scale factor and volume increases as the cube of the scale factor to help them interpret these graphs.

d. The pressure on the femur is directly proportional to the scale factor.

Assignment (page 227)

6.1 Answers will vary. Sample response: The pressure on each femur in Figure **6** was 42.9 N/cm^2. Since I am 160 cm tall and the pressure on each femur is directly proportional to the scale factor:

$$\frac{\text{my pressure}}{42.9 \text{ N/cm}^2} = \frac{160 \text{ cm}}{180 \text{ cm}}$$

$$\text{my pressure} \approx 38.1 \text{ N/cm}^2$$

***6.2** Answers will vary. Students may use height to estimate the cross-sectional area of each femur. Sample response: Since area increases as the square of the scale factor and my height is 160 cm, then the cross-sectional area of my femur is:

$$\frac{\text{area}}{7 \text{ cm}^2} = \left(\frac{160 \text{ cm}}{180 \text{ cm}}\right)^2$$

$$\text{area} \approx 5.5 \text{ cm}^2$$

Assuming that human bone can withstand a pressure of 1200 N/cm^2, the maximum weight can be found by multiplying the cross-sectional area by 1200. Therefore the maximum weight my femur will hold is:

$$5.5 \text{ cm}^2 \bullet \frac{1200 \text{ N}}{\text{cm}^2} = 6600 \text{ N}$$

6.3 Answers will vary. Students may mention limits on blood circulation, muscle strength, and nutrition needed to sustain growth, as well as other biological and environmental factors.

* * * * *

6.4 **a.** 1960 N

b. 19,600 N

c. The ratio of the heights of King Kong and an average gorilla is about $5.4/1$. Since weight increases by the cube of the scale factor, the ratio of the weights would be:

$$\left(\frac{5.4}{1}\right)^3 \approx \frac{160}{1}$$

Therefore, King Kong would weigh about 310,000 N, which is much more than the femur could support.

6.5 The cross-sectional area of the dinosaur femur is $12.5^2 \bullet \pi \approx 491 \text{ cm}^2$. The Tyrannosaurus has two femurs, so the total cross-sectional area is $2(491 \text{ cm}^2)$ or 982 cm^2. For a weight of 35,000 N, the pressure is:

$$\frac{35000 \text{ N}}{982 \text{ cm}^2} \approx 36 \text{ N/cm}^2$$

For a weight of 62,000 N, the pressure is about 63 N/cm^2.

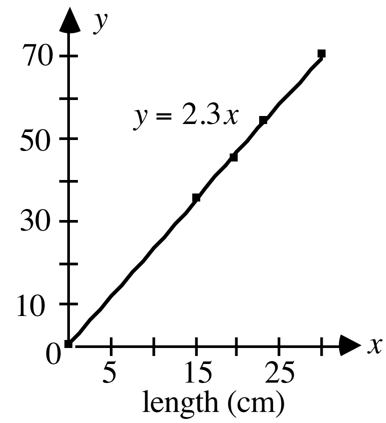
From Part **c** of the exploration, the pressure on a human femur for a 600-N person is 42.9 N/cm^2 —a little less than the average of the two Tyrannosaurus calculations.

* * * * * * * * * *

Answers to Summary Assessment (page 229)

If students collect their own data, they will probably conclude that siblings are not similar. The data in the table, however, can be modeled very well by equations that indicate similarity. As shown in the graph below, for example, the equation $y = 2.3x$ models the relationship between foot length and circumference for all four siblings.

Circumference versus Length



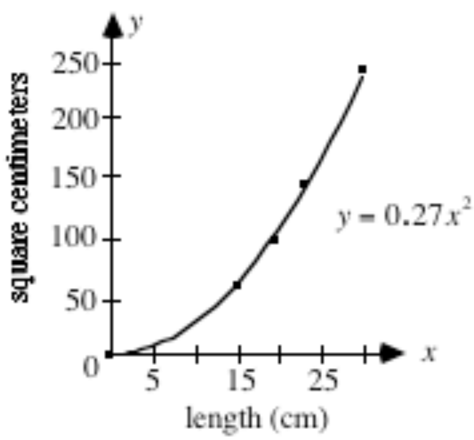
The table below shows the sum of the absolute values of the residuals for this model.

Sibling	**Length of Foot (x)**	**Circumference of Foot (y)**	**Predicted Circumference**	**Absolute Value of Residual**
A	15	35	34.5	0.5
B	19.5	45	44.85	0.15
C	23	54	52.9	1.1
D	30	70	69	1
			Sum	2.75

The fit appears to be good both from the graph end and the small residual sum.

Similarly, the equation $y = 0.27x^2$ models the relationship between foot length and footprint area.

Area versus Length

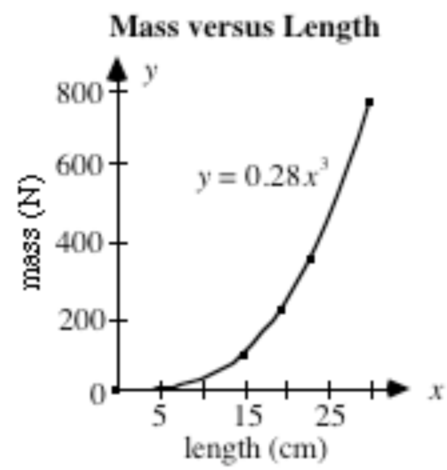


The table below shows the sum of the absolute values of the residuals for this model.

Sibling	Length of Foot (x)	Area of Footprint (y)	Predicted Area	Absolute Value of Residual
A	15	61	60.75	0.25
B	19.5	101	102.67	1.67
C	23	149	142.83	6.17
D	30	251	243	8
			Sum	16.09

The fit appears to be good both from the graph end and the small residual sum.

The equation $y = 0.28x^3$ models the relationship between foot length and weight.



The table below shows the sum of the absolute values of the residuals for this model.

Sibling	Length of Foot (x)	Body Weight (y)	Predicted Area	Absolute Value of Residual
A	15	93	94.5	1.5
B	19.5	208	207.6	0.4
C	23	340	340.7	0.7
D	30	756	756	0
			Sum	2.6

The fit appears to be good both from the graph end and the small residual sum.

Module Assessment

1. Robert Wadlow's height was 272 cm and his hand length was 32 cm. Predict the height of a similar person who has a hand length of 22 cm.

2. Two dolls are similar to each other. The larger one is 36 cm tall and the smaller one is 24 cm tall.

a. What is the ratio of their lengths?

b. What is the ratio of the areas of their footprints?

c. What is the ratio of their volumes?

d. Which of the ratios in Parts **a–c** most closely corresponds to the ratio of the masses of the two dolls? Explain your reasoning.

3. Willie caught a rainbow trout that was 30 cm long and had a mass of 1200 g. Assuming that all rainbow trout are similar,

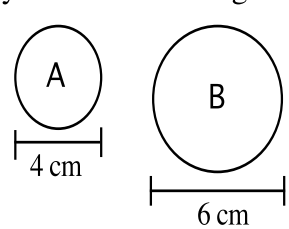
a. what is the mass (to the nearest 100 g) of a rainbow trout that is 37.5 cm long?

b. what is the length (to the nearest centimeter) of a rainbow trout that has a mass of 850 g?

4. The table below shows information on hand length and footprint area. Find an equation that models the data reasonably well. Use residuals to determine how will your model fits the data.

Hand Length (cm)	**Footprint Area (cm^2)**
0	0
9	55
14	115
18	190
21	260
22	300
32	640

5. The diagrams below represent cross sections of the femurs of two different four-legged animals. Animal A weighs 400 N, while animal B weighs 800 N. Which femur bears more weight per unit area? Carefully support your answer with good mathematical reasoning.



Teacher Note

If students did not complete Activities **5** and **6**, Problem **5** should be omitted.

Answers to Module Assessment

1. Since corresponding lengths are proportional,

$$\frac{\text{height}}{272\text{ cm}} = \frac{22\text{ cm}}{32\text{ cm}}$$
$$\text{height} = 187\text{ cm}$$

2.
a. 3:2

b. 9:4

c. 27:8

d. Sample response: The ratio of the volumes is closest to the ratio of the masses. Volume is proportional to the cube of the scale factor. Since mass is proportional to volume (for constant density), it also changes as the cube of the scale factor.

3.
a. Since mass is proportional to the cube of the scale factor,

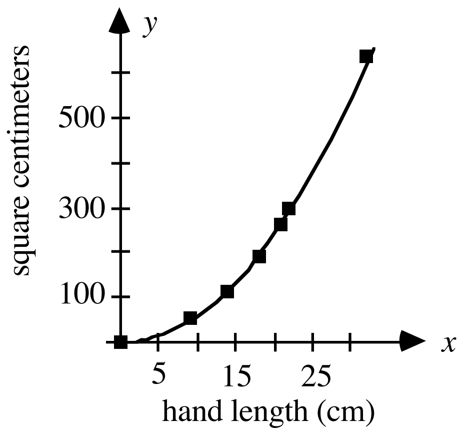
$$\frac{\text{mass}}{1200\text{ g}} = \left(\frac{37.5\text{ cm}}{30\text{ cm}}\right)^3$$
$$\text{mass} \approx 2300\text{ g}$$

b. Since length is proportional to the cube root of the mass,

$$\frac{\text{length}}{30\text{ cm}} = \sqrt[3]{\frac{850\text{ g}}{1200\text{ g}}}$$
$$\text{length} \approx 27\text{ cm}$$

4. As shown in the following graph, the equation $y = 0.6x^2$ is a very good model of the data. The low residual sum (53.2; mean of 7.6) also supports $y = 0.6x^2$ as a very good model of the data.

Foot Area versus Hand Length



5. The cross-sectional area of femur A is $\pi r^2 = 3.14 \bullet 2^2 = 12.56 \text{ cm}^2$. Since the animal has four legs, the pressure is:

$$\frac{400 \text{ N}}{4 \bullet 12.56 \text{ cm}^2} \approx 8 \text{ N/cm}^2$$

The cross-sectional area of femur B is $\pi r^2 = 3.14 \bullet 3^2 = 28.26 \text{ cm}^2$. The pressure is:

$$\frac{800 \text{ N}}{4 \bullet 28.26 \text{ cm}^2} \approx 7 \text{ N/cm}^2$$

Femur A supports more weight per unit area.

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Consortium for Mathematics and Its Applications (COMAP). *For All Practical Purposes*. New York: W. H. Freeman and Company, 1991.

Haldane, J. *Possible Worlds*. Boston: Harper and Brothers, 1955.

McMahon, T., and J. T. Bonner. *On Size and Life*. New York: Scientific American Books, 1983.

McFarlan, D., ed. *Guinness Book of World Records*. Toronto: Bantam Books, 1990.

Reiss, M. *The Allometry of Growth and Reproduction*. Cambridge: Cambridge University Press, 1989.

Schmidt-Nielsen, K. *Scaling, Why Is Animal Size so Important?* Cambridge: Cambridge University Press, 1984.

Flashbacks

Activity 1

1.1 **a.** What are integers?

b. What are rational numbers?

1.2 Describe several ways to write ratios.

1.3 Write the following as ratios in the form a/b where a and b are integers:

a. $1.5/2$

b. $0.75/2.3$

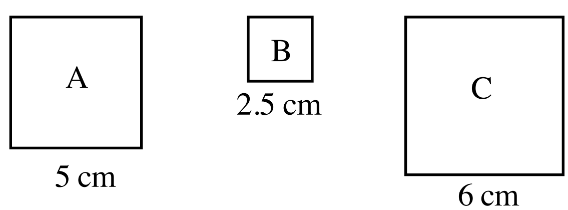
1.4 Solve each of the following proportions for x.

a. $\frac{4}{5} = \frac{x}{20}$ **b.** $\frac{2}{x} = \frac{7}{21}$

c. $\frac{1}{4} = \frac{5}{x}$ **d.** $\frac{x}{6} = \frac{7}{2}$

Activity 2

2.1 Find the area of each square in the diagram below:



2.2 What is the ratio of the areas of:

a. square A to square C?

b. square B to square C?

2.3 What is the ratio of the side lengths of:

a. square A to square C?

b. square B to square C?

2.4 Simplify each of the following expressions:

a. 3^2

b. -3^2

c. $(-3)^2$

Activity 3

3.1 Find the value of each of the following expressions:

a. 3^3

b. -3^3

c. $(-3)^3$

3.2 Find the volume of a cube with each of the following edge lengths:

a. 4 cm

b. *n* cm

Activity 4

4.1 **a.** Sketch the general shape of a linear graph.

b. Write the general equation of a line in slope-intercept form.

c. Write a specific equation of a line in slope-intercept form, then graph the equation.

4.2 **a.** Sketch the general shape of an exponential graph.

b. Write the general form of an exponential equation.

c. Write a specific exponential equation, then graph it.

4.3 Simplify each of the following expressions:

a. $|15 - 6|$

b. $|6 - 15|$

4.4 If the scale factor of two similar figures is a/b, what is the ratio of their areas?

Activity 5

5.1 Convert 4000 g to kilograms.

5.2 Write a sentence to describe each of the following equations:

a. $\frac{\$5.00}{\text{hr}} \bullet 6 \text{ hr} = \$30.00$

b. $\frac{30 \text{ words}}{\text{min}} \bullet \frac{60 \text{ min}}{\text{hr}} = \frac{1800 \text{ words}}{\text{hr}}$

Activity 6

6.1 Describe how you could determine if two objects were similar.

6.2 Given two similar objects with a scale factor of x/y, describe the following:

a. the ratio of their areas

b. the ratio of their volumes.

6.3 Find the area and circumference of a circle with a radius of:

a. 3 cm

b. 4.5 cm.

Answers to Flashbacks

Activity 1

1.1 **a.** The integers are the whole numbers (0, 1, 2, 3, ...) and their opposites (–1, –2, –3, ...).

b. Rational numbers are numbers that can be written in the form a/b where a and b are integers and $b \neq 0$.

1.2 Ratios can be written as decimals, fractions, percentages, using colons, or using the word *to*. For example, the ratio between 3 and 5 can be written as 0.6, $3/5$, 60%, 3:5 and 3 to 5.

1.3 **a.** $\frac{1.5}{2} = \frac{15}{20} = \frac{3}{4}$

b. $\frac{0.75}{2.3} = \frac{75}{230} = \frac{15}{46}$

1.4 **a.** $x = 16$ **b.** $x = 6$

c. $x = 20$ **d.** $x = 21$

Activity 2

2.1 The area of square A is 25 cm^2, the area of square B is 6.25 cm^2, and the area of square C is 36 cm^2.

2.2 **a.** $25/36$

b. $\frac{6.25}{36} = \frac{625}{3600} = \frac{25}{144}$

2.3 **a.** $5/6$

b. $\frac{2.5}{6} = \frac{25}{60} = \frac{5}{12}$

2.4 **a.** 9

b. –9

c. 9

Activity 3

3.1 **a.** 27

b. –27

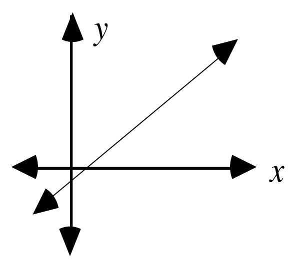
c. –27

3.2 **a.** 64 cm^3

b. n cm^3

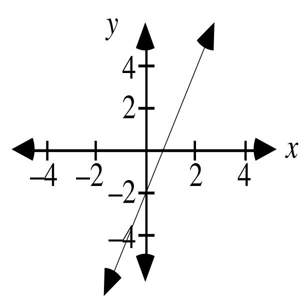
Activity 4

4.1 **a.** The following sample graph shows a linear equation in which the slope is positive.

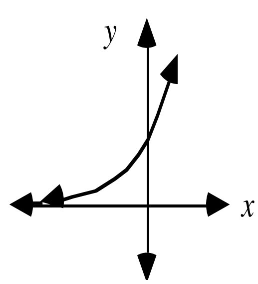


b. $y = mx + b$

c. The following sample response shows a graph of $y = 3x - 2$.

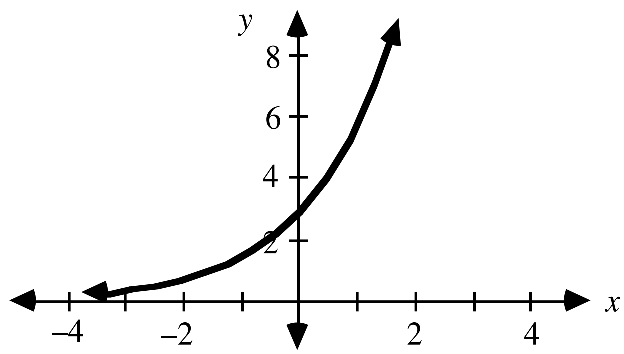


4.2 **a.** The following sample graph shows an exponential equation of the form $y = a \bullet b^x$ in which $b > 0$.



b. $y = a \bullet b^x$

c. The following sample response shows a graph of $y = 3 \bullet 2^x$.



4.3 **a.** 9

b. 9

4.4 The ratio of the areas is a^2/b^2.

Activity 5

5.1 $4000 \text{ g} \bullet \dfrac{1 \text{ kg}}{1000 \text{ g}} = 4 \text{ kg}$

5.2 **a.** Sample response: If I make $5.00 an hour and work for 6 hr, I will make $30.00.

b. Sample response: I can type 30 words a minute. Assuming I can continue this rate, since there are 60 min in an hour, I can type 1800 words in an hour.

Activity 6

6.1 Sample response: Two objects are similar when they have the same shape and the ratios of corresponding sides are equal.

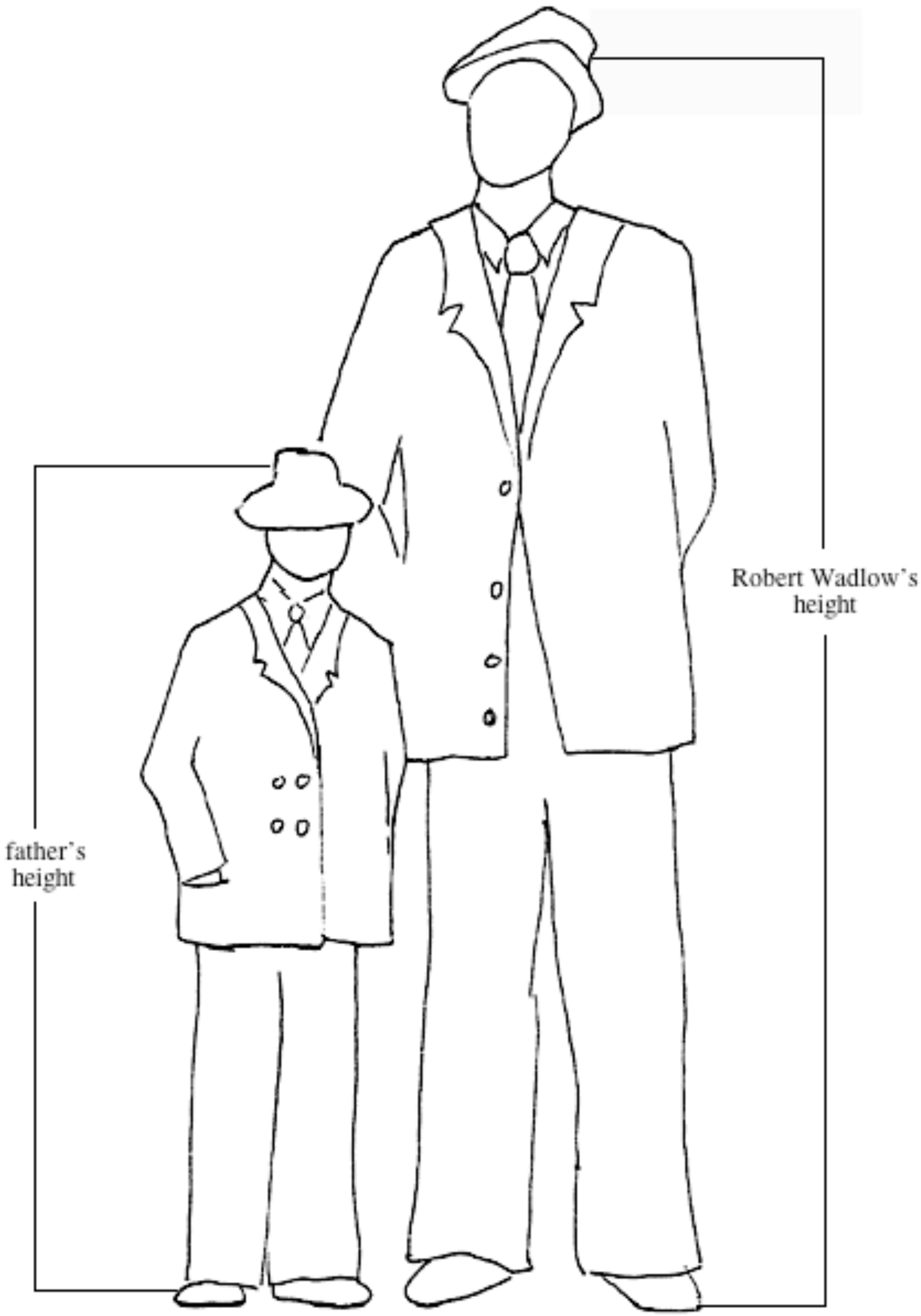
6.2 **a.** x^2/y^2

b. x^3/y^3

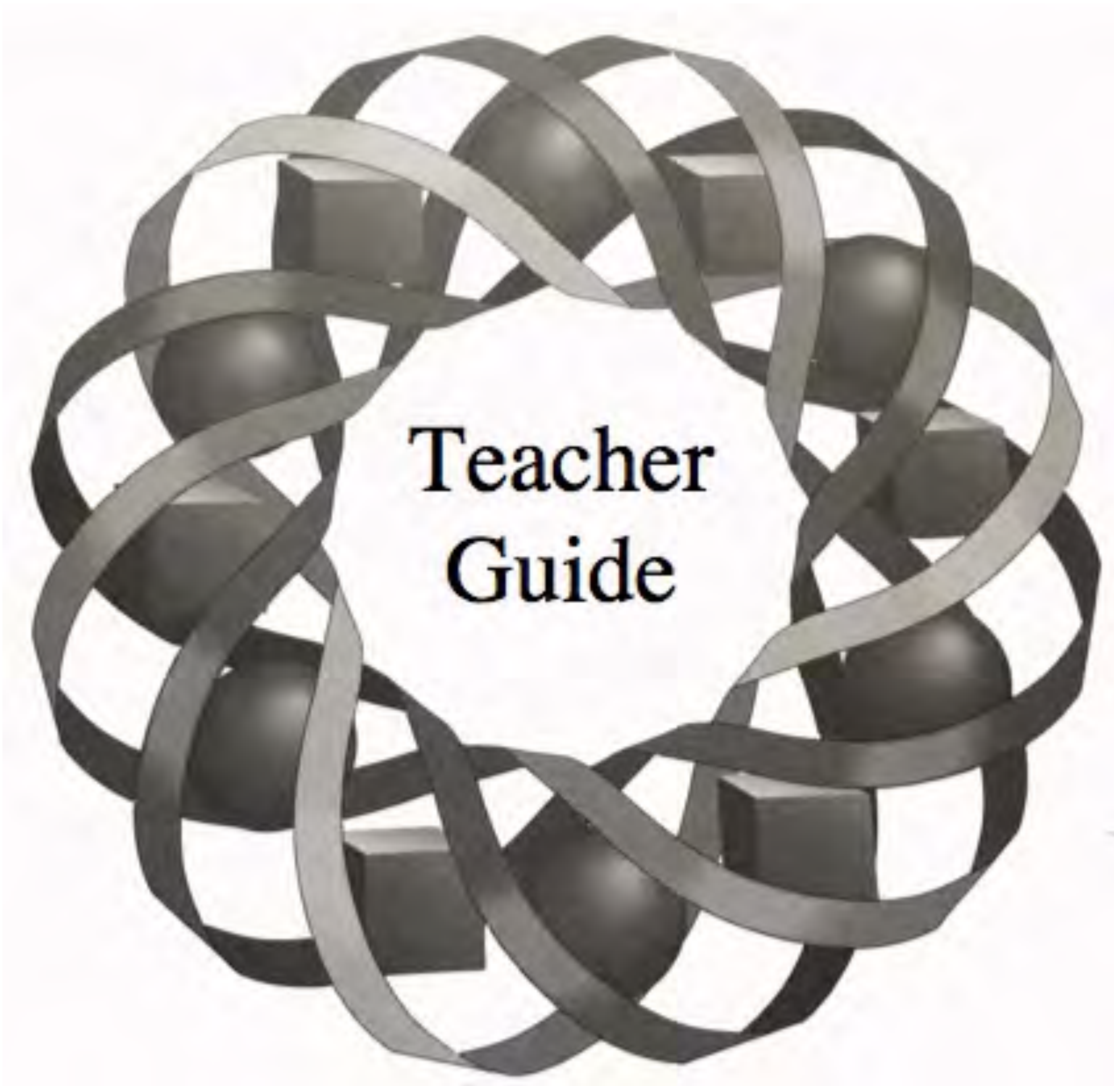
6.3 **a.** The area is approximately 28.3 cm^2; the circumference is approximately 18.8 cm.

b. The area is approximately 63.6 cm^2; the circumference is approximately 28.3 cm.

Wadlow Template



AIDS: The Preventable Epidemic



A recent congressional study reported that AIDS and HIV infections among teenagers rose an alarming 70% between 1990 and 1992. Are teens playing Russian roulette with their sex lives?

Shirley Bagwell • Patricia Bean • Karen Longhart • Michael Trudnowski

Teacher Edition

AIDS: The Preventable Epidemic

Overview

Students explore the graphical representation and interpretation of data collected on the spread of the AIDS virus and continue their investigations of exponential growth. They also use Venn diagrams and tree diagrams to organize information, determine sample spaces, and calculate probabilities.

Objectives

In this module, students will:

- collect and analyze data
- model the spread of disease using exponential equations
- use Venn diagrams to organize data and determine probabilities
- use the fundamental counting principle to determine the number of elements in a sample space
- use tree diagrams to determine sample spaces and calculate probabilities.

Prerequisites

For this module, students should know:

- how to make scatterplots
- how to graph equations
- how to calculate percent increase and percent decrease
- the characteristics of the graph of an exponential equation
- how to model exponential growth using the general equation $y = ab^x$
- how to calculate the residuals for a model
- how to find the sample space of an experiment
- how to calculate simple probabilities.

Time Line

Activity	1	2	3	Summary Assessment	Total
Days	3	2	2	2	9

Materials Required

Materials	Activity			
	1	**2**	**3**	**Summary Assessment**
red beans	X			
white beans	X			
dimes	X			
large, flat containers	X			
survey template		X		

Teacher Note

A blackline master of the template for the survey of childhood diseases appears at the end of the teacher edition for this module. Students must conduct this survey before beginning Activity **2**. You may wish to assign the survey on the first day of the module, then review the data to ensure that the exploration is appropriate for the survey results. Some sample data appears in Activity **2**.

For Activity **1**, cafeteria trays or pizza boxes provide suitable containers.

Technology

Software	Activity			
	1	**2**	**3**	**Summary Assessment**
spreadsheet	X			
graphing utility	X			

AIDS: The Preventable Epidemic

Introduction

(page 235)

The introduction describes how the AIDS epidemic is affecting the U.S. population, particularly teenagers.

Teacher Note

This module uses information available from the Centers for Disease Control (CDC) and the World Health Organization (WHO) to deal with the sensitive and important issue of AIDS. You may find it helpful to invite concerned parents to school to work through Activity **1**, then discuss their concerns. A sample letter to parents, as well as more information about AIDS and HIV, appears at the end of this teacher edition.

The CDC is the primary source for AIDS data in the United States. Because data for previous years is updated as more knowledge becomes available, you may notice some discrepancies between the information given in this module and the most current data. For example, the *HIV/AIDS Surveillance Report* for April 1992 identifies 86 AIDS cases before 1981, while the 1995 edition of the *Report* mentions 104 cases.

Discussion

(page 235)

a. Students may mention the dangers of contracting the disease and the resulting loss of productive lives. In addition to the devastating impact HIV has on individuals, the social and economic costs of caring for AIDS patients and children orphaned by AIDS are also high.

b. Sample response: People should become informed about how HIV is spread, then act responsibly.

c. Answers will vary. In a school with a senior class of 500 students, for example, approximately 100 students may be particularly at risk for HIV infection.

(page 235)

Activity 1

In this activity, students simulate the spread of an epidemic, then model the collected data with exponential equations.

Materials List

- small red beans (approximately 1 cup per group)
- small white beans (approximately 1 cup per group)
- dimes (one per group)
- large, flat containers such as cafeteria trays or pizza boxes (one per group)

Technology

- graphing utility
- spreadsheet

Exploration 1

(page 236)

The exploration simulates the spread of an infectious disease.

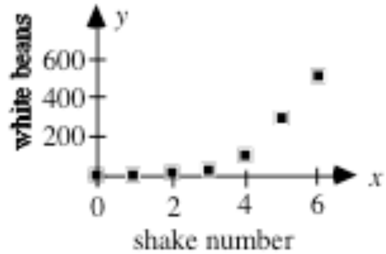
a–b. Students place a cup of red beans in the container, along with one white bean.

c. A dime is approximately 1 mm thick. If the dime touches both beans, then the red bean is near enough to contract the disease.

d–e. Sample data:

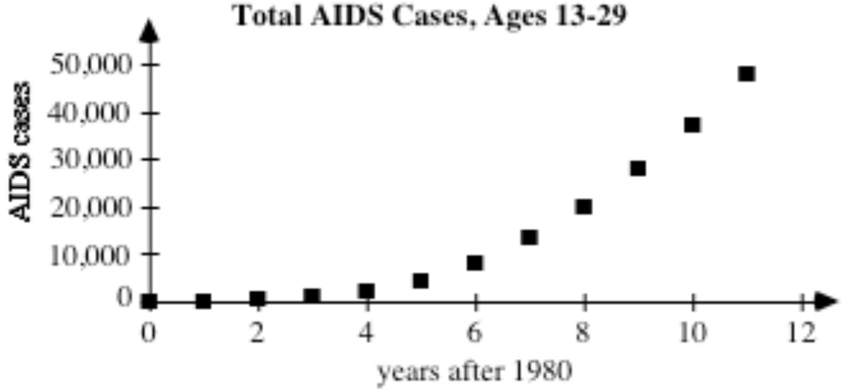
Shake No. (x)	No. of White Beans (y)
0	1
1	2
2	7
3	25
4	104
5	293
6	508

f. **1.** Sample scatterplot:



2. Sample response: The graph curves upward. As the shake number increases, the number of white beans increases.

g. Sample scatterplot:



h. The two scatterplots should have similar shapes.

Discussion 1

(page 237)

a. **1.** Sample response: The scatterplots all look similar and curve upward.

2. Answers may vary. Because the population of red beans is fixed, the growth rate between shakes may decrease as the shake number increases. (As more red beans are replaced with white beans, it becomes harder to find red beans to infect.) If the number of red beans were unlimited, the graph would continue to curve upward.

3. Answers may vary. The two scatterplots should have similar shapes.

b. Sample response: Because the number of people who can be infected is finite, the scatterplot of AIDS cases eventually must level out. As more people become infected, it becomes harder to find healthy people to infect.

c. **1.** Sample response: This data cannot be modeled well by linear equations because the scatterplots do not look linear and do not have constant slopes.

2. Sample response: This data could be modeled by exponential equations because the scatterplots look like exponential curves.

d. **1.** Sample response: The data collected during the simulation resembles the actual spread of HIV, as indicated by the similar shapes of the scatterplots.

2. Sample response: In the simulation, the spread of the disease was random and basically unchecked. The spread of HIV is not random and efforts continue to stop it. People can take precautions to prevent infection.

Teacher Note

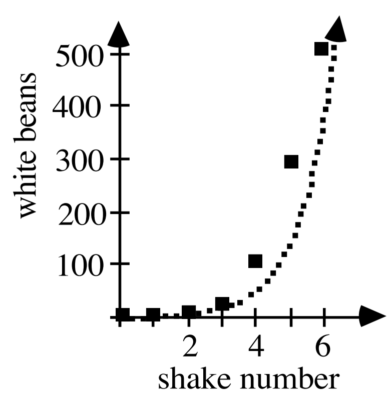
Throughout this module, students are asked to evaluate how well they believe an equation models a set of data. At this point in the curriculum, they should respond either in terms of the graph or by using the sum of the absolute values of the residuals. When comparing two different models, the smaller sum indicates the better fit. When evaluating a single model, the mean absolute value of the residual can be used. If this mean is relatively small when compared to the data, the model can be said to fit the data reasonably well. Students will be introduced to the principle of least squares in later modules.

Exploration 2 (page 238)

Students use exponential equations to model the data in Tables **1** and **2**. **Note:** Because epidemics usually reach a peak, then level out, they are typically modeled with logistic curves. For brief time periods, however, the spread of a disease often can be described exponentially.

a. **1.** Answers will vary. Using the sample data given in Exploration **1**, the equation $y = 1(2.7)^x$ provides a reasonable model. Students may find similar equations by using 1 as the initial population, then adjusting *b* until a reasonable fit is obtained.

2. Sample scatterplot:

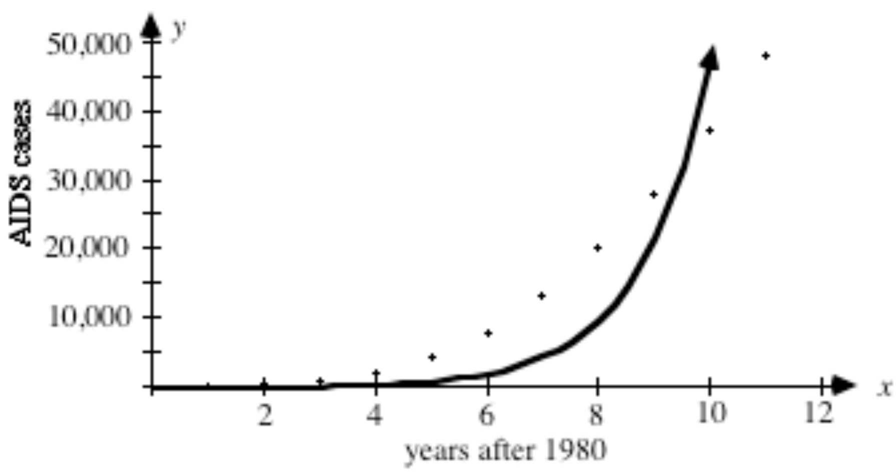


3. Using the sample data and the equation $y = 1(2.7)^x$, the sum of the absolute value of the residuals is 328.

Shake No. (x)	**Actual Value (y)**	**Predicted Value**	**Absolute Value of Residual**
0	1	1	0
1	2	3	1
2	7	7	0
3	25	20	5
4	104	53	51
5	293	143	150
6	508	387	121
		Sum	328

b. **1.** One possible model for the data in Table **2** is $y = 18(2.2)^x$. Students may find similar equations by using 18 as the initial population, then adjusting *b* until a reasonable fit is obtained. (Using a TI-92 calculator, the equation of the form $y = ab^x$ that best fits is $y = 35.6(2)^x$.)

2. Sample graph:



3. Using the data in Table **2** and the equation $y = 18(2.2)^x$, the sum of the absolute value of the residuals is 104,974.

Year after 1980 (*x*)	**Actual Value (*y*)**	**Predicted Value**	**Absolute Value of Residual**
0	18	18	0
1	79	40	39
2	301	87	214
3	886	192	694
4	2064	422	1642
5	4296	928	3368
6	7897	2041	5856
7	13,307	4490	8817
8	19,998	9878	10,120
9	27,999	21,731	6268
10	37,022	47,808	10,786
11	48,007	105,177	57,170
		Sum	104,974

Discussion 2

(page 239)

a. **1.** Sample response: Since the initial population is the number of white beans at shake 0, we substituted 1 for *a* in the general exponential equation $y = ab^x$. The value of *b* was found using the average growth rate from shake to shake.

2. Answers may vary. An exponential model appears to fit the sample data reasonably well. Both the sum of the absolute values of the residuals and the graphs appear to support this conclusion.

b. **1.** Sample response: Since the initial population is the number of cases in 1980, we substituted 18 for *a* in the general exponential equation $y = ab^x$. The value of *b* was found by trial and error.

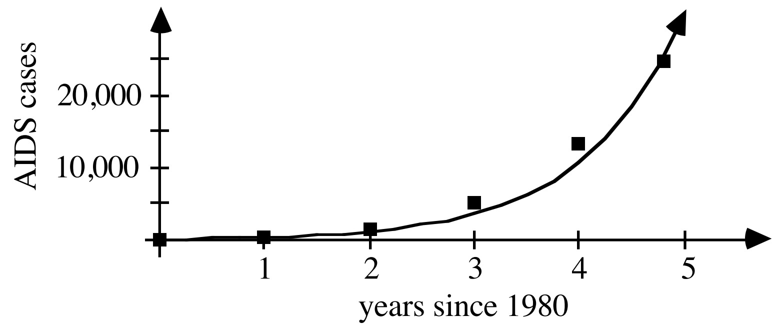
2. An exponential equation does not appear to model the data well, as shown by the sum of the absolute values of the residuals and the shapes of the graphs. Most of the points are far from the model and there are many more points below the curve than above it.

As noted in Discussion **1**, populations that grow unchecked are usually modeled well by exponential equations. However, the population in Table **2** is not growing unchecked. Although the number of AIDS cases continues to increase, the growth rate from year to year appears to be decreasing.

Assignment (page 239)

1.1 When $x = 0$, the population is 140. This can be calculated by dividing 210 by 1.5.

1.2 **a.** Sample graph:



b. One equation that models the data is $y = 94 \bullet 3^x$.

c. The sample equation given in Part **b** appears to fit the data reasonably well. The sum of the absolute value of the residuals is 6502.

d. The following predictions were made using the sample equation given in Part **b**.

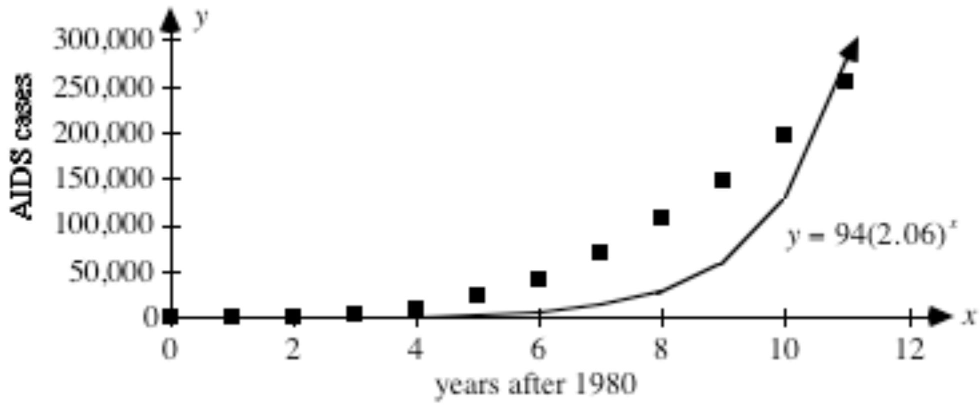
Year	No. of Cases
1986	68,526
1987	205,578
1988	616,734
1989	1,850,202
1990	5,550,606
1991	16,651,818

e. Sample response: The predictions seem high for the later years. In the exploration, the number of AIDS cases for the 13–29 age group started to level off after 1987.

1.3 **a.** Answers may vary. The predictions made using the sample equation in Problem **1.2d** are too high.

b. Sample response: There has been more education and public awareness about the disease. Since people are more careful about engaging in risky behavior, the number of AIDS cases is leveling off.

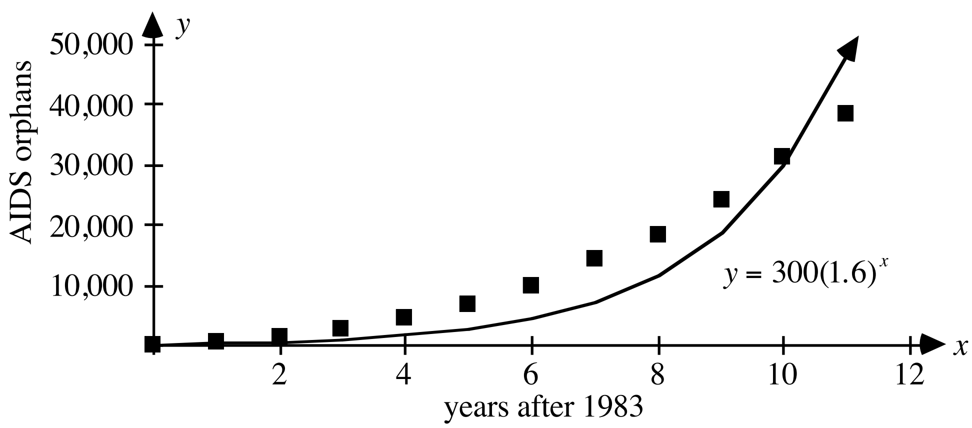
c. Sample graph:



d. One exponential equation that models the data is $y = 94(2.06)^x$.

e. The sample exponential equation does not model the data well. (The sum of the absolute values of the residuals is 366,148.) Students should justify their responses using residuals and graphs.

1.4 **a.** Sample scatterplot:

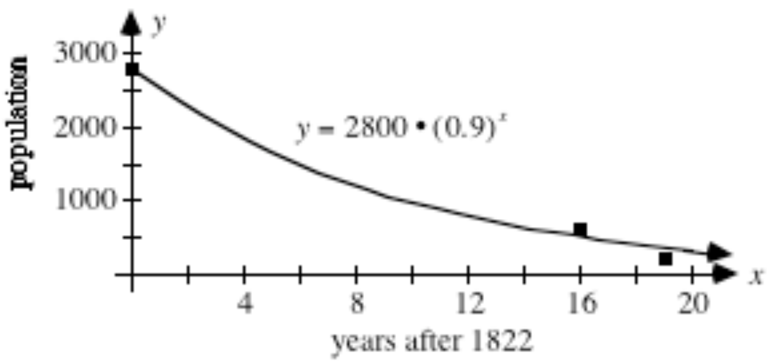


b. One exponential equation that models the data is $y = 300(1.6)^x$.

c. Answers will vary. Using the sample equation given in Part **b**, the sum of the absolute values of the residuals is 45,288.

d. The sample equation does not appear to model the data well. Students should use residuals and graphs to justify their responses.

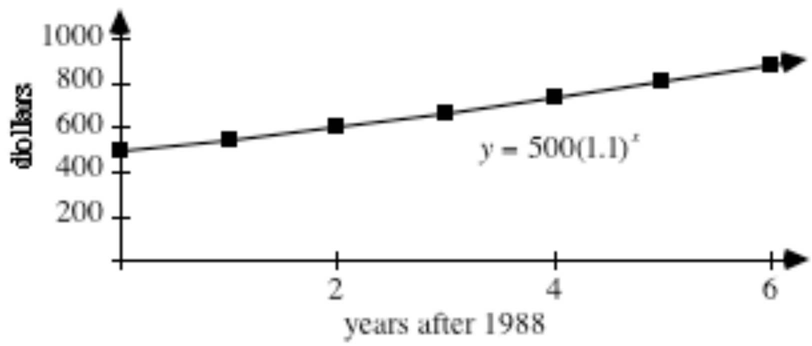
1.5 **a.** Answers may vary. An exponential equation that models the data is $y = 2800(0.9)^x$. This appears to be a reasonable model. (The sum of the absolute values of the residuals is 259.) Sample graph:



b. Three data points are typically not enough to determine an accurate model, unless more knowledge about the general trend is available.

* * * * *

***1.6** **a.** Sample scatterplot:



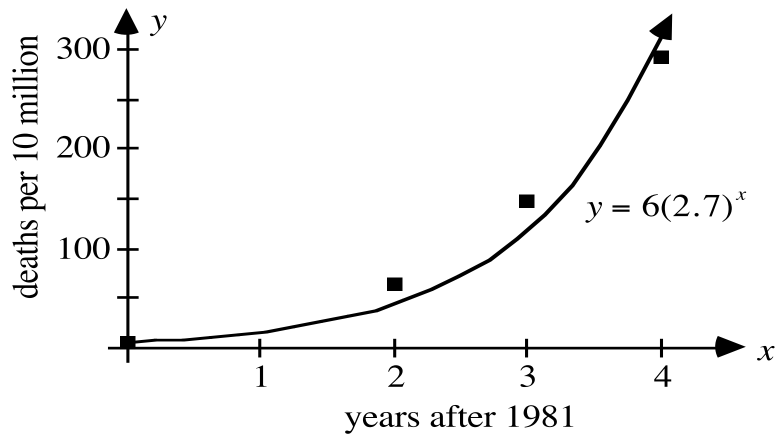
b. One exponential equation that models the data is $y = 500(1.1)^x$.

c. Answers may vary. The table below shows the predictions made using the sample equation given in Part **b**.

Year	Amount
1995	$974
1996	$1072
1997	$1179
1998	$1297
1999	$1427
2000	$1569

d. The sample equation models the data very well. The sum of the absolute values of the residuals is 1.

***1.7** **a.** Sample scatterplot:



b. One exponential equation that models the data is $y = 6(2.7)^x$.

c. The sample equation given in Part **b** appears to be a reasonable model. The sum of the absolute values of the residuals is approximately 77.

d. **1.** Using the sample equation given in Part **b**, the estimate is 16 deaths per 10 million.

2. Using the sample equation given in Part **b**, the estimate is 2325 deaths per 10 million.

e. The sample equation given in Part **b** provides a reasonable estimate for 1982 but not for 1987. You may wish to point out that using a model to make predictions outside the domain of the data can produce unreliable results.

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(page 243)

Activity 2

In this activity, students use Venn diagrams to organize data and determine probabilities. **Note:** Since the sources of HIV infection are often misunderstood, you may wish to invite a guest from the medical community to speak to your class.

Materials List

- survey template (one per student; a blackline master appears at the end of the teacher edition for this module)

Exploration

(page 244)

Students compile and organize the data collected in the childhood disease survey. **Note:** The survey must be completed before beginning the exploration.

a. Some sample data appears in the following table

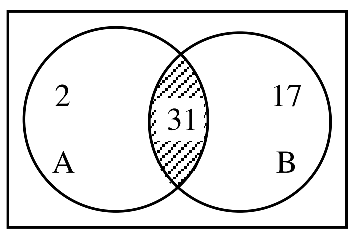
Individual	Disease				
	Mumps	Measles	Strep Throat	Chicken Pox	Whooping Cough
1			X	X	
2				X	
3			X	X	
4				X	
5			X	X	
6				X	
7			X	X	
8			X	X	
9			X	X	
10			X	X	
11			X	X	
12			X	X	
13			X	X	
14				X	
15			X	X	X
16				X	X
17			X	X	
18			X	X	
19			X	X	
20	X		X		
21			X	X	
22				X	
23			X	X	
24				X	
25			X	X	
26			X	X	
27				X	
28			X	X	
29			X	X	
30				X	
31				X	
32				X	
33		X	X	X	
34			X	X	
35			X	X	
36			X	X	
37			X	X	
38			X	X	
39			X	X	
40				X	
41				X	
42	X		X	X	
43	X		X	X	
44			X	X	
45			X		
46			X	X	X
47		X		X	
48				X	
49				X	
50				X	

b. The following responses are based on the sample data given in Part **a**.

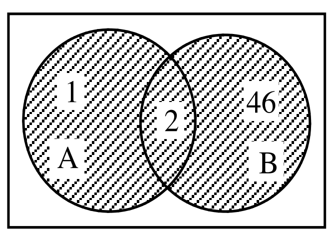
1. The number of individuals who have had strep throat is 33.
2. The number of individuals who have had chicken pox is 48.
3. The number of individuals who have had strep throat and chicken pox is 31.
4. The number of individuals who have had mumps or chicken pox is 49.
5. The number of individuals who have had strep throat, mumps, and chicken pox is 2.
6. The number of individuals who have had whooping cough, but have not had strep throat is 1.
7. No individuals in the survey have had whooping cough and measles.

c. The following Venn diagrams are based on the sample data given in Part **a**.

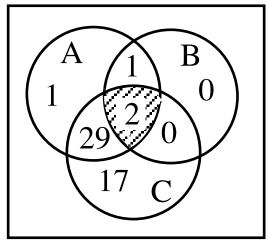
1. In this diagram, set A contains those who have had strep throat and set B contains those who have had chicken pox. The shaded area represents an intersection of sets.



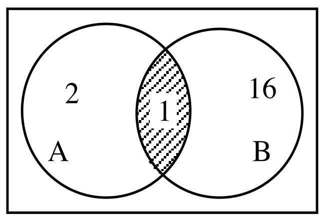
2. In this diagram, set A contains those who have had mumps and set B contains those who have had chicken pox. The shaded area represents a union of sets.



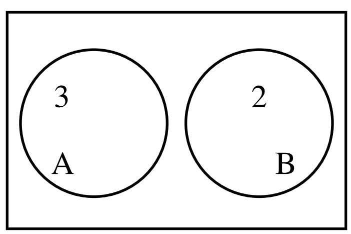
3. In this diagram, set A contains those who have had strep throat, set B those who have had mumps, and set C those who have had chicken pox. The shaded area represents an intersection of sets. **Note:** This can be written as $(A \cap B) \cap C$ or $A \cap B \cap C$. Parentheses are not necessary to show order.



4. In this diagram, set A contains those who have had whooping cough and set B contains those who have not had strep throat. The shaded area represents an intersection of sets.



5. In this diagram, set A contains those who have had whooping cough and set B contains those who have had measles. The intersection of these disjoint sets is the empty set.



d. 1. Using the sample data and the Venn diagram in Part **c1**, there are 31 individuals who have had both strep throat and chicken pox. The sample space contains 50 people. Therefore, the probability is $31/50 = 0.62 = 62\%$.

2. Using the sample data and the Venn diagram in Part **c5**, there are no individuals who have had both whooping cough and measles. Therefore, the probability is 0.

Discussion

(page 246)

a. Like AIDS, these childhood diseases are transferred from person to person and pose a threat to the public health. Some of these diseases are also caused by viruses. However, these viruses are not nearly as deadly as HIV and the modes of transmission are different.

b. Sample response: A Venn diagram is better because it shows more information visually. There is a large amount of information in the table that is not readily accessible.

c. Since interpreting information from a table for 50 individuals is difficult, a table for 250 million people would be almost impossible to understand. A Venn diagram makes the job much easier. **Note:** The logical processes used to create and analyze Venn diagrams resemble those used by computer databases.

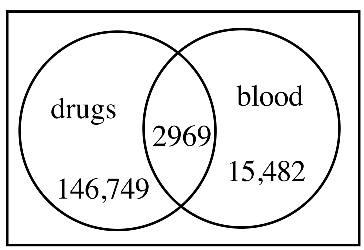
d. Sample response: A Venn diagram would have provided the necessary information in a more useful format.

e. By separating the data into categories, Venn diagrams facilitate counting the number of outcomes in both the event and the sample space.

Assignment

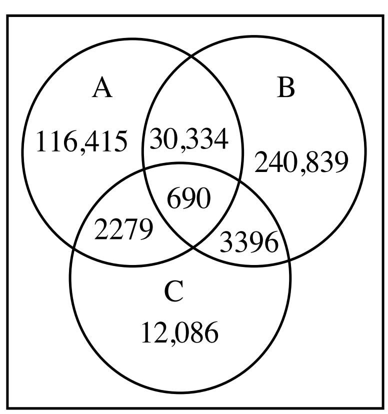
(page 247)

2.1 **a.** Sample diagram:



b. The probability is $146,749/165,200 \approx 0.89$ or 89%.

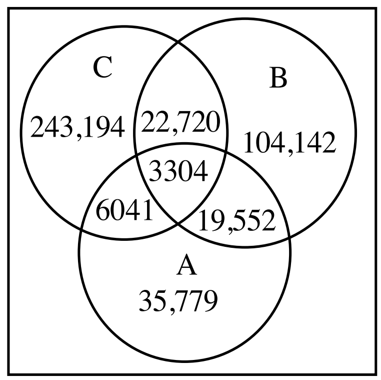
***2.2** **a.** Sample diagram:



b. The total number of AIDS cases is 406,039. This can be calculated by adding all the numbers shown in the Venn diagram. (This insures that every category is counted without duplication.)

c. The probability is $31{,}024/406{,}039 \approx 0.08$ or 8%.

***2.3** **a.** In the following sample diagram, set A contains those who contracted the disease through heterosexual contact, set B those who contracted the disease through intravenous drug use, and set C those who contracted the disease through male homosexual contact.



b. The probability is $22,856/434,732 \approx 0.05$ or 5%.

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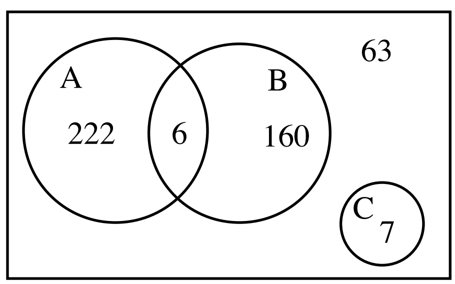
***2.4** **a.** Sample diagram:



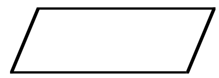
b. **1.** $6/458 \approx 0.01 \approx 1\%$.

2. $63/458 \approx 0.14 = 14\%$

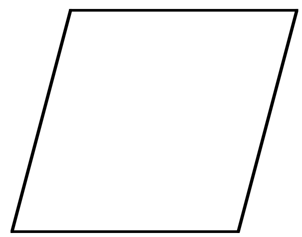
3. 0

2.5 **a.** Sample trapezoids:

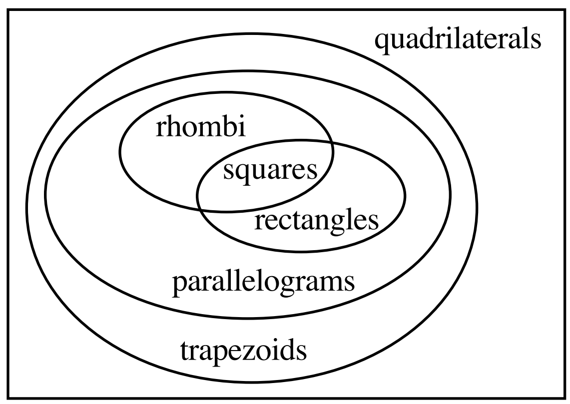
b. The sample parallelogram shown below is neither a rhombus nor a rectangle.



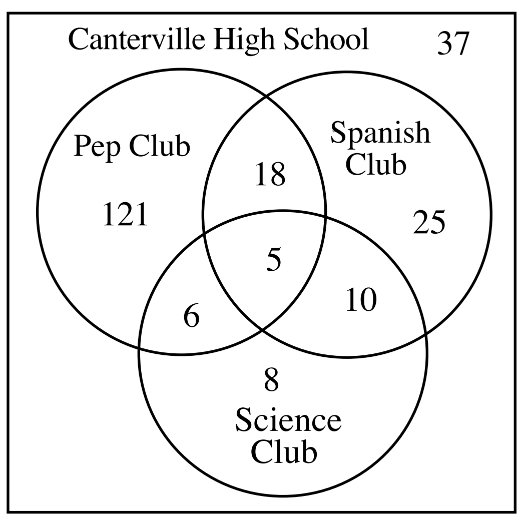
c. Sample rhombus:



d. Sample Venn diagram:



2.6 a–b. Sample Venn diagram:



c. The probability is $15/193 \approx 0.08 = 8\%$.

d. The probability is $37/230 \approx 0.16 = 16\%$.

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Research Project (page 250)

As research on HIV and AIDS continues, the health organizations responsible for monitoring the disease constantly update their information. This project should give students an opportunity to identify good sources of recent data.

(page 250)

Activity 3

In this activity, students use tree diagrams and the fundamental counting principle to determine sample sizes and calculate probabilities.

Materials List

- none

Exploration (page 250)

While playing the game "Epidemic," students review the concepts of sample size and theoretical probability and investigate tree diagrams and the fundamental counting principle.

a-b. Students should read the rules described in Part **a** before beginning the exporation.

1. You may wish to ask the "players" to stand at the front of the class.

2. Outcomes must be randomly selected. You may wish to write each outcome on a slip of paper, place the appropriate slips in three different containers, then ask each player to draw one slip from the appropriate container. For example, player 2 must choose either "measles" or "healthy."

3–4. Players should not reveal their selections until each member of the class has recorded a prediction.

5. You may wish to record the numbers of correct and incorrect predictions on a blackboard.

c. **1.** Students may use the tree diagram in Figure **3** to list the possible outcomes.

2. There are 24 outcomes in the sample space. This can be determined by counting the number of terminal branches in the tree diagram.

3. Sample response: Using a tree diagram helps organize the information and makes it easier to keep track of all the possibilities.

d. **1.** $1/4 = 0.25 = 25\%$

2. The $18/24 = 0.75 = 75\%$.

e. There are $4 \bullet 2 \bullet 3 = 24$ possible outcomes.

f. **1.** There are $3 \bullet 3 \bullet 3 = 27$ possible outcomes.

2. $1/27 \approx 0.04 = 4\%$

g. **1.** There are $4 \bullet 2 \bullet 3 \bullet 5 = 120$ possible outcomes.

2. $18/120 = 0.15 = 15\%$

Discussion

(page 252)

a. The probability of making a correct prediction is $1/24 \approx 0.04 = 4\%$.

b. Sample response: You would expect one correct prediction. **Note:** The actual expected value is:

$$30 \bullet \frac{1}{24} = \frac{30}{24} = 1.25$$

c. Answers may vary. Sample response: Yes, it is possible, since one combination is "healthy, healthy, healthy." However, since it is only 1 out of 24 possible combinations, it is not very likely.

d. Answers may vary. Sample response: Yes, it is possible, since there are 6 outcomes in which all three players choose a disease. It is not very likely, however, because the probability is only $6/24 = 25\%$.

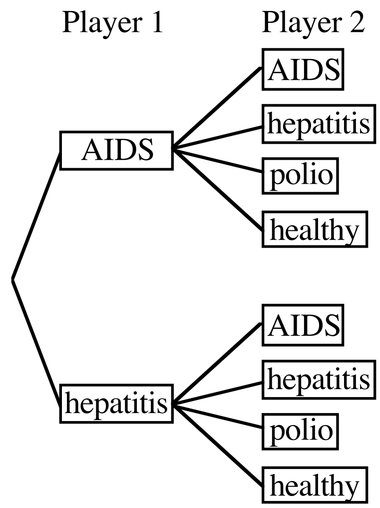
e. Sample response: Tree diagrams help to organize the information so that the sample space can be more easily determined.

f. Answers may vary. For problems with large sample spaces, the fundamental counting principle provides a quick way to calculate the total number of outcomes.

Assignment

(page 253)

***3.1** **a.** Sample tree diagram:



b. If A represents AIDS, h represents hepatitis, P represents polio, and H represents healthy, the possible outcomes are AA, Ah, AP, AH, hA, hh, hP and hH.

c. Using the fundamental counting principle, the number of outcomes is $2 \bullet 4 = 8$.

d. **1.** $1/8 \approx 0.13 = 13\%$

2. $2/8 = 25\%$

3.2 Answers may vary. Using the fundamental counting principle, the number of outcomes is $2 \bullet 2 \bullet 2 = 8$. If students use a tree diagram, they may list the outcomes as follows: player 1 has choices A and B, player 2 has choices C and D, while player 3 has choices E and F. The 8 possible outcomes are ACE, ACF, ADE, ADF, BCE, BCF, BDE, and BDF.

***3.3** The formula for the number of outcomes is $5kn$.

3.4 **a.** $98/100 = 0.98 = 98\%$

b. $90/100 = 0.90 = 90\%$

c. Answers may vary. Sample response: Yes, there is a significant difference. For every 1,000,000 times condoms are used, a 98% probability of effectiveness corresponds to 20,000 failures. However, a 90% probability of effectiveness corresponds to 100,000 failures. A difference of 80,000 in the number of failures could have a significant impact on the spread of AIDS and other diseases.

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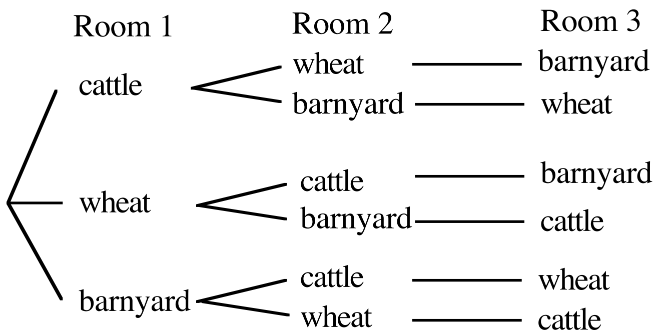
3.5 Using the fundamental counting principle, there are $2 \bullet 2 \bullet 1 \bullet 5 \bullet 5 \bullet 3 = 300$ possible schedules.

***3.6** **a.** Sample tree diagram:



b. $1/6 \approx 0.17 = 17\%$

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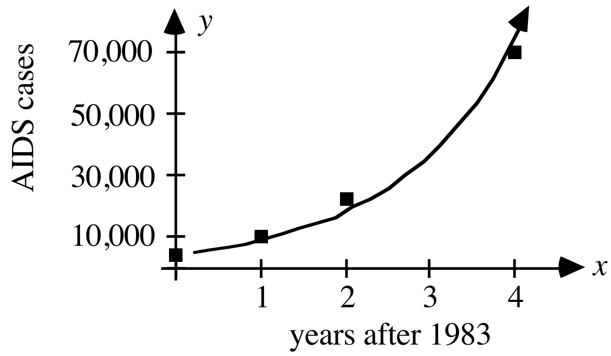
Answers to Summary Assessment

(page 255)

1. **a.** Sample scatterplot:



b. One exponential equation that models the data is $y = 4589(2)^x$.

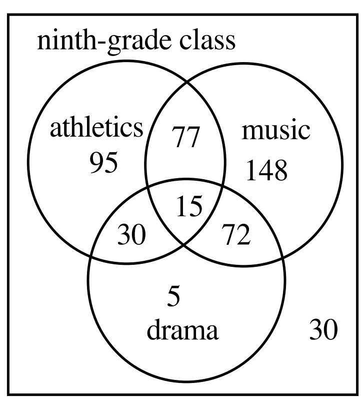
c. **1.** Using the sample equation from Part **b**, the estimated number of AIDS cases is 36,712.

2. Using the sample equation from Part **b**, the estimate differs from the actual value by approximately 11%.

d. **1.** Students may determine their predictions by tracing the graph. Using the sample equation from Part **b**, the predicted number of cases reaches 500,000 approximately 6.8 years after 1983.

2. Sample response: This prediction does not seem reasonable. It means that a seven-fold increase would occur from 1987 to 1990, despite improved efforts to control the spread of AIDS. **Note:** The actual number of cases in 1990 was 193,878.

2. **a.** Sample Venn diagram:

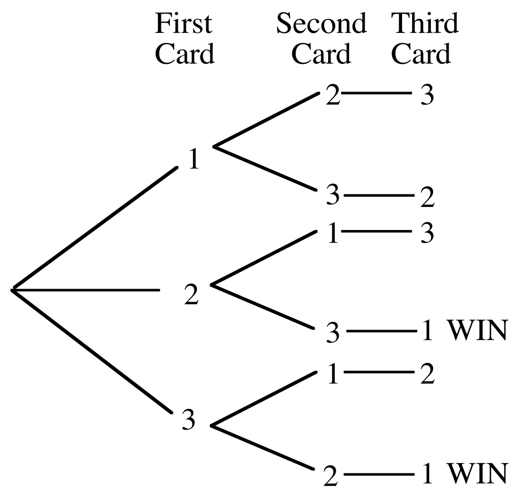


b. There are 122 students involved in drama. This can be calculated by adding all the numbers in the appropriate circle in the Venn diagram: $30 + 15 + 72 + 5 = 122$.

c. There are 95 ninth graders involved only in athletics only. The probability, therefore, is $95/472 \approx 0.20 = 20\%$.

d. Sample response: There are 437 students involved in athletics or music. This can be calculated by adding the numbers in the appropriate two circles: $95 + 30 + 15 + 77 + 148 + 72 = 437$.

3. Answers may vary. In this game, there are 2 ways out of 6 to win: by turning over a 2, then a 3, then a 1, or by turning over a 3, then a 2, then a 1. Students may justify their responses using a tree diagram that shows all possible outcomes.



Module Assessment

1. Under certain conditions, a population of the bacteria that causes cholera can quadruple every hour. The table below shows some data collected from a laboratory culture of this bacteria.

Time (hours)	Number of Bacteria
0	25
1	100
2	400
3	1600
4	6400
5	25,600

a. Make a scatterplot of the data. Let y represent the number of bacteria and x represent time in hours.

b. Find an exponential equation that models the data.

c. Does your equation appear to be a good model for the data? Explain your response.

d. Use your equation to predict when the number of bacteria will exceed 1 million.

2. The table below shows some data on the spread of a flu virus on a college campus.

Day	No. of Cases	Day	No. of Cases
0	1	5	54
1	2	6	119
2	5	7	260
3	11	8	540
4	24		

a. Make a scatterplot of the data. Let y represent the number of cases of flu and x represent the number of days.

b. Find an exponential equation that models the data.

c. Does your equation appear to be a good model for the data? Explain your response.

d. Use your equation to predict the number of cases on day 10.

3. In 1990, a population of elk contained 1500 animals. Since then, the herd has declined as shown in the table below.

Year	**Elk**
1991	1050
1992	735
1993	484
1994	350
1995	265

a. Make a scatterplot of the data. Let y represent the number of elk and x represent the number of years after 1990.

b. Will an exponential equation provide a good model for this data? Justify your response.

c. Predict the number of elk in the population in the year 2000.

4. The table below shows one high school's enrollment for three science courses.

Course	**Students**
chemistry	46
physics	22
biology	42
chemistry and biology	7
physics and biology	6
chemistry and physics	4
chemistry, biology, and physics	4

a. Organize this information in a Venn diagram.

b. Identify the numbers of students enrolled only in chemistry, only in physics, and only in biology.

c. If a student is selected at random from the three science courses, what is the probability that the student is enrolled in chemistry or physics?

5. Consider an experiment in which a coin is flipped four times.

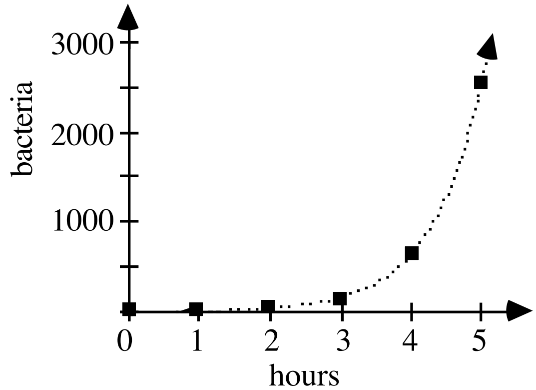
a. Draw a tree diagram that models all the possible outcomes.

b. Determine the size of the sample space.

c. Determine the probability of obtaining four heads.

Answers to Module Assessment

1. **a.** Sample scatterplot:

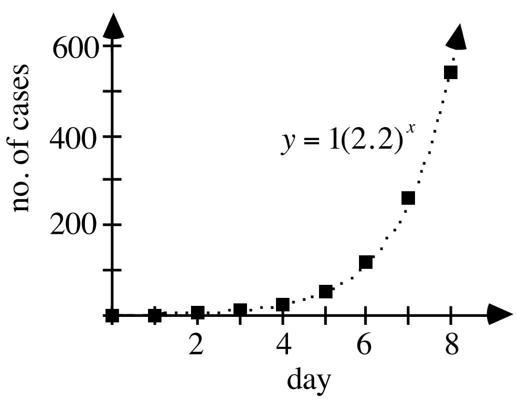


b. An exponential equation that models the data is $y = 25(4)^x$.

c. The sample equation models the data points exactly. The sum of the absolute values of the residuals is 0.

d. Using the equation in Part **b**, the number of bacteria will exceed 1 million after 8 hr.

2. **a.** Sample scatterplot:

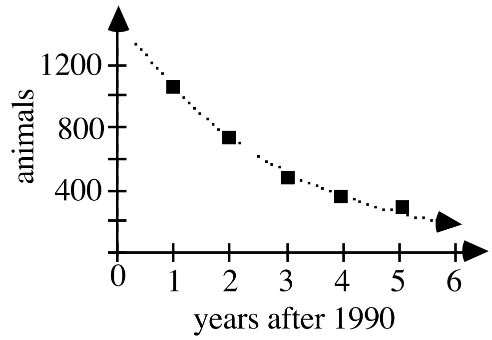


b. An exponential equation that models the data is $y = 1(2.2)^x$.

c. The sample equation models the data very well. The sum of the absolute values of the residuals is 29.

d. Using the sample equation given in Part **b**, the predicted number of cases on day 10 is 2656.

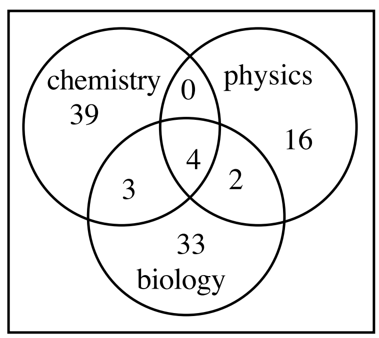
3. **a.** Sample scatterplot:



b. One exponential equation that models the data well is $y = 1500(0.7)^x$. The sum of the absolute value of the residuals is 54.

c. Replacing x with 10 in the sample equation given in Part **b**, the predicted number of animals in the year 2000 is 30.

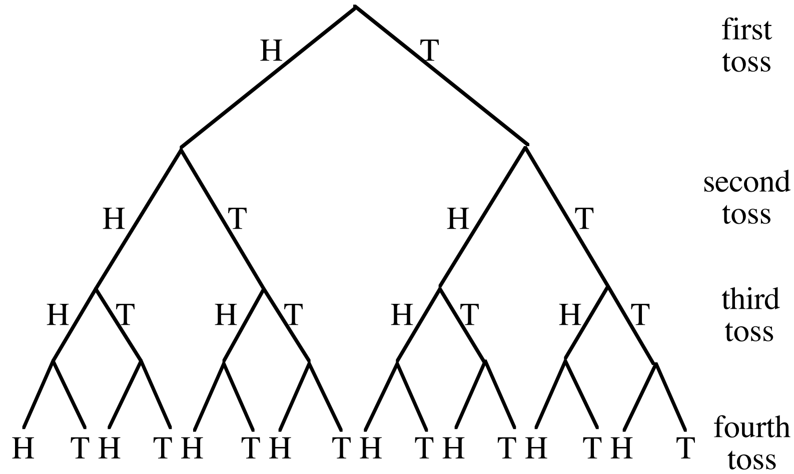
4. **a.** Sample Venn diagram:



b. The number of students enrolled only in chemistry is 39, only in physics is 16, and only in biology is 33.

c. The total population is 97. The number of students enrolled in either chemistry or physics is 64. Therefore, the probability is $64/97 \approx 0.66 = 66\%$.

5. **a.** Sample tree diagram:



b. The sample space has 16 equally likely outcomes.

c. Since there is only 1 way of getting all heads in the 16 possible outcomes, the probability is $1/16 \approx 0.063 = 6.3\%$.

Selected References

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Michaels, D., and C. Levine, "Estimates of the Number of Motherless Youth Orphaned by AIDS in the United States." *Journal of the American Medical Association* 268 (December 23/30, 1992): 3456–3461.

Oregon State Health Division. *AIDS: The Preventable Epidemic*. Portland, OR: Oregon State Health Division, 1987.

Stearn, E. W., and A. E. Stearn. *The Effect of Smallpox on the Destiny of the Amerindian*. Boston: Bruce Humphries Publishers, 1945.

U.S. Bureau of the Census. *Statistical Abstract of the United States*, 1995. Washington, D.C.: U.S. Government Printing Office, 1995.

For more information on AIDS and HIV, call the Centers for Disease Control (CDC) National AIDS Hotline: 1-800-342-2437.

Flashbacks

Activity 1

1.1 Draw the general shape of the graph of an exponential equation in the form $y = ab^x$ when $a > 0$, $b > 1$, and $x \geq 0$.

1.2 Calculate the average percent increase or decrease, per decade, for the data in the table below.

Year	Population	Year	Population
1940	450	1970	2375
1950	1034	1980	3104
1960	1682	1990	3864

1.3 **a.** Determine the absolute value of each of the following:

1. –2

2. 3

3. –7

4. 4

b. Find the sum of the absolute values you determined in Part **a**.

1.4 Use the following table to complete Parts **a** and **b**.

Day	Population	Day	Population
1	57	4	22
2	42	5	19
3	31	6	18

a. Make a scatterplot of the data. Let *y* represent the population and *x* represent the days.

b. Graph the exponential equation $y = 71(0.8)^x$ on the same set of axes as the scatterplot.

c. Assume that $y = 71(0.8)^x$ is used to model the data. Determine the sum of the absolute values of the residuals.

Activity 2

2.1 **a.** Consider an experiment which involves rolling two ordinary dice.

1. What is the probability of obtaining a 3 on one die and a 6 on the other?

2. What is the probability that the sum of the two dice is 7?

b. What is the sample space for the roll of one die?

2.2 At a football game, 10,432 of the fans have blond hair, 4996 have red hair, and 12,688 have dark hair.

a. What percentage of the fans have blond hair?

b. What percentage of the fans have dark hair?

c. What percentage of the fans have red hair?

2.3 **a.** What number is 86% of 120?

b. What percentage of 360 is 30?

2.4 A store sells red, yellow, and green apples. Of the 820 apples in stock, 20% are yellow, 65% are red, and the rest are green. How many of each type of apple are there?

Activity 3

3.1 Consider the set of two-digit numbers formed from the digits 3, 4, 5, 6, and 7.

a. List all the elements in the set.

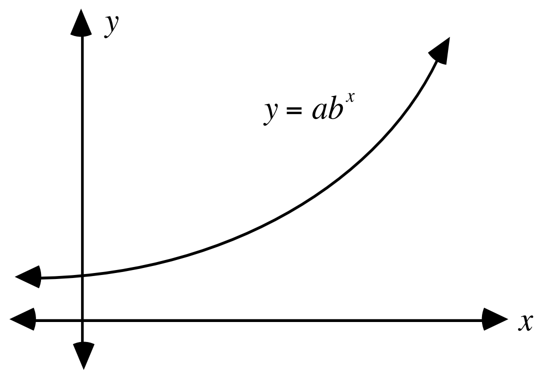
b. What is the probability that a number in this set includes a 3?

c. What is the probability that a number in this set includes a 4 or a 5?

Answers to Flashbacks

Activity 1

1.1 Sample graph:



1.2 The average percent increase per decade is approximately 58%. This can be calculated as follows:

$$[(1034 - 450)/450 + (1082 - 1034)/1034 + (2375 - 1682)/1682 + (3104 - 2375)/2375 + (3864 - 3104)/3104]/5$$

1.3 **a.** **1.** 2

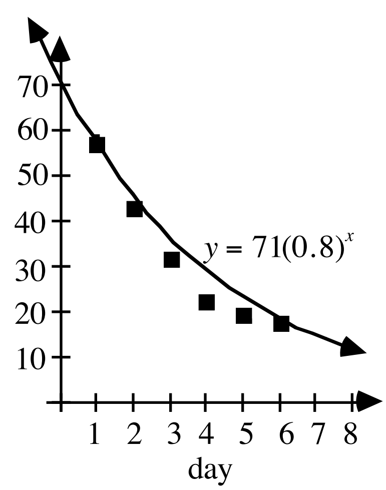
2. 3

3. 7

4. 4

b. $2 + 3 + 7 + 4 = 16$

1.4 **a–b.** Sample graph:



b. The sum of the absolute values of the residuals is approximately

21.

Activity 2

2.1 **a.** **1.** The probability is $2/36 \approx 6\%$.

2. The probability is $6/36 \approx 17\%$.

b. The sample space for the roll of one die is $\{1,2,3,4,5,6\}$.

2.2 **a.** $10,432/28,116 \approx 0.37 = 37\%$

b. $12,688/28,116 \approx 0.45 = 45\%$

c. $4996/28,116 \approx 0.18 \approx 18\%$

2.3 **a.** $0.86(120) = 103.2$

b. $30/360 \approx 8\%$

2.4 The number of red apples is $0.65(820) = 533$. The number of yellow apples is $0.20(820) = 164$. The number of green apples is $0.15(820) = 123$.

Activity 3

3.1 **a.** The set contains the following 25 numbers: {33, 34, 35, 36, 37, 43, 44, 45, 46, 47, 53, 54, 55, 56, 57, 63, 64, 65, 66, 67, 73, 74, 75, 76, 77}.

b. $9/25 = 36\%$

c. $16/25 = 64\%$

Sample Letter to Parents

Date:

Dear Parents:

On (date), your student will begin a new module in math class. The title of the module is "AIDS: The Preventable Epidemic." This module deals with a sensitive and important issue. I would like to invite you to the school on (date and time) to read the module and participate in its first activity. Following the activity, we will discuss the technology used in the class as well as any of your concerns or questions. Please sign and return this letter.

Sincerely,

Teacher's Name

____
Student Name

I will attend the meeting.	I do not wish to attend.
_________________________	_________________________
Parent's or guardian's signature	Parent's or guardian's signature

Survey Template

Conduct a survey of the incidence of childhood diseases in your family. Complete the table below, marking an X in the appropriate cell for each member of your family, including yourself, who has had a particular disease. **Note**: Save this data for use in Activity **2**.

Disease	**Family Member**						
	Self	**2**	**3**	**4**	**5**	**6**	**7**
mumps							
measles							
strep throat							
chicken pox							
whooping cough (pertussis)							

Conduct a survey of the incidence of childhood diseases in your family. Complete the table below, marking an X in the appropriate cell for each member of your family, including yourself, who has had a particular disease. **Note**: Save this data for use in Activity **2**.

Disease	**Family Member**						
	Self	**2**	**3**	**4**	**5**	**6**	**7**
mumps							
measles							
strep throat							
chicken pox							
whooping cough (pertussis)							

More Information about Viruses, HIV, and AIDS

Viruses are minute, disease-causing agents that depend on host cells for growth and reproduction. They can affect plants, animals, and humans. Human viral diseases include polio, herpes, smallpox, mononucleosis, measles, chicken pox, rabies, and influenza.

The human immune system provides one defense against viruses. It acts both to prevent infection and reduce the severity of the illness if infection occurs. The skin and mucous membranes form the first layer of defense. They make it more difficult for disease-causing agents to enter the body.

If a virus does invade the body, white blood cells—another component of the immune system—provide an internal defense. B-lymphocytes and T-lymphocytes are two types of white blood cells. T-lymphocytes, more commonly called T-cells, help activate B-lymphocytes when an infection is present. The B-lymphocytes, or B-cells, are responsible for the production of antibodies—the molecules that attack and destroy viruses and bacteria. The production of antibodies increases the ability of the body to prevent re-infection by the same agent.

Medical laboratories provide another form of disease prevention: vaccines. Vaccines contain a weakened or dead strain of an actual disease-causing agent. The immune system responds to the vaccine by producing the antibodies necessary to fight the disease. These antibodies remain in the body to fight re-infection. In the United States, vaccines for smallpox and polio have greatly reduced the number of cases of these serious diseases.

AIDS is caused by the Human Immunodeficiency Virus (HIV). Transmission of HIV occurs through the exchange of body fluids. Exchange can occur through high-risk behaviors such as intravenous drug use or sexual intercourse. It can also occur through the birthing process and blood transfusions. No documented HIV infections have occurred through casual contact, including contact with tears or saliva.

When HIV infection occurs, the virus enters a type of T-cells known as T4 cells. It may remain inactive for a time, often 8 to 11 years. During this time, infected persons may not be aware of their condition unless specifically tested. Consequently, a person with no symptoms at all may unknowingly spread the disease to many others.

Once the virus becomes active, it begins to reproduce. This results in the death of T4 cells and the spread of the infection. If enough T4 cells die, the immune system begins to fail. Other diseases that the body would normally fight off become killers.

People are diagnosed with AIDS when they develop conditions which indicate that their immune system is damaged. Some of these symptoms include:

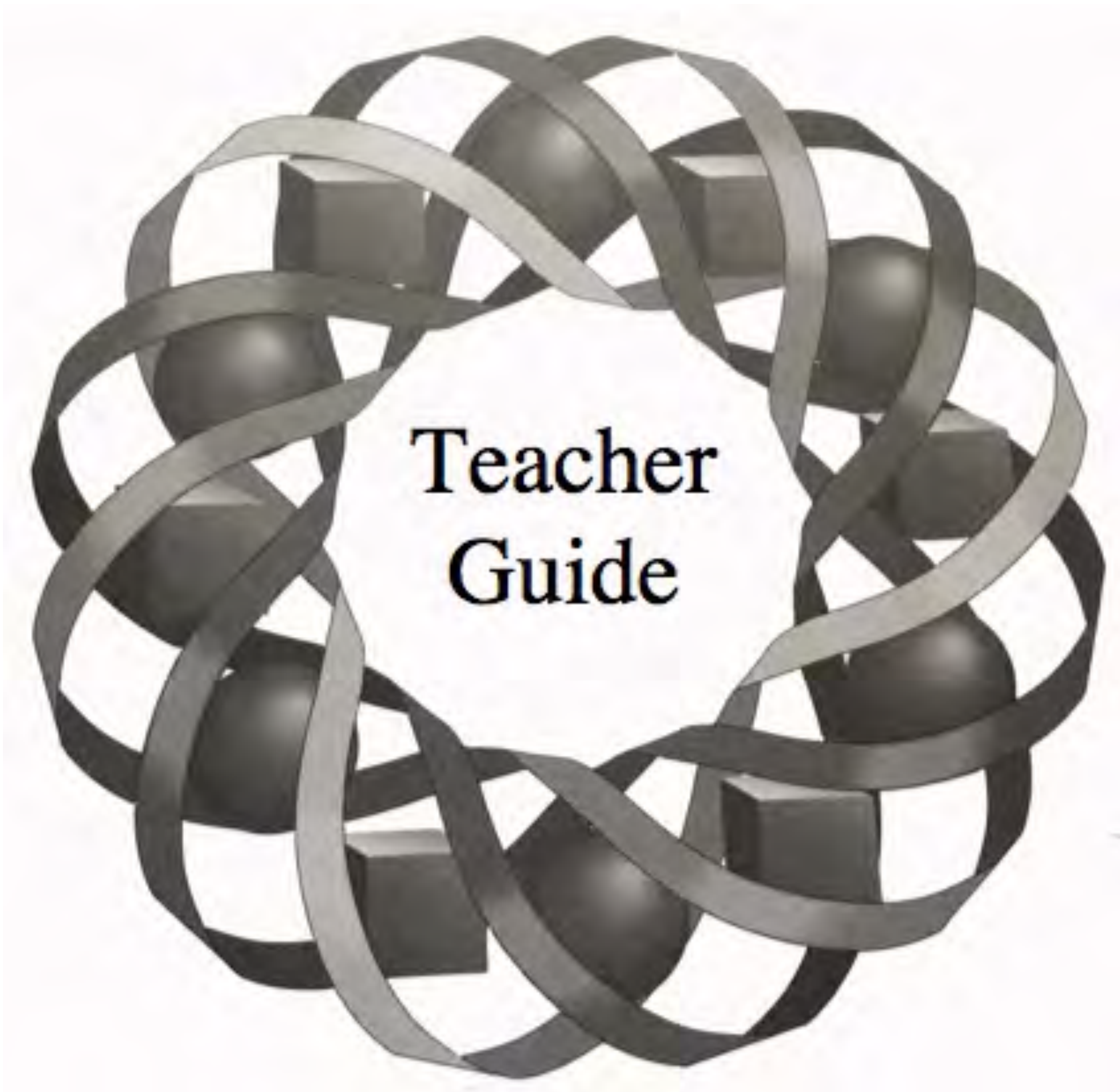
- swollen glands for two months or longer
- continuing fever, chills, and night sweats for two or more months
- weight loss, in excess of 10 pounds, not caused by diet or exercise
- unexplained and continued fatigue
- continuing diarrhea
- continuing dry cough and shortness of breath
- continuing infections
- white spots in the mouth
- reddish-purple blotches on the skin, mouth, nose, eyelids, and rectum.

Currently, there is no vaccine for HIV. When a virus reproduces, it may change, or mutate. Since HIV mutates much faster than any other known virus, very few individual viruses are exactly alike. Some mutations are severe enough to allow HIV to escape the antibodies produced to fight it. Unless scientists find a new way to combat the disease, people now infected with HIV will eventually develop AIDS and die.

The following table compares HIV with the virus that causes the common cold:

Common Cold	**AIDS**
• cold virus invades • cold symptoms appear • B-lymphocytes produce antibodies • cold virus destroyed by antibodies • immune system still intact • person becomes well	• HIV invades • B-lymphocytes produce antibodies • HIV attacks T4 cells • HIV reproduces • immune system breaks down • AIDS develops • susceptibility to disease increases • person dies

Going in Circuits



After leaving for work in the morning, a bus driver must make six stops before returning home. There are more than 500 ways to complete this route. Which ones are best? In this module, you explore some ways to find an efficient route without testing all the possibilities.

John Carter • Janet Higgins • Paul Swenson • Steve Yockim

Teacher Edition

Going in Circuits

Overview

This module encourages students to analyze complicated problems involving many choices in a systematic manner. Students explore Hamiltonian circuits and use algorithms to plan a tour for a hypothetical rock band.

Objectives

In this module, students will:

- use tree diagrams to organize information and solve problems
- use the fundamental counting principle
- use factorial notation
- solve problems involving Hamiltonian circuits
- develop algorithms for solving problems.

Prerequisites

For this module, students should know:

- how to use tree diagrams.

Time Line

Activity	**1**	**2**	**3**	**Summary Assessment**	**Total**
Days	3	2	3	2	10

Materials Required

Materials	**Activity**			
	1	**2**	**3**	**Summary Assessment**
metric rulers	X	X		
Hamilton High template	X			
U.S. map		X		
state map			X	

Going in Circuits

Introduction
(page 261)

This introduction describes the constraints involved in a scheduling problem. Students use the information given to complete the exploration in Activity **1**.

Note: To introduce students to some other applications of the mathematics in this module, you may wish to use the video *Trains, Planes and Critical Paths*, Module I.3 in the video series *For All Practical Purposes*.

(page 262)

Activity 1

This activity introduces students to some fundamentals of graph theory.

Materials List

- metric rulers (one per student)
- Hamilton High School template (one copy per group)

Teacher Note

Since the exploration requires many measurements and calculations, students should work in pairs or small groups. A blackline master of the Hamilton High School template appears at the end of the teacher edition for this module.

Exploration
(page 263)

Students plan a route through the high school that takes less than 55 min and satisfies all the requirements described in the introduction. All paths must follow hallways. Students should measure distances on the template provided, convert these measurements to actual distances using the given scale, then calculate the corresponding walking times.

The table below shows the distances and times used to calculate the sample responses. (You may wish to initiate a discussion about precision and accuracy of measurement.)

Mr. Chang	**Ms. Garcia**	**Locker**	**Library**	**Guidance Office**	**Main Office**	
63.8 m 47.8 sec	112.5 m 84.4 sec	120 m 90 sec	150 m 112.5 sec	150 m 112.5 sec	105 m 78.8 sec	**Study Hall**
	48.8 m 36.6 sec	56.3 m 42.2 sec	86.3 m 64.7 sec	75 m 56.3 sec	71.3 m 53.4 sec	**Mr. Chang**
		37.5 m 28.1 sec	60 m 45 sec	48.8 m 36.6 sec	75 m 56.3 sec	**Ms. Garcia**
			45 m 33.8 sec	67.5 m 50.6 sec	112.5 m 84.4 sec	**Locker**
				60 m 45 sec	105 m 78.8 sec	**Library**
					45 m 33.8 sec	**Guidance Office**

a.

1. The distance from the Guidance Office to Mr. Chang's room is 75 m. The distance from Mr. Chang's room to Study Hall is 63.8 m.

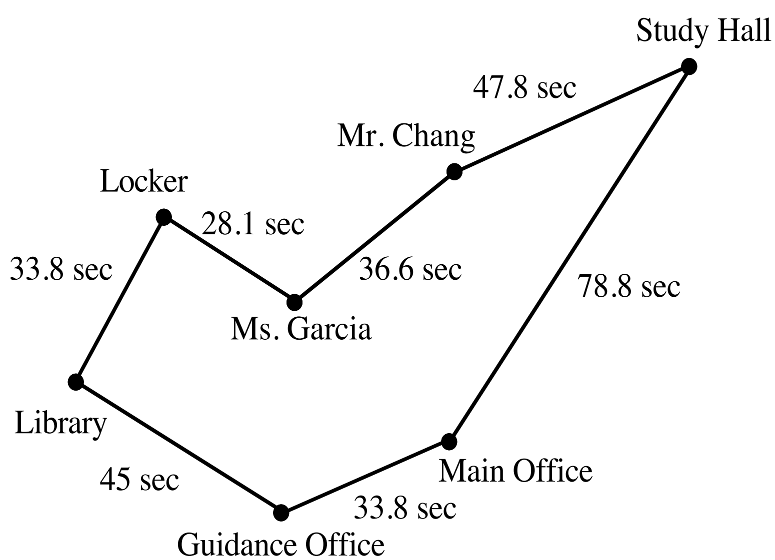
2. The walk from the Guidance Office to Mr. Chang's takes 56.3 sec. The walk from Mr. Chang's room to Study Hall takes 47.8 sec.

3. The total walking time for the route in Figure **2** is approximately 6 min.

4. The total time for the route after all tasks are completed is 50 min.

b.

1–2. Students may use a variety of techniques to select a route. One possible route is shown in the following graph.



3. The total time for the sample route is 49.1 min.

Discussion

(page 264)

a. Sample response: The better route of the two is the one in Part **b**, because it takes less time to complete.

b. Answers may vary. Sample response: The route that satisfies all the requirements described in the introduction and takes the least amount of time is the best solution.

c. To be assured of finding the best solution to the problem (no matter what the criteria), students would have to try all possible solutions.

d. The vertices of the graphs in the exploration represent locations in Hamilton High School.

e. The edges of the graphs in the exploration represent distances or times between locations in Hamilton High School.

f. The graph from Part **b** of the exploration is a Hamiltonian circuit because it is a path that starts and stops at the same vertex, and visits each vertex exactly once.

g. Sample response: The sequence of vertices *A–B–D–C–E–B–A* is not a Hamiltonian circuit because vertex *B* is visited twice.

Assignment

(page 265)

1.1

a. Any number of vertices connected by edges is acceptable as long as no edge is repeated. Sample response: *A–B–C*.

b. Any circuit that does not visit all the points in the graph is acceptable. Sample response: *A–B–D–E–F–A*.

c. Any path that does not begin and end at the same vertex is acceptable. Sample response: *A–B*.

d. Since a circuit must be a path by definition, there are no circuits that are not paths.

e. The path must visit each vertex exactly once and return to the starting vertex. Sample response: *D–F–A–B–C–E–D*.

1.2

a. The path from *A* to *D* that takes the least amount of time is *A–E–D*. This path requires 50 min.

b. One possible Hamiltonian circuit is *A–B–C–D–E–A*. The total time for this route is 180 min.

c. Another Hamiltonian circuit is *A–E–B–C–D–A*. The total time for this route is 240 min.

d. Of the two Hamiltonian circuits, the one with the lesser total time would be more efficient.

***1.3** **a–b.** Sample graph:

c. The shortest path on the graph is from Fairbanks to Tok.

d. Answers will vary. Sample response: *F–V–A–F*.

e. Sample response: Two Hamiltonian circuits that yield different total distances are: *F–T–A–V–F* (1915 km) and *T–F–A–V–T* (1801 km).

* * * * *

1.4 **a.** The six possible Hamiltonian circuits, where *S* represents the start, are: *S–J–K–L–S*, *S–L–K–J–S*, *S–J–L–K–S*, *S–L–J–K–S*, *S–K–L–J–S*, and *S–K–J–L–S*.

b. In order to make a selection, students should find the total time for each circuit identified in Part **a**. The table below shows the time in hours for each edge of the graph. The rate of 7 km/hr was used to estimate walking time on edges *S–K* and *J–K*, since one path goes over a mountain and the other goes through thick brush. The time for the path around the lake, *S–L*, was calculated using the rate of 9.6 km/hr.

	L	***K***	***J***
S	0.27	0.35	0.22
J	0.28	0.27	
K	0.22		

The following table shows the total time for each circuit.

Circuit	**Total Time (hr)**
S–J–K–L–S	0.97
S–L–K–J–S	0.97
S–J–L–K–S	1.07
S–L–J–K–S	1.17
S–K–L–J–S	1.07
S–K–J–L–S	1.17

The scouts in the competition should hope to be assigned one of the first two routes listed in the table above, as these should require less time to complete.

* * * * * * * * * *

(page 267)

Activity 2

In this activity, students use the fundamental counting principle and factorials to determine the number of possibilities in several different problem settings.

Materials List

- map of the United States (optional)

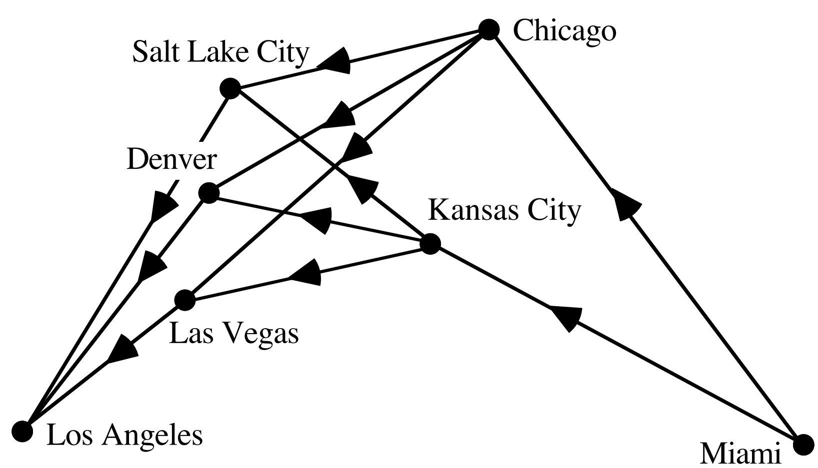
Teacher Note

To help students visualize the relative positions of the cities described in the exploration, you may wish to display a map of the United States.

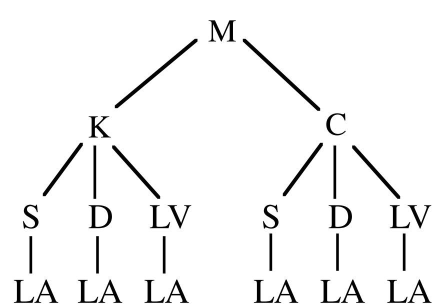
Exploration

(page 267)

a. Sample digraph:



b. Sample tree diagram:



c. From the tree diagram above, there are 6 possible routes.

Discussion

(page 268)

a. Methods may vary. Sample response: The total was determined by counting the number of different routes in the tree diagram.

b. Sample response: Using a tree diagram would not work well. Adding more cities would add many more branches.

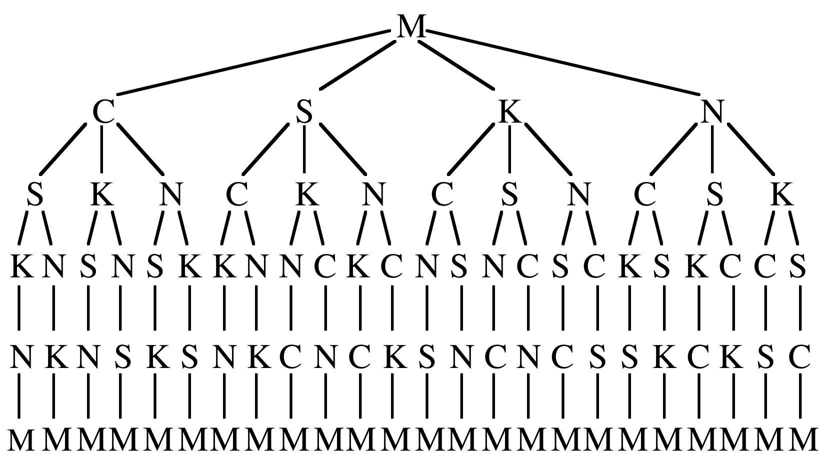
c. Since there are 2 routes from Miami, 3 each from Chicago and Kansas City, and 1 each from Salt Lake City, Las Vegas, and Denver, using the fundamental counting principle gives $2 \bullet 3 \bullet 1$ or 6 routes.

Assignment

(page 268)

***2.1** **a.** Using the fundamental counting principle, there are $4 \bullet 3 \bullet 2 \bullet 1 \bullet 1 = 24$ possible routes.

b. Sample tree diagram:



c. Sample response: From a point representing Miami, draw a branch to each of the four remaining cities. From each of these four cities, draw a branch to each of the three cities not yet visited along the route. Continue this process until each city is connected to every other city. From each of the last points, draw a single branch to represent the trip back to Miami. The total number of routes can be found by counting the number of terminal branches.

2.2 **a.** $7 \bullet 6 \bullet 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1 = 5040$

b. Sample response: It would take too much time to list 5040 different schedules.

2.3 **a.** Using the fundamental counting principle, there are $4 \bullet 3 \bullet 2 \bullet 1 = 24$ possible orders for the band members.

b. $4 \bullet 3 \bullet 2 \bullet 1 = 4!$

***2.4** **a.** There are $10 \bullet 9 \bullet 8 \bullet 7 \bullet 6 \bullet 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1 = 10! = 3{,}628{,}800$ possible orders.

b. Since the band knows the first song they will play, there are $9 \bullet 8 \bullet 7 \bullet 6 \bullet 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1 = 9! = 362{,}880$ possible orders.

c. The order in which the numbers are multiplied makes no difference because multiplication is commutative. Playing the hit song fourth makes no difference in the number of possibilities.

d. Since the band knows the first and second song they will play, there are $8 \bullet 7 \bullet 6 \bullet 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1 = 8! = 40{,}320$ possible orders.

e. There are $n!$ possible orders.

2.5 **a.** **1.** $1 = 1! = 1$

2. $2 \bullet 1 = 2! = 2$

3. $3 \bullet 2 \bullet 1 = 3! = 6$

4. $9 \bullet 8 \bullet 7 \bullet 6 \bullet 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1 = 9! = 362{,}880$

b. If there are n cities in a tour and the first city is known, there are $(n-1)!$ possibilities. **Note:** If students have trouble seeing this pattern, encourage them to examine more examples, such as 5-, 6-, 7-, and 8-city tours.

2.6 **a.** $6! = 720$

b. Sample response: Listing 720 routes would be very time consuming and the chance of making mistakes is high.

* * * * *

2.7 $9 \bullet 8 \bullet 7 \bullet 6 \bullet 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1 = 9! = 362{,}880$

2.8 **a.** $7! = 5040$

b. $\dfrac{9!}{7!} = \dfrac{362{,}880}{5040} = 72$ or $\dfrac{9!}{7!} = 9 \bullet 8 = 72$

c. Sample response:

$$\begin{aligned} 200! &= 200 \bullet 199 \bullet 198 \bullet 197 \bullet \cdots \bullet 1 \\ &= 200(199 \bullet 198 \bullet 197 \bullet \cdots \bullet 1) \\ &= 200(199!) \end{aligned}$$

d. Sample response:

$$\begin{aligned} \frac{100!}{95!} &= \frac{100 \bullet 99 \bullet 98 \bullet 97 \bullet 96 \bullet 95!}{95!} \\ &= 100 \bullet 99 \bullet 98 \bullet 97 \bullet 96 \\ &= 9{,}034{,}502{,}400 \end{aligned}$$

e. $\dfrac{n!}{(n-1)!} = \dfrac{n(n-1)!}{(n-1)!} = n$

2.9 **a.** $40 \bullet 39 \bullet 38 = 59,280$

b. It would take over 10 days to try all the possible combinations.

* * * * * * * * * *

(page 270)

Activity 3

In this activity, students use algorithms to find reasonable solutions to optimization problems.

Materials List

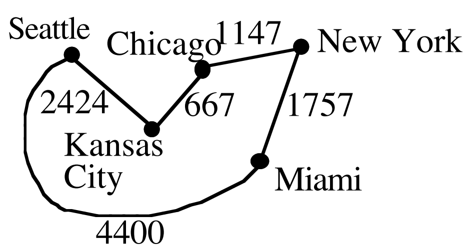
- U.S. map (optional)
- state map (one per student)

Exploration

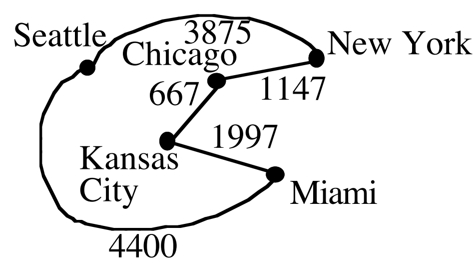
(page 271)

a. Sample response: Start in Miami, travel to Kansas City, Seattle, Chicago, New York, and then back to Miami. The total distance traveled along this route is 10,120 km.

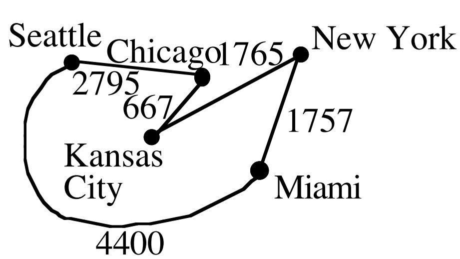
b. The figure shows the five Hamiltonian circuits created by the nearest neighbor algorithm.



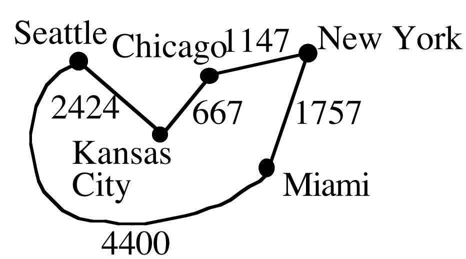
1. Starting in Miami: 10,395 km



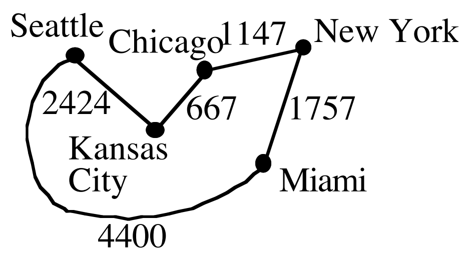
2. Starting in New York: 12,186 km



3. Starting in Chicago: 11,384 km



4. Starting in Kansas City: 10,395 k



5. Starting in Seattle: 10,395 km

c. **1.** The five circuits created using the nearest neighbor algorithm produce only three different graphs.

2. The graphs produced by starting in either Seattle, Miami, or Kansas City give the shortest total distance.

d. Answers may vary. For example, the tour described in the sample response to Part **a** results in a shorter total distance than the shortest one produced by the nearest neighbor algorithm.

Students should realize that although the nearest neighbor algorithm yields a tour with a reasonably short distance, it may not produce the shortest possible tour.

Discussion

(page 272)

a. **1.** The nearest neighbor provides a relatively quick method to solve an optimization problem when there are too many possibilities to list.

2. By checking all possible routes, a brute force algorithm guarantees finding the shortest distance.

b. The nearest neighbor algorithm should be used when a reasonable solution is acceptable or when it is not feasible to use a brute force algorithm.

c. A brute force algorithm should be used when it is important to find the best possible solution.

d. Sample response: Any of the five circuits may be used. Since a Hamiltonian circuit visits every vertex in a graph and starts and stops at the same point, it creates a closed path. Therefore, you can start and stop at any point on the circuit and still have a Hamiltonian circuit.

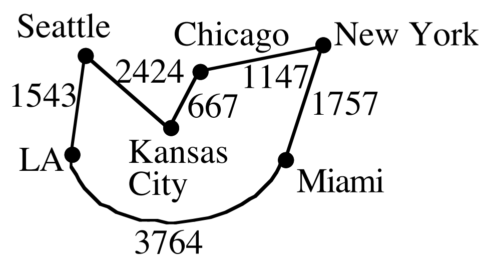
Teacher Note

Problems **3.1–3.3** demonstrate the usefulness of algorithms that yield a good solution in a reasonable amount of time, but may not yield the best possible solution.

To complete Problem **3.7**, students may require a map of your state.

Assignment (page 272)

3.1 **a–b.** The following graph shows the weighted Hamiltonian circuit created by starting from Los Angeles and using the nearest neighbor algorithm. Of the six circuits created, this graph results in the shortest total distance: 11,302 km.



c. Students check six routes, one for each graph in Part **a**.

d. Since the tour must begin and end in Miami, there are $5! = 120$ possible routes.

3.2 **a.** Sample response: It took about 50 sec to generate and calculate the total distance.

b. Sample response: $120(50 \text{ sec}) = 6000 \text{ sec}$ or 1 hr, 40 min.

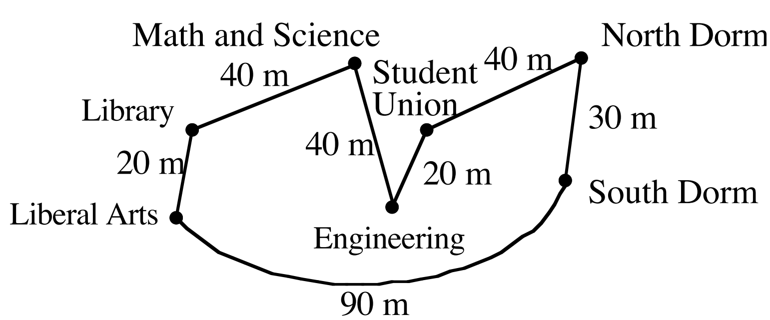
3.3 **a.** The number of possible 25-city tours that begin and end in the same town is 24! or approximately $6.2 \bullet 10^{23}$.

b. Using an estimate time of 50 sec, $6.2 \bullet 10^{23}(50 \text{ sec}) = 3.1 \bullet 10^{25}$ sec or approximately $9.8 \bullet 10^{17}$ years.

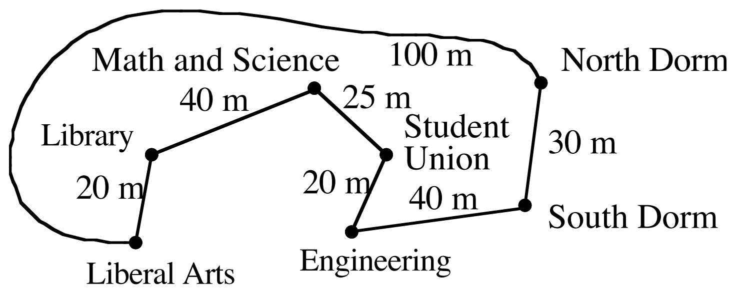
3.4 **a.** There are 7! or 5040 ways to organize the walking tour.

b. There are 6! or 720 ways to organize this walking tour.

c. The following graph shows the route created by beginning and ending with South Dorm. The total distance traveled with this tour is 280 m.



3.5 **a.** Using the cheapest link algorithm produces the route shown below. The total distance traveled is 275 m.



b. The distance traveled using this route is 5 m less than in the sample response given in Problem **3.4c**.

***3.6** **a.** The circuit found using the cheapest link algorithm follows a route from Miami to New York to Chicago to Kansas City to Los Angeles to Seattle to Miami.

b. The total distance is 11,696 km.

c. Using the nearest neighbor algorithm resulted in a route with a total distance of 11,302 km, 394 km less than the distance in Part **b** above.

* * * * *

3.7 Answers will vary. The following sample response uses the Montana cities of Kalispell, Missoula, Great Falls, Bozeman, and Havre. The tour begins and ends in Kalispell.

a. In the following sample table, distances are reported in kilometers.

	Havre	**Bozeman**	**Great Falls**	**Missoula**
Kalispell	420	468	367	185
Missoula	452	323	270	
Great Falls	182	298		
Bozeman	467			

b. There are $4! = 24$ possible routes.

c–d. In this case, the routes produced by the two algorithms are the same: *K–M–G–H–B–K*. The total distance is 1572 km.

e. In general, both the nearest neighbor and cheapest link algorithms yield good solutions, but not necessarily the best one.

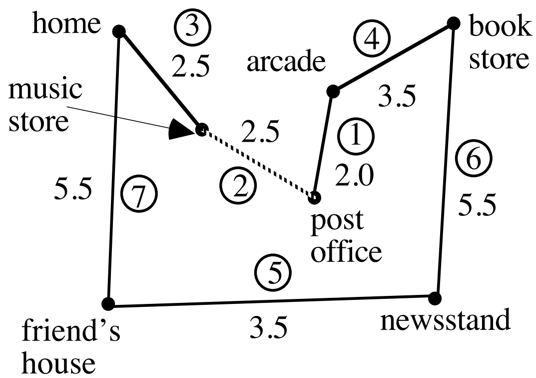
f. Using a brute force algorithm, the shortest possible route is 1408 km long. **Note:** Only 12 of the 24 circuits are unique. For example, *K–M–G–B–H–K* is the same as *K–H–B–G–M–K* because circuits do not have a direction.

Circuit	**Distance**	**Circuit**	**Distance**
K–M–G–B–H–K	1640	*K–G–B–M–H–K*	1860
K–M–G–H–B–K	1572	*K–G–H–M–B–K*	1792
K–M–B–G–H–K	1408	*K–G–M–H–B–K*	2024
K–M–B–H–G–K	1524	*K–G–M–B–H–K*	1847
K–M–H–G–B–K	1585	*K–H–G–M–B–K*	1663
K–M–H–B–G–K	1769	*K–H–M–G–B–K*	1908

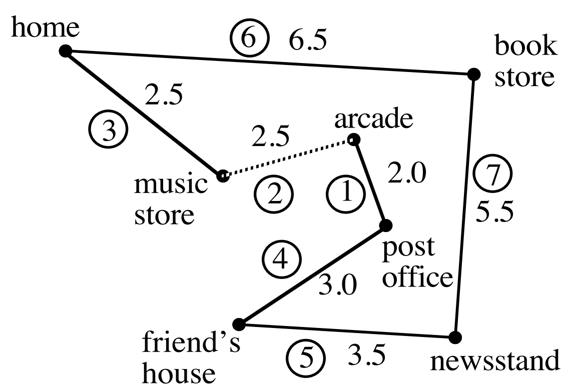
* * * * * * * * * *

Answers to Summary Assessment (page 276)

1. The cheapest link algorithm gives two possible routes. This occurs because three of the edges have the same time (2.5 min). In the sample graphs below, the dotted segments indicate the two choices at this step.

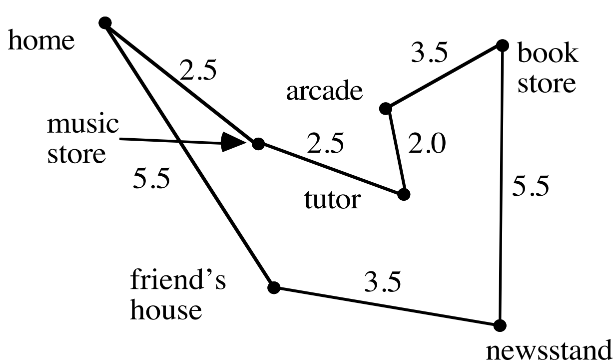


25.0 min



25.5 min

The nearest neighbor algorithm gives several possible routes. Student graphs will vary, depending on the decisions they make when two or more edges have the same time. The sample graph below was drawn by starting at Jack and Jill's home.

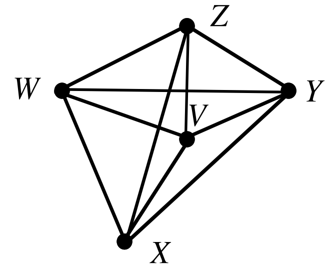


25 min

2. Sample response: Since there are 5 errands and each requires 5 min, this accounts for 25 min. Adding 25 min of travel time results in a total of 50 min. Jack and Jill have a total of 90 min for their trip. Therefore, they have 40 min remaining for the arcade, or enough for 8 games.
3. Answers will vary. Both the cheapest link and nearest neighbor algorithms yield reasonable solutions.
4. If using a brute force algorithm, there are a total of $6!/2 = 360$ possibilities.

Module Assessment

1. In the following graph, every vertex is connected to every other vertex.



a. Use this graph to describe an example, if possible, of each of the following:

1. a path containing all five vertices

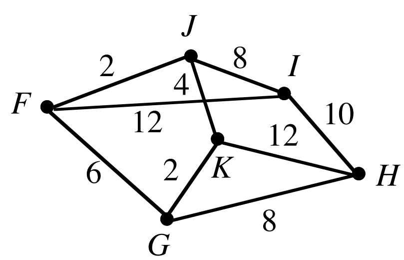
2. a circuit containing at least four vertices

3. a Hamiltonian circuit.

b. How many Hamiltonian circuits are there in the graph?

2. A bus company offers service to six cities. On each route, the bus stops once at each city. The company would like to minimize the number of hours required to complete a route.

In the weighted graph below, the vertices represent the cities and the numbers on the edges represent the time in hours for each trip.



a. The company's headquarters are located in the city represented by the letter *I*. Use the cheapest link algorithm to find a route that begins and ends at *I*.

b. Using *I* as the starting point and the nearest neighbor algorithm, find another possible route.

c. Which of the two greedy algorithms produces the faster route? Do you think that this is the fastest possible route? Explain your response.

d. Explain why it is not feasible to use the fundamental counting principle to determine the number of possible routes that begin and end in *I*.

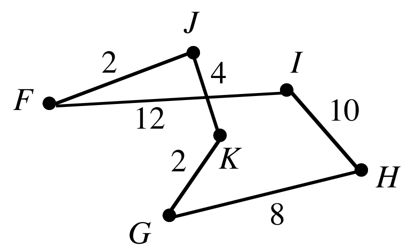
Answers to Module Assessment

1. **a.** Answers will vary.

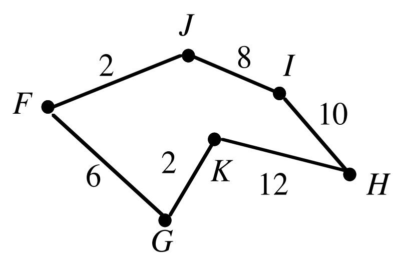
1. One path that contains all five vertices is *W–V–X–Y–Z*.
2. One circuit that contains at least four vertices is *Y–Z–W–V–Y*.
3. One Hamiltonian circuit is *Y–Z–W–X–V–Y*.

b. There are 5! or 120 Hamiltonian circuits.

2. **a.** The cheapest link algorithm yields the route *I–F–J–K–G–H–I*, for a total travel time of 38 hr.



b. Starting from *I*, the nearest neighbor algorithm yields the route *I–J–F–G–K–H–I,* for a total travel time of 40 hr.



c. Sample response: The cheapest link algorithm produces the faster route. However, since neither algorithm guarantees the best solution, this is not necessarily the fastest possible route. The only way to verify that it is the fastest would be to check all possible routes.

d. It is not feasible to use the fundamental counting principle to determine the total number of possible routes because some pairs of cities have no direct path between them. For example, there is no direct path from *F* to *K*.

Selected References

Chartrand, G. *Introductory Graph Theory*. New York: Dover Publications, 1977.

de Lange, J. *Matrices*. The Netherlands: University of Utrecht, 1990.

Cozzens, M. B., and R. Porter. "Problem Solving Using Graphs." High School Mathematics and Its Applications Project (HiMAP). Module 6. Arlington, MA: COMAP, 1987.

National Council of Teachers of Mathematics (NCTM). *Discrete Mathematics Across the Curriculum K–12*. Reston, VA: NCTM, 1991.

Roman, S. *An Introduction to Discrete Mathematics*. New York: Saunders College Publishing, 1986.

Trains, Planes and Critical Paths. Videocassette. Produced by the Consortium for Mathematics and Its Applications (COMAP). Module I.3 in the series *For All Practical Purposes: Introduction to Contemporary Mathematics*. Santa Barbara, CA: The Annenberg/CPB Project, 1989. 30 min.

Flashbacks

Activity 1

1.1 If you can run at an average speed of 14 km/hr, how long would it take you to run 18 km?

1.2 The following table shows the driving distances in kilometers among five cities.

	A	**B**	**C**	**D**
E	44 km	19 km	19 km	17 km
D	38 km	17 km	11 km	
C	27 km	6 km		
B	24 km			

a. What is the distance between city D and city B?

b. If you drove from city A to city D to city E then back to city A, what is the total distance traveled?

1.3 Find the value of each of the following expressions when $n = 10$.

a. $3(n-1)$

b. $n(n-1)$

c. $n(n-1)(n-2)$

d. $(n+2)(n-4)$

Activity 2

2.1 Anthony, Jesse, Cassy, and Pavlo are playing a game of catch. Jesse has the ball. Using a tree diagram, show all the possible ways in which the ball can be passed so that each of the other three players catches it once before it returns to Jesse.

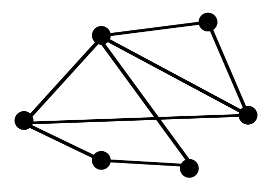
2.2 Determine the value of n in each of the following equations.

a. $n + 1 = 12$

b. $n - 10 = 20$

c. $\frac{n}{2} = 8$

2.3 Use the graph below to complete Parts **a–c**.



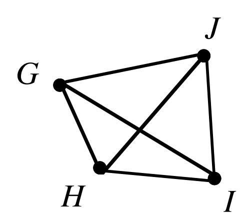
a. How many edges are there in this graph?

b. How many vertices are there in this graph?

c. Does this graph contain a Hamiltonian circuit? Explain your response.

Activity 3

3.1 Use the graph below to complete Parts **a** and **b**.



a. Determine the number of Hamiltonian circuits in the graph.

b. List two Hamiltonian circuits in the graph.

3.2 Find the value of each of the following expressions:

a. $12!/10!$

b. $5!$

3.3 A costume wardrobe contains 5 hats, 7 jackets, 3 pairs of gloves, and 2 pairs of boots. If the stage manager selects one article of clothing from each group, how many different combinations are possible?

Answers to Flashbacks

Activity 1

1.1 approximately 1.3 hr

1.2 **a.** 17 km

b. 99 km

1.3 **a.** 27

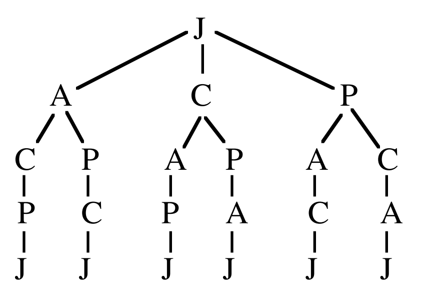
b. 90

c. 720

d. 72

Activity 2

2.1 Sample tree diagram:



2.2 **a.** 11

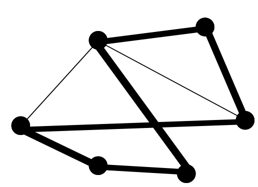
b. 30

c. 16

2.3 **a.** 8

b. 6

c. This graph contains a Hamiltonian circuit, as shown below.



Activity 3

3.1 **a.** There are 4! or 24 Hamiltonian circuits.

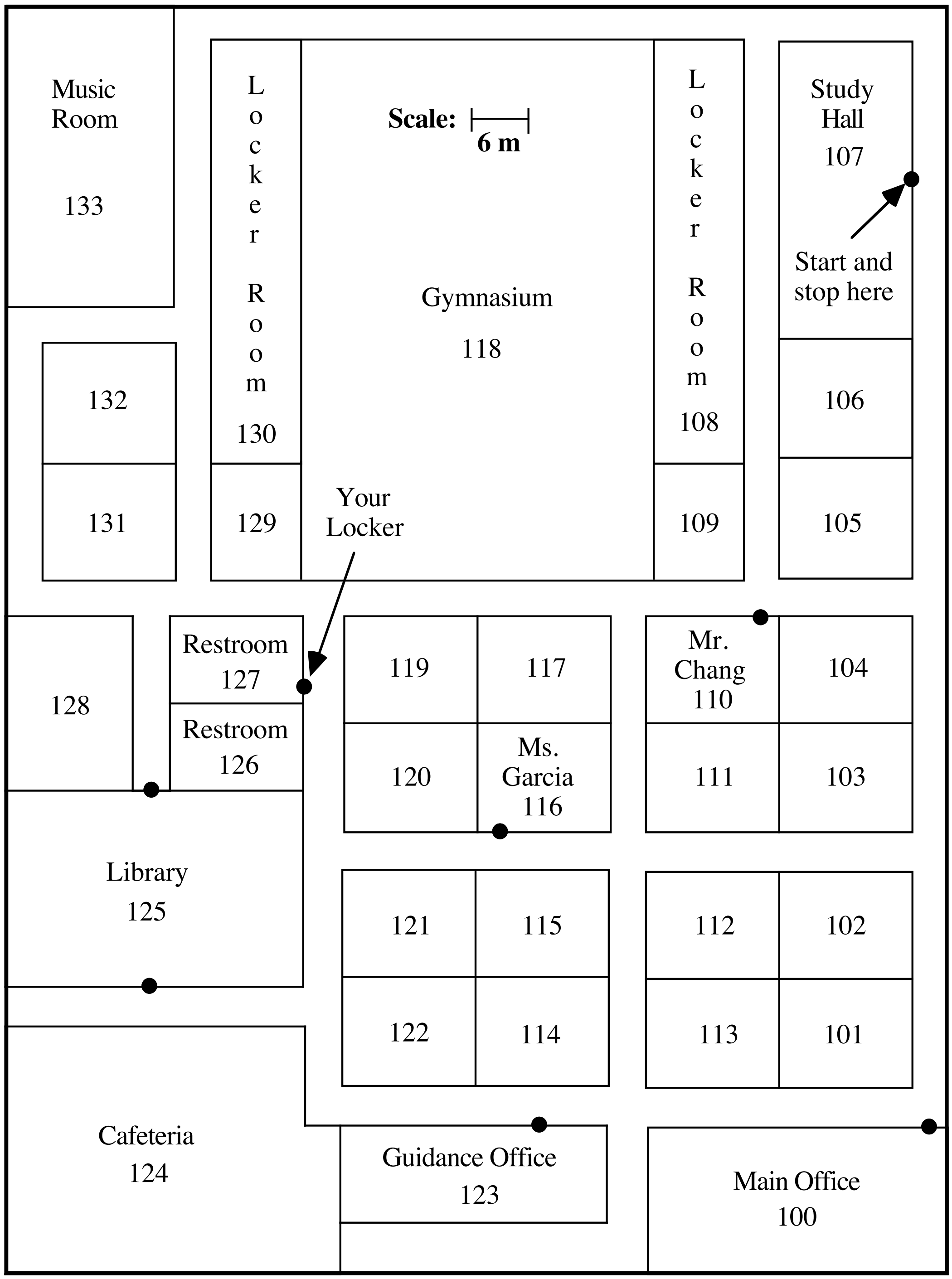
b. Sample response: Two of these Hamiltonian circuits are *G–H–I–J–G* and *H–G–I–J–H*.

3.2 **a.** 132

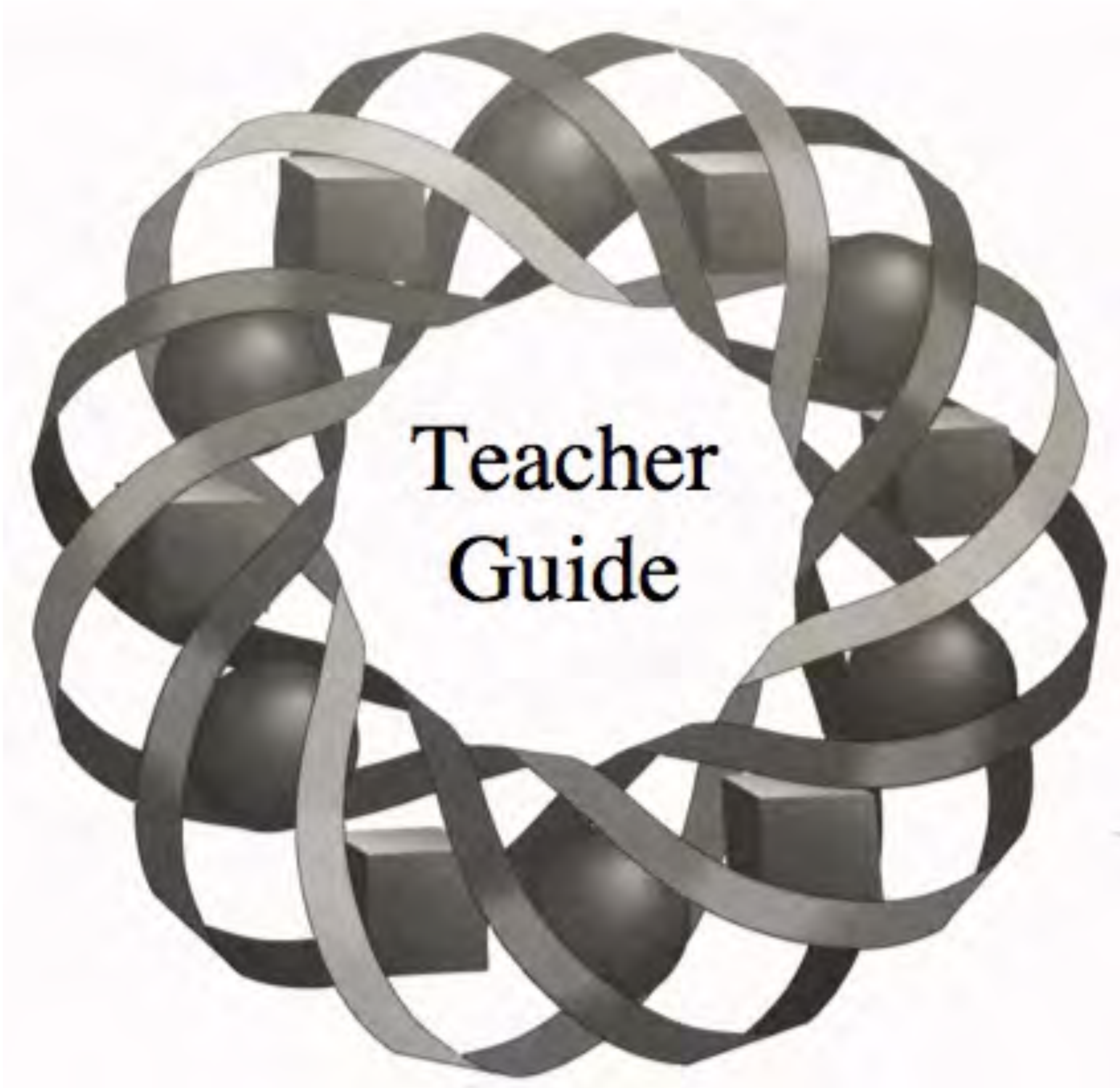
b. 120

3.3 $5 \bullet 7 \bullet 3 \bullet 2 = 210$

Hamilton High School Template



One Step Beyond



How much did you earn last week? How much was that long-distance phone call? And what did it cost to send that package to Pawtucket? Every time you cash a paycheck, dial a telephone, or mail a postcard, you're stepping off into the unknown.

Bill Chalgren • Bonnie Eichenberger • Paul Swenson • Karen Umbaugh

Teacher Edition
One Step Beyond

Overview
In this module, students investigate step functions in general and three different types of rounding functions in particular: round down, round up, and round to the nearest integer. The greatest integer (round down) function is used to write equations for many other step functions.

Objectives
In this module, students will:

- represent compound inequalities on a number line
- represent compound inequalities algebraically
- use interval notation to represent inequalities
- graph and interpret step functions
- use the greatest integer function to write equations of step functions.

Prerequisites
For this module, students should know:

- how to represent simple inequalities algebraically
- how to represent simple inequalities on a number line
- how to graph ordered pairs on a two-dimensional coordinate system
- how to round numbers.

Time Line

Activity	1	2	3	Summary Assessment	Total
Days	3	3	2	2	10

Materials Required

Materials	Activity			
	1	**2**	**3**	**Summary Assessment**
tape or glue	X			
freezer paper	X			
pay template	X			
graph paper	X	X	X	

Teacher Note

Other types of paper available in rolls may be substituted for freezer paper. A blackline master of the pay template appears at the end of the teacher edition for this module.

Technology

Materials	Activity			
	1	**2**	**3**	**Summary Assessment**
graphing utility	X	X	X	
spreadsheet			X	

Teacher Note

When graphing step functions, some graphing utilities may connect the steps. In such cases, it may be desirable to plot individual points instead. Regardless of the technology used, students should be alerted to the possible pitfalls of relying only on the graphs to interpret step functions. For example, few graphing utilities display the open endpoints on each step.

One Step Beyond

Introduction

(page 281)

Step functions can be used to model many real-world situations, including long-distance telephone rates, postal delivery charges, and pay schedules.

(page 281)

Activity 1

In this activity, students create and interpret graphs of different rounding methods. Inequalities and interval notation are used to express intervals.

Materials List

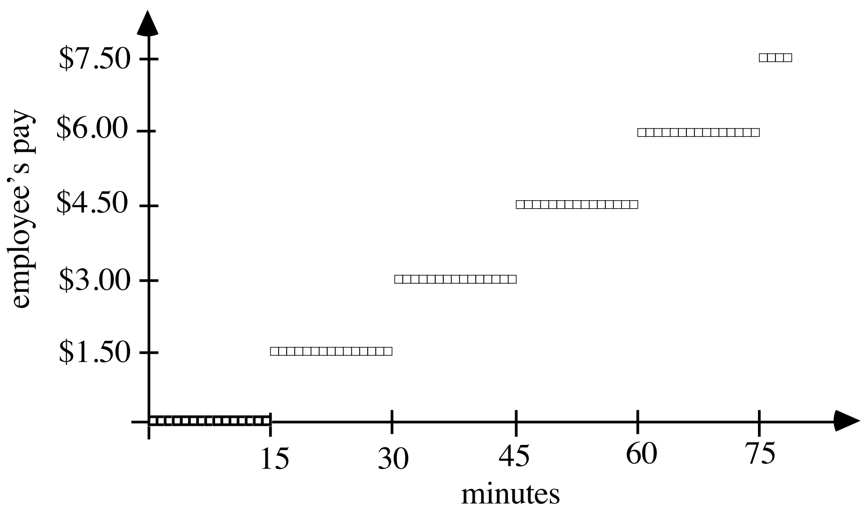
- pay template (one per group; a blackline master appears at the end of the teacher edition for this module)
- tape or glue
- freezer paper (about 1.5 m per group)
- graph paper (one sheet per student)

Exploration

(page 281)

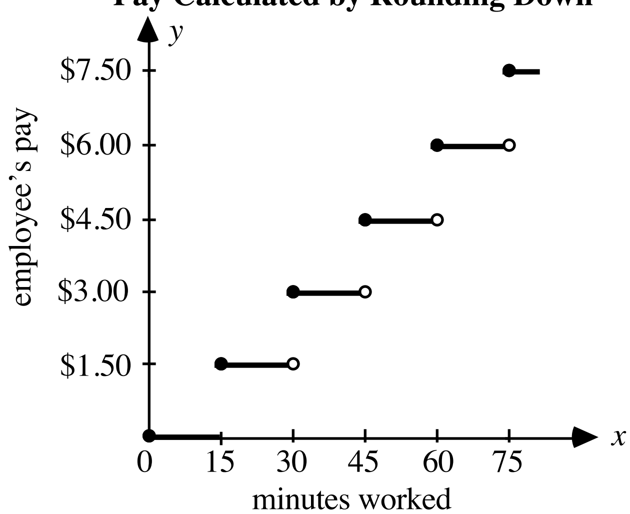
Note: To save time, you may wish to complete Part **a** as a class, assigning several ordered pairs to each student. However, creating at least two different graphs allows students to compare results.

a. Students should tape or glue all ordered pairs on a large set of coordinate axes, starting with 0 min and proceeding sequentially. You may wish to display the freezer paper on a wall or bulletin board. The finished graphs should resemble the graph of a step function, with 15 squares of paper forming each step. Sample graph:



Note: For classroom reference, you may wish to display at least one of the graphs created in this activity during the entire module.

b. Sample graph:



Discussion

(page 283)

a. Both of the graphs from the exploration should form a series of steps.

b. Sample response: The graph made of paper squares shows time only in whole numbers of minutes. The graph created using intervals represents all the real numbers in each interval.

c. Sample response: The graph in Part **b** of the exploration uses open and closed endpoints to show the exact time when the graph jumps from one step to another. **Note:** Graphs created using technology may not show these endpoints.

d. This graph is a function because every element in the domain is paired with exactly one element in the range.

e. This graph passes the vertical line test. No vertical line can be drawn that passes through more than one point on the graph. **Note:** Since the vertical line test does not determine if every element in the domain is paired with an element in the range, it does not guarantee that a graph which passes the test is a function.

f.
1. A time of exactly 15 min is represented by a solid endpoint at the coordinates (15,1.50).
2. The length of each step represents the time interval for which an employee receives the same amount of pay.
3. Sample response: The vertical distance between successive steps represents a jump of \$1.50 in pay.

g. Using multiples of 10 minutes, employee earnings are likely to rise. For example, an employee who works 21 min would earn \$2.00 if time is rounded to the nearest multiple of 10. If time is rounded to the nearest multiple of 15, the employee would earn only \$1.50.

h. Since the domain is all non-negative real numbers, it can be written as the interval $[0,+\infty)$.

Teacher Note

You may wish to discuss some of the different ways in which functions can be represented (lists of ordered pairs, function mappings, graphs, etc.). You may also wish to describe some everyday situations that can be interpreted as functions (time in toaster versus darkness of bread), as well as some examples of relationships that are not functions (numbers displayed on the face of a clock and hours after midnight).

Assignment

(page 285)

1.1 A completed table is shown below.

Time	Wages Earned
14 min	\$0.00
14 min, 20 sec	\$0.00
14 min, 57 sec	\$0.00
15 min	\$1.50
15 min, 2 sec	\$1.50
16 min	\$1.50

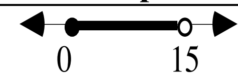
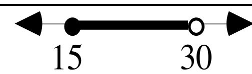
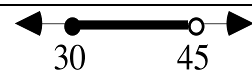
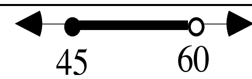
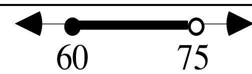
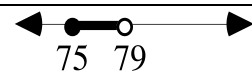
1.2 **a.** $0 \leq x < 15$

b. Sample response: This interval is the time period between 0 min and 15 min, where 0 min is included and 15 min is excluded.

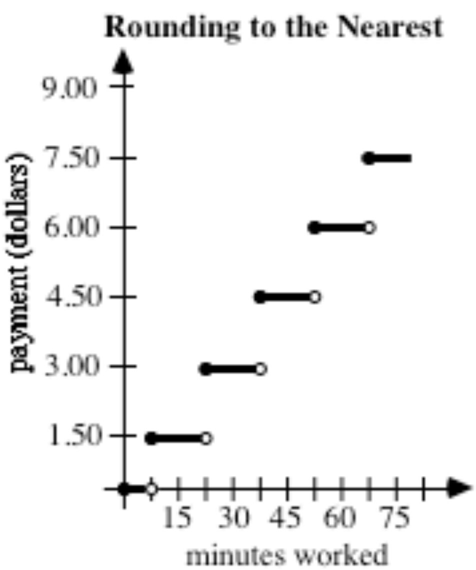
c. Sample graph:

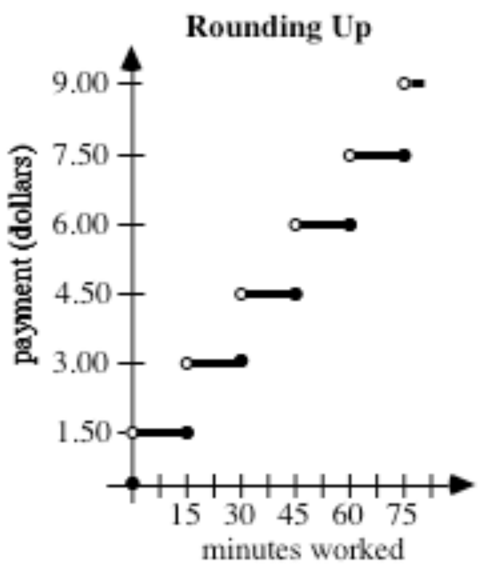
d. [0, 15)

1.3 **a–b.** A completed table is shown below.

Time Interval (min)	Interval Notation	Number-line Graph	Wages Earned
$0 \leq t < 15$	[0, 15)		$0.00
$15 \leq t < 30$	[15, 30)		$1.50
$30 \leq t < 45$	[30, 45)		$3.00
$45 \leq t < 60$	[45, 60)		$4.50
$60 \leq t < 75$	[60, 75)		$6.00
$75 \leq t < 79$	[75, 79)		$7.50

1.4 **a–b.** Sample graphs:





1.5 **a.** \$0, \$1.50, \$3.00, \$4.50, \$6.00

b. $22.5 \le x < 37.5$ or $[22.5, 37.5)$

c. $15 < x \le 30$ or $(15, 30]$

d. A completed table is shown below.

Money Earned	**Time Worked in Minutes**	
	Rounding Up	**Rounding to the Nearest**
\$0.00	$x = 0$	$0 \le x < 7.5$
\$1.50	$0 < x \le 15$	$7.5 \le x < 22.5$
\$3.00	$15 < x \le 30$	$22.5 \le x < 37.5$
\$4.50	$30 < x \le 45$	$37.5 \le x < 52.5$
\$6.00	$45 < x \le 60$	$52.5 \le x \le 67.5$

e. The intervals for which the two rounding methods pay the same amount are: $x = 0$, $7.5 \le x \le 15$, $22.5 \le x \le 30$, $37.5 \le x \le 45$, and $52.5 \le x \le 60$.

1.6 **a.** The minimum amount of time an employee can work to earn \$1.50 when rounding down is 15 min. The minimum time when rounding to the nearest is 7.5 min. When rounding up, any time greater than 0 min will do.

b. Sample response: Rounding up favors employees since the time is never rounded to a lesser value.

c. Sample response: Rounding down favors the employer since time is usually rounded to a lesser value.

d. Sample response: Rounding to the nearest might serve as a compromise since time can be rounded both to greater and lesser values.

1.7 The following table shows the times for which all three rounding methods result in the same pay. **Note:** You may wish to discuss the solutions of systems of simple inequalities with only one variable. Overlaying the graphs of the inequalities may help students visualize the intersection.

Money Earned	**Time Worked (min)**
\$0.00	$x = 0$
\$1.50	$x = 15$
\$3.00	$x = 30$
\$4.50	$x = 45$
\$6.00	$x = 60$

1.8 **a.** Shelly would pay her employees \$8.00 per hour.

b. Sample response: Using this method, Shelly rounds parts of hours to the nearest multiple of 30 minutes. Employee pay is then calculated using a rate of \$8.00 per hour.

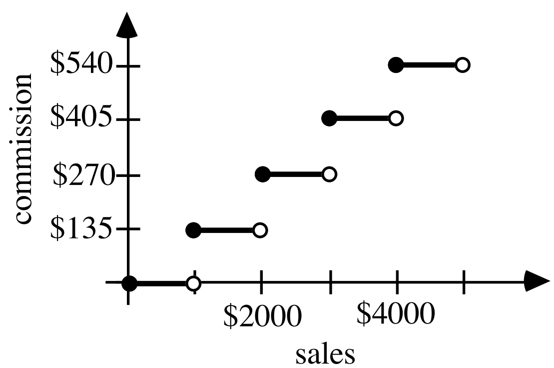
* * * * *

1.9 **a.** The store rounds \$4800 down to \$4000. Since the salesperson receives \$135 for each \$1000 in sales, she would earn $4(\$135) = \$540$.

b. The following table shows the appropriate intervals.

Sales (s)	**Commission**
$\$0 \le s < \$1000$	\$0
$\$1000 \le s < \$2000$	\$135
$\$2000 \le s < \$3000$	\$270
$\$3000 \le s < \$4000$	\$405
$\$4000 \le s < \$5000$	\$540

c. Sample graph:



1.10 **a.** Sample response: Time in days is "rounded up" to the next week. For example, a book is considered 1 week overdue if it is 1 day late. It is considered 2 weeks overdue if it is 8 days late.

b. Some students may argue that a vertical line drawn anywhere on the graph intercepts the graph in only one place. However, it should be pointed out that this is only part of the requirements for a function. It must also be shown that every day or part of a day is assigned a late fine.

c. The domain is d days where $0 \le d \le 42$. The range is {\$0, \$1.50, \$3.00, \$4.50, \$6.00, \$7.50, \$9.00}.

d. $14 < d \le 21$ or $(14, 21]$.

* * * * * * * * * *

(page 288)

Activity 2

In this activity, students explore the greatest integer function, $y=[x]$, which rounds non-integers down to the previous integer. The parameters a, b, c, and d in $y=a[bx+c]+d$ are introduced in the context of parking costs.

Materials List

- graph paper

Technology

- graphing utility

Exploration 1

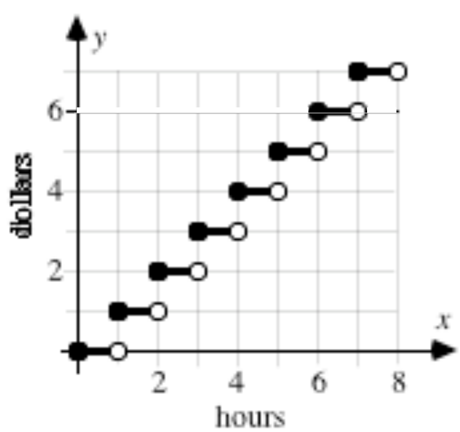
(page 288)

Students explore the greatest integer function $y=[x]$.

a. Students should complete the bottom row as shown:

Hours Parked (interval notation)	Number of Hours Charged	Parking Cost
$[x,\ (x+1))$	x	$x \bullet \$1.00$

b. Sample graph:



c. **1.** $1.00

2. $5.00

3. $7.00

d. **1.** $[2.2] = 2$

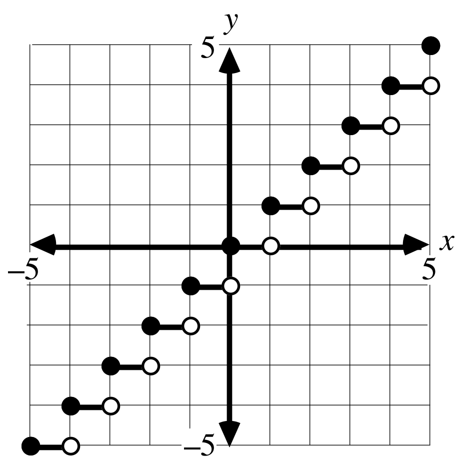
2. $[2.9] = 2$

3. $[-2.2] = -3$

4. $[-2.9] = -3$

e. Students should determine the keystrokes necessary to enter $[x]$ in their graphing utility. Using the TI-92, for example, the command is int(x).

f. **1.** The sample graph below correctly shows both open and closed endpoints.



2. **Note:** Some forms of technology may not show the appropriate endpoints when graphing the greatest integer function. Others may connect the endpoints. Sample response: The domain of $y = [x]$ is all real numbers, while the domain of the graph in Part **b** is [0, 8).

Discussion 1

(page 289)

a. **1.** When parking for 7.5 hr, the cost can be found as follows: $y = [7.5] = 7$. Therefore, 7.5 hr of parking costs $7.00.

2. When parking for 0.9 hr, $y = [0.9] = 0$. Therefore, 0.9 hr of parking costs $0.00.

b. Sample response: The graph in Part **b** of the exploration shows this using an open endpoint at (1 hr, $0) and a closed endpoint at (1 hr, $1). In Part **f**, our graphing utility does not clearly represent this point in time. **Note:** The graph created using paper squares in Activity **1** has the same limitation in representing domain values where the function “steps” to the next level of range values.

c. If there is no limit to the amount of time somebody can park, it might be argued the domain is $[0, +\infty)$. If a time limit exists, such as 7 days, the domain would be $[0,168]$.

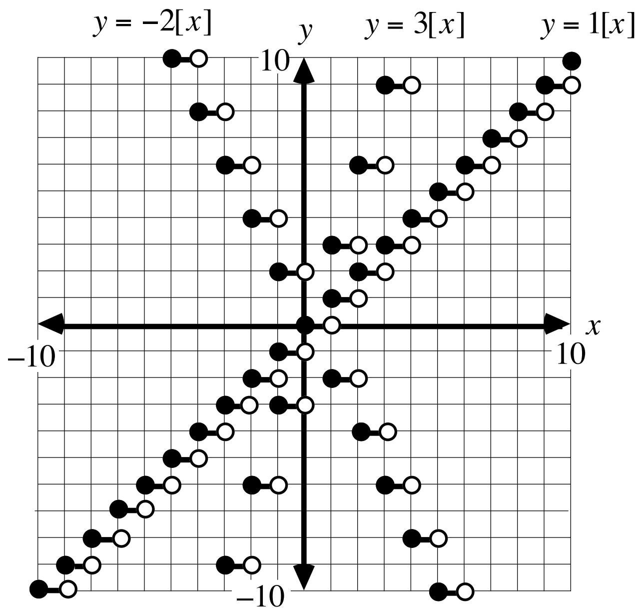
Exploration 2

(page 290)

a. **1.** A completed table appears below.

x	$y = 1[x]$	$y = 3[x]$	$y = -2[x]$
0	0	0	0
0.4	0	0	0
1.6	1	3	–2
2.4	2	6	–4

2. Sample graph:

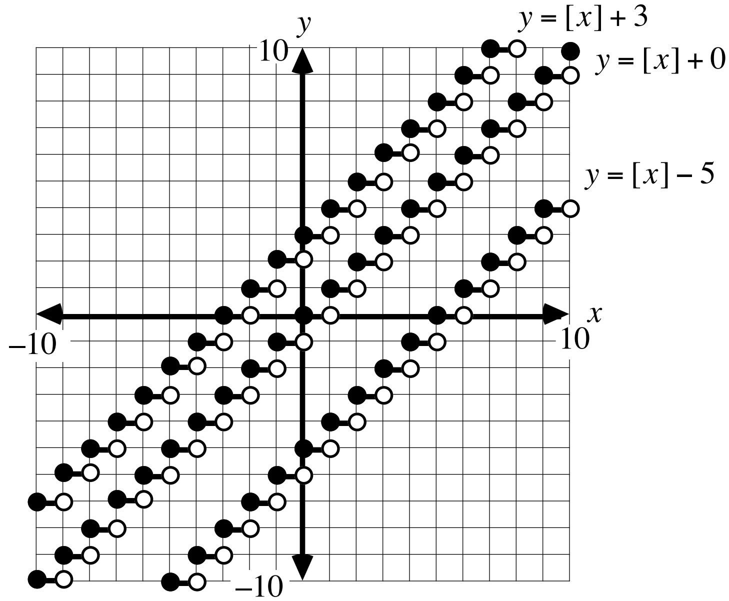


3. When $0 \le x < 1$, the graphs coincide. For other values of x, the graph of $y = 3[x]$ shows range values that are 3 times those for the graph of $y = [x]$. Similarly, the graph of $y = -2[x]$ shows range values that are –2 times those for the graph of $y = [x]$.

b. **1.** A completed table appears below.

x	$y = [x] + 0$	$y = [x] + 3$	$y = [x] - 5$
0	0	3	–5
0.4	0	3	–5
1.6	1	4	–4
2.4	2	5	–3

2. Sample graph:

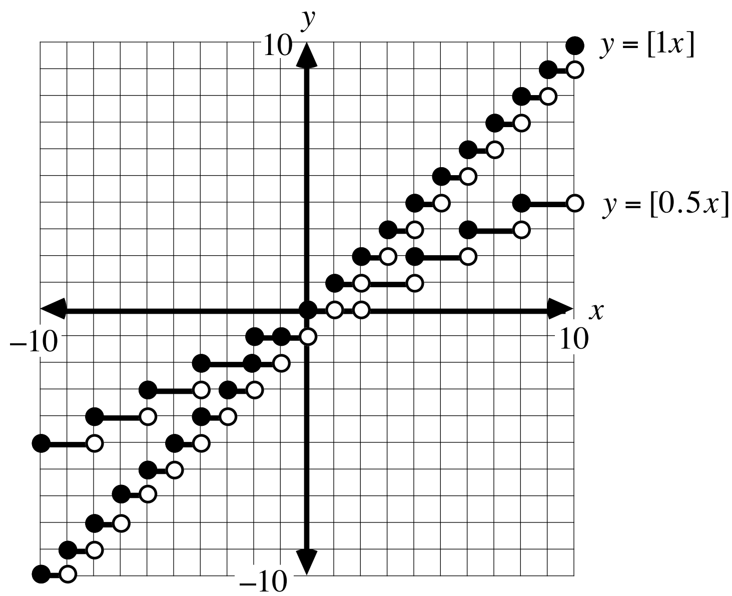


3. Adding the constant 3 to the greatest integer function increases each range value by 3 for every domain value. Adding the constant –5 decreases each range value by 5 for every domain value.

c. **1.** A completed table appears below.

x	$y = [1x]$	$y = [0.5x]$
0	0	0
0.4	0	0
1.6	1	0
2.4	2	1
3.6	3	1
4.2	4	2

2. Sample graph:



3. When the constant b is 0.5, each interval on the graph appears to stretch horizontally and the "incline" of the steps appears to decrease.

Discussion 2

(page 291)

a. When $0 \leq x < 1$, the graphs coincide. For other values of x, the graph of $y = 3[x]$ shows range values that are 3 times those for the graph of $y = [x]$.

b. As $|a|$ increases, the "incline" of the steps appears to increase. When a is negative, a negative association results.

c. Adding the constant 3 to the greatest integer function increases each range value by 3 for every domain value.

d. The constant d causes a vertical shift in the graph of d units.

e. When the constant is 0.5, each interval on the graph appears to stretch horizontally and the "incline" of the steps appears to decrease. Some students may notice that the intervals appear to double in length. That is the stretch by a factor of 2 which is the reciprocal of 0.5.

f. Sample response: The constant b appears both to stretch the steps and affect the incline of the steps.

g. **1.** The parking lot with costs modeled by $y = [0.5x]$ is less expensive. For example, it would cost $2 to park in this lot for 5 hr. It would cost $5 to park in Jeff's Lot for 5 hr.

2. Sample response: The graph of the function that models the cheaper lot will have steps with a smaller "incline." This indicates that the cost rises more slowly over time.

Assignment

(page 292)

2.1 $[A] = -4$; $[B] = -3$; $[C] = -1$; $[D] = 0$; $[E] = 1$; $[F] = 2$

2.2 **a.** $[-2.3] = -3$

b. $[0] = 0$

c. $[1.5] + 0.5 = 1.5$

d. $3[(0.45) \bullet 10] + 7 = 3[4.5] + 7 = 3(4) + 7 = 12 + 7 = 19$

2.3 **a.** **1.** Sample response: The solution set for the equation $[x] = 9$ is all numbers that round down to the integer 9. For example, numbers like 9.2 and 9.9 round down to 9.

2. $9 \le x < 10$

b. **1.** $5 \le x < 6$

2. $-3 \le x < -2$

3. $n \le x < n + 1$

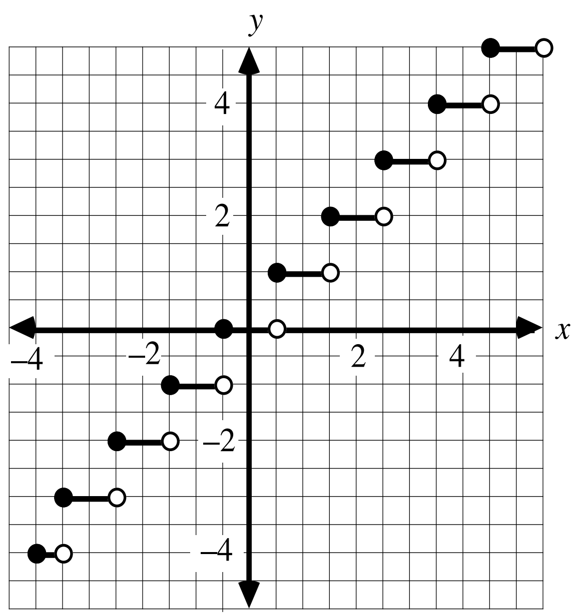
2.4 **a.** Two possibilities are $y = \$1.00[x] + \$1.00$ and $y = \$1.00[x+1]$, where y represents cost to park and x represents time in hours.

b. Two possibilities are $y = \$1.50[x] + \$1.50$, and $y = \$1.50[x+1]$ where y represents cost to park and x represents time in hours.

c. In the following sample response, y represents cost to park and x represents time in minutes:

$$y = \$0.75\left[\frac{1}{30}x\right]$$

2.5 **a.** Sample graph:



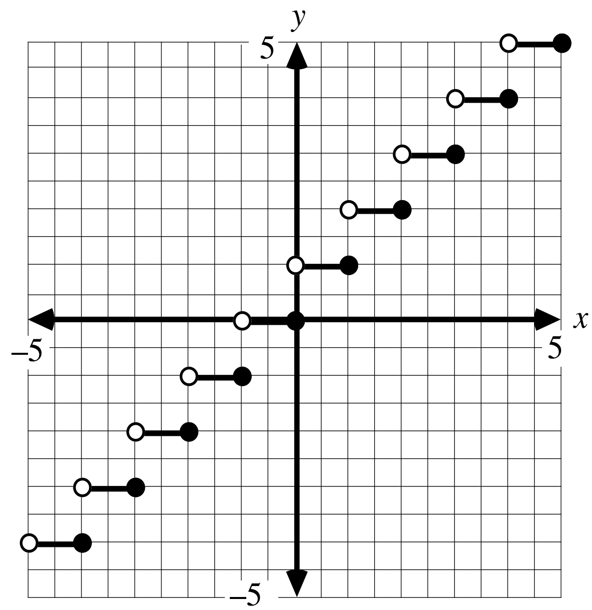
b. Sample response: The graph is a step function. It is a function because it passes the vertical line test and every real number x is paired with a number y.

c. $y = [x + 0.5]$

d. This function rounds to the nearest integer. For domain values which are exactly between two integers, such as 1.5 and –1.5, the function rounds up to the next integer. **Note:** Some graphing utilities may not show endpoints on the intervals.

e. Many built-in rounding functions on calculators and computers round numbers like –1.5 and –2.5 down to –2 and –3, respectively.

2.6 **a.** As shown in the following graph, the open endpoints are on the left of each interval and the closed endpoints are on the right.



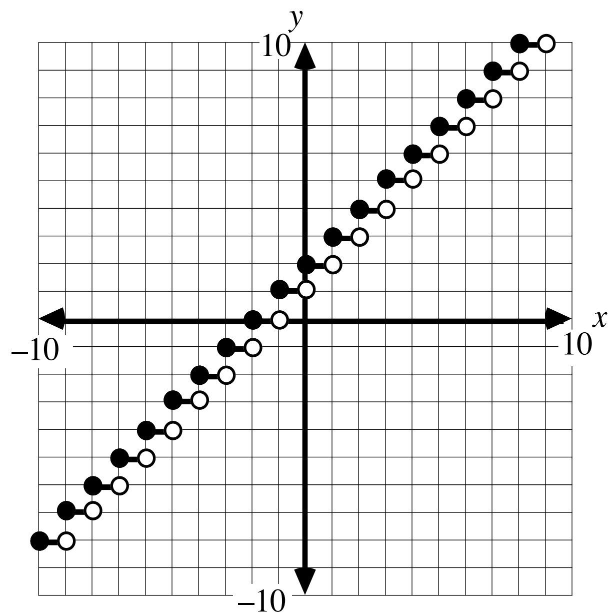
b. This graph should look the same as the graph in Part **a**. **Note:** Some graphing utilities may not show endpoints on the intervals.

c. Sample response: This could be called the round-up function.

2.7 **a.** A completed table is shown below.

x	$y = [x + 2]$	$y = [x] + 2$	$y = [x + 2.5]$	$y = [x] + 2.5$
–4	–2	–2	–2	–1.5
–3	–1	–1	–1	–0.5
–2	0	0	0	0.5
–1	1	1	1	1.5
0	2	2	2	2.5
1	3	3	3	3.5
2	4	4	4	4.5
3	5	5	5	5.5
4	6	5	6	6.5

b. As shown in the following graph, the graphs of $y = [x+2]$ and $y = [x]+2$ are identical.



c. From the table in Part **a**, it appears that $y = [x+c]$ and $y = [x]+c$ yield identical range values when c is an integer. When c is a non-integer, the functions yield different range values.

2.8 **a.** Two possibilities are $y = [x]+3$ and $y = [x+3]$.

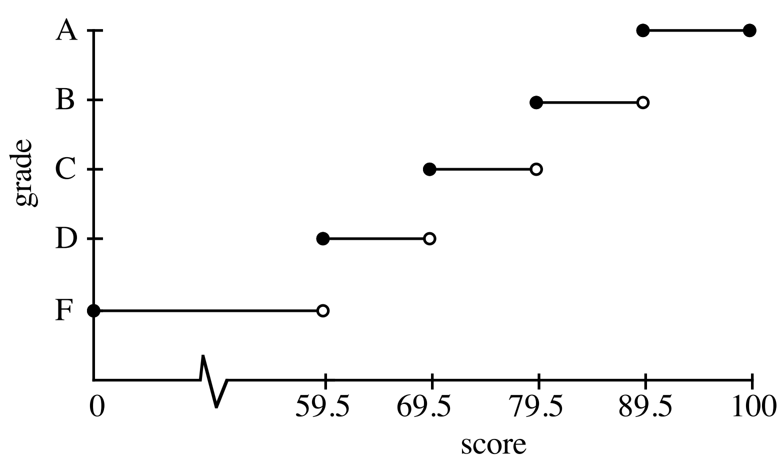
b. One possibility is $y = [x]+0.5$.

* * * * *

2.9 **a.** Sample response: Since the grading scale does not indicate letter grades for scores between 89 and 90, 79 and 80, 69 and 70, or 59 and 60, the teacher must round to an integer.

b. $89.5 \le x \le 100$

c. Sample graph:



d. Sample response: Yes. Each possible score (x-value) is assigned a letter grade (y-value) and there is exactly one letter grade assigned to each score. **Note:** Unless the domain is defined carefully, some students may argue that not all points of the domain are used.

e. Answers may vary. Some may argue that this method is unfair since, for example, a score of 89.49 receives a B. Others may argue that it is fair because a score of 89.5 is rounded up—and receives the higher grade.

2.10 **a.** **1.** The domain is the integers in the interval (0, 200].

2. Each bus can carry up to 40 students.

3. Two possibilities are:

$$y = \left[\frac{1}{41}x\right] + 1 \text{ and } y = \left[\frac{1}{41}x + 1\right]$$

b. **1.** Two possibilities are:

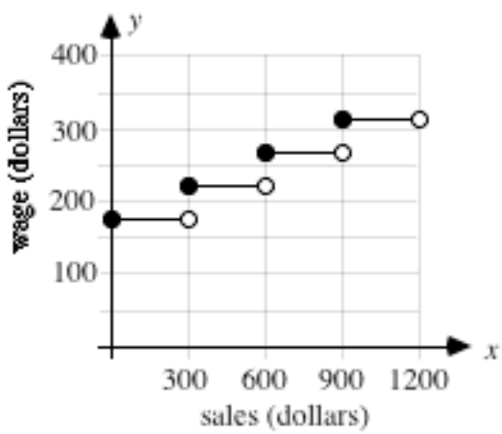
$$y = \left[\frac{1}{80}x\right] + 1 \text{ and } y = \left[\frac{1}{80}x + 1\right]$$

2. Sample response:

$$\begin{aligned} y &= \left[\frac{1}{80} \bullet 1543\right] + 1 \\ &= 19 + 1 = 20 \end{aligned}$$

2.11 **a.** Since Dana gets \$45 for each \$300 of sales, for \$1000 of sales she gets $3 \bullet \$45 = \$135$. Adding the \$175, she gets a total of \$310.

b. Sample graph:



c. In the following sample response, y represents wages in dollars and x represents sales in dollars:

$$y = 45 \bullet \left[\frac{1}{300}x\right] + 175$$

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(page 296)

Activity 3

In this activity, students examine greatest integer functions with negative associations in the context of long-distance telephone rates.

Materials List

- none

Technology

- spreadsheet
- graphing utility

Exploration

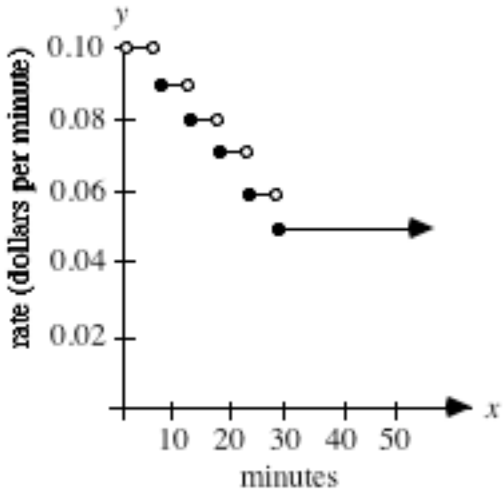
(page 296)

Students use a spreadsheet to examine long-distance telephone rates.

a. Sample spreadsheet:

Interval of Minutes	Decrease in Initial Rate	Discounted Rate (Cost per Minute)	Cost of Call
(0, 1)	\$0.00	\$0.10	\$0.10
[1, 2)	\$0.00	\$0.10	\$0.20
[2, 3)	\$0.00	\$0.10	\$0.30
[3, 4)	\$0.00	\$0.10	\$0.40
⋮	⋮	⋮	
[23, 24)	\$0.04	\$0.06	\$1.94
[24, 25)	\$0.04	\$0.06	\$2.00
[25, 26)	\$0.05	\$0.05	\$2.05

b. Sample graph:



Discussion

(page 297)

a. The association is negative since the rate decreases as the number of minutes increases.

b. In this graph, the association is negative. In most of the previous graphs, the association was positive.

c. Sample response: The graph of the phone company's rates is a function because for every possible number of minutes there is a rate and for each number of minutes there is only one rate.

d. Sample response: The cost of a call in a given interval of minutes is the sum of the entries in the "Discounted Rate" column.

Assignment

(page 297)

3.1 a–b. Sample spreadsheet:

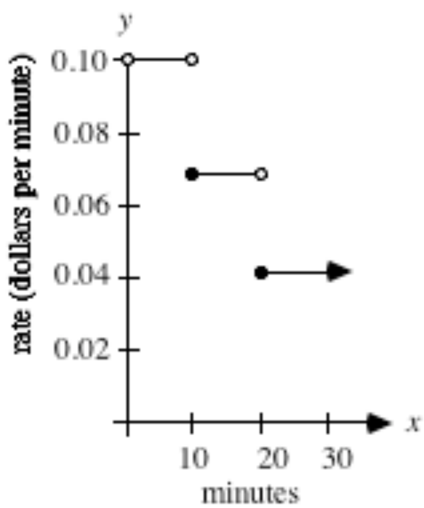
Interval of Minutes	Decrease in Initial Rate	Discounted Rate (Cost per Minute)	Cost of Call
(0, 1)	$0.00	$0.10	$0.10
[1, 2)	$0.00	$0.10	$0.20
[2, 3)	$0.00	$0.10	$0.30
[3, 4)	$0.00	$0.10	$0.40
⋮	⋮	⋮	⋮
[28, 29)	$0.06	$0.04	$2.06
[29, 30)	$0.06	$0.04	$2.10
[30, 31)	$0.06	$0.04	$2.14

c–d. The following table compares the costs of a call for the two companies.

Minutes	**Total Cost (A)**	**Total Cost (B)**
(0, 1)	\$0.10	\$0.10
[1, 2)	\$0.20	\$0.20
[2, 3)	\$0.30	\$0.30
[3, 4)	\$0.40	\$0.40
⋮	⋮	⋮
[28, 29)	\$2.20	\$2.06
[29, 30)	\$2.25	\$2.10
[30, 31)	\$2.30	\$2.14

The costs are the same for calls less than 5 min long. They are also the same for calls in the interval $[14, 20)$. Company A is cheaper for calls in the interval [5, 14) and more expensive for calls in the interval $[20, +\infty)$.

3.2 **a.** Sample graph:



b. **1.** The vertical distance between steps for company B is \$0.03, while the same distance for company A is \$0.01.

2. For company B, the rates change every 10 min for the first 20 min. For company A, the rates change every 5 min for the first 25 min.

3. The rates are equal for calls less than 5 min. They are also equal over the interval [14, 20).

c. Sample response: These graphs are meaningful only in the first quadrant because they represent rates and times. A negative rate would mean that the phone company would be paying you. A negative amount of time means that the company keeps track of time before you make the call. Neither makes sense in this context.

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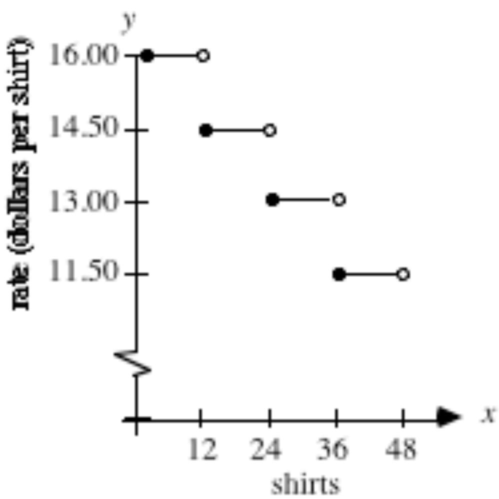
3.3 **a.** The domain is the natural numbers in the interval $(0, 48)$.

b. Sample graph:



c. In the following sample equation, y represents the cost per shirt and x represents the number of shirts:

$$y = -1.5 \bullet \left\lfloor \frac{1}{12}x \right\rfloor + 16$$

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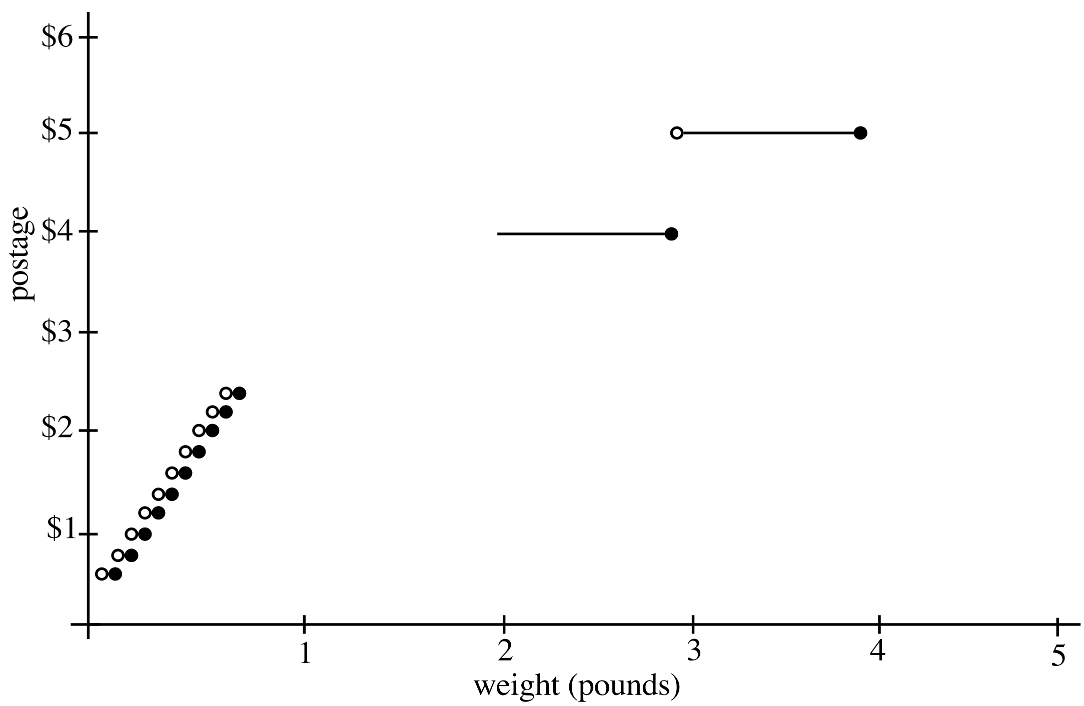
Answers to Summary Assessment (page 299)

1. **a.** Sample response: For letters that weigh up to and including 1 oz, the Postal Service charges $0.32 cents. For each additional ounce (or fraction of an ounce), they charge $0.23 cents more.

b. Sample response:

Weight (w) in Pounds	**Cost**
$11/16 < w \le 2$	$3.00
$2 < w \le 3$	$4.00
$3 < w \le 4$	$5.00
$4 < w \le 5$	$6.00

c. Sample graph:



d. Answers will vary. Sample response: Charts may enable customers to see the whole rate structure at a glance. Explanations (written or verbal) may be better for answering specific questions. Inequalities can display a large amount of information in a limited amount of space. Graphs may more clearly represent the steps in price for each weight.

2. **a.** The integer part function is the same as the greatest integer function for $x \geq 0$ and the same as the function $y = -[-x]$ for $x < 0$. Students may use graphs or tables to demonstrate these facts.

b. **1.** Sample response: $0 \leq x < 0.5$; $-0.5 \leq x < 0$.

2. Sample response: $0 \leq x < 0.5$. **Note:** There are no negative values of *x* that give the same result.

Module Assessment

1. Tim is planning a 240-mile trip. Depending on the driving conditions, his car can get as little as 16 miles per gallon and or as much as 23 miles per gallon.

a. Write an inequality that represents the number of gallons of fuel Tim may need for the trip.

b. Graph this inequality on a number line.

2. Rounding up, rounding down, and rounding to the nearest are three different methods of rounding.

a. Draw the graph of a step function to illustrate each method.

b. Write equations for two of the three functions.

3. The following chart is used to determine the cost of a speeding ticket.

Speed (miles per hour)	**Fine (dollars)**
$55 < s \le 60$	35.00
$60 < s \le 65$	45.00
$65 < s \le 70$	55.00
$70 < s \le 75$	65.00
$75 < s \le 80$	75.00
$80 < s \le 85$	85.00
$85 < s \le 90$	95.00
$90 < s \le 95$	315.00
$95 < s$	500.00

a. Create a graph of this function.

b. If this chart did not represent a function, what difficulties might arise in determining the cost of a speeding ticket?

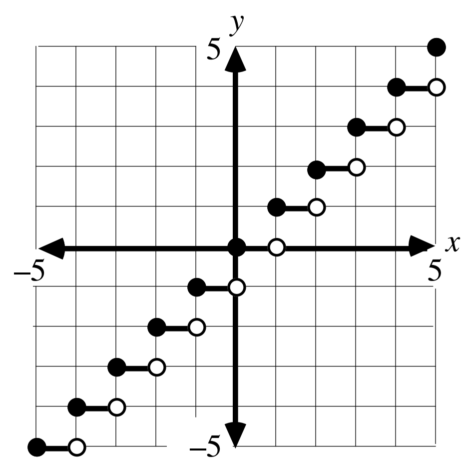
c. Peggy received a speeding ticket for traveling 75 miles per hour. Using the chart above, the officer fined her $75.00. Peggy argued that the fine should be only $65.00 and took her case to court. What should the judge's decision be? Explain your response.

Answers to Module Assessment

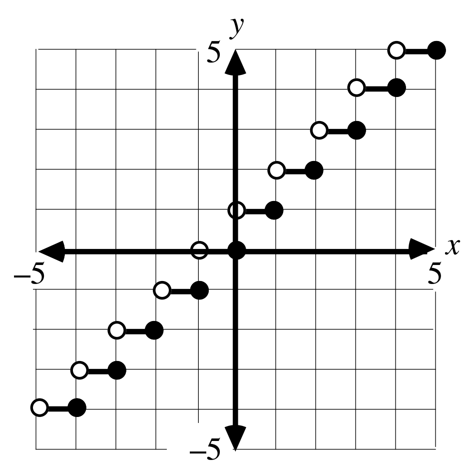
1. **a.** Answers will vary, depending on how students round the number of gallons. If students use the minimum number of whole gallons required, the interval is $11 \le g \le 15$. To find the endpoints of this interval, 240 miles is divided by 16 and 23, respectively.

b. Sample graph:

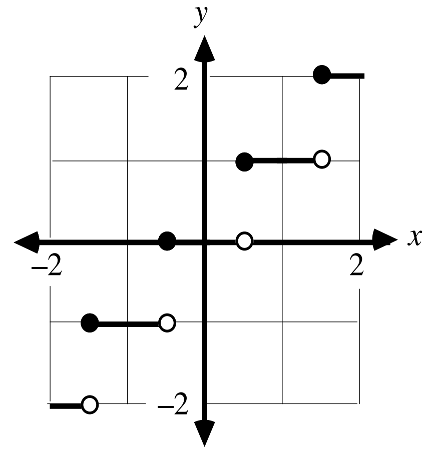
2. **a.** Sample graph for rounding down to the previous integer:



Sample graph for rounding up to the next integer:



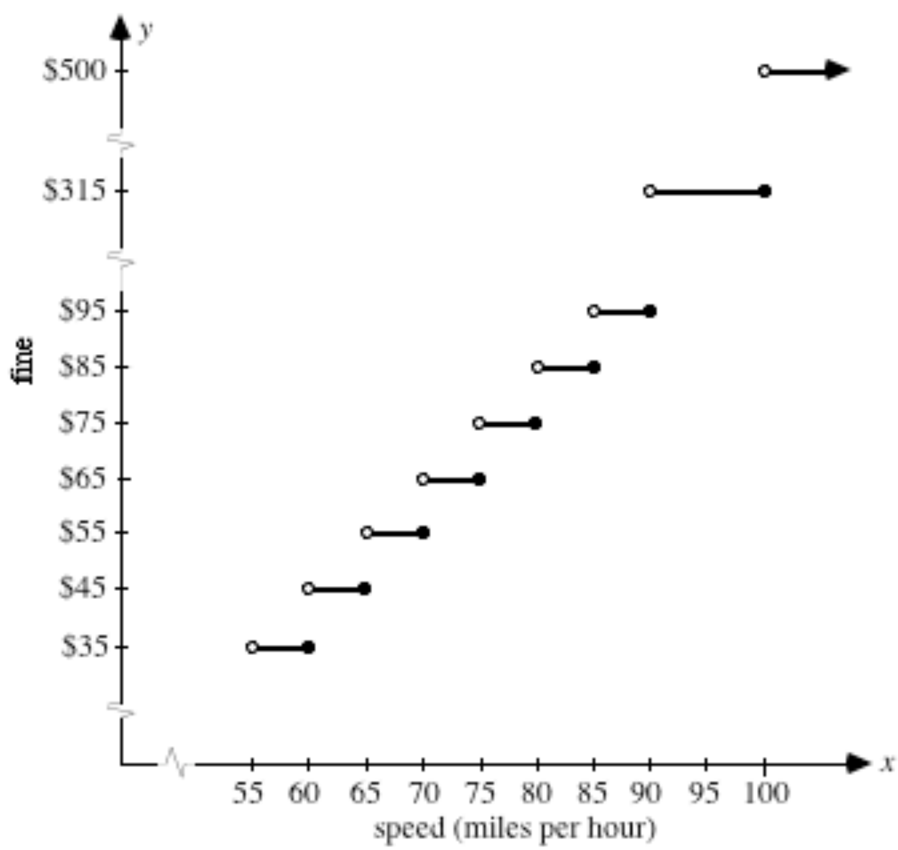
Sample graph for rounding to the nearest integer:



b. The following equations correspond to the sample graphs in Part **a**.

Rounding Method	Equation
round down	$y = [x]$
round up	$y = -[-x]$
round to the nearest	$y = [x + 0.5]$

3. **a.** Sample graph:



b. Sample response: If this were not a function, there would be more than one fine for a given speed.

c. The judge should decide in Peggy's favor since 75 is in the interval $70 < s \leq 75$ and not in the interval $75 < s \leq 80$.

Selected References

Lott, J. W., and A. W. Wilson. "Applications of the Step Function." *The Illinois Mathematics Teacher* 30 (January 1979): 2–7.

Mathematical Association of America (MAA) and National Council of Teachers of Mathematics (NCTM). *A Sourcebook of Applications of School Mathematics*. Reston, VA: NCTM, 1980.

Sharron, Sidney, ed. *Applications in School Mathematics*. 1979 Yearbook. Reston, VA: NCTM, 1979.

Usiskin, Zalman. "The Greatest Integer Symbol—An Applications Approach." *The Mathematics Teacher* 70 (December 1977): 739–743.

Flashbacks

Activity 1

1.1 Identify two values of x that satisfy each of the following inequalities.

a. $x < 5$

b. $x \geq 2.3$

1.2 Graph each of the following inequalities on a number line.

a. $x < 5$

b. $x \geq 2.3$

1.3 If Lisa earns $10.00 per hour and is paid for every minute she works, determine how much she earns in each of the following time periods.

a. 30 min

b. 45 min

c. 3 hr, 20 min

d. x min

1.4 Round each of the following to the nearest dollar:

a. $3.85

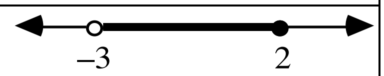
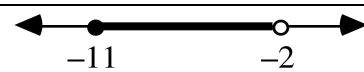
b. $6.45

c. $0.75

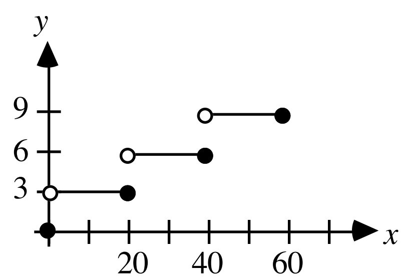
d. $1.39

Activity 2

2.1 Complete the following table:

Inequality	**Interval Notation**	**Graph**
$-3 \le x \le 5$		
	$[4, 6)$	
		
$3 \le x < 7$		
	$(-4, -2)$	
		

2.2 Use the following graph to complete Parts **a** and **b**.

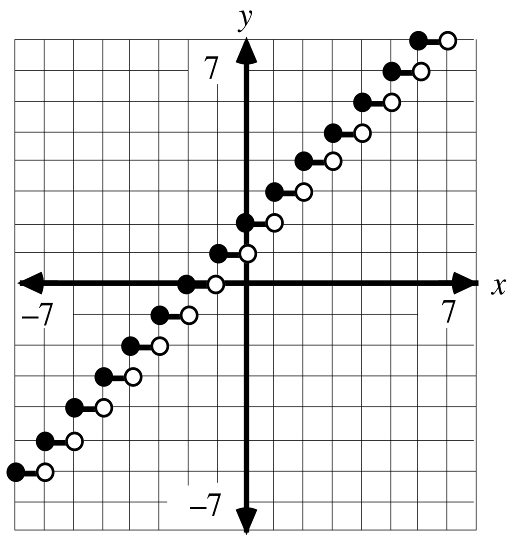


a. Identify the domain and the range for the graph.

b. Is this a graph of a function? Explain your response.

Activity 3

Use the following graph to complete Flashbacks **3.1–3.3**.



3.1 What is the *y*-intercept of this graph?

3.2 Determine an equation that describes the graph.

3.3 Find the value of *y* for each of the following values of *x*:

a. $x = 2$

b. $x = 3.5$

c. $x = 0$

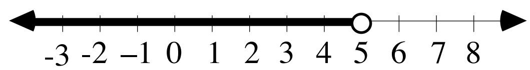
Answers to Flashbacks

Activity 1

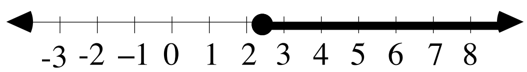
1.1 **a.** Sample response: 2, –4.

b. Sample response: 2.3, 5.

1.2 **a.** Sample graph:



b. Sample graph:



1.3 **a.** \$5.00

b. \$7.50

c. \$33.33

d. $\$10 \bullet \frac{x}{60}$

1.4 **a.** \$4.00

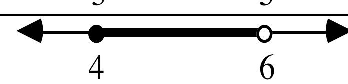
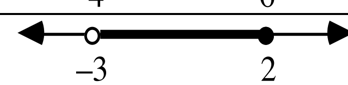
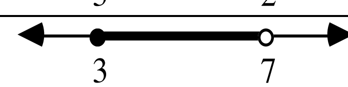
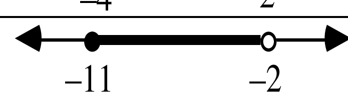
b. \$6.00

c. \$1.00

d. \$1.00

Activity 2

2.1 A completed table is shown below.

Inequality	**Interval Notation**	**Graph**
$-3 \le x \le 5$	$[-3,5]$	
$4 \le x < 6$	$[4,6)$	
$-3 < x \le 2$	$(-3,2]$	
$3 \le x < 7$	$[3,7)$	
$-4 < x < -2$	$(-4,-2)$	
$-11 \le x < -2$	$[-11,-2)$	

2.2 **a.** The domain is $0 \le x \le 60$ and the range is $\{0, 3, 6, 9\}$.

b. This is the graph of a function because each element in the domain is mapped to exactly one element of the range.

Activity 3

3.1 The y-intercept is 2.

3.2 One equation that describes the graph is $y = [x] + 2$.

3.3 **a.** $y = 4$

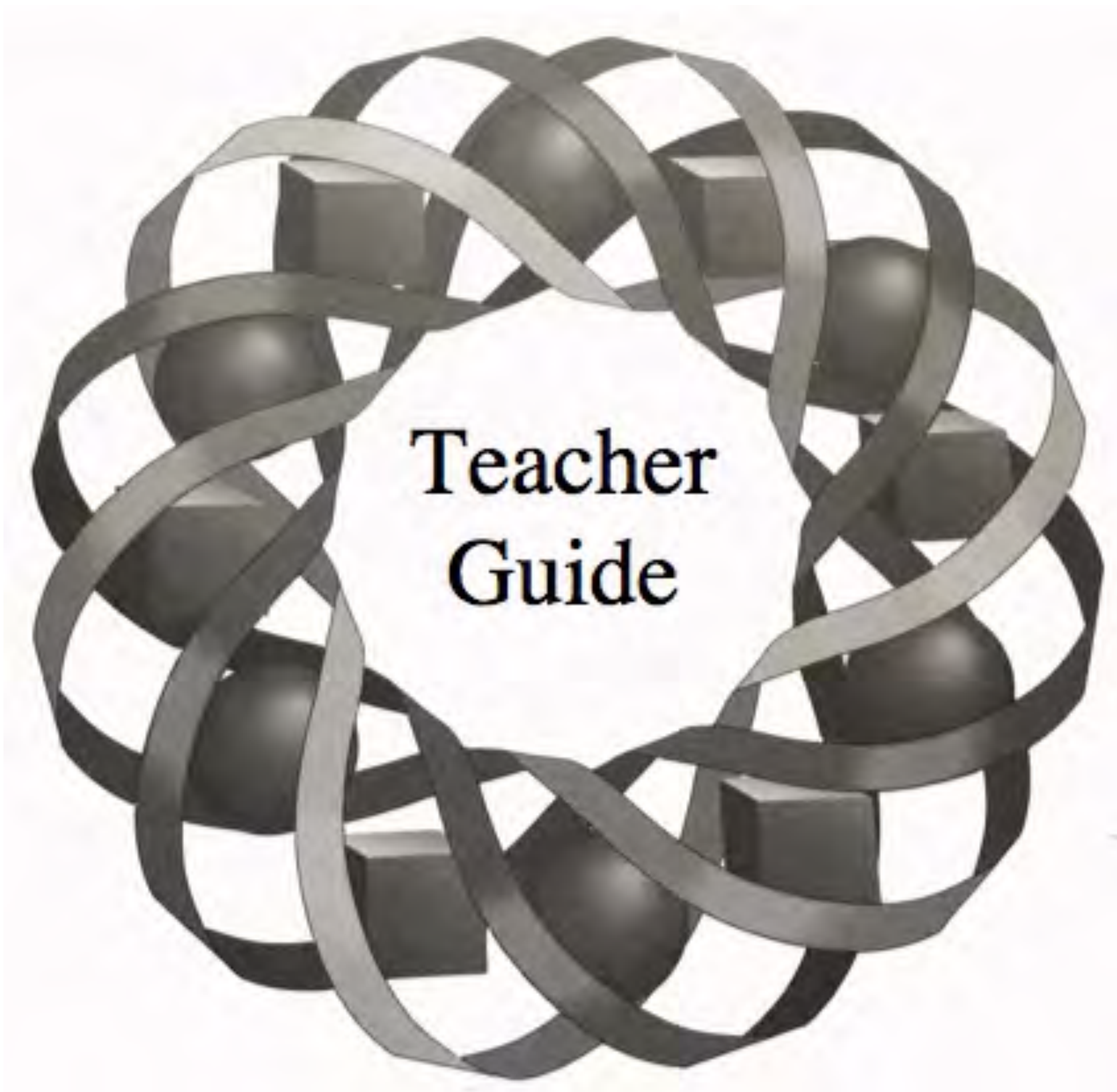
b. $y = 5$

c. $y = 2$

Employees' Pay Template

(0,)	(1,)	(2,)	(3,)	(4,)	(5,)	(6,)	(7,)
(8,)	(9,)	(10,)	(11,)	(12,)	(13,)	(14,)	(15,)
(16,)	(17,)	(18,)	(19,)	(20,)	(21,)	(22,)	(23,)
(24,)	(25,)	(26,)	(27,)	(28,)	(29,)	(30,)	(31,)
(32,)	(33,)	(34,)	(35,)	(36,)	(37,)	(38,)	(39,)
(40,)	(41,)	(42,)	(43,)	(44,)	(45,)	(46,)	(47,)
(48,)	(49,)	(50,)	(51,)	(52,)	(53,)	(54,)	(55,)
(56,)	(57,)	(58,)	(59,)	(60,)	(61,)	(62,)	(63,)
(64,)	(65,)	(66,)	(67,)	(68,)	(69,)	(70,)	(71,)
(72,)	(73,)	(74,)	(75,)	(76,)	(77,)	(78,)	(79,)

From Rock Bands to Recursion



You've endured hours on the highway, rows of seats in an empty amphitheater, lagging ticket sales, crowded soft drink concessions, bouncing beach balls, and—finally—the notes of an electric guitar. Is this a pattern worth repeating?

Shirley Bagwell • Janet Higgins • Sandy Johnson • Terry Souhrada

Teacher Edition

From Rock Bands to Recursion

Overview

In this module, students use spreadsheets to investigate arithmetic and geometric sequences.

Objectives

In this module, students will:

- analyze number patterns
- develop arithmetic and geometric sequences
- compare linear equations and explicit formulas for arithmetic sequences
- compare the graphs of linear equations and arithmetic sequences
- compare exponential equations and explicit formulas for geometric sequences
- compare the graphs of exponential equations and geometric sequences
- compare the graphs of arithmetic and geometric sequences
- evaluate series

Prerequisites

For this module, students should be able to:

- recognize the graphs of linear equations and exponential equations
- write and graph linear equations in slope-intercept form
- represent multiplication as repeated addition
- represent repeated multiplication using exponents.

Time Line

Activity	1	2	3	4	Summary Assessment	Total
Days	3	2	2	2	2	11

Materials Required

Materials	Activity				
	1	2	3	4	Summary Assessment
graph paper (optional)	X	X	X	X	X

Technology

Software	Activity				
	1	2	3	4	Summary Assessment
spreadsheet	X	X	X	X	X
graphing utility	X	X	X	X	X

From Rock Bands to Recursion

Introduction

(page 305)

The sales, performances, and travels of a rock band provide a context for investigating sequences throughout the module. (Students revisit the patterns created by stacks of blocks in Problem **2.8**.)

(page 305)

Activity 1

Students use tables and spreadsheets to explore patterns found in the number of performances made by a rock band. These patterns are arithmetic sequences.

Materials List

- graph paper (optional)

Technology

- spreadsheet
- graphing utility

Exploration

(page 305)

a. A completed table appears below.

No. of Weeks on Tour	Total No. of Performances
1	8
2	10
3	12
4	14
5	16
6	18
7	20
8	22
9	24
10	26

b. Sample response: In the left-hand column, the pattern begins with 1, then adds 1 to each successive cell. In the right-hand column, the pattern begins with 8, then adds 2 to each successive cell.

c. 1. 9 weeks

2. 30 performances

3. 42 performances

d. Sample spreadsheet:

No. of weeks (n)	Total No. of Performances (p_n)
1	8
2	10
3	12
⋮	⋮
48	102
49	104
50	106

e. The recursive formula for the sequence is:

$$\begin{cases} p_1 = 8 \\ p_n = p_{n-1} + 2, \ \ n > 1 \end{cases}$$

Discussion

(page 307)

a. The numbers in the left-hand column are found by starting with 1, then adding 1 to the previous number. The numbers in the right-hand column are found by starting with 8, then adding 2 to the previous number.

b. 1. The value of n is 20.

2. The value of p_{20} is 46.

3. The value of n is 7.

4. The value of p_7 is 20.

c. This can be expressed as $p_{30} = 66$.

d. The sequences are both arithmetic because they both have a common difference between any two successive terms. The first column has a common difference of 1, and the second column has a common difference of 2.

Assignment

(page 308)

1.1 **a.** Sample spreadsheet:

Hours Driven	Kilometers Remaining	Hours Driven	Kilometers Remaining
1	3190	18	1490
2	3090	19	1390
3	2990	20	1290
4	2890	21	1190
5	2790	22	1090
6	2690	23	990
7	2590	24	890
8	2490	25	790
9	2390	26	690
10	2290	27	590
11	2190	28	490
12	2090	29	390
13	1990	30	290
14	1890	31	190
15	1790	32	90
16	1690	33	–10
17	1590		

b. They arrive in Portland during hour 33.

c. $k_5 = 2790$

d. Since 1990 km remain after 13 hr, $k_{13} = 1990$.

e. When they are 1390 km from Portland, the band has traveled 19 hr: $k_{19} = 1390$.

f. Since the term before 2290 is 2390, $k_9 = 2390$.

g. The recursive formula for kilometers remaining to Portland is:

$$\begin{cases} k_1 = 3190 \\ k_n = k_{n-1} + (-100), \ n > 1 \end{cases}$$

1.2 **a.** $t_1 = 790$, $t_2 = 1003$, $t_3 = 1216$, $t_4 = 1429$, $t_5 = 1642$

b. Ticket sales exceed 2200 for the first time on the eighth day; $t_8 = 2281$.

c. $t_{n+1} = t_{10} = 2707$

d. The recursive formula is:

$$\begin{cases} t_1 = 790 \\ t_n = t_{n-1} + 213, \;\; n > 1 \end{cases}$$

e. Sample response: This is not a good assumption. Although the average daily sales was 213 tickets, this does not mean that 213 tickets were sold every day.

***1.3** A positive common difference produces an increasing sequence. A negative common difference produces a decreasing sequence.

1.4 **a.** The first term is 9.0.

b. The next four terms are 9.5, 10.0, 10.5, and 11.0.

1.5 The recursive formula is:

$$\begin{cases} t_1 = 3 \\ t_n = t_{n-1} + 4, \;\; n > 1 \end{cases}$$

***1.6** **a.** Answers will vary. Sample response: 6, 17, 28, 39, ...

b. For the sample sequence in Part **a**, the common difference is 11.

c. The recursive formula for the sample sequence in Part **a** is:

$$\begin{cases} t_1 = 6 \\ t_n = t_{n-1} + 11, \;\; n > 1 \end{cases}$$

1.7 Sample response: For $t_1 = 5$, the sequence is 5, 8, 11, 14, 17, … . For $r_1 = -4$, the sequence is –4, –1, 2, 5, 8, 11, 14, … . The two sequences are similar because they both increase by a constant of 3. The first sequence is a subset of the second. The two sequences are different since they have different first terms.

* * * * *

1.8 Answers will vary, depending on student ages.

a. The sample response given below is for a 14-year-old.

$$\frac{0.8 \text{ cm}}{1 \text{ issue}}\left(\frac{12 \text{ issues}}{1 \text{ yr}}\right)(14 \text{ yr}) \approx 134 \text{ cm}$$

b. The shelf space needed for each year (the common difference) is $0.8 \bullet 12 = 9.6 \text{ cm}$. In the following sample response, the student is 5 years old in kindergarten.

$$\begin{cases} w_1 = 9.6(5) = 48 \\ w_n = w_{n-1} + 9.6, n > 1 \end{cases}$$

c. Students may use a spreadsheet or table to answer this question. It will take 576 cm of shelf space for 60 yr of *National Geographic*.

Years	Shelf Space (cm)
1	9.6
2	19.2
3	28.8
⋮	⋮
58	556.8
59	566.4
60	576

1.9 **a.** The recursive formula is:

$$\begin{cases} m_1 = 51 \\ m_n = m_{n-1} - 3, \ n > 1 \end{cases}$$

b. The recursive formula is:

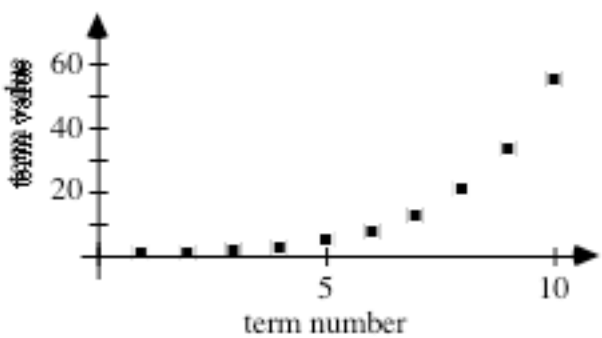
$$\begin{cases} k_1 = 11 \\ k_n = k_{n-1} + 2, \ n > 1 \end{cases}$$

c. Melinda and Kris will both have $27.00 at the beginning of the ninth week. Using subscript notation, $m_9 = k_9 = 27$.

1.10 **a.** Using the recursive formula, $t_3 = t_1 + t_2 = 1 + 1 = 2$.

b. The recursive formula generates the Fibonacci sequence. The first 10 terms of the sequence are 1, 1, 2, 3, 5, 8, 13, 21, 34, and 55.

c. Sample scatterplot:



d. The sequence is not an arithmetic sequence. Although the terms are generated by addition, the difference between terms is not constant.

* * * * * * * * * *

(page 310)

Activity 2

This activity continues the exploration of arithmetic sequences through the experiences of the rock band.

Materials List

- graph paper (optional)

Technology

- spreadsheet
- graphing utility

Exploration

(page 311)

Students develop an explicit formula for an arithmetic sequence.

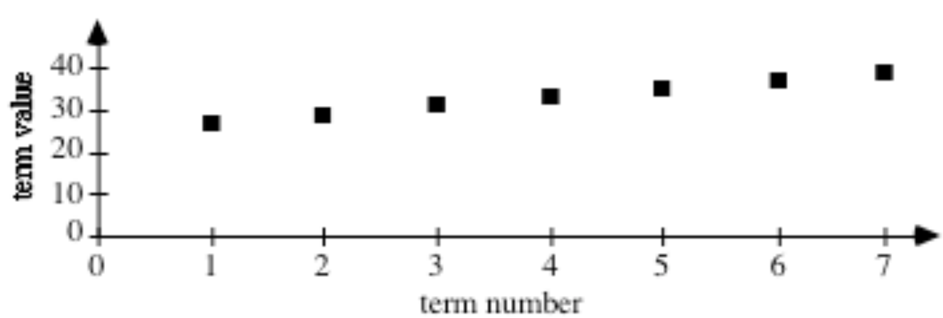
a. A completed table appears below.

Term No. (n)	**Term (p_n)**	**Recursive Form of p_n**	**Another Form of p_n**
1	27	27	27 + (0 • 2)
2	29	27 + 2	27 + (1 • 2)
3	31	27 + 2 + 2	27 + (2 • 2)
4	33	27 + 2 + 2 + 2	27 + (3 • 2)
5	35	27 + 2 + 2 + 2 + 2	27 + (4 • 2)
6	37	27 + 2 + 2 + 2 + 2 + 2	27 + (5 • 2)
7	39	27 + 2 + 2 + 2 + 2 + 2 + 2	27 + (6 • 2)

b. $(n-1)$

c. $p_n = p_1 + (n-1)2 = 27 + (n-1)2$

d. Sample scatterplot:



e. The linear equation $y = 2x + 25$ models the scatterplot well. This equation is equivalent to the one found in Part **c**.

Discussion

(page 311)

a. **1.** The number of 2s is 1 less than 21, or 20.

2. The number of 2s is 1 less than n; or $n-1$.

b. **1.** One possible formula is $p_n = 27 + (n-1)2$.

2. $p_{47} = 119$; $p_{100} = 225$

c. In general, the x-values (or domain) for a linear equation are the set of all real numbers, while the n-values (or domain) for an arithmetic sequence are the set of natural numbers. (The context of a problem may limit the domain of a linear equation.) Therefore, the graph of a linear equation is a set of connected points (or continuous line), while the graph of a sequence is a set of disconnected points.

d. Sample response: The scatterplot of an arithmetic sequence can always be modeled well by a linear equation because an arithmetic sequence always has a constant difference and a line always has a constant slope.

e. **1.** Both a_n and y represent the values on the y-axis, or the range.

2. Both n and x represent the values on the x-axis, or the domain.

3. The difference d and the slope m are equal because they both represent the ratio of the change in y-values to the change in x-values:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{a_n - a_{n-1}}{n - (n-1)} = \frac{d}{1}$$

4. The y-intercept b is the value of the linear equation at $x = 0$. The value of the explicit formula at $n = 0$ would be the difference between a_1 and d. Therefore, $a_1 - d = b$.

f. An explicit formula is easier to use when the previous term is not known. For example, the 100th term of the arithmetic sequence in Table **4** would be easier to find using an explicit formula rather than a recursive formula.

g. Answers may vary. The following proof substitutes b for $(a_1 - d)$, m for d, and x for n:

$$\begin{aligned} a_n &= a_1 + d(n-1) \\ &= dn + (a_1 - d) \\ &= dn + b \\ &= mx + b \end{aligned}$$

Assignment

(page 313)

2.1 **a.** The data for weekly sales form an arithmetic sequence. The explicit formula for this sequence is: $S_n = 9050 + 2353(n-1)$, Since the week of December 23–29 is week 29:

$$S_{29} = 9050 + 2353(29 - 1) = 74,934$$

b. Sales exceed 1 million during the 26th week. Students may use a spreadsheet to solve this problem as shown below:

n	S_n	Total States
1	9050	9050
2	11,403	20,453
3	13,756	34,209
⋮	⋮	⋮
25	65,522	932,150
26	67,875	1,000,025

2.2 **a.** Sample response: $t_n = 120 + 16(n-1)$.

b. There are 360 seats in the 16th row.

c. There are 30 rows in the theater.

d. There are 10,560 seats in the theater. Students may use a spreadsheet to solve this problem.

Row	Seats	Total Seats
1	120	120
2	136	256
3	152	408
⋮	⋮	⋮
28	552	9408
29	568	9976
30	584	10,560

***2.3** Sample response: $w_n = 4.25 + 0.10(n-1)$.

***2.4** **a.** **1.** The first five terms are 94, 103, 112, 121, and 130.

2. The common difference $d = 9$.

3. An explicit formula is $p_n = 94 + 9(n-1)$.

4. 8185 people

b. **1.** The first five terms are 8500, 8390, 8280, 8170, and 8060.

2. The common difference $d = -110$.

3. An explicit formula is $l_n = 8500 + (-110)(n-1)$ or $l_n = 8500 - 110(n-1)$.

4. The supply will last approximately 78 min.

2.5 **a.** In the following recursive formula, c_1 is the amount of money that the cashier started with and c_n represents the total amount of money in the cash register after n minutes.

$$\begin{cases} c_1 = \$50 \\ c_n = c_{n-1} + \$2.15, \ \ n > 1 \end{cases}$$

b. In the explicit formula below, c_n represents the amount of money in the cash register after n minutes, \$50 is the amount that the cashier started with, and \$2.15 is the amount taken in each minute.

$$c_n = \$50 + \$2.15(n-1)$$

c. In the following linear equation, y represents the amount of money in the cash register after x minutes, x represents the number of minutes that the cash register has been open, and 47.85 is the y-intercept.

$$y = 2.15x + 47.85$$

Note: You may wish to point out that, in this situation, the possible values for x are the natural numbers.

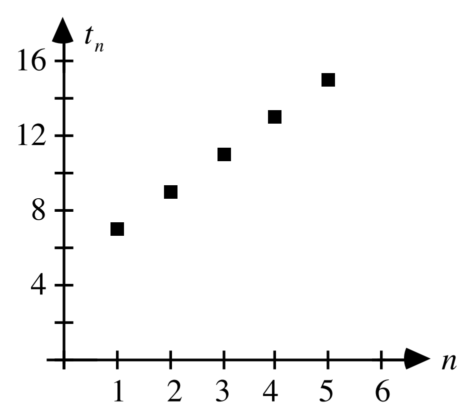
* * * * *

2.6 **a.** There will be 93 members after 12 months. This can be calculated using the following explicit formula:

$$a_{12} = 38 + 5(12-1) = 93$$

b. Using the corresponding arithmetic series, $38 + 43 + 48 + \cdots + 93 = 786$ stamps.

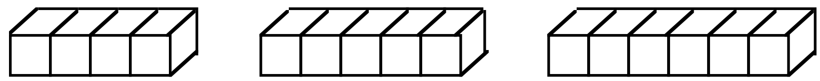
***2.7** **a.** Sample scatterplot:



b. An equation of the line is $y = 2x + 5$ or $t_n = 2n + 5$.

c. The explicit formula for the arithmetic sequence is $t_n = 7 + 2(n - 1)$, which simplifies to $t_n = 2n + 5$. This is the same as the linear equation in Part **b**.

***2.8** **a.** The next three layers are shown below.



b. The first six terms of the sequence are 1, 2, 3, 4, 5, and 6.

c. The sequence is an arithmetic sequence since there is a constant difference of 1.

d. Students can describe the sequence recursively or explicitly. The recursive formula is:

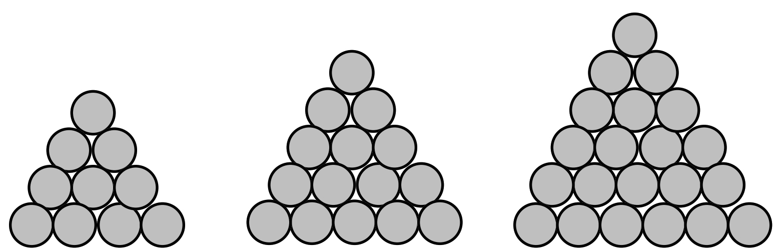
$$\begin{cases} l_1 = 1 \\ l_n = l_{n-1} + 1, \; n > 1 \end{cases}$$

The explicit formula is:

$$\begin{aligned} l_n &= 1 + 1(n - 1) \\ &= 1 + (n - 1) \\ &= n \end{aligned}$$

e. $1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$ blocks of cheese

2.9 **a.** The next three levels in the display are shown below.



b. The first six terms of the sequence are 1, 3, 6, 10, 15, and 21.

c. The sequence is not an arithmetic sequence because the difference between successive terms is not constant.

d. A recursive formula is:

$$\begin{cases} l_1 = 1 \\ l_n = l_{n-1} + (n-1), \ \ n > 1 \end{cases}$$

An explicit formula is:

$$l_n = \frac{n(n+1)}{2}$$

e. Using the corresponding series,
$1 + 3 + 6 + 10 + 15 + 21 + 28 + 36 + 45 + 55 = 220$ oranges.

* * * * * * * * * *

(page 315)

Activity 3

In this activity, students explore geometric sequences using recursion and graphs.

Materials List

- graph paper (optional)

Technology

- spreadsheet
- graphing utility

Exploration

(page 316)

a. A completed table for weeks 10–12 appears below.

Week	Weekly Sales	Total Sales
1	500	500
2	1000	1500
3	2000	3500
⋮	⋮	⋮
10	256,000	511,500
11	512,000	1,023,500
12	1,024,000	2,047,500

b. Sample response: The pattern in the "Weekly Sales" column begins with 500. The value for each successive cell is the value in the previous cell multiplied by 2. The pattern in the "Total Sales" column also begins with 500. The value for each successive cell is the value in the previous cell plus the weekly sales for the next week.

c. **1.** The common ratio is $2/1$ or 2.

2. Sample formula:

$$\begin{cases} g_1 = 500 \\ g_n = g_{n-1}(2),\ n > 1 \end{cases}$$

3. The geometric series is 2,047,500.

Discussion

(page 317)

a. See sample response to Part **b** of the exploration.

b. **1.** 2,047,500

2. 256,000

3. week 11

c. The numbers form a geometric sequence because they have a common ratio of 2 between any two successive terms.

d. The numbers do not form a geometric sequence because they do not have a common ratio. For example, the ratio between the first two terms is $1500/500 = 3/2$, while the ratio between the second and third terms is $3500/1500 = 7/3$.

e. Sample response: No. If the pattern continues, the total sales would be unrealistically high—over 8 billion. This is more than the world population.

Assignment

(page 317)

3.1 **a.** Sample recursive formula:

$$\begin{cases} t_1 = 4 \\ t_n = t_{n-1} \bullet 7,\ \ n > 1 \end{cases}$$

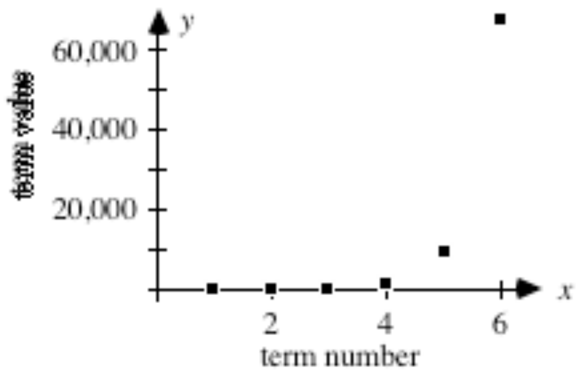
b. Sample recursive formula:

$$\begin{cases} t_1 = 9 \\ t_n = t_{n-1} \bullet 2.1,\ \ n > 1 \end{cases}$$

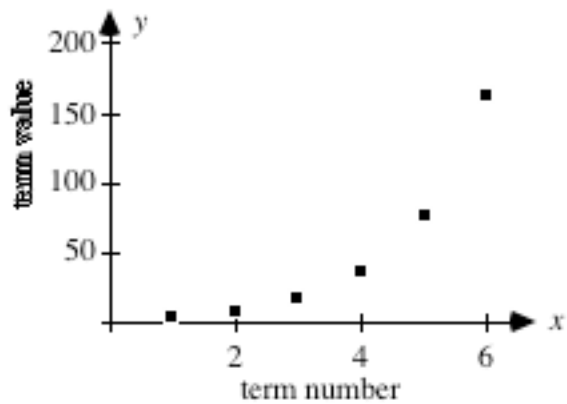
c. Sample recursive formula:

$$\begin{cases} t_1 = 144 \\ t_n = t_{n-1} \bullet \dfrac{1}{4},\ \ n > 1 \end{cases}$$

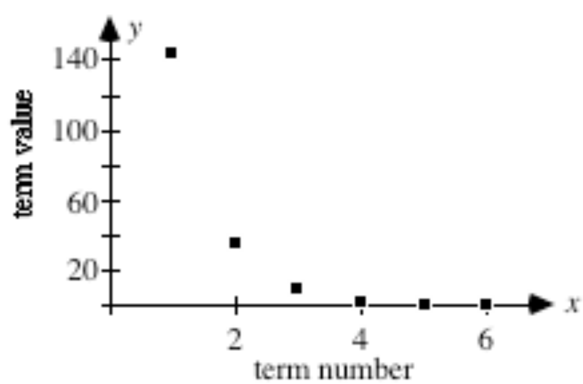
***3.2** **a.** Sample graph of the sequence in Problem **3.1a**:



Sample graph of the sequence in Problem **3.1b**:



Sample graph of the sequence in Problem **3.1c**:



b. Sample response: All three graphs appear to be exponential. Graphs **a** and **b** increase. Graph **c** decreases.

3.3 **a.** The following table is an extension of Table **5** (created in the exploration).

Week	Weekly Sales	Total Sales
12	1,024,000	2,047,500
13	256,000	2,303,500
14	64,000	2,367,500
15	16,000	2,383,500
16	4000	2,387,500
17	1000	2,388,500
18	250	2,388,750
19	62.5	2,388,812.5
20	15.625	2,388,828.125

b. Sample recursive formula:

$$\begin{cases} t_{12} = 1{,}024{,}000 \\ t_n = t_{n-1} \bullet 0.25,\ n > 12 \end{cases}$$

c. Sample response: After week 18, the formula yields fractional CDs sold. If these values are treated as approximations, however, the formula is still valid.

d. There will be 15 or 16 CDs sold during week 20.

e. The predicted total sales are 2,388,828.

***3.4** **a.** Sample table:

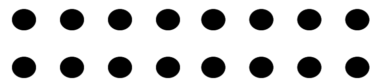
Barrel Number	Charge per Barrel	Total Charge
1	$0.01	$0.01
2	$0.02	$0.03
3	$0.04	$0.07
4	$0.08	$0.15
⋮	⋮	⋮
15	$163.84	$327.67
16	$327.68	$655.35
17	$655.36	$1310.71
18	$1310.72	$2621.43
19	$2621.44	$5242.87
20	$5242.88	$10,485.75

b. To earn at least $500.00, the students must collect 16 barrels of garbage.

c. When the students collect 17 barrels of garbage, the manager will be $310.71 over budget.

* * * * *

3.5 **a.** The next picture in this pattern is shown below.



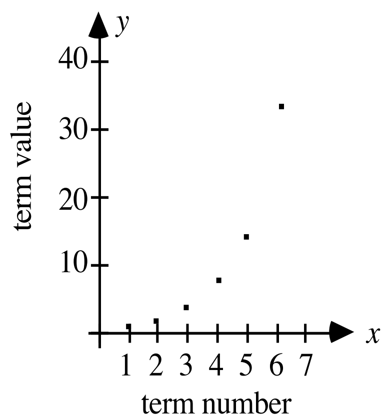
b. This pattern can be represented as the sequence 1, 2, 4, 8, 16, … .

c. Sample response: This is a geometric sequence because each term after the first is formed by multiplying the preceding term by the common ratio of 2.

d. A recursive formula for the sequence from Part **b** is:

$$\begin{cases} t_1 = 1 \\ t_n = 2 \bullet t_{n-1},\ n > 1 \end{cases}$$

e. Sample graph:



3.6 **a.** Sample response: Yes, the sale prices will form a geometric sequence. The sale price for each week is found by multiplying the previous week's price by a common ratio of 0.9.

b. In week 6, the sales price would be \$11.81.

c. $\$20.00 + \$18.00 + \$16.20 + \$14.58 + \$13.12 = \$81.90$

3.7 **a.** Sample recursive formula:

$$\begin{cases} y_1 = 350 \\ y_n = y_{n-1} \bullet 1.02,\ n > 1 \end{cases}$$

b. The monthly payment will be more than \$500.00 during year 20.

c. The loan will be paid in full after 27 years, 3 months.

* * * * * * * * * *

(page 319)

Activity 4

In this activity, students continue their investigation of geometric sequences using explicit formulas and exponential equations.

Materials List

- graph paper (optional)

Technology

- spreadsheet
- graphing utility

Exploration

(page 320)

a. $t_n = 500 \bullet 2^{(n-1)}$

b. Students should use the explicit formula from Part **a** in their spreadsheets. Sample spreadsheet:

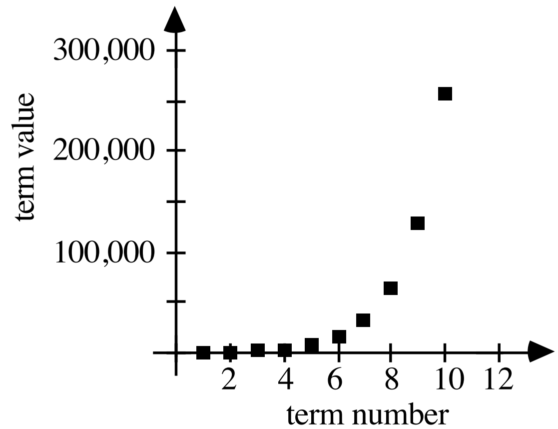
	A	B	C
1	**Term No. (*n*)**	**Recursive Form of t_n**	**Explicit Form of t_n**
2	1	500	500*2^(A2-1)
3	2	(B2)*2	500*2^(A3-1)
4	3	(B3)*2	500*2^(A4-1)
5	4	(B4)*2	500*2^(A5-1)
⋮	⋮	⋮	⋮

	A	B	C
1	**Term No. (*n*)**	**Recursive Form of t_n**	**Explicit Form of t_n**
2	1	500	500
3	2	1000	1000
4	3	2000	2000
5	4	4000	4000
⋮	⋮	⋮	⋮
9	8	64000	64000
10	9	128000	128000

c. In the sample spreadsheet shown above, the values in columns B and C are equal to t_n.

d. The fourth column produces a constant ratio of 2.

e. Sample scatterplot:



f. The exponential equation $y = 250 \bullet 2^x$ models the scatterplot exactly. Using the laws of exponents, this equation can be rewritten as $y = 250 \bullet 2^1 \bullet 2^{(x-1)}$, which equals $y = 500 \bullet 2^{(x-1)}$. This equation is equivalent to the formula found in Part **a**.

Discussion

(page 320)

a. **1.** $t_{19} = 500 \bullet 2^{18}$

2. The power of 2 equals $n - 1$.

b. **1.** $t_n = 500 \bullet 2^{n-1}, n \geq 1$

2. $t_7 = 500 \bullet 2^6 = 32,000$

c. The ratio is the same for each pair of successive terms.

d. The common ratio is the base of the exponent in the explicit formula.

e. The scatterplot of a geometric sequence can always be modeled by an exponential equation because a geometric sequence has a constant ratio between successive terms. This constant ratio corresponds with the value of b in an exponential equation of the form $y = ab^x$.

f. In general, the x-values (or domain) for an exponential equation are the set of all real numbers, while the n-values (or domain) for a geometric sequence are the set of natural numbers. (The context of a problem may limit the domain of an exponential equation.) Therefore, the graph of an exponential equation is a set of connected points (or a continuous curve), while the graph of a geometric sequence is a set of disconnected points.

g. **1.** Both g_n and y represent the values on the y-axis, or the range.

2. Both n and x represent the values on the x-axis, or the domain.

3. The common ratio r and the value of b are the same because b represents the amount by which y is multiplied for every unit increase in x.

4. The first term g_1 equals the y-value when $x = 1$, while a is the y-value when $x = 0$. Therefore, $g_1 = ab$. Since b equals the common ratio r, $g_1 = ar$.

h. The explicit formula is easier to use when the previous term is not known. For example, the 20th term of the geometric sequence in Table **5** would be easier to find by using the explicit formula rather than the recursive formula.

i. Answers may vary. The following proof substitutes ar for g_1, b for r, and x for n:

$$\begin{aligned} g_n &= g_1 r^{n-1} \\ &= ar(r)^{n-1} \\ &= ar^n \\ &= ab^x \end{aligned}$$

Assignment

(page 322)

***4.1** **a**. The sequence is geometric. The explicit formula is $t_n = 0.5(5)^{n-1}$. The 10th term is 976,562.5.

b. The sequence is not geometric because there is no common ratio between successive terms.

c. The sequence is geometric. The explicit formula is $t_n = 4(-3)^{n-1}$. The 10th term is –78,732.

d. The sequence is geometric. The explicit formula is $t_n = 1000(1/4)^{n-1}$.The 10th term is approximately 0.00381.

4.2 **a.** $1 + 3 + 9 + 27 + 81 = 121$

b. $2 + 5 + 12.5 + 31.25 + 78.125 = 128.875$

c. $0.5 + 1 + 2 + 4 + 8 + 16 + 32 + 64 = 127.5$

d. $125 + 25 + 5 + 1 + 0.2 + 0.04 = 156.24$

***4.3** Sample response: The ball will come to rest after about the eighth bounce. This was calculated using the following table, with heights rounded to 2 decimal places.

Bounce	Height (m)
Start	24.00
1	8.00
2	2.67
3	0.89
4	0.30
5	0.10
6	0.03
7	0.01
8	0.00
9	0.00
10	0.00

4.4 **a.** In increasing geometric sequences, the common ratio is greater than 1.

b. In decreasing geometric sequences, the common ratio is between 0 and 1.

* * * * *

4.5 **a.** If this pattern is expressed as a geometric sequence, $t_1 = 1$ and $r = 2$.

b. An explicit formula that describes this pattern is $t_n = 2^{n-1}$.

c. $t_{14} = 8192$

4.6 **a.** A completed table appears below

Note	Frequency (Hz)
A	3520
A	1760
A	880
A	440
middle C	
A	220
A	110
A	55
A	27.5
A	13.75

b. Two possible formulas are shown below:

$$\begin{cases} t_1 = 13.75 \\ t_n = t_{n-1}(2),\ n > 1 \end{cases} \qquad \begin{cases} t_1 = 3520 \\ t_n = t_{n-1}(0.5),\ n > 1 \end{cases}$$

c. The fans would hear all of the notes in the table except the fifth A below middle C.

***4.7** **a.** A geometric sequence best describes the wages offered by Plouvier's Pottery because each hourly wage after the first is formed by multiplying the preceding wage by the common ratio of 1.10.

b. An arithmetic sequence best describes the wages offered by Brocklebank's Bakery because every hourly wage after the first is formed by adding the constant value of 0.10 to the preceding wage.

c. An explicit formula that describes the wages for Plouvier's Pottery is $P_n = 5(1.10)^{n-1}$. An explicit formula that describes the wages for Brocklebank's Bakery is $B_n = 6.5 + 0.10(n-1)$.

d. If Annaborg wants to make as much money as possible, she should work for Brocklebank's Bakery. If she works 1 hr every 2 weeks, her total salary after 12 weeks for Plouvier's Pottery would be:

$$\$5.00 + \$5.50 + \$6.05 + \$6.66 + \$7.32 + \$8.05 = \$38.58$$

Her corresponding salary for Brocklebank's Bakery would be:

$$\$6.50 + \$6.60 + \$6.70 + \$6.80 + \$6.90 + \$7.00 = \$40.50$$

Therefore, her total salary at Brocklebank's Bakery will always exceed her total salary at Plouvier's Pottery, as long as she can work the same number of hours at either job.

* * * * * * * * * *

Answers to Summary Assessment (page 324)

1. Answers will vary. Sample response: Both residents used one year's data to predict the future. Mrs. Stephens assumed that the growth would continue arithmetically with a common difference of 35. She used the recursive formula:

$$\begin{cases} p_1 = 350 \\ p_n = p_{n-1} + 35, n > 1 \end{cases}$$

or the explicit formula $p_n = 350 + 35(n-1)$. Mr. Aloishan assumed that the growth would continue geometrically with a common ratio of 1.1. He used the recursive formula:

$$\begin{cases} p_1 = 350 \\ p_n = p_{n-1}(1.1), n > 1 \end{cases}$$

or the explicit formula $p_n = 350(1.1)^{n-1}$. Mr. Aloishan is more concerned than Mrs. Stephens because the geometric sequence results in faster growth than the arithmetic sequence.

2. Answers will vary. Sample response: It will take between over 126,839 years years to cancel the debt using Plan A. This is calculated by:

$4,000,000,000,000/(365 \bullet 24 \bullet 60 \bullet 60) \approx 126,839.2$ years.

Under Plan B, the debt will be paid off during the 42nd year. This is found by using the following table:

Year	Debt (dollars)
1	3,999,999,999,999
2	3,999,999,999,997
3	3,999,999,999,993
⋮	⋮
41	1,800,976,744,449
42	–398,046,511,103

Plan A would be affordable but it takes too long. Plan B pays off faster, but the last payments are not feasible. Interest payments are not considered under either plan.

Module Assessment

1. Consider the arithmetic sequence 2, 6, 10, 14, ... and the geometric sequence 2, 6, 18, 54, ...

a. Write an explicit formula for each sequence.

b. Write a recursive formula for each sequence.

c. Graph each sequence.

d. Determine a linear or exponential equation that models each sequence.

e. Find the corresponding series for the first eight terms of each sequence.

2. Determine the first five terms of an arithmetic sequence in which 14 is the third term, 30 is the seventh term, and 34 is the eighth term.

3. Describe how each of the following affects a geometric sequence:

a. a negative common ratio

b. a common ratio greater than 1

c. a positive common ratio between 0 and 1.

4. Imagine that your job grants a raise of *d* dollars on a regular basis. If your beginning salary is *a* dollars, write an explicit formula to calculate your salary after *n* raises.

5. Write a recursive formula for the sequence whose explicit formula is $t_n = 9.8(3)^{n-1}$.

Answers to Module Assessment

1. **a.** The explicit formula for the arithmetic sequence is $a_n = 2 + 4(n-1)$.
The explicit formula for the geometric sequence is $g_n = 2(3)^{n-1}$.

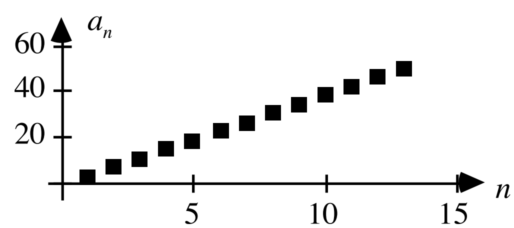
b. The recursive formula for the arithmetic sequence is:

$$\begin{cases} a_1 = 2 \\ a_n = a_{n-1} + 4, \ n > 1 \end{cases}$$

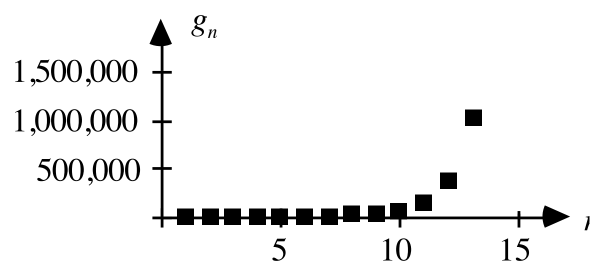
The recursive formula for the geometric sequence is:

$$\begin{cases} g_1 = 2 \\ g_n = g_{n-1}(3), n > 1 \end{cases}$$

c. Sample graph of the arithmetic sequence:



Sample graph of the geometric sequence:



d. The graph of the arithmetic sequence can be modeled by the linear equation $y = 4x - 2$. The graph of the geometric sequence can be modeled by the exponential equation $y = 2 \bullet 3^{x-1}$.

e. The arithmetic series is:

$$2 + 6 + 10 + 14 + 18 + 22 + 26 + 30 = 128$$

The geometric series is:

$$2 + 6 + 18 + 54 + 162 + 486 + 1458 + 4374 = 6560$$

2. The first five terms are 6, 10, 14, 18, and 22.

3. **a.** A negative common ratio causes consecutive terms to alternate between positive and negative.

b. A common ratio greater than 1 causes the geometric sequence to increase if $g_1 > 0$.

c. A common ratio less than 1 but greater than 0 causes the geometric sequence to decrease if $g_1 > 0$.

4. An explicit formula using the variable *w* is $w_n = a + d(n)$.

5. The recursive formula is:

$$\begin{cases} t_1 = 9.8 \\ t_n = t_{n-1}(3), \; n > 1 \end{cases}$$

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Flashbacks

Activity 1

1.1 Identify a possible pattern in each of the following sets of numbers:

a. $1, 3, 5, 7, 9, \ldots$

b. $11, 7, 3, -1, -5, \ldots$

c. $2, 6, 18, 54, 162, \ldots$

d. $100, 50, 25, 12.5, 6.25, \ldots$

1.2 **a.** Create a scatterplot of the data in the table below.

x	y
1	11
2	7
3	3
4	–1
5	–5
6	–9
7	–13

b. What type of equation might provide a good model of the data in Part **a**? Explain your response.

Activity 2

2.1 **a.** Create a scatterplot of the data in the table below.

x	y
1	14
2	11
3	8
4	5
5	2

b. Find a linear equation that models the data.

c. Identify the slope of the equation.

d. Identify the y-intercept of the equation.

2.2 Consider the arithmetic sequence $15, 12, 9, 6, 3, 0, -3, \ldots$.

a. What is the value of a_{n-1} when $n = 3$?

b. What is the value of a_{n-1} when $a_n = 3$?

c. What is the value of a_{n-2} when $n = 3$?

d. What is the value of a_{n+2} when $a_n = 3$?

Activity 3

3.1 Write the first five terms of the arithmetic sequence described by each of the following formulas:

a. $\begin{cases} a_1 = 2 \\ a_n = a_{n-1} - 3, \; n > 1 \end{cases}$

b. $a_n = -3 + 2(n-1)$

3.2 Consider the arithmetic sequence $8, 6, 4, 2, 0, -2, -4, -6, -8$.

a. Write a recursive formula for this sequence.

b. Write an explicit formula for this sequence.

c. Determine the corresponding series.

Activity 4

4.1 **a.** Create a scatterplot of the data in the table below.

x	***y***
1	7.5
2	18.8
3	46.9
4	117.2
5	293.0

b. Find an equation that models the data.

Answers to Flashbacks

Activity 1

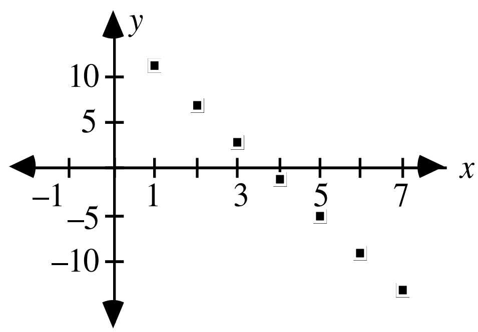
1.1 **a.** Each successive term in this set is 2 more than the preceding term.

b. Each successive term in this set is 4 less than the preceding term.

c. Each successive term in this set is 3 times the preceding term.

d. Each successive term in this set is half the preceding term.

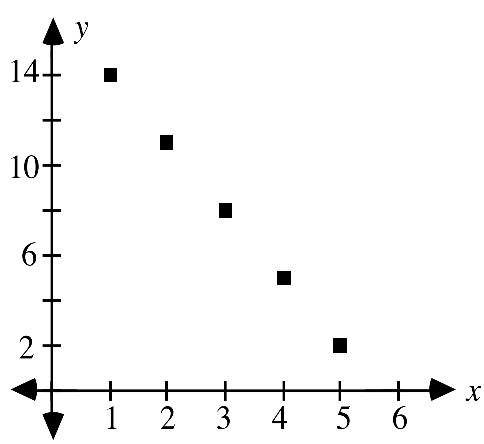
1.2 **a.** Sample scatterplot:



b. Sample response: A linear equation appears to model the data because the scatterplot has a constant slope of approximately –4. The equation that appears to fit is $y = -4x + 15$.

Activity 2

2.1 **a.** Sample scatterplot:



b. One equation that models the data is $y = -3x + 17$.

c. The slope is –3.

d. The *y*-intercept is 17.

2.2 **a.** 12

b. 6

c. 15

d. –3

Activity 3

3.1 **a.** The first five terms are 2, –1, –4, –7, and –10.

b. The first five terms are –3, –1, 1, 3, and 5.

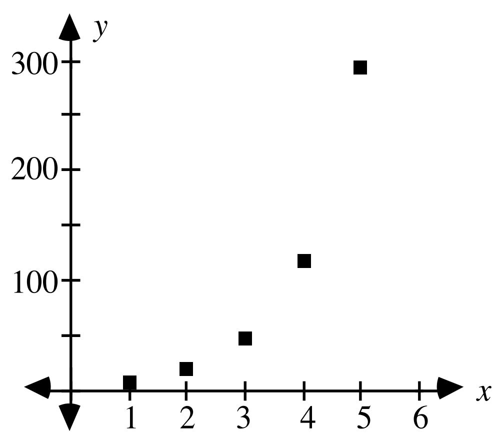
3.2 **a.** $\begin{cases} a_1 = 8 \\ a_n = a_{n-1} - 2, \quad n > 1 \end{cases}$

b. $a_n = 8 - 2(n-1)$

c. 8 + 6 + 4 + 2 + 0 + (–2) + (–4) + (–6) + (–8) = 0

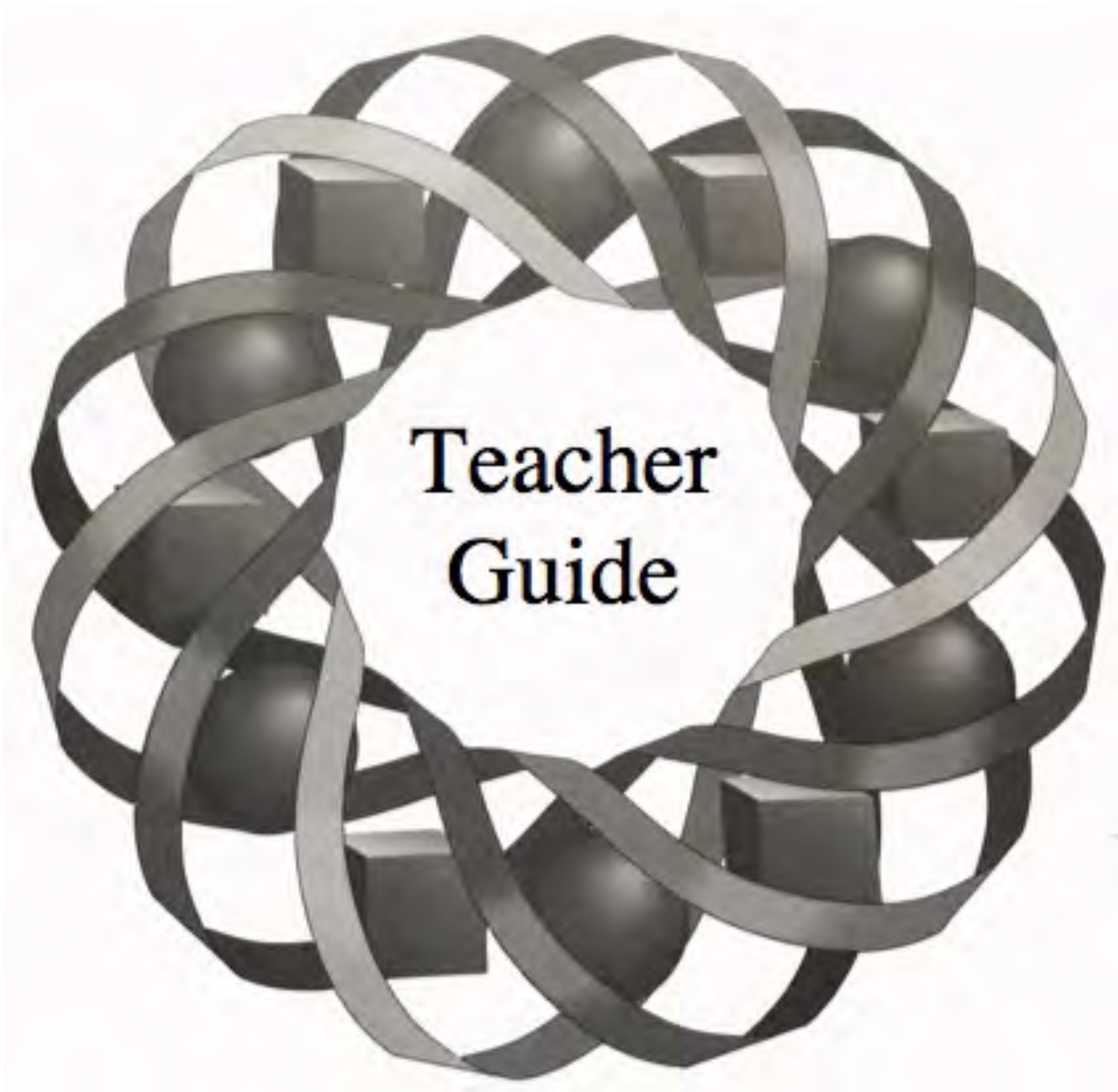
Activity 4

4.1 **a.** Sample graph:



b. One equation that models the data is $y = 3(2.5)^x$.

Under the Big Top But Above the Floor



It's carnival time at Dantzig High School. Before you play "Guess My Number" or "Roll-a-rama," however, you'll need some practice. In this module, you'll develop the skills necessary to model and solve problems involving linear inequalities.

Bonnie Eichenberger • Paul Swenson • Teri Willard

Teacher Edition

Under the Big Top but Above the Floor

Overview

In this module, students are introduced to linear programming. They define feasible regions by graphing inequalities determined by vertical, horizontal, and oblique lines. They find the coordinates of corner points both graphically and algebraically. They then use the corner principle to find optimum values for linear objective equations.

Objectives

In this module, students will:

- graph linear inequalities
- solve systems of linear equations
- use linear inequalities to define regions graphically
- determine optimum values for linear objective equations.

Prerequisites

For this module, students should know:

- how to write and graph inequalities with one variable
- how to determine the coordinates of points which satisfy equations and inequalities
- how to determine the slope of a line
- how to find the equation of a line given the coordinates of two points
- how to graph equations of the form $y = mx + b$
- how to solve an equation for one variable in terms of another
- how to determine simple probabilities.

Time Line

Activity	Intro.	1	2	3	4	Summary Assessment	Total
Days	1	2	2	2	2	1	10

Materials Required

Materials	Activity					
	Intro.	**1**	**2**	**3**	**4**	**Summary Assessment**
graph paper		X	X	X	X	X
red dice				X		
white dice				X		
rulers		X	X	X	X	X
Roll-a-rama template					X	

Teacher Note

A blackline master of the template appears at the end of the teacher edition FOR THIS MODULE.

Technology

Software	Activity					
	Intro.	**1**	**2**	**3**	**4**	**Summary Assessment**
graphing utility		X	X	X	X	X

Under the Big Top but Above the Floor

Introduction

(page 331)

Students play the game "Guess My Number" to review writing and graphing inequalities on a number line.

Materials List

- none

Teacher Note

The game Guess My Number is best played in pairs. Students should read Part **a** only, then play the game. This should encourage them to experiment with winning strategies. Part **b** of the exploration guides them toward a specific strategy and reviews mathematical symbolism. Make sure students understand the significance of inclusive and non-inclusive inequalities ($\leq$ and $\geq$ versus $<$ and $>$). You may wish to compare strategies discovered in Part **a** of the exploration (in pairs or as a class) before proceeding to Part **b**.

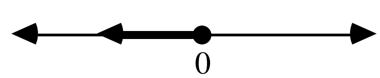
Exploration

(page 331)

a. Students should record both the questions asked and the information they received from each answer. As errors or arguments occur, ask students to use their records to identify mistakes or settle disputes.

One possible strategy is to ask questions which reduce the range of possible numbers by half each time.

b–c. Students write their questions as inequalities and record the corresponding responses on a number line. For example, one question might be "Is $x > 0$?" If the response is "No," the following graph may be drawn to represent the remaining possibilities for the opponent's number:



Discussion

(page 331)

a. It is possible to identify the number using seven questions or less. Each question divides the set of integers into two subsets. Reducing the original set of integers by "halves" with each question results in the following maximum cardinal numbers for successive subsets: 101 possibilities, 51 possibilities, 26 possibilities, 13 possibilities, 7 possibilities, 4 possibilities, 2 possibilities, and 1 possibility.

b. Using the same reasoning as in Part **a** above, the minimum number of questions is eight. The successive numbers of possibilities are 151, 76, 38, 19, 10, 5, 3, 2, and 1.

(page 332)

Activity 1

In this activity, students play a game that depends on their ability to write and graph systems of linear inequalities.

Materials List

- graph paper (five sheets per student)
- rulers (one per student)

Technology

- graphing utility

Discussion 1

(page 333)

a.

1. Sample response: The points in the shaded region all have *y*-coordinates greater than –3.

2. $y \geq -3$

b.

1. The graph of $x < 5$ is shaded to the left of the line $x = 5$ since all the *x*-coordinates to the left of the line are less than 5.

2. The points lying on $x = 5$ should not be included since the *x*-values that satisfy the inequality must be less than 5. This can be shown by representing the boundary as a dashed line.

c. For $x \geq 2$, any point with an x-coordinate of 2 or more is included in the solution set. For $y < -7$, only points with y-coordinates less than –7 are included.

Inequalities which use the symbols ≤ and ≥ have graphs whose borders are included. Inequalities which use the symbols > and < have borders that are not included.

Exploration 1

(page 334)

Students should read the example and the rules before playing "Guess My Location." You may wish to model some questions and responses. Students should record both their questions and the information received. A pair of students can play against each other, one pair can play another pair, or one group can play another group. You may wish to serve as moderator for group play.

Discussion 2

(page 335)

a. Answers will vary. In one possible strategy, students may ask questions that eliminate possible regions for the point until only one point is left. In another strategy, students may identify the quadrant in which the point is located, then repeatedly subdivide that quadrant until the point is identified.

b. $x < 0$ and $y < 0$

c. Yes, the conjunction $x > 0$ and $y > 0$ represents all points in the first quadrant. (The points on the x and y-axes are not included in any quadrant.)

d.

1. If Lisa says "yes", her point would be in Quadrant IV. Since she answered "no", it must be located in Quadrants I, II, or III.
2. Lisa's point must be located in Quadrant IV since it is the only quadrant where $x > 0$ and $y < 0$.

e. $x \geq -1$ and $y > -2$

Exploration 2

(page 336)

In this version of Guess My Location, students must ask questions in the form of conjunctions. One part of the question must refer to x and the other to y. **Note:** One game should take 10–15 minutes.

Discussion 3

(page 337)

a. Sample response: With two-part questions, a "yes" answer makes it possible to eliminate a larger portion of the playing grid. If the answer is "no," however, you do not know to which part of the question the answer refers.

b. The intersection of the two boundaries, $x = 0$ and $y = 0$, is the point with coordinates (0,0). This point is not in the solution set since it does not satisfy the two inequalities. If the conjunction were $x \geq 0$ and $y \leq 0$, however, this point would be included in the solution set.

c. Any point in the first quadrant, such as (4,5), has a positive *x*-coordinate and a positive *y*-coordinate. Since any positive *x*-coordinate is greater than 0 and any positive *y*-coordinate is greater than 0, all the ordered pairs in the first quadrant are solutions to this conjunction.

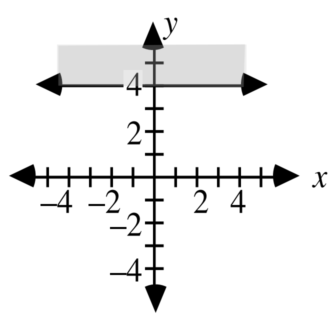
d. The *y*-coordinates are "constrained" or restricted to positive real numbers.

e. Using simple inequalities, this region can be described as $x \geq -3$ and $y \geq -4$ and $y < 2$. It can also be described by the conjunction $x \geq -3$ and $-4 \leq y < 2$.

Assignment

(page 337)

1.1 **a.** Sample graph:



b. $y \geq 4$

c. See sample graph in Part **a**.

d. Answers will vary. Sample response: (–5,4.75), (0,8), (12,12).

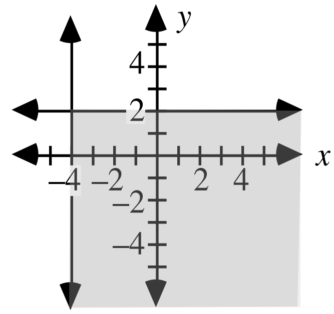
e. There are no constraints on the *x*-coordinates.

f. All *y*-coordinates must be greater than or equal to 4.

1.2

Quadrant	Conjunction
I	$x > 0$ and $y > 0$
II	$x < 0$ and $y > 0$
III	$x < 0$ and $y < 0$
IV	$x > 0$ and $y < 0$

***1.3** **a**. Sample graph:



b. Answers will vary. Students should substitute the *x*- and *y*-values of their selected points in the inequalities. These values should make both inequalities true. For example, the point (0,1) satisfies both inequalities since $0 \geq -4$ and $1 \leq 2$.

c. Answers will vary. For example, the point (0,3) does not satisfy both inequalities since 3 is not less than or equal to 2.

d. Answers will vary. For example, the point (–4,1) satisfies both inequalities because $-4 \geq -4$ and $1 \leq 2$.

e. The corner point is the intersection of the two boundaries, (–4,2).

***1.4** **a.** **1.** $y \leq -4$

2. $x > 1$ and $y \geq 1$

3. $x \geq -1, x \leq 4, y \geq -1,$ and $y \leq 2$ (this can also be written as $-1 \leq x \leq 4$ and $-1 \leq y \leq 2$)

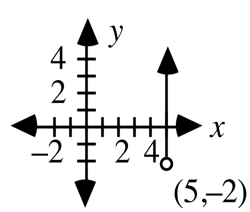
b. **1.** There are no corner points on this region.

2. (1,1) by using the corner point definition in Discussion **3** Part **b**.

3. (–1,2), (4,2), (–1,–1), and (4,–1)

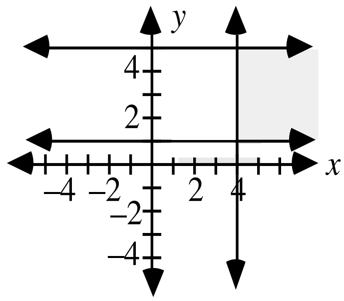
1.5 Answers will vary. Students should test their inequalities with the point (0,–2). Sample response: “Is $x \geq 0$ and $y < 0$?” and “Is $x \geq -1$ and $y < -1$?”

1.6 **a.** Sample graph:

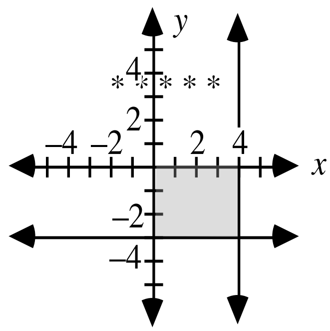


b. The graph formed is a vertical ray pointed upwards, with an open endpoint at (5,–2).

***1.7** **a.** The point is located in the region defined by the following system of inequalities: $x \geq 4$, $y \geq 1$, and $y \leq 5$. Sample graph:

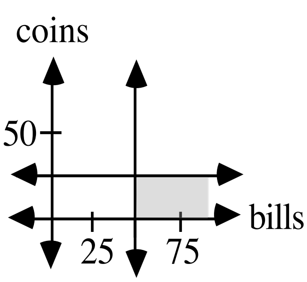


b. The point is located in the region defined by the following system of inequalities: $x \geq 0$, $x \leq 4$, $y \geq -3$, and $y \leq 0$ (this may also be written as $0 \leq x \leq 4$ and $-3 \leq y \leq 0$). Sample graph:

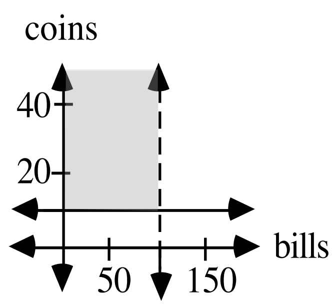


1.8 Sample graphs:

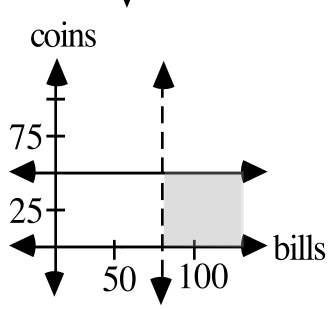
a.



b.



c.



1.9 **a.** $-3 \leq x \leq -1$ and $y \geq -2$

b. $1 \leq x \leq 2$ and $-2 \leq y \leq 1$

c. $x \geq -2$ and $-1 \leq y < 2$

* * * * * * * * * *

(page 340)

Activity 2

In this activity, students are introduced to the substitution method of solving a system of linear equations. Conjunctions are used to describe regions bounded by oblique lines.

Materials List

- graph paper (several sheets per student)

Technology

- graphing utility

Exploration 1

(page 340)

Students are introduced to systems of linear equations.

a. The equations of the boundary lines are $y = 7$, $y = 2x - 3$, $x = 10$, and $y = -10$.

b. One possible question is: "Is $y < 7$ and $y \leq 2x - 3$?" For this question, Sharline must answer "Yes."

c. From the graph, the coordinates appear to be approximately (–3.5,–10).

Discussion 1

(page 342)

a. **1.** Sample response: "Is $y < 2x - 3$ and $y < 7$?"

2. Sample response: "Is $y \leq 2x - 3$ and $y \leq 7$?"

b. A region bordered by dashed lines excludes all the points along those lines.

c. Points *C* and *D* are included in the solution set since they occur at the intersections of two solid lines.

d. Sample response: The coordinates of *C* appear to be approximately (–3.5,–10). From the graph there is no way to determine the exact value of the *x*-coordinate.

e. **1.** The corner point *C* is defined by the intersection of the lines $y = -10$ and $y = 2x - 3$. According to the mathematics note, a solution to a system of linear equations is the point where the lines intersect. The coordinates of point *C* satisfy the equations of both lines.

2. Since *C* is on the line $y = -10$, the *y*-coordinate is –10. Substituting this value in the equation $y = 2x - 3$ results in the following value for *x*:

$$-10 = 2x - 3$$
$$-7 = 2x$$
$$-3.5 = x$$

Therefore, the coordinates of point *C* are (–3.5,–10).

Exploration 2

(page 343)

To find the coordinates of the corner points of a region, students use substitution to solve several systems of linear equations.

a. Sample response: The coordinate of point *B* are approximately (2.3,1.5).

b. **1.** Sample response: Solve the equation $3x + y = 8$ for *y* to get $y = -3x + 8$. Then substitute this expression for *y* in the other equation:

$$5x - 5y = 4$$
$$5x - 5(-3x + 8) = 4$$
$$5x + 15x - 40 = 4$$
$$20x = 44$$
$$x = 2.2$$

Substituting 2.2 for *x* in $3x + y = 8$ gives a *y*-value of 1.4. The exact coordinates of point *B* are (2.2,1.4).

2. Student estimates should be fairly close to the exact values.

3. Students should substitute as follows:

$$3x + y = 8 \qquad\qquad 5x - 5y = 4$$
$$3(2.2) + 1.4 \stackrel{?}{=} 8 \quad \text{and} \quad 5(2.2) - 5(1.4) \stackrel{?}{=} 4$$
$$8 = 8 \qquad\qquad 4 = 4$$

c. Point *A* is the intersection of the line $3x + y = 8$ and the line $3y - x = 6$. Solving this system of equations by substitution yields the coordinates (1.8,2.6).

Point *C* is the intersection of the line $5x - 5y = 4$ and the line $3y - x = 6$. Solving this system of equations by substitution yields the coordinates (4.2.3.4).

Discussion 2

(page 344)

a. Sample response: Yes, the coordinates estimated by looking at the graph were close to the actual values.

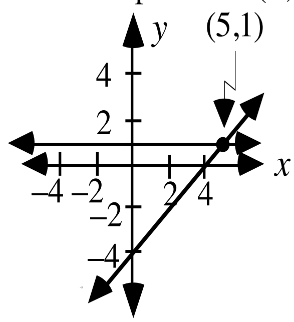
b. Sample response: One way to confirm the solution to a system of linear equations is to graph the lines and estimate the coordinates of the point of intersection. Another, more exact method is to substitute the coordinates of the solution into each equation then check to see that a true statement results.

c. The system of linear inequalities for region *ABC* is: $5x - 5y \leq 4$, $3x + y \geq 8$, and $3y - x \leq 6$.

Assignment

(page 345)

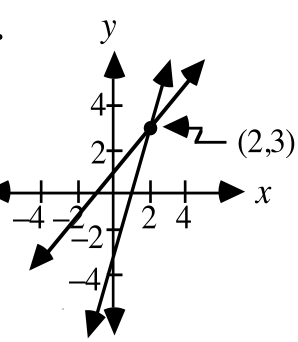
2.1 The coordinates of Anton's point are (5,1). Sample graph:



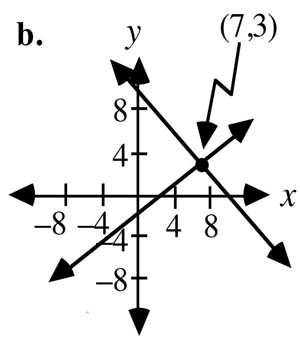
2.2 The coordinates of Sharline's point are $(1/3, -4)$.

2.3 The solutions to these systems of equations are shown in the graphs below.

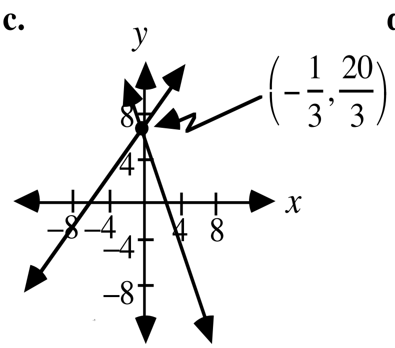
a.



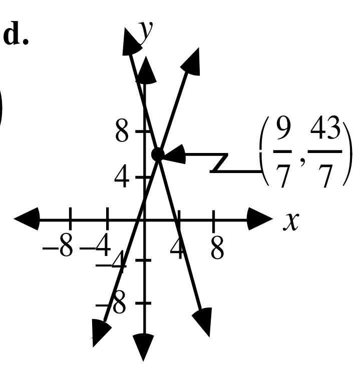
b.



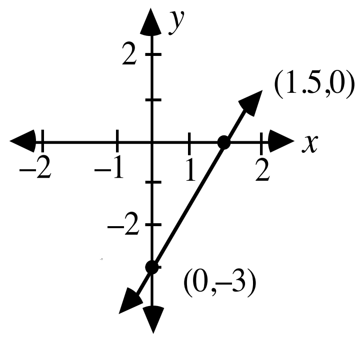
c.



d.



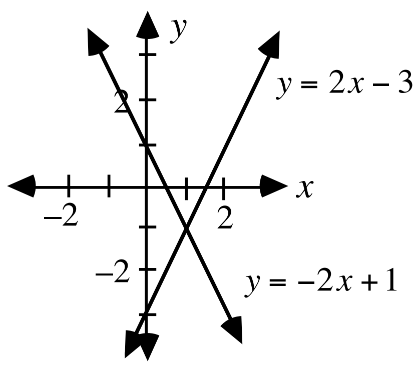
2.4 **a.** Sample graph:



b. $y \le 2x - 3$

c. Sample response: (0,–3) and (5,0).

2.5 **a.** Sample graph:



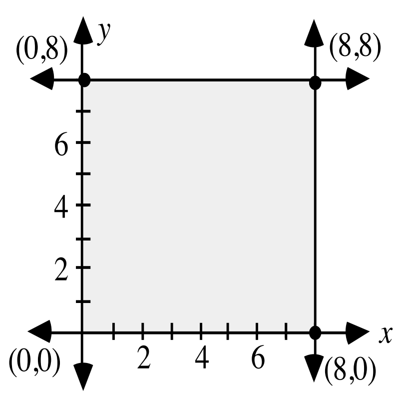
b. $y \ge -2x + 1$

c. Sample response: (3,1), (3,–2).

2.6 **a.** The point of intersection is (1,–1).

b. Students should substitute (1,–1) in the two equations to verify that each equality is true for that point.

2.7 **a.** Sample graph:



b. The shape of this region is a square.

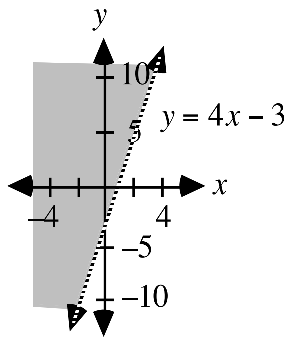
c. (0,8), (8,8), (0,0), (8,0)

d. $x > 0,\ y > 0,\ x < 8,\ y < 8$

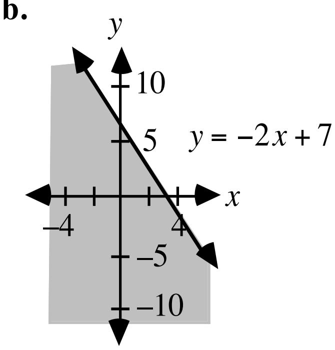
e. Sample response: $y = 4$, $x = 4$, and $y = x$.

2.8 Sample graphs:

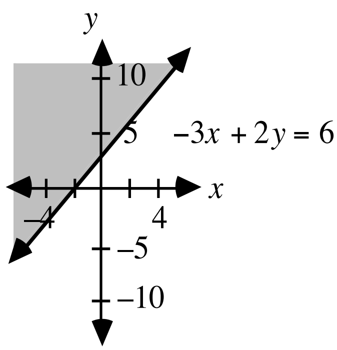
a.



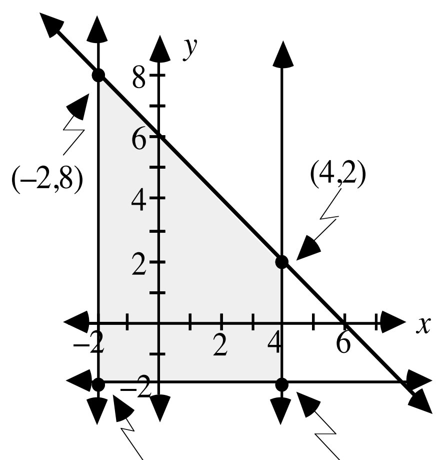
b.



c.



***2.9** **a.** Sample graph:



b. Students may describe the shape as a quadrilateral or a trapezoid.

c. (–2,8), (–2,–2), (4,–2), (4,2)

***2.10** The shaded region may be described by the following system of inequalities:

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ y \leq -2x + 6 \text{ or } y + 2x \leq 6 \end{cases}$$

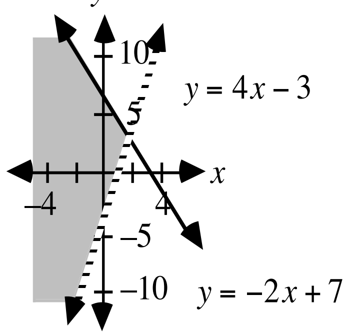
* * * * *

2.11 **a.** (–7,–4)

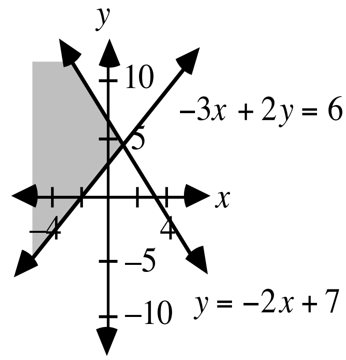
b. (–2,–4)

2.12 Sample graphs:

a.



b.



2.13 **a.** Sample response: The equation $j = 290 + 5w$ represents the amount of money for Jasmine. The equation $a = 200 + 8w$ represents the amount of money for Alan.

b. As shown below, the two must save for 30 weeks to have the same amount of money:

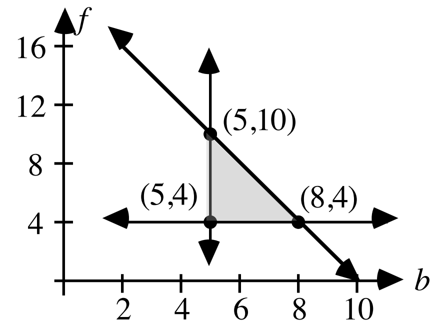
$$200 + 8w = 290 + 5w$$
$$w = 30$$

Note: You may wish to remind students that a given system of equations may not have integer solutions.

2.14 **a.** This situation can be described by the following system of three inequalities, where *b* represents the number of baskets and *f* represents the number of free throws:

$$\begin{cases} b \geq 5 \\ f \geq 4 \\ 2b + f \leq 20 \end{cases}$$

b. **Note:** In this context, both *b* and *f* must be integers. Sample graph:



c. The set of ordered pairs that satisfy the system of inequalities is: (5,4), (6,4), (7,4), (8,4), (5,5), (6,5), (7,5), (5,6), (6,6), (7,6), (5,7), (6,7), (5,8), (6,8), (5,9), and (5,10).

* * * * * * * * * *

(page 347)

Activity 3

In this activity, students are introduced to feasible regions defined by constraints.

Materials List

- graph paper (several sheets per student)
- rulers (one per student)
- red dice (one per group)
- white dice (one per group)

Technology

- graphing utility

Exploration

(page 347)

a. Sample data:

Roll No.	White Die	Red Die	Sum	Win?
1	2	6	8	no
2	2	2	4	yes
3	2	3	5	yes
4	5	6	11	no
5	3	5	8	no
6	5	3	8	no
7	4	5	9	no
8	4	1	5	no
9	2	2	4	yes
10	2	6	8	no
			Total Wins	3

b. Using the sample data given in Part **a**, the experimental probability of winning is $3/10 = 30\%$.

c. The following table shows one method for determining the sample space for the game.

		White Die					
		1	**2**	**3**	**4**	**5**	**6**
	1	lose	lose	lose	lose	lose	lose
	2	win	win	win	win	win	lose
Red	**3**	win	win	win	win	lose	lose
Die	**4**	win	win	win	lose	lose	lose
	5	win	win	lose	lose	lose	lose
	6	lose	lose	lose	lose	lose	lose

d. As shown in Part **c**, there are 36 possible outcomes, 14 of which result in a win. The theoretical probability of winning is $14/36 \approx 0.39$.

Discussion

(page 347)

a. Since there are more possible losing rolls than winning rolls, players are more likely to lose. The probability of losing is $22/36$; the probability of winning is $14/36$.

b. Since rolls are determined by chance, there is no strategy that increases the probability of winning.

c. The graph is a set of 36 discrete points whose coordinates are the integers from 1 to 6.

d. **Note:** Students should realize that this graph will be a set of discrete points rather than a continuous region bordered by lines.

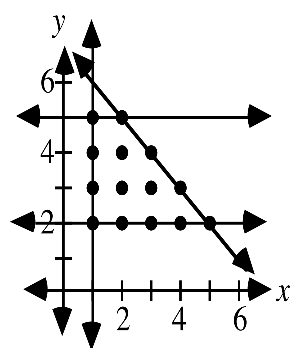
1. Sample response: Shading the feasible region would imply that points with decimal coordinates are possible outcomes in Roll-a-rama, when only those points with integer coordinates are possible.
2. Sample response: The feasible region can be represented by discrete dots at the points with integer coordinates [lattice points].

Assignment
(page 348)

***3.1** **a.** In these inequalities, x represents the number on the white die and y represents the number on the red die.

b. The constraint $x \geq 1$ corresponds with the fact that the roll of the white die cannot be less than 1. The constraints $y \geq 2$ and $y \leq 5$ correspond with the rule that specifies the range of winning numbers for the red die. The constraint $x + y \leq 7$ corresponds with the rule for the sum of a winning roll.

c. As shown in the following sample graph, the feasible region consists of discrete points:



d. There are 14 possible winning rolls.

3.2 There are 4 winning rolls in these 10 games. Students should identify these ordered pairs using the graph from Problem **3.1c**.

3.3 **a.** There are 22 possible losing rolls.

b. Sample response: The combinations (1,1), (6,5), and (4,4) are all losing rolls. These points are not included in the feasible region graphed in Problem **3.1c**.

***3.4** Sample response: No. The y-intercept has coordinates (0,7) which indicates a roll of 0 on the white die and 7 on red die. Since these values are not included on either die, this point cannot be in the feasible region.

***3.5** **a.** The number rolled on each die must be an integer from 1 to 6.

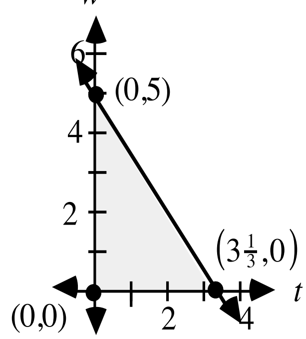
b. These constraints can be described by the following system of inequalities: $x \geq 1$, $x \leq 6$, $y \geq 1$, and $y \leq 6$, where x and y are integers.

3.6 Sample response: If the constraint on the sum of the two dice is changed to $x + y \le 8$, there will be a 50% chance of winning.

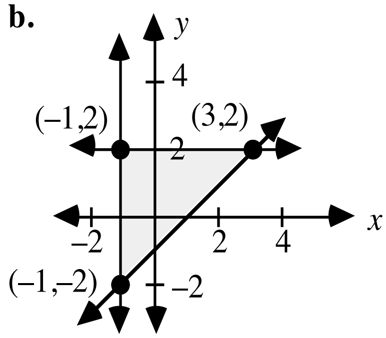
* * * * *

3.7 Sample graphs:

a.



b.



3.8 **a.** **1.**

$$\begin{cases} x \ge 0 \\ y \ge 0 \\ y \le -2.5x + 5 \end{cases}$$

2.

$$\begin{cases} x \ge 0 \\ y \ge 0 \\ y \ge -2.5x + 5 \\ y \le -2.5x + 10 \end{cases}$$

3.

$$\begin{cases} y \ge -4 \\ y \le \dfrac{5}{3}x + 5 \\ y \le -2.5x + 5 \end{cases}$$

b. **1.** (0,0), (2,0), and (0,5)

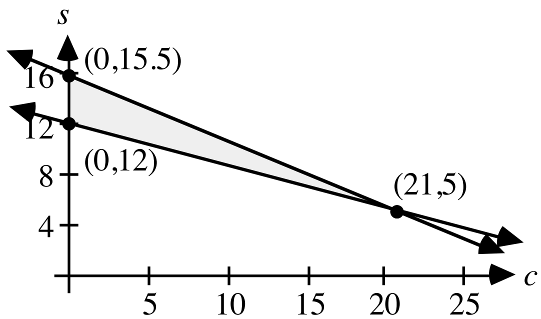
2. (0,5), (0,10), (4,0), and (2,0)

3. (0,5), (–5.4,–4), and (3.6,–4)

3.9 **a.** In the following system of inequalities, c represents the number of chairs and s represents the number of sofas:

$$\begin{cases} c \geq 0 \\ s \geq 0 \\ 4c + 8s \leq 124 \\ 2c + 6s \leq 72 \end{cases}$$

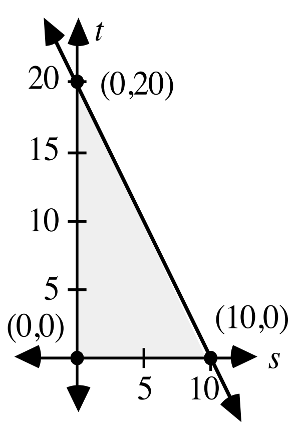
b. The following sample graph identifies the coordinates of the corner points. Students should save this graph for use in Problem **4.3**.



3.10 **a.** The constraints can be described by the following system of inequalities, where t represents the number of toppings and s represents the number of soft drinks. **Note:** In this context, both t and s must be integers.

$$\begin{cases} s \geq 0 \\ t \geq 0 \\ s + 0.5t \leq 10 \end{cases}$$

Sample graph:



b. The coordinates of the corner points are (10,0), (0,0), and (0,20).

* * * * * * * * * *

(page 351)

Activity 4

In this activity, students write a system of inequalities to represent the constraints on a problem. Using the corner principle, they then find a solution that optimizes an objective equation.

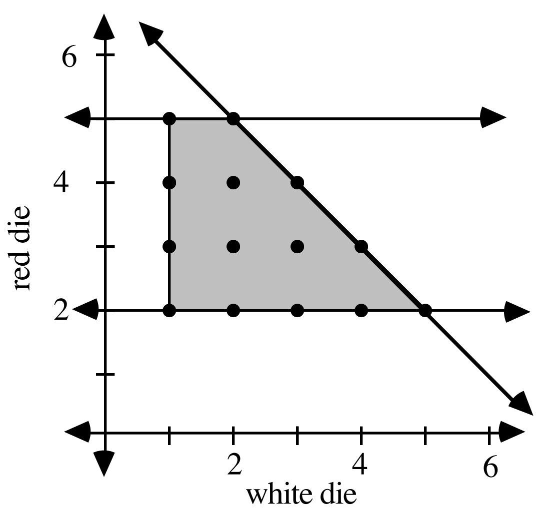
Materials List

- graph paper (several sheets per student)
- rulers (one per group)
- Roll-a-rama template (one per group; a blackline master appears at the end of the teacher edition for this module)

Exploration

(page 351)

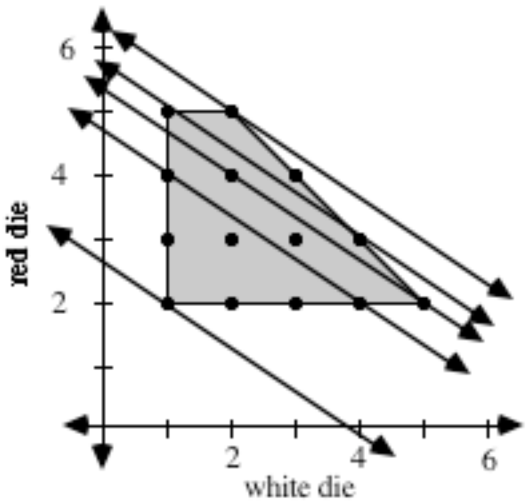
a. Sample graph:



b. A completed table is show below.

Roll	Score	Roll	Score	Roll	Score
(1,2)	8	(2,2)	10	(3,2)	12
(4,2)	14	(5,2)	16	(1,3)	11
(2,3)	13	(3,3)	15	(4,3)	17
(1,4)	14	(2,4)	16	(3,4)	18
(1,5)	17	(2,5)	19		

c. **Note:** In Parts **c–e**, students graph five lines, each with a slope of $-2/3$. The middle three lines represent the solutions to Part **c**. The line with the greatest *y*-intercept is the solution to Part **d**. The line with the least *y*-intercept is the solution to Part **e**. Sample graph:



The score is 14 for the points with coordinates (4,2) and (1,4). An equation of the line through these points in slope-intercept form is:

$$y = -\frac{2}{3}x + \frac{14}{3}$$

The score is 16 for the points (2,4) and (5,2). An equation of the line through these points in slope-intercept form is:

$$y = -\frac{2}{3}x + \frac{16}{3}$$

The score is 17 for the points (1,5) and (4,3). An equation of the line through these points in slope-intercept form is:

$$y = -\frac{2}{3}x + \frac{17}{3}$$

d. **1.** The maximum score of 19 occurs with a roll of (2,5).

2. Students should obtain the following equation:

$$y = -\frac{2}{3}x + \frac{19}{3}$$

e. **1.** The minimum score of 8 occurs with a roll of (1,2).

2. Students should obtain the following equation:

$$y = -\frac{2}{3}x + \frac{8}{3}$$

Discussion

(page 352)

a. The location of the maximum is (2,5). The location of the minimum is (1,2). Both of the points are corners of the feasible region.

b. **1.** The lines are parallel (they have the same slope).

2. Each equation can be written in the form of the objective equation ($S = 2x + 3y$) with a value substituted for S.

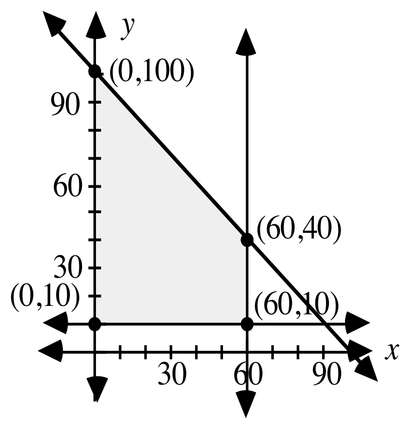
c. A player must roll a 2 on the white die and a 5 on the red die.

Assignment

(page 353)

4.1 **a.** The constraints can be represented as follows: $x + y \leq 100$, $x \leq 60$, $y \geq 10$, and $x \geq 0$. **Note:** Students should include the constraint $x \geq 0$ as part of the system, since a negative number of prizes cannot be purchased.

b. Sample graph:



c. Answers will vary. Since prizes cannot be bought in fractional amounts, the coordinates of these points must be integers. The x-values represent the number of small prizes, while the y-values represent the number of large prizes.

d. **1.** The profit on each small prize is $0.75.

2. The profit on each large prize is $0.25.

e. $P = 0.75x + 0.25y$

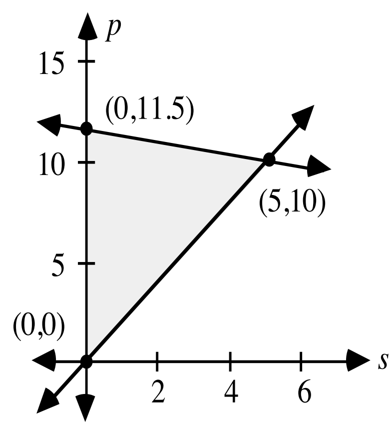
f. The maximum profit ($55.00) occurs at the point with coordinates (60,40). Anton and Sharline should buy 60 of the small prizes and 40 of the large prizes. **Note:** When applying the corner principle to locate this point, students should evaluate the objective equation at every vertex.

4.2 **a.** One possible equation is $P = 3s + 8p$, where P represents profit in dollars, s represents the number of six-packs and p represents number of pizzas. **Note:** Students should recognize that this equation must be expressed in terms of six-packs and whole pizzas. Profit is described in terms of single soft drinks and single servings of pizza.

b. The system of inequalities that describes the constraints in this situation is shown below:

$$\begin{cases} s \geq 0 \\ p \geq 0 \\ 3s + 10p \leq 115 \\ 12s \leq 6p \end{cases}$$

The following sample graph shows the feasible region described by this system.



The maximum profit of $95.00 occurs at the corner point (5,10). This indicates that 5 six-packs and 10 pizzas should be ordered.

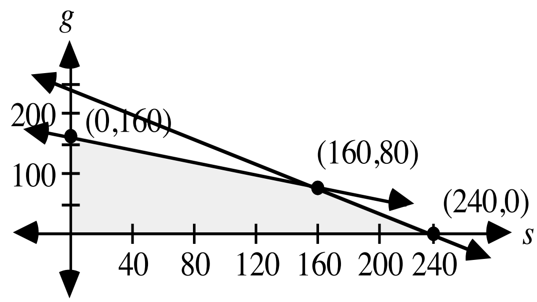
4.3 **a.** One possible equation is $P = 50c + 80s$, where P represents profit in dollars, c represents the number of chairs and s represents the number of sofas.

b. The maximum profit of $1450 occurs when the company makes 21 chairs and 5 sofas.

4.4 **a.** The system of inequalities that describes the constraints in this situation is shown below, where *s* represents the number of scientific calculators and *g* represents the number of graphing calculators:

$$\begin{cases} s \geq 0 \\ g \geq 0 \\ s + g \leq 240 \\ 10s + 20g \leq 3200 \end{cases}$$

Sample graph:



b. $P = 9s + 15g$

c. The maximum profit of $2640 occurs at the corner point (160,80). The company should make 160 scientific calculators and 80 graphing calculators.

* * * * * * * * * *

Answers to Summary Assessment

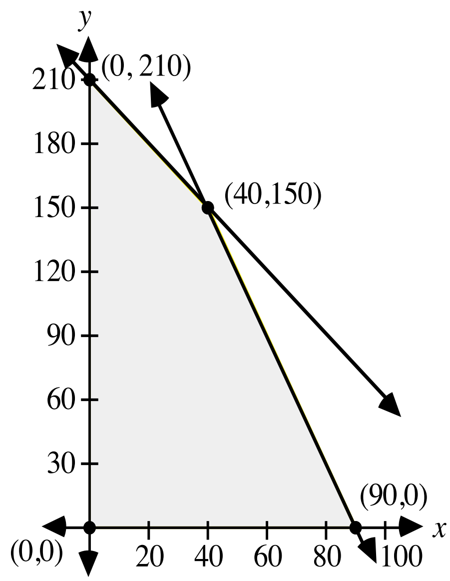
(page 355)

1. **Note:** The assignment of *x* and *y* to the two sizes of Spirit Animals will affect the appearance of the system of inequalities but will not change the maximum profit. In the following sample responses, *x* represents the number of large animals and *y* represents the number of small animals.

2. $3x + y \leq 270$, $x \geq 0$, and $y \geq 0$

3. $1.5x + y \leq 210$, $x \geq 0$, and $y \geq 0$

4. Sample graph:



5. The profit for large animals is $\$0.90 - 3(\$0.05) = \$0.75$ each. The profit for small animals is $\$0.65 - \$0.05 = \$0.60$ each.

6. $P = 0.75x + 0.60y$

7. The maximum profit of $126.00 occurs at (0,210). The Pep Club should make 210 small animals and no large animals.

8. Answers will vary. Students should describe generating inequalities to define a feasible region, identifying the vertices of the feasible region, evaluating the objective equation for each of the vertices, identifying the vertex that produces the maximum profit, and interpreting the meaning of the coordinates of that vertex.

Module Assessment

1. **a.** On the same set of coordinate axes, graph and label the lines $y = 3$, $x = -2$, and $y = 2x - 3$.

b. Shade the triangular region bordered by, and including, the three lines in Part **a**.

c. Write a system of inequalities that completely describes the region.

d. Verify that your system is correct.

2. Is (4,–5) a solution for the following system of equations? Explain your response.

$$\begin{cases} y = 3x - 17 \\ 2x + y = 3 \end{cases}$$

3. Solve each of the following systems of equations using the substitution method. Verify each solution by graphing.

a. $\begin{cases} x = -4 \\ y = 2x + 10 \end{cases}$

b. $\begin{cases} y + 12 = x \\ y = -x - 6 \end{cases}$

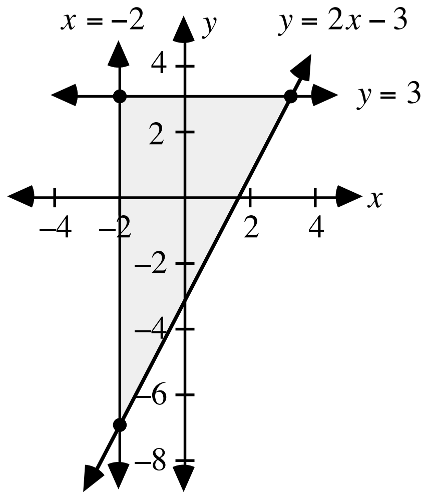
c. $\begin{cases} x - 2y = 4 \\ 7x + 2y = 8 \end{cases}$

4. To raise money for its activities, the Dantzig High Key Club sells candy at the carnival. The Mint Lover's Pack contains 4 g of chocolates and 12 g of mints. The 50-50 Pack contains 8 g of chocolates and 8 g of mints.

A local business has donated 1200 g of chocolates and 1920 g of mints. The Key Club sells each Mint Lover's Pack for \$0.25 and each 50-50 Pack for \$0.45. Assuming that all the candy is sold, how many of each pack should the club make to earn the maximum profit?

Answers to Module Assessment

1. **a–b.** Sample graph:



c. The region can be described by the following system of inequalities:

$$\begin{cases} y \le 3 \\ x \ge -2 \\ y \ge 2x - 3 \end{cases}$$

d. Students should verify their system by substituting the coordinates of several points in the shaded region into the inequalities.

2. Sample response: Yes, (4,–5) is a solution for the system because when 4 and –5 are substituted for *x* and *y*, respectively, they produce true mathematical statements.

3. To verify the solutions given below, students should graph each system of equations and estimate the coordinates of the point of intersection.

a. (–4,2)

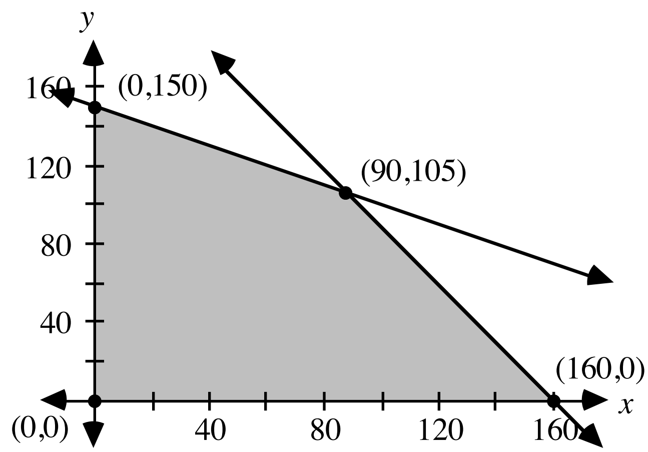
b. (3,–9)

c. (1.5,–1.25)

4. In the following system of inequalities, *x* represents the number of Mint Lover's Packs and *y* represents the number of 50-50 Packs.

$$\begin{cases} x \ge 0 \\ y \ge 0 \\ 4x + 8y \le 1200 \\ 12x + 8y \le 1920 \end{cases}$$

The objective equation, where P represents profit, is $P = 0.25x + 0.45y$. As shown in the following graph, the feasible region has four corner points.



The maximum profit of $69.75 occurs at the point (90,105). This means that the club should make 90 Mint Lover's Packs and 105 50-50 Packs.

Selected References

Farlow, S. J., and G. Haggard. *Finite Mathematics and Its Applications*. New York: Random House, 1988.

Flashbacks

Activity 1

1.1 Graph each of the following relationships on a separate number line.

a. $x \leq 7$

b. $x \geq -2$

1.2 **a.** Graph and label several points on an *xy*-coordinate system with a *y*-coordinate of –1.

b. Write an equation for the line that passes through these points.

c. What is the slope of this line?

1.3 Graph the equation $y = x - 1$ on an *xy*-coordinate system.

Activity 2

2.1 Solve each of the following equations for *y*.

a. $8x + 2y = 4$

b. $2x + \frac{1}{2}y = 5$

2.2 Sketch the region formed by the intersection of $5 \geq x$ and $y \geq -5$.

2.3 Find the point of intersection of $x + 3y = 9$ and $4x + 5y = 1$.

Activity 3

3.1 Find the intersection of $x = 9$ and $y = 1 - x$.

3.2 List the coordinates of three points in the region formed by the intersection of $y < 4$ and $y + x \geq 1$.

3.3 When rolling a fair die, what is the theoretical probability of rolling a 6?

Activity 4

4.1 Determine an equation for the line that contains each of the following pairs of points:

a. (0,4) and (–5,0)

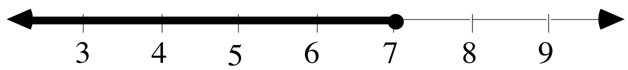
a. (2,7) and (0,–6)

a. (–2,3) and (4,7)

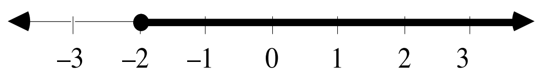
Answers to Flashbacks

Activity 1

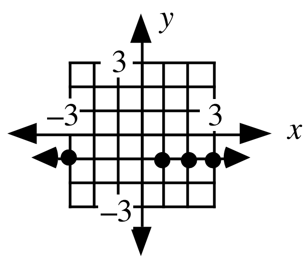
1.1 **a.**



b.



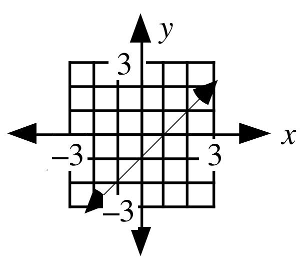
1.2 **a.** Sample graph:



b. $y = -1$

c. The line has a slope of 0.

1.3 Sample graph:

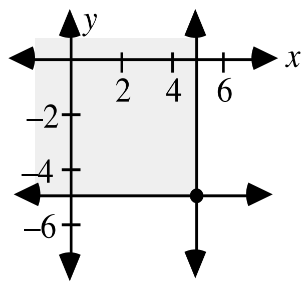


Activity 2

2.1 **a.** $y = 2 - 4x$

b. $y = 10 - 4x$

2.2 Sample graph:



2.3 (–6,5)

Activity 3

3.1 (9,–8)

3.2 Sample response: (9,2), (8,3), and (7,2).

3.3 1/6

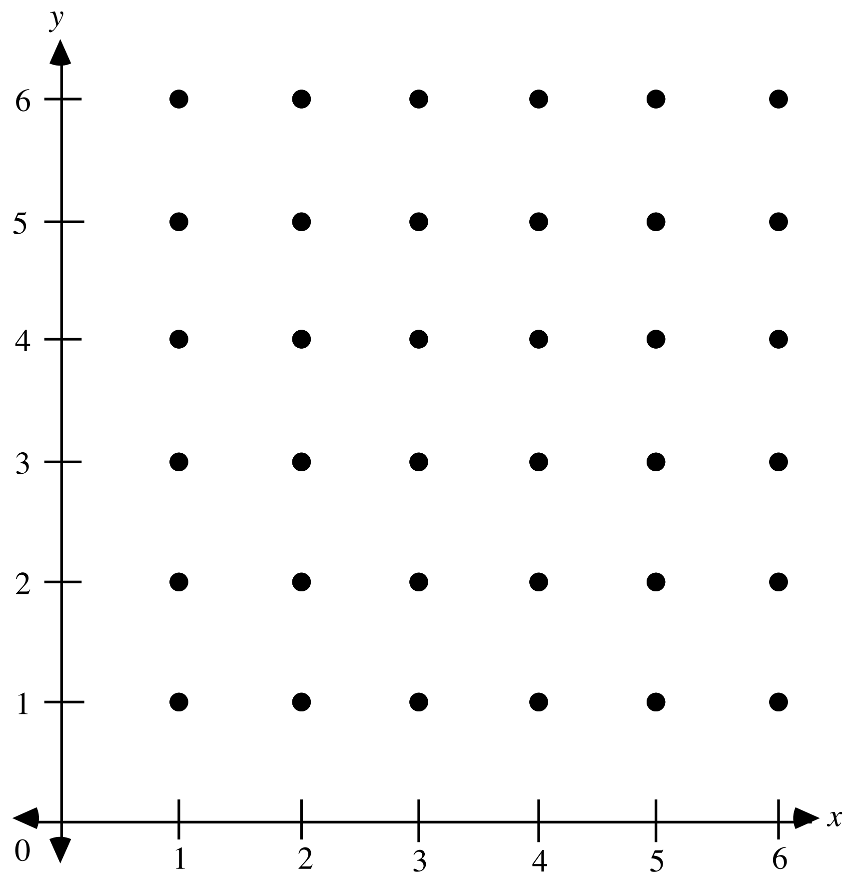
Activity 4

4.1 **a.** $y = \frac{4}{5}x + 4$

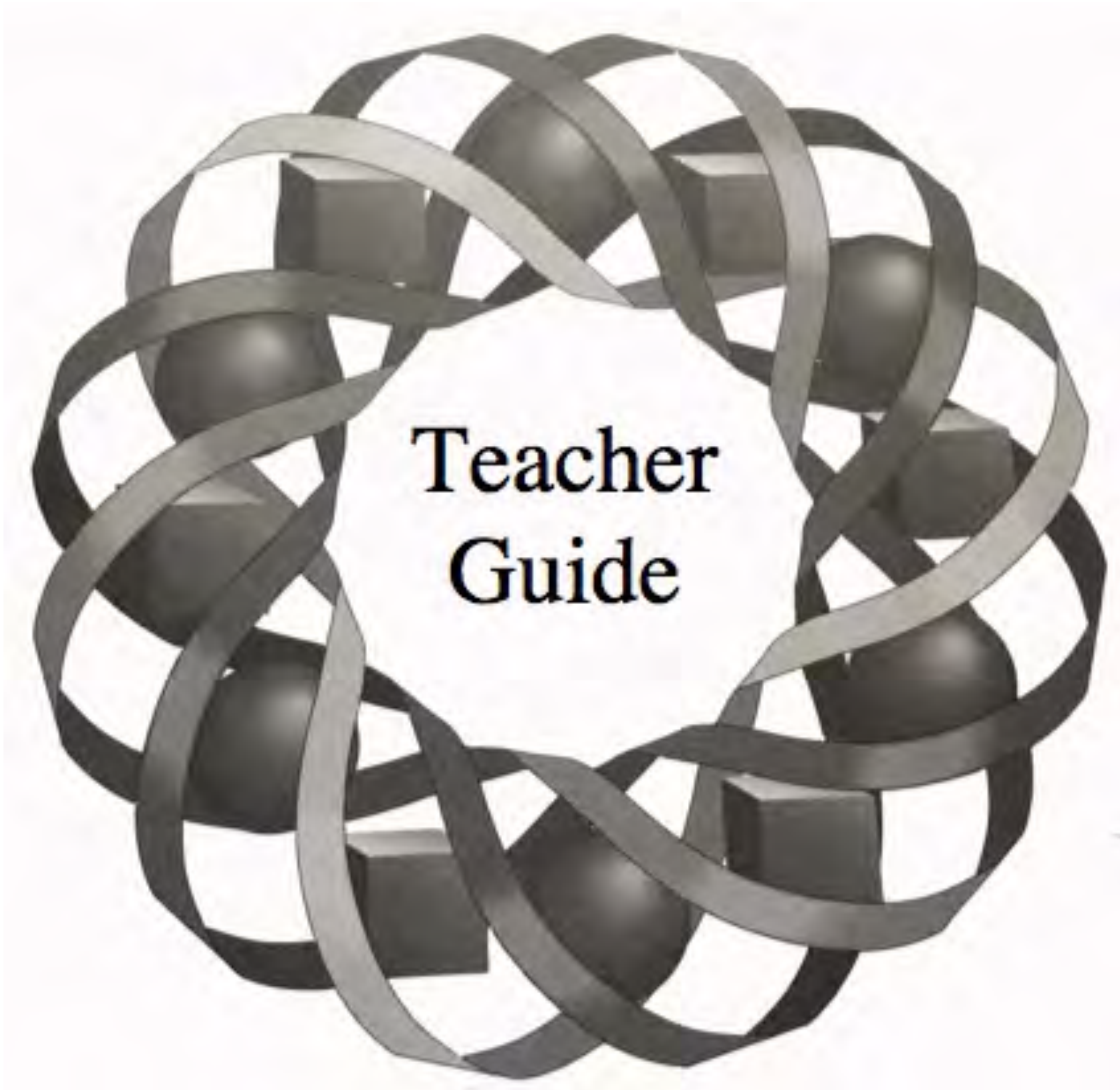
b. $y = \frac{13}{2}x - 6$

c. $y = \frac{2}{3}x + \frac{13}{3}$

Roll-a-Rama Template



Digging into 3-D



How do you record the position of a sunken treasure? How do you plot the site of a buried dinosaur bone? How can you describe locations in the world around, above, and below you? The answers to these questions lie in another dimension.

Gary Bauer • Sherry Horyna • Jeff Hostetter • Margaret Plouvier

Teacher Edition

Digging into 3-D

Overview

Students investigate the three-dimensional coordinate system within two contexts: sunken treasure and hidden artifacts. They also use a symbolic manipulator to create surface plots.

Objectives

In this module, students will:

- model a three-dimensional coordinate system
- graph in a three-dimensional coordinate system
- draw surface plots
- describe three-dimensional objects using correct mathematical language.

Prerequisites

For this module, students should know:

- how to plot and label points on a coordinate plane
- how to recognize pyramids and prisms.

Time Line

Activity	1	2	Summary Assessment	Total
Days	2	3	2	7

Materials Required

Materials	Activity		
	1	2	Summary Assessment
metersticks	X		
rulers	X	X	
scissors	X	X	
tape or glue	X	X	
template for 3-D model	X		
template for 3-D coordinate system	X		
template of sunken ship	X		
centimeter graph paper	X	X	
cardboard	X	X	
toothpicks	X		
plastic foam	X	X	
maps	X		
cardboard boxes		X	
stiff wire		X	
models of solids		X	

Teacher Note

Blackline masters for the templates appear at the end of the teacher edition FOR THIS MODULE. The cardboard boxes for Activity **2** should be no larger than a shoe box. The plastic foam should be approximately 1 cm thick. (This can often be purchased in large sheets at building supply stores.)

Maps for the research project should show the elevations of key sites.

Technology

Software	Activity		
	1	2	Summary Assessment
symbolic manipulator		X	X

Digging into 3-D

Introduction

(page 361)

This module uses sunken treasure to introduce students to three-dimensional graphing.

(page 361)

Activity 1

Students develop a method for identifying the location of items in the classroom and build a model of a three-dimensional coordinate system. **Note:** Throughout this module, three-dimensional coordinate systems are oriented as shown in Figure **1** of the student edition. This orientation is often referred to as "the left-hand rule." Students responses may correctly use other orientations.

Materials List

- rulers (one per group)
- meterstick (one per group)
- tape or glue
- scissors (one pair per group)
- template for 3-D model (one per group; a blackline master appears at the end of the teacher edition for this module)
- template for 3-D coordinate system (one per student; a blackline master appears at the end of the teacher edition for this module)
- template for sunken ship (one per student; a blackline master appears at the end of this teacher edition)
- cardboard (four sheets per group)
- centimeter graph paper (one sheet per group)
- toothpicks
- sheets of thick cardboard or plastic foam
- maps

Teacher Note

The toothpicks, maps, and sheets of thick cardboard or plastic foam are required for the research project at the end of this activity.

Exploration (page 362)

a. To build the model, each group will need the appropriate template, cardboard, scissors, and tape or glue.

b. Students use the classroom to model one octant of a three-dimensional coordinate system. **Note:** The room should be shaped like a rectangular prism.

In this situation, the floor represents the *xy*-plane, with the *z*-axis along the intersection of two adjacent walls. You may wish to ask students to label the planes *xy*, *yz*, and *xz* by taping pieces of paper onto the walls and floor.

c–d. Students may work in teams to identify the selected points.

Discussion (page 365)

a. Students may have concerns about the order in which to list coordinates or about the placement of the origin.

b. Sample response: If a point is on the floor, its *z*-coordinate is 0.

c. Sample response: When points are located below the floor, their *z*-coordinates could be given negative numbers.

d.

1. Sample response: The coordinates in the room are in the first octant since they are all positive.
2. Answers may vary. Sample response: No. If you use the same location for the origin, a point in the room next door would be in another octant.

e. **Note:** The numbering system outlined in the following sample response represents one of several possible methods. Sample response: In the first octant, the *x*-, *y*-, and *z*-coordinates are positive. Going clockwise, the second octant has positive *x*- and *z*-coordinates and negative *y*-coordinates. In the third octant, *z* is positive, and *x* and *y* are negative. In the fourth octant, *z* and *y* are positive, while *x* is negative. The fifth octant (beneath the first) has positive *x*- and *y*-coordinates and negative *z*-coordinates. In the sixth octant, *x* is positive and *z* and *y* negative. In the seventh octant, all coordinates are negative. In the eighth octant, *y* is positive while *x* and *z* are negative.

f. Sample response: There are four quadrants and eight octants. Two coordinates determine a point in a quadrant while three coordinates determine a point in an octant. A quadrant is bounded by two rays, while an octant is bounded by three quarter-planes. Quadrants are regions of a plane while octants are regions of space.

g. Students may respond that it makes no difference, as long as the axes and coordinates are matched consistently and accurately.

h. Sample response: The location of an object in a room can be described by a set of points. For example, the corner points of a desk could define the surface of the desktop.

Teacher Note

Each student will need a copy of the coordinate system template for Problem **1.5** and a copy of the sunken ship template for Problem **1.8**.

Assignment

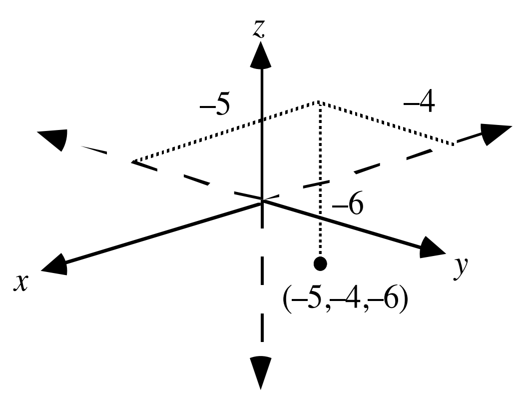
(page 365)

1.1 The following responses assume that the axes are oriented as shown in Figure **1**.

- **a.** When a point is located above the *xy*-plane, its *z*-coordinate is positive.
- **b.** The *x*-coordinate of a point on the *yz*-plane is 0.
- **c.** If the *y*-coordinate of a point is positive, it is located to the right of the *xz*-plane.

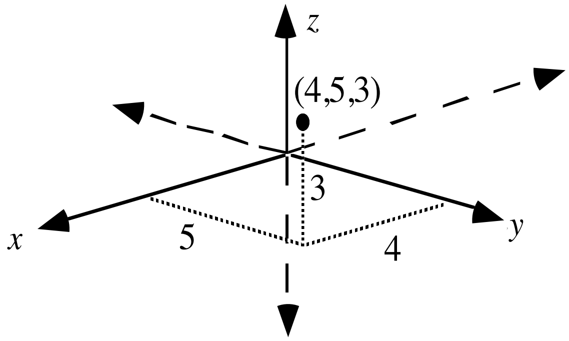
1.2 *A* (1,2,3); *B* (–3,3,4); *C* (5,4,–2); *D* (6,–5,1)

1.3 Sample response: When viewed from the first octant, the point $(-5,-4,-6)$ is below the *xy*-plane, behind the *yz*-plane, and to the left of the *xz*-plane. A sample sketch follows.

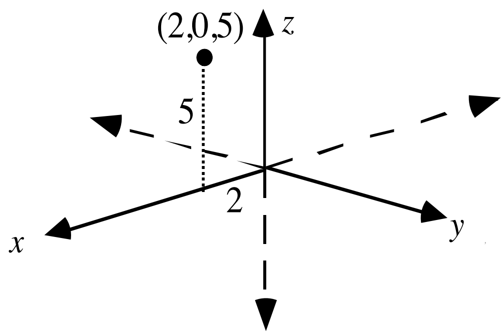


1.4
- **a.** *A* (5,0,3); *B* (0,0,3); *C* (0,0,0); *D* (5,0,0)
- **b.** *A* (4,0,2); *B* (0,0,2); *C* (0,3,2); *D* (4,3,2); *E* (4,0,0); *F* (0,0,0); *G*(0,3,0); *H* (4,3,0)
- **c.** *A* (0,0,3); *B* (–2,0,3); *C* (–2,5,3); *D* (0,5,3); *E* (0,0,0); *F* (–2,0,0), *G*(–2,5,0); *H* (0,5,0)

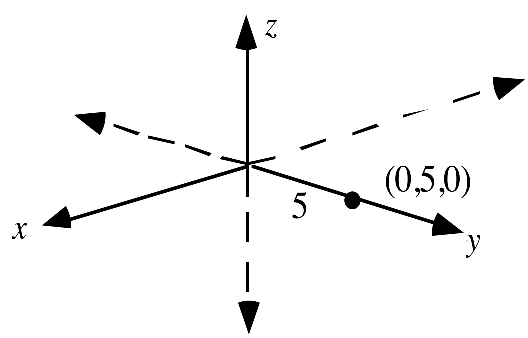
1.5 **a.** Sample graph:



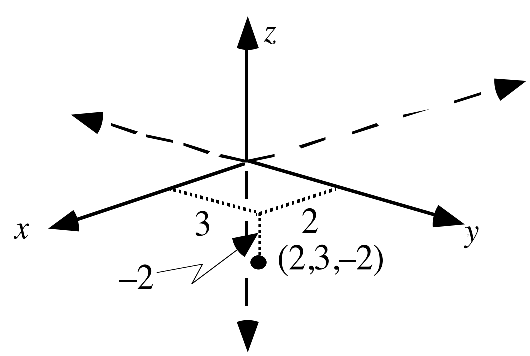
b. Sample graph:



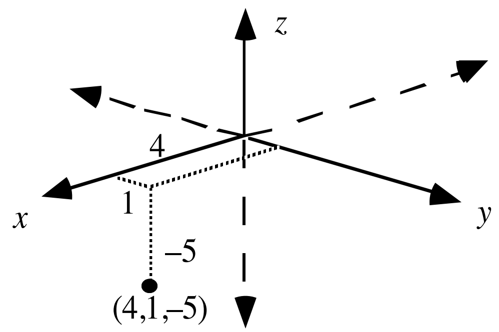
c. Sample graph:



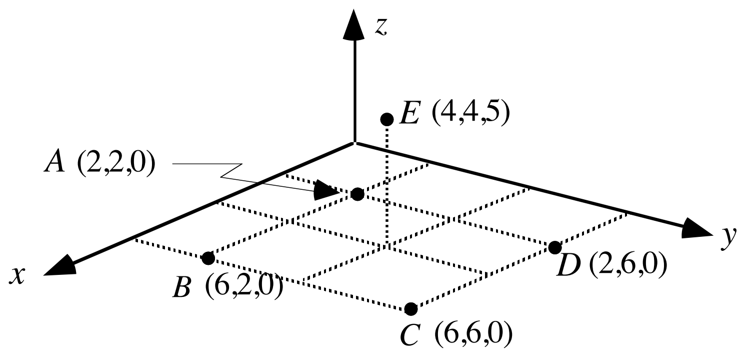
d. Sample graph:



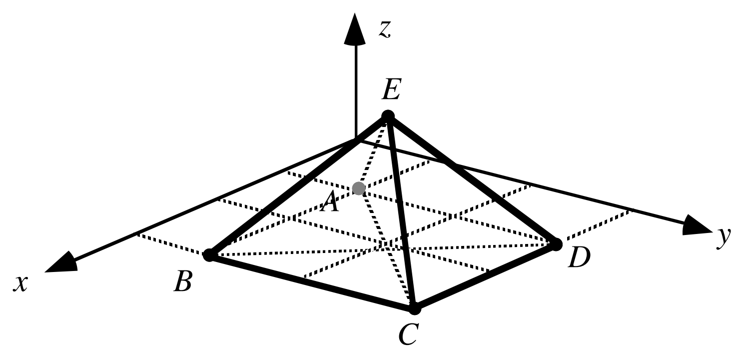
e. Sample graph:



1.6 **a.** Sample graph:



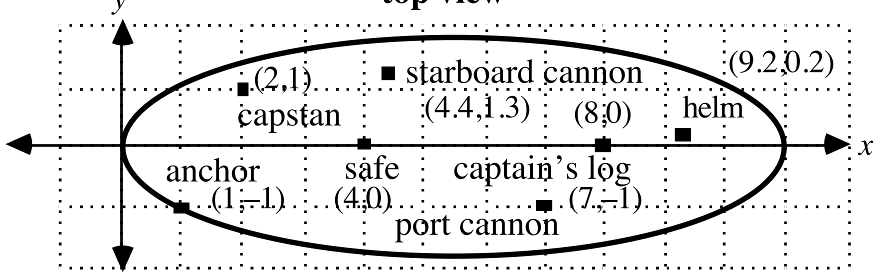
b. Sample graph:

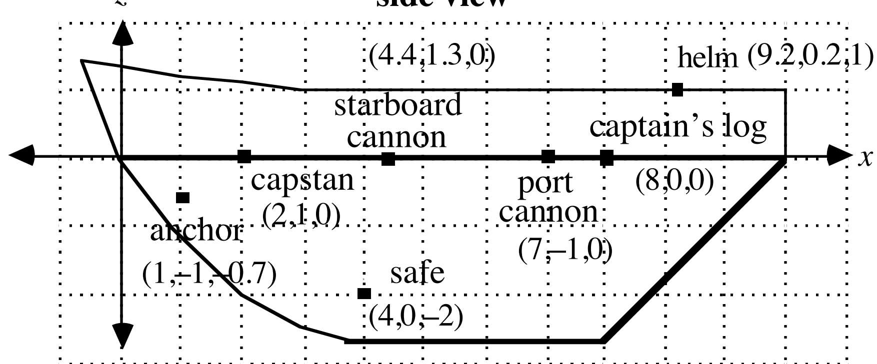


c. Sample response: I drew a pyramid with a square base (along with its diagonals).

1.7 *A* (2,0,0), *B* (4,4,0), *C* (0,4,0), *D* (0,0,0), *E* (2,0,4), *F* (0,0,4), *G*(0,4,4), *H* (4,4,4)

***1.8** **a.** Answers will vary, depending on the location of the origin. In the following sample response, the origin was placed near the bow of the ship. Each unit on the axes represents 4 m. All the *z*-coorinates are 0.





b. Sample response: Yes. Any point could have been selected as the origin. However, the location of the origin affects the coordinates of the treasures. Using a visible landmark like the front of the deck as the origin may make it easier to locate other points.

* * * * *

1.9 **a.** The set of points is a sphere with a radius of 3 cm and center at the given point.

b. The set of points is a right circular cylinder whose height corresponds to the length of the segment and whose base has a radius of 3 cm. At each end of the cylinder is a hemisphere with a radius of 3 cm.

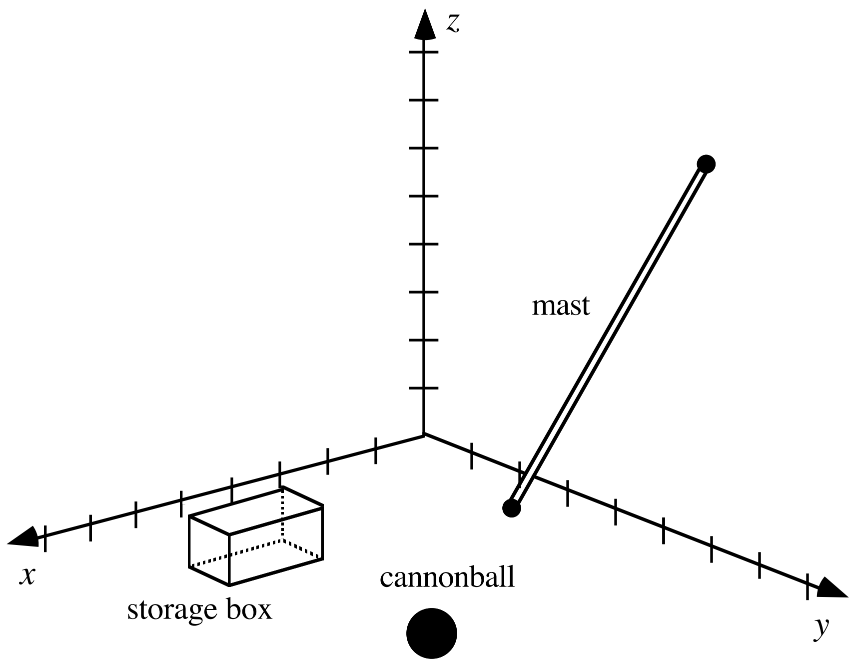
c. The set of points is a plane perpendicular to the plane containing the two parallel lines. It intersects the plane containing the two lines half way between them.

d. The set of points is two parallel planes 6 cm apart, one on each side of the given plane.

1.10 **a.** The set of points is a hemisphere with a radius of 180 cm and center at the center of the floor.

b. The set of points is a line segment perpendicular to the floor which has as its endpoints the centers of the ceiling and the floor.

1.11 Sample sketch:



* * * * * * * * * *

Research Project (page 369)

Students review graphing in the coordinate plane by locating points on a map. By adding the elevation to each site, they change a two-dimensional model into a three-dimensional one. This process should help students visualize ordered triples in space. **Note:** Since students may need help determining an appropriate vertical scale and the corresponding length of each toothpick, you may wish to conduct Part **e** as a demonstration. Scaling on the z-axis is easier when students use longer toothpicks.

The following list provides some historical information for each numbered site in Figure **7**, as well as the corresponding elevation.

1.	Absarokee	Keogh Buffalo Jump (Pishkin); 1545 m
2.	Bannack	First territorial capital; 2060 m
3.	Billings	Indian pictograph cave; 950 m
4.	Butte	"Richest Hill on Earth"; 1758 m
5.	Choteau	Dinosaur dig; 1161 m
6.	Ekalaka	Dinosaur dig; 898 m
7.	Fort Peck	*Tyrannosaurus rex*; 639 m
8.	Glendive	Badlands area; 631 m
9.	Granite Peak	Highest elevation in Montana; 3901 m
10.	Great Falls	Home of Charles M. Russell, Western artist;1009 m

11.	Havre	Wahkpa-Chu'gn Buffalo Jump; 870 m
12.	Helena	Home of Dr. Maria M. Dean, pioneer physician; 1267 m
13.	Lewistown	Geographical center of Montana; 1208 m
14.	Lowest Point	Where Kootenai River crosses the Idaho border; 555 m
15.	Maiden	Clara McAdow operated Spotted Horse Mine; 1512 m
16.	Martinsdale	Home of Alberta Bair, philanthropist; 1294 m
17.	Miles City	Trailhead for early cattle drives; 723 m
18.	Missoula	Home of Jeannette Rankin, first woman elected to U.S. House of Representatives; 978 m
19.	Pryor	Wild horse refuge; 1102 m
20.	Scobey	Archetypical Pioneer Museum; 754 m

(page 371)

Activity 2

Students simulate the type of sonar mapping described in the introduction to this module. Although the data is collected by a mechanical process, the actual map may be created by a symbolic manipulator.

Materials List

- rulers (one per group)
- scissors (one pair per group)
- tape or glue
- cardboard box with lid (one per group)
- plastic foam (cut to fit box lid; one sheet per group)
- equal lengths of stiff, straight wire (one for each lattice point on the lid)
- centimeter graph paper (one sheet per group)
- three-dimensional models of common geometric solids
- cardboard (optional)

Teacher Note

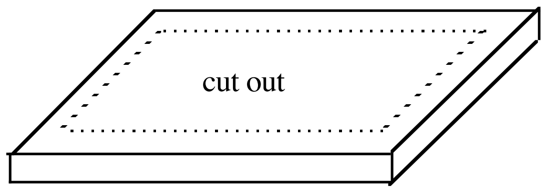
Cardboard boxes should be no larger than a shoe box. (When centimeter graph paper is taped to the lid, there should be no more than 100 lattice points.)

Lengths of stiff wire can be cut from coat hangers or welding rods. Bamboo skewers will also work. The length of each wire should equal the depth of the box.

Models of simple geometric solids, such as cubes and triangular prisms, provide good results.

Before beginning the exploration, you should prepare one box for each group of students.

1. Cut out the center of the lid as shown below.



2. Cut a sheet of plastic foam the same size as the box lid.

3. Glue or tape the foam to the inside of the lid.

4. Tape a three-dimensional solid to the bottom of each box. (The model should not touch the sides of the box.)

5. Tape the lid shut.

Technology

- symbolic manipulator

Teacher Note

To complete Part **c** of the exploration, students require a symbolic manipulator that creates three-dimensional surface plots defined as matrices (such as Mathcad or Mathematica). Some graphing utilities may only generate surface plots using equations.

Exploration

(page 372)

Students collect data for a surface plot by a mechanical process that simulates sonic mapping.

a. Students create coordinate systems on the lids of their boxes.

b. **1.** Lattice points have coordinates with integer values.

2. Students should produce a data table like the one in Figure **10**.

c. Students create a surface plot on a symbolic manipulator using the table created in Part **b**. (Directions for completing this step using Mathcad and Mathematica are included at the end of this teacher edition.)

d–e. Students use the surface plot to guess the shape of the artifact, then open the box to verify their predictions.

Discussion

(page 373)

a. The technology works on the assumption that a surface is linearly continuous between data points in a matrix. Depending on the distance between data points and the shape of the object, a surface plot may not closely resemble the actual object.

b. The tops of the wires should describe the approximate shape of the surface of the artifact.

c. Sample response: The numbers in the matrix correspond to the elevations of different lattice points. A value of 0 indicates that the wire is touching the bottom of the box. In one sense, the matrix is like a "numerical" contour map.

Teacher Note

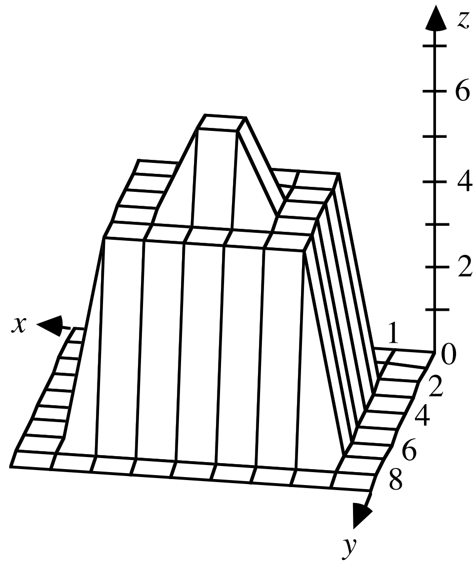
You may wish to ask students to build models for Problems **2.1** and **2.2**. Using a chart like the one below taped to a piece of cardboard, for example, students may insert toothpicks of the appropriate length into the center of each square. Unit cubes also work well.

		5	5	5	5	5	5		
		5	5	5	5	5	5		
		5	5	7	7	5	5		
		5	5	7	7	5	5		
		5	5	5	5	5	5		
		5	5	5	5	5	5		

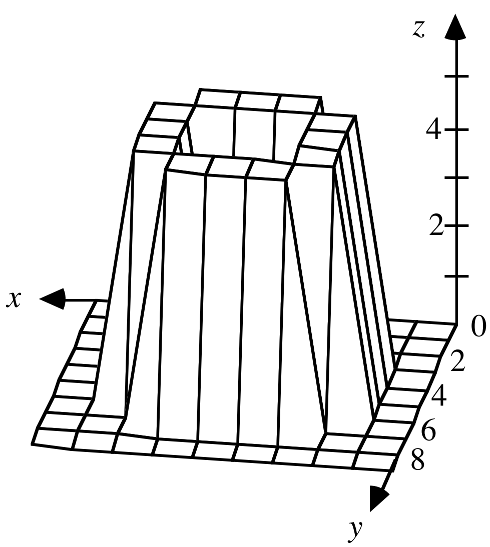
Assignment

(page 374)

2.1 **a.** Sample graph:



b. Sample graph:



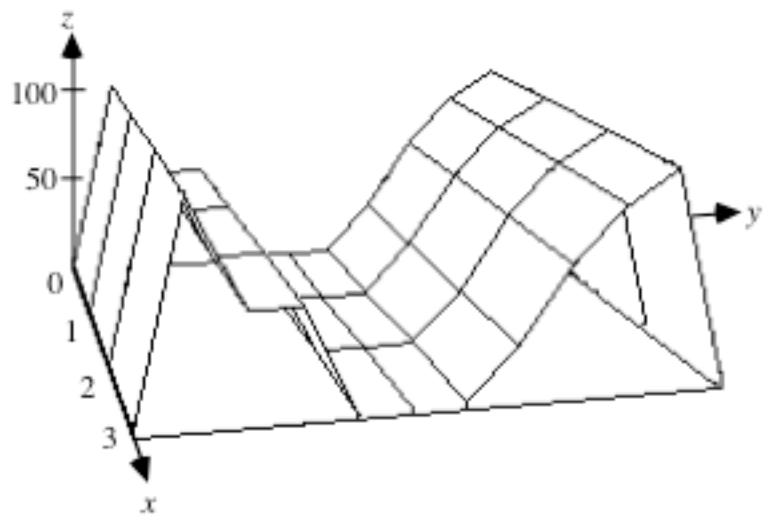
2.2 **a.**

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 2 & 2 & 0 \\ 0 & 2 & 4 & 4 & 2 & 0 \\ 0 & 2 & 4 & 4 & 2 & 0 \\ 0 & 2 & 2 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

b.

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 25 & 25 & 25 & 25 & 25 & 25 & 25 & 25 & 0 \\ 0 & 25 & 50 & 50 & 50 & 50 & 50 & 50 & 25 & 0 \\ 0 & 25 & 50 & 75 & 75 & 75 & 75 & 50 & 25 & 0 \\ 0 & 25 & 50 & 75 & 100 & 100 & 75 & 50 & 25 & 0 \\ 0 & 25 & 50 & 75 & 100 & 100 & 75 & 50 & 25 & 0 \\ 0 & 25 & 50 & 75 & 75 & 75 & 75 & 50 & 25 & 0 \\ 0 & 25 & 50 & 50 & 50 & 50 & 50 & 50 & 25 & 0 \\ 0 & 25 & 25 & 25 & 25 & 25 & 25 & 25 & 25 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2.3 The following sample plot was generated using the matrix shown below.



$$\begin{bmatrix} 0 & 100 & 50 & 50 & 0 & 0 & 0 & 25 & 60 & 85 & 100 & 0 \\ 0 & 100 & 50 & 50 & 0 & 0 & 0 & 25 & 60 & 85 & 100 & 0 \\ 0 & 100 & 50 & 50 & 0 & 0 & 0 & 25 & 60 & 85 & 100 & 0 \\ 0 & 100 & 50 & 50 & 0 & 0 & 0 & 25 & 60 & 85 & 100 & 0 \end{bmatrix}$$

2.4 **a.** The value of each entry in the table would be 2 units greater than the original value.

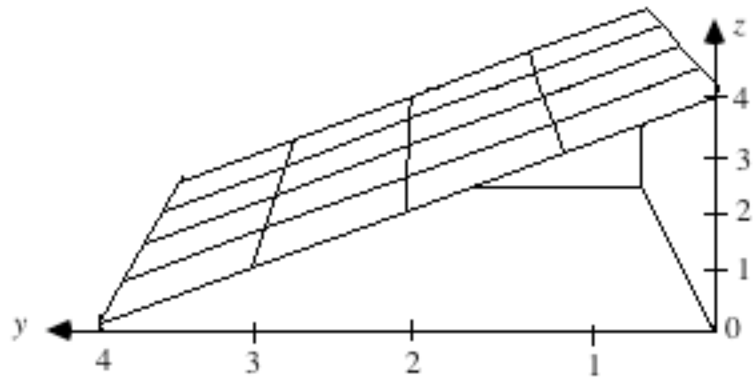
b. The value of each entry in the table would be 3 times the original value.

c. The value of each entry in the table would be the additive inverse of the original value.

2.5 The surface plot is a plane parallel to the *xy*-plane which intersects the *z*-axis at $(0,0,C)$, where C is the constant value in the table.

* * * * *

2.6 **a.** Sample sketch:



b. The surface plot makes a 45˚ angle with the *xy*-plane.

2.7 **a.** The graph is a plane parallel to the *yz*-plane which intersects the *x*-axis at (3,0,0).

b. The graph is a plane parallel to the *xz*-plane which intersects the *y*-axis at (0,3,0).

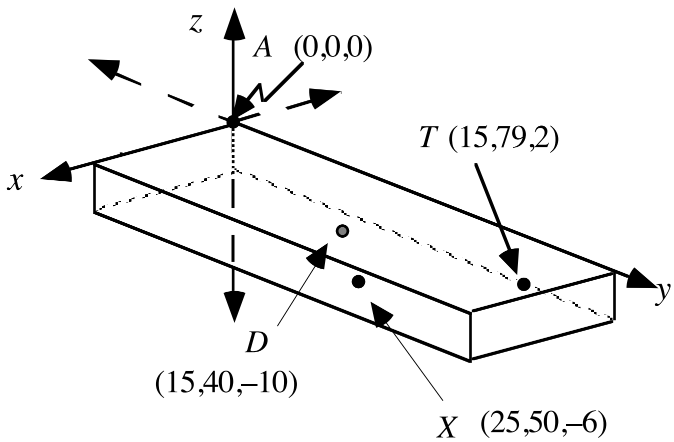
c. The graph is a plane parallel to the *xy*-plane which intersects the *z*-axis at (0,0,3).

Answers to Summary Assessment (page 377)

1. **a.** Answers will vary. The following sample response uses the location of the penny as the origin: penny (0,0); nickel (0,–4); dime (1,–1).

b. Using the sample response given in Part **a** and assuming that the thickness of the coin is not a consideration, the ordered triples are: penny (0,0,–0.5); nickel (0,–4,–0.5); dime (1,–1,–0.5).

2. **a–c.** In the following sample response, *A* represents the location of the announcer, *T* represents the location of the trainer, and *D* represents the location of the drain.

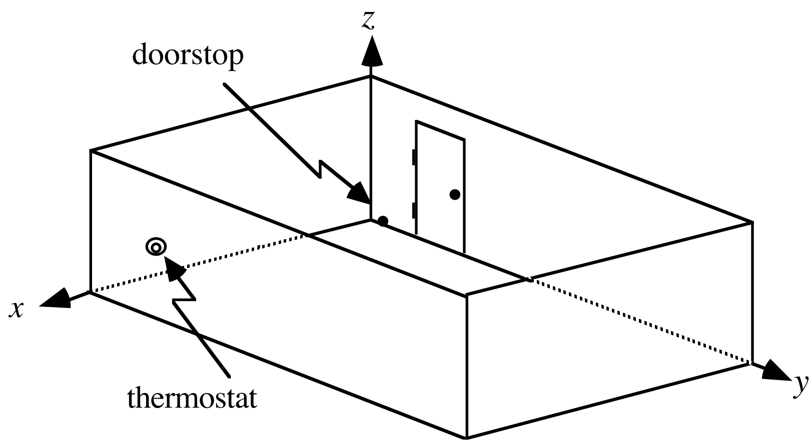


d. (25,50,0)

3. **a–b.** The following matrix organizes the data as shown in Figure **11**, with the origin placed at the upper right-hand corner of the map.

$$\begin{bmatrix} 0 & 0 & 0 & 7 & 0 & 0 \\ 0 & 60 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 7 & -6 & 0 \\ 0 & 0 & 4 & 0 & 15 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

4. **a.** Sample sketch:



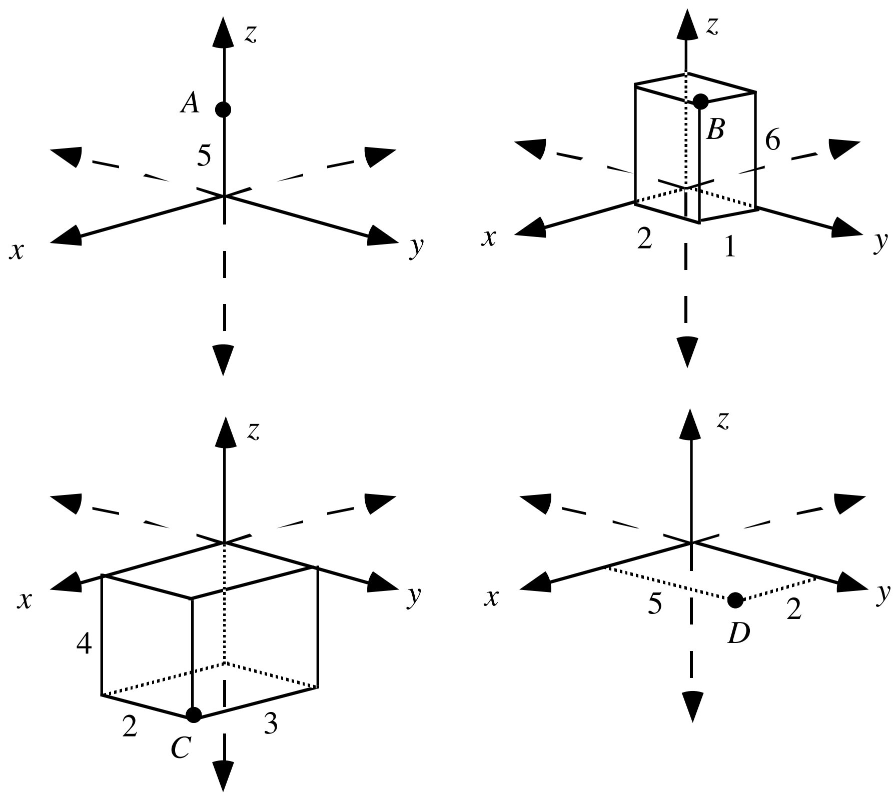
b. Using the sample sketch shown above, the coordinates are (3,4,4).

c. Answers will vary. Using the sample sketch shown above, the coordinates are approximately (0,0.2,0).

d. Using the sample sketch shown above, the coordinates are approximately (6,1.5,1.5).

Module Assessment

1. Identify the coordinates of each labeled point in the following graphs.



2. Graph each ordered triple below on a three-dimensional coordinate system.

a. (2,5,3)

b. (4,1,–2)

c. (4,–3,0)

3. In the town of Simmsville, the avenues run north and south, while the streets run east and west.

a. **1.** Create a two-dimensional map of Simmsville that shows First through Fifth Streets and Avenues A through F.

2. City Hall is located at the intersection of 1st Street and Avenue A. Label this intersection as (1st,A,0).

b. Imagine that you are standing on the fourth floor of City Hall. How could you use the mathematics in this module to represent your location?

c. The public library is located on the corner of 3rd Street and Avenue D. If you are on the fifth floor of the library, what are your coordinates?

d. Samantha's Auto Shop is located in the basement of the building at the corner of 2nd Street and Avenue E. What are the coordinates of the shop?

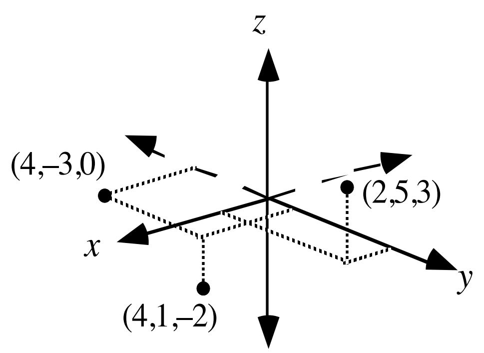
4. Use the following matrix to sketch a surface plot on a three-dimensional coordinate system.

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 4 & 0 \\ 0 & 2 & 2 & 4 & 0 \\ 0 & 2 & 2 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

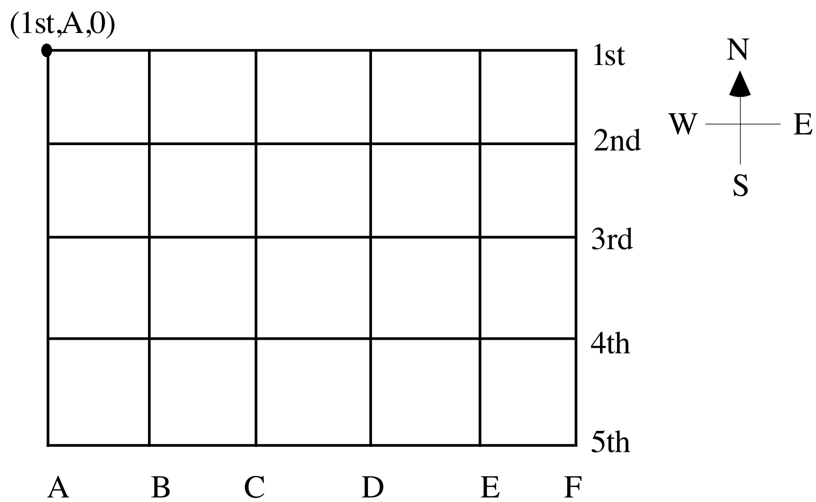
Answers to Module Assessment

1. A (0,0,5), B (1,2,6), C (3,2,–4), D (2,5,0)

2. **a–c.** Sample graph:



3. **a.** Sample map:

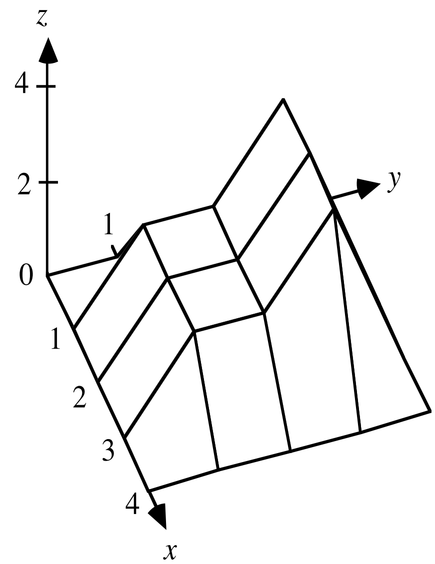


b. Sample response: This location could be described with an ordered triple with a z-coordinate of 4, where each unit on the z-axis represents 1 floor.

c. (3rd,D,5)

d. (2nd,E,–1)

4. Sample graph:



Selected References

Marx, R. F. "In Search of the Perfect Wreck." *Sea Frontiers* (September/October 1990): 47–51.

Petrik, P. *No Step Backward: Women and Family on the Rocky Mountain Mining Frontier*. Helena, MT: Montana Historical Society Press, 1987.

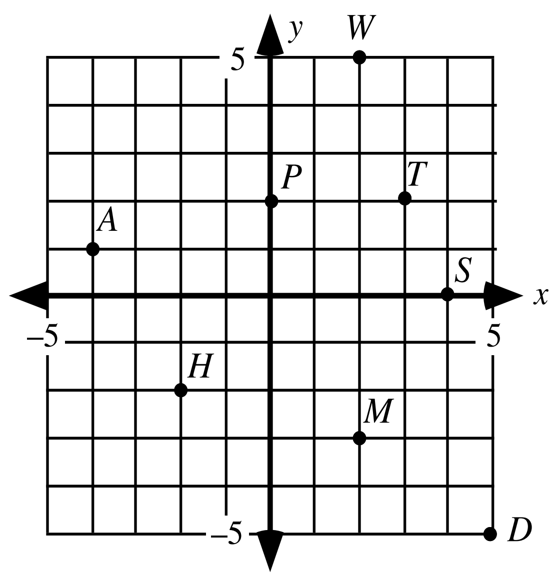
Seanor, D. "The Case with the Midas Touch." *ABA Journal* (May 1990): 50–55.

Flashbacks

Activity 1

1.1 Draw an *xy*-coordinate system for the treasure map below. Clearly label the origin and list the coordinates of the five features on the map.

1.2 Identify the coordinates of each labeled point in the graph below.



1.3 **a.** In what quadrant are all *x*-coordinates negative and *y*-coordinates positive?

b. In what quadrant are all *x*-coordinates positive and *y*-coordinates negative?

c. In what quadrant are all *x*-coordinates and *y*-coordinates negative?

Activity 2

2.1 Plot and label each of the following points on a three-dimensional coordinate system.

a. *A* (0,3,0)

b. *B* (5,3,6)

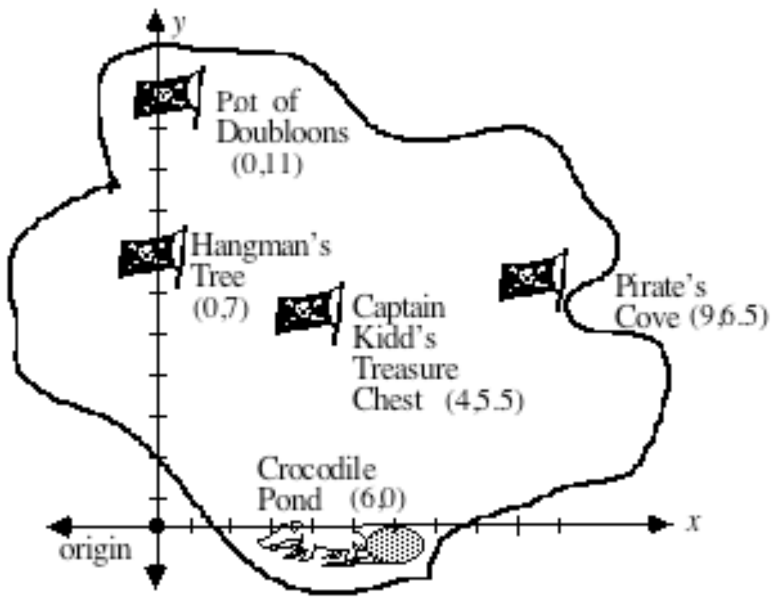
c. *C* (5,3,–4)

d. *D* (–6,–2,3)

Answer to Flashbacks

Activity 1

1.1 Answers will vary, depending on the placement of the origin, the scale used on axes, and the point placement for each symbol. In the following sample response, the center of the flag is the point of placement.

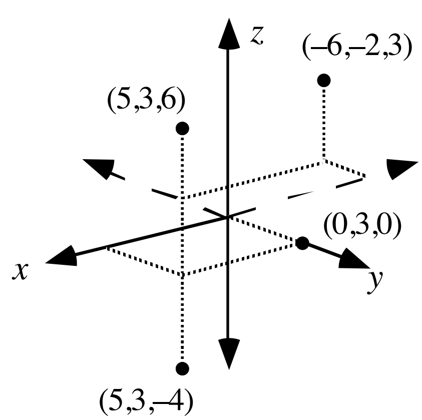


1.2 $A(-4,1)$; $P(0,2)$; $H(-2,-2)$; $W(2,5)$; $M(2,-3)$; $T(3,2)$; $S(4,0)$; $D(5,-5)$

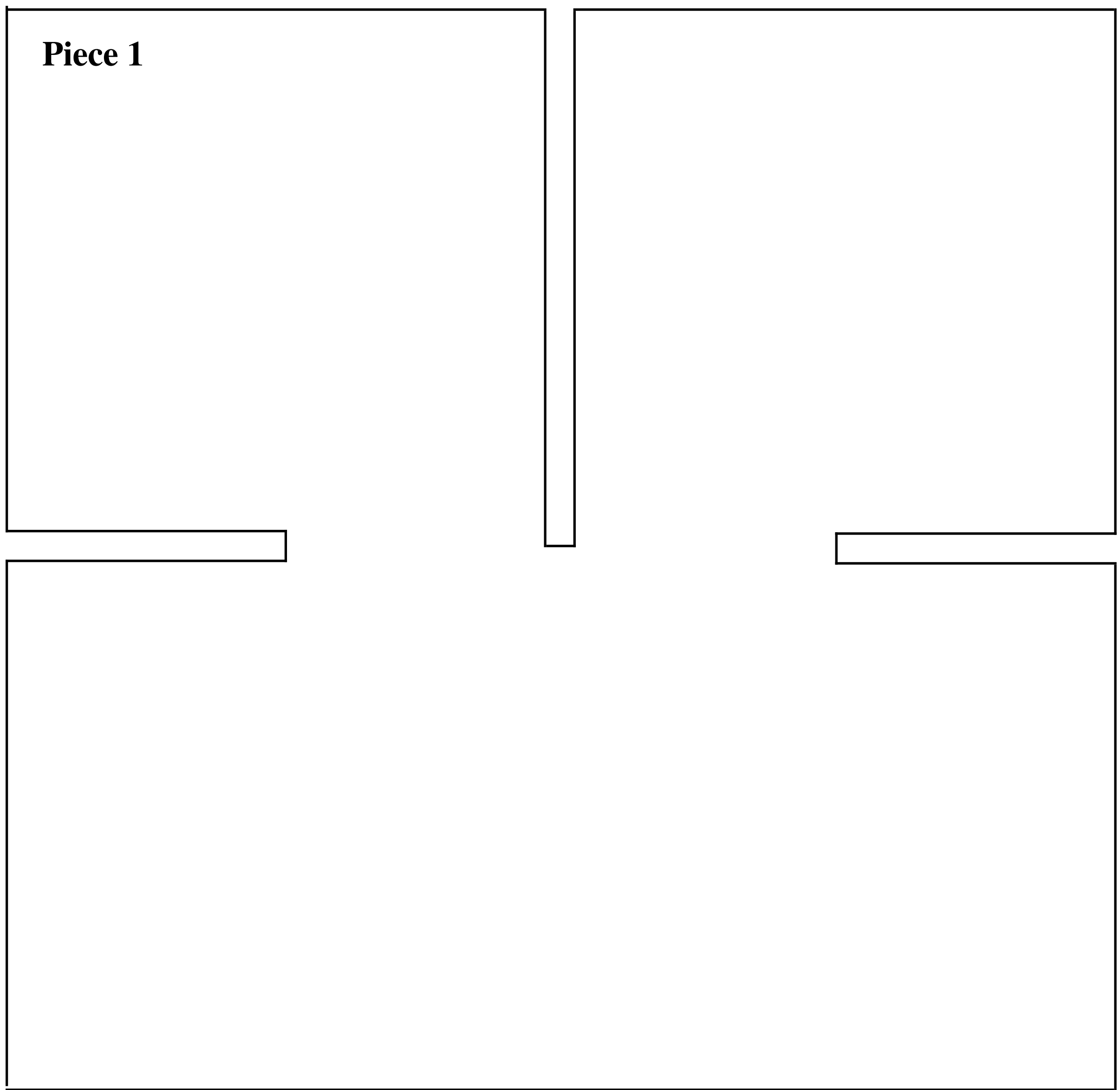
1.3
- **a.** quadrant II
- **b.** quadrant IV
- **c.** quadrant III

Activity 2

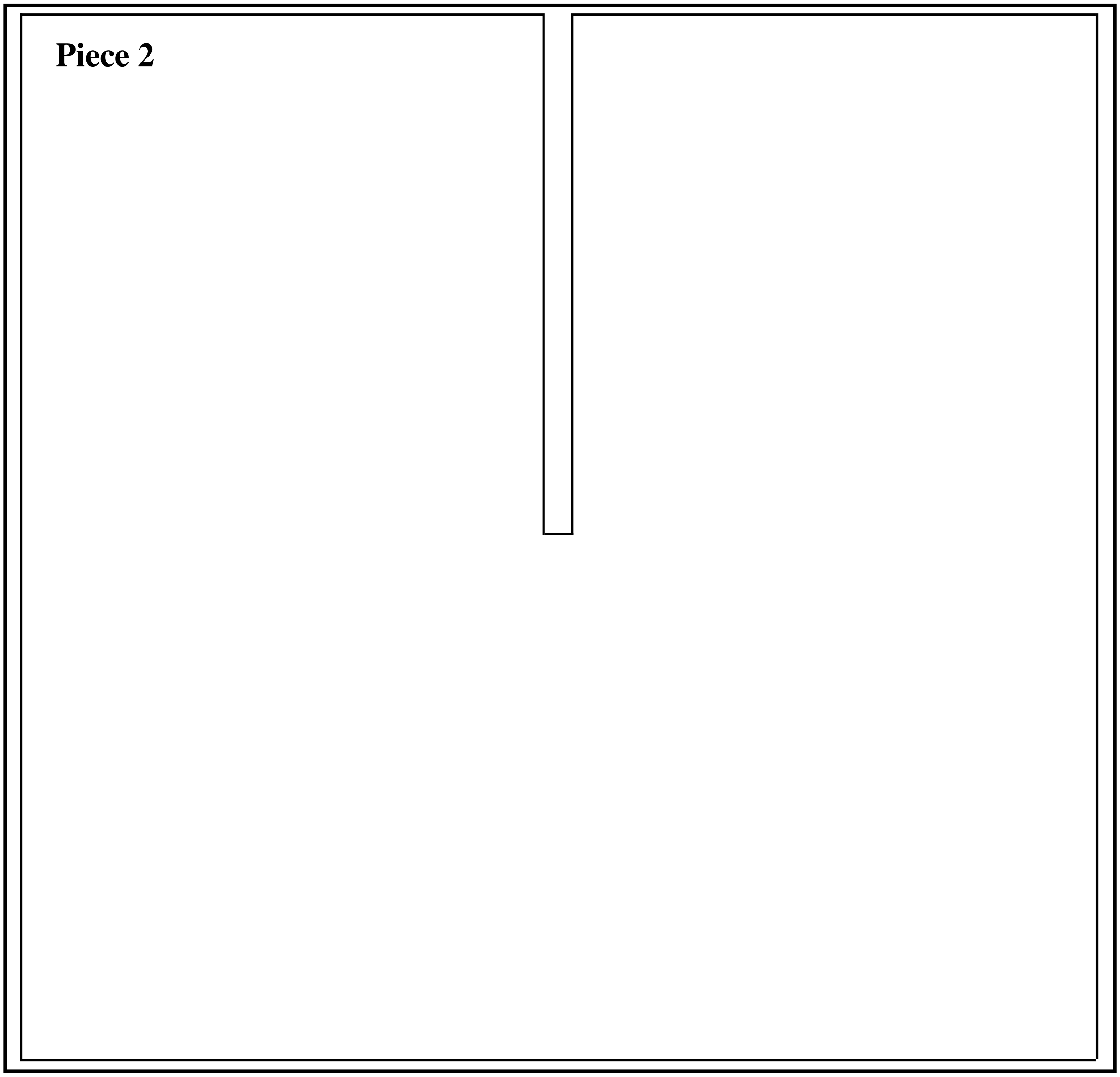
2.1 Sample graph:

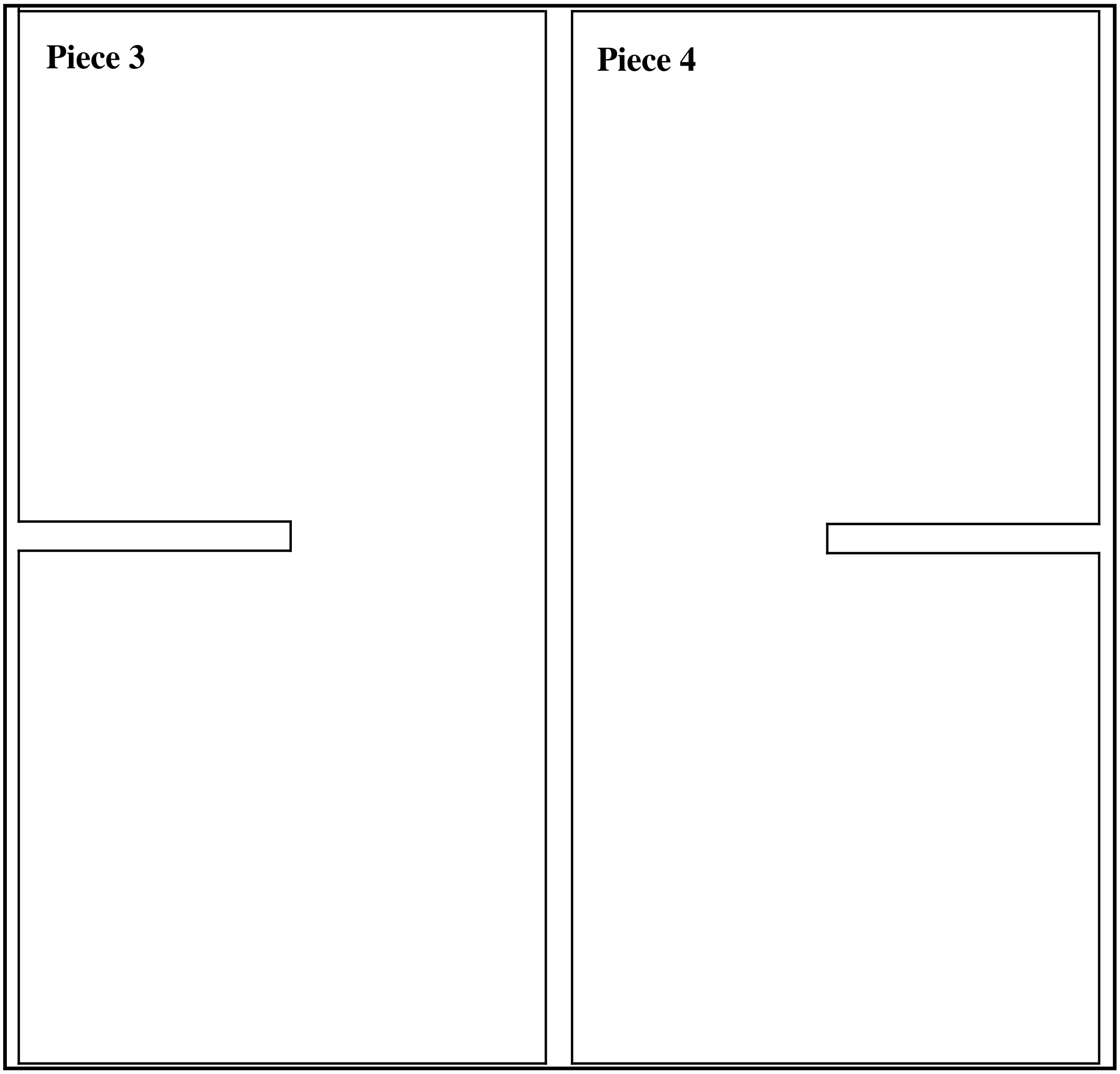


Template for 3-D Model

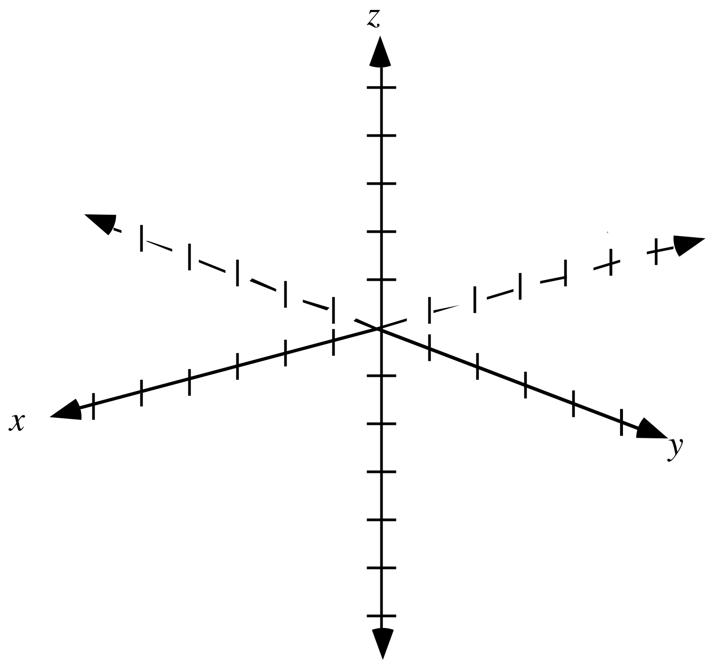


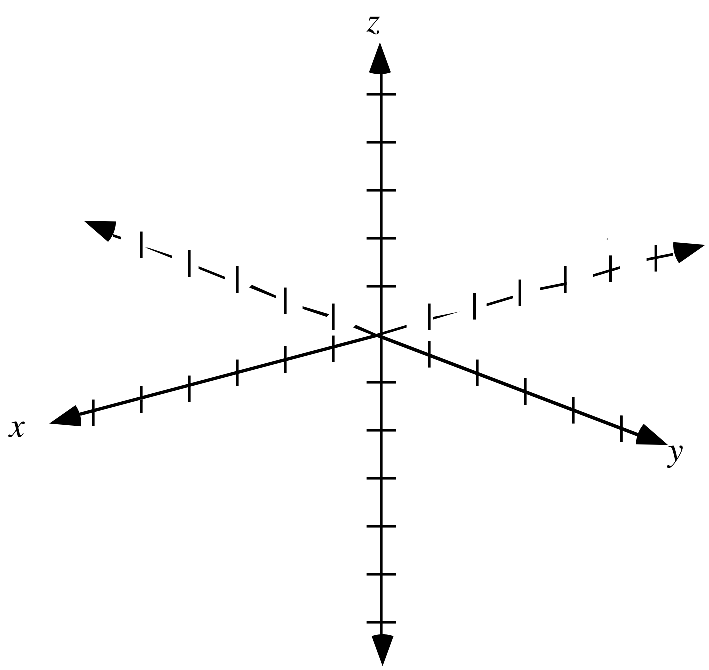
Piece 2




Piece 3
Piece 4

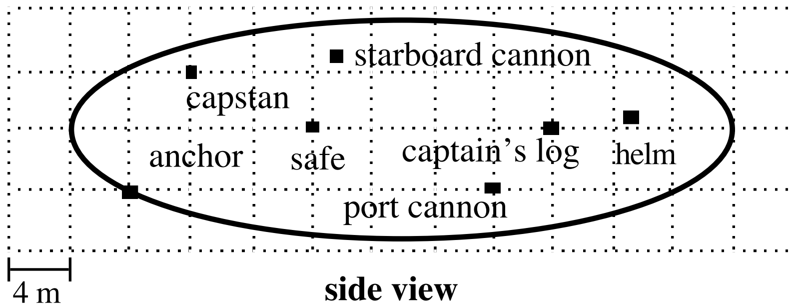
3-D Coordinate System Template



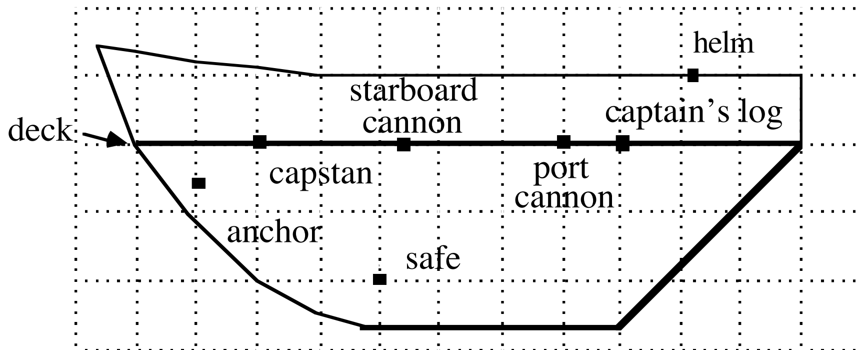


Sunken Ship Template

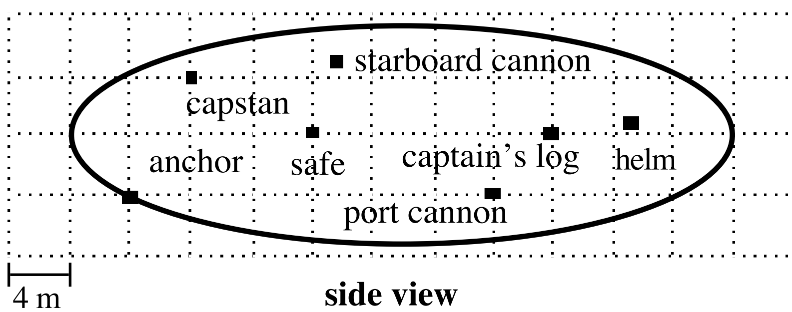
top view



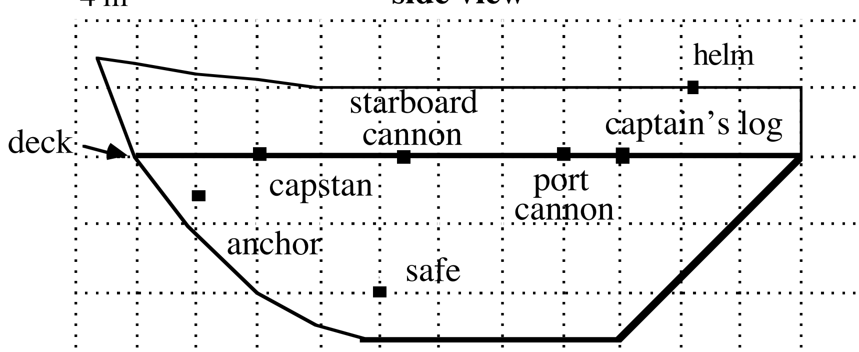
side view



top view



side view



Making Surface Plots with Mathcad

1. Open the **Mathcad** computer program.

2. *Entering table data into a matrix*

- **a.** Type **M:** (Do not press **Return. M:=** will appear on the screen.)
- **b.** Choose the **Matrices** command from the **Math** menu.
- **c.** Enter the number of rows and columns in your table in the appropriate boxes.
- **d.** Click **Create**. (An empty table called a **matrix** will appear on the screen.)
- **e.** Enter the *z*-values from your table into the matrix. (To enter the next *z*-value, move the cursor to the next entry point and click the mouse.)
- **f.** After making the last entry, move the cursor outside the matrix and click the mouse.

3. *Making your data into a three-dimensional graph*

- **a.** Choose the **Create Surface Plot** command from the **Graphics** menu.
- **b.** Type **M**.
- **c.** Move the cursor outside the empty box and click the mouse. (A three-dimensional graph will appear.)

4. *Formatting your graph*

- **a.** Move the cursor to the surface plot and double-click with the mouse. (The **Surface Plot Format…** dialogue box will appear.)
- **b.** Change **Shading** to either **Color spectrum** or **Grayscale**.
- **c.** Experiment with various **Rotation** and **Tilt** angles to obtain the clearest picture of the object.

5. Print a copy of your **Mathcad** worksheet. Make sure to set the printer to **Grayscale**.

Making Surface Plots with Mathematica*

1. Open the **Mathematica** program.

2. *Entering table data into a matrix*

   a. Type **m={**

   Press **Return.**

   b. Enter data in the following way:

      1. Type **{**first line of numbers from table, each separated by a comma**},** (Be sure to type a comma after the bracket.)

      2. Press **Return.**

      3. Type **{**second line of numbers from table, each separated by a comma**},**

      4. Press **Return.**

      5. Continue until all lines of numbers from table are entered. Do not type a comma after the last bracket.

   c. Type **}**

   d. On a DOS or Windows machine, hold down the **Shift** key and press **Return**. On a Macintosh, press **Enter**. (A list of numbers will appear after a few moments; be patient.)

   e. Type **MatrixForm[m]**

   f. Hold down the **Shift** key and press **Return.** (**Mathematica** will print a copy of the matrix you entered on the screen. Make sure that the numbers in the matrix match the numbers in your data table.)

3. *Making your table into a three-dimensional graph*

   a. Type **ListPlot3D[m]**

   b. Hold down the **Shift** key and press **Return**. (**Mathematica** will create a surface plot of your matrix.)

4. Follow these steps to rotate the surface plot and explore the picture from different viewpoints.

a. Press **Enter** or **Return** to open a new cell.

b. Under the **Action** menu, open the **Prepare Input** submenu, and choose **3D ViewPoint Selector**.

c. Click the **Cartesian** button.

d. Drag the cube in the upper left corner until you get the viewpoint you want. (Notice the **\z**, **\x**, **\y** boxes directly below the **Cartesian** button. As you drag the cube, the numbers in these boxes change.)

e. After you have rotated the cube to the desired position, click in the box numbered at the bottom of the 3-D viewpoint selector box.

f. Select **Show[%,ViewPoint->{\x,\y,\z}]**.

g. Click the **Paste** button.

h. Hold down the **Shift** key and press **Return**.

i. Try this with a number of different **ViewPoint** settings.

5. Print a copy of your **Mathematica** worksheet. Make sure to set the printer to **Grayscale**.

*To avoid accidental confusion or interference with built-in Mathematica definitions, programmers strongly suggest that user variables and definitions use lowercase letters *only*.