# AIDS: The Preventable Epidemic

A recent congressional study reported that AIDS and HIV infections among teenagers rose an alarming 70% between 1990 and 1992. Are teens playing Russian roulette with their sex lives?

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# AIDS: The Preventable Epidemic

## Introduction

Acquired Immune Deficiency Syndrome (AIDS) is a fatal, communicable disease. By the end of 1992, approximately 200,000 cases of AIDS had been diagnosed in the United States. The AIDS epidemic represents a particular danger to America's youth. People under age 25 account for at least a quarter of the estimated 40,000 newly infected Americans each year, and a congressional study reported that AIDS and Human Immunodeficiency Virus (HIV) infections among teenagers rose an alarming 70% between 1990 and 1992. Dr. Lloyd Kold, director of the Centers for Disease Control's Division of Adolescent and School Health, noted that "Many teens are playing Russian roulette with their sex lives."

As the population of AIDS patients continues to grow, our health care system has been struggling to provide hospital space, medication, and other services. In the United States, the yearly cost of treating one AIDS victim is about $38,000 (as of July 1992). The projected lifetime cost is at least $102,000. While insurance companies and the federal government are the initial sources of payment, higher taxes and insurance premiums may ultimately shift the financial burden to the American public.

## Discussion

**a.** Why should the AIDS epidemic be a concern for everyone?

**b.** What steps may be taken to help prevent the spread of HIV?

**c.** Approximately 20% of American teenagers have had four or more sexual partners by their senior year of high school. This group is particularly susceptible to HIV infection. What do these numbers mean in terms of your school population?

## *Activity 1*

Due to the unusual nature of the AIDS virus, health professionals did not immediately recognize the manner of transmission. In 1987, several massive educational programs began to inform people about the dangers of AIDS.

With improved screening of the nation's blood supply, the number of new cases of AIDS among recipients of blood transfusions has decreased substantially. In addition, those who engaged in risky behavior began to make wiser choices.

Consequently, the number of new cases in two particularly high-risk populations—intravenous drug users and homosexuals—also began to level off. However, because of a mistaken sense of security, the number of AIDS cases among heterosexuals is on the rise.

## Exploration 1

In this exploration, you use a model to examine the spread of infectious diseases.

**a.** Pour one cup of red beans into a flat box. Each red bean represents a healthy, disease-free individual.

**b.** Place one white bean in the container. This bean represents an individual with an infectious disease.

**c.** Gently shake the container. Replace every red bean that is within 1 mm of the white bean with a white bean. These beans represent individuals who came into contact with the infectious individual and have contracted the disease. (You can use the edge of a dime to check the distance between beans.)

**d.** Count the number of white beans in the container. Write this number in the appropriate column of Table **1**.

**Table 1: Simulated spread of a disease**

| Shake No. ($x$) | No. of White Beans ($y$) |
|---|---|
| 0 | 1 |
| 1 | |
| 2 | |
| ⋮ | |
| 6 | |

**e.** Repeat Parts **c** and **d** for six shakes.

**f.**
1. Create a scatterplot of your data in Table **1**. **Note:** Save this scatterplot for use in Exploration **2**.
2. Describe any patterns you observe in the scatterplot.

**g.** Table **2** shows the total U.S. AIDS cases reported for people ages 13–29 from 1980 to 1991.

**Table 2: Total U.S. AIDS cases reported for ages 13-29**

| Year | No. of Cases | Year | No. of Cases |
|---|---|---|---|
| 1980 | 18 | 1986 | 7897 |
| 1981 | 79 | 1987 | 13,307 |
| 1982 | 301 | 1988 | 19,998 |
| 1983 | 886 | 1989 | 27,999 |
| 1984 | 2064 | 1990 | 37,022 |
| 1985 | 4296 | 1991 | 48,007 |

**Source:** Centers for Disease Control, 1995.

Create a scatterplot of the data in Table **2**. Let $y$ represent the number of AIDS cases and $x$ represent the number of years after 1980 (for 1980, $x = 0$). **Note:** Save this scatterplot for use in Exploration **2**.

**h.** Compare the two scatterplots you created in Parts **f** and **g**.

## Discussion 1

**a.**
1. Compare your scatterplot from Part **f** of Exploration **1** with those of others in the class. Describe any differences or similarities you observe.
2. As the shake number increases, does your scatterplot level off? If so, explain why.
3. How does this scatterplot compare with the scatterplot of actual AIDS cases?

**b.** Do you think that the scatterplot of AIDS cases will level off in the years after 1991? Explain your response.

**c.**
1. Can the data in Tables **1** and **2** be modeled well by linear equations? Why or why not?
2. Can the data in Tables **1** and **2** be modeled well by exponential equations? Why or why not?

**d.**
1. How does the data collected during the simulated spread of an infectious disease resemble the actual spread of HIV?
2. How does the simulation differ from the actual spread of HIV?

## Exploration 2

In this exploration, you use exponential equations to model the data in Tables **1** and **2**.

### Mathematics Note

The equation $y = a \bullet b^x$ describes a pattern of **exponential growth**. When describing population growth, $a$ is the initial population and $x$ is the number of time periods. The value of $b$ is the sum of two percentages: 100 (representing the initial population) and $r$, the growth rate from one time period to the next.

For example, consider the sample data in Table **3**.

**Table 3: Sample population data**

| Year | Population |
|---|---|
| 1940 | 250 |
| 1950 | 450 |
| 1960 | 810 |
| 1970 | 1458 |
| 1980 | 2624 |
| 1990 | 4724 |

A scatterplot of this data is shown in Figure **1** below.

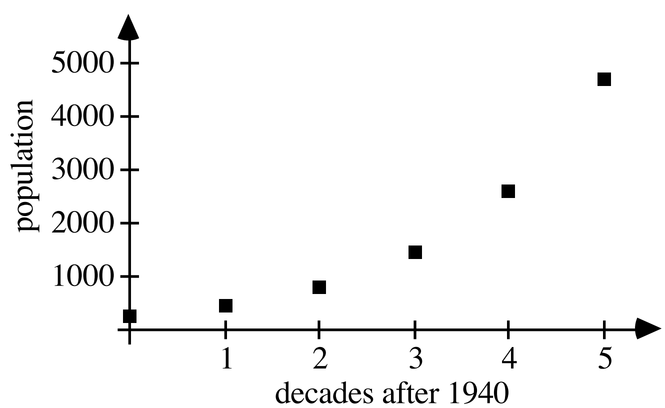


**Figure 1: Scatterplot of population data**

Because this data shows a pattern of exponential growth, it can be modeled by an exponential equation. In this case, the initial population ($a$) is 250. The growth rate ($r$) from one decade to the next can be found by determining the percent increase from 1940 to 1950:

$$\frac{450 - 250}{250} = 0.8 = 80\%$$

Since $r = 0.8$, $b = 1 + 0.8 = 1.8$. Therefore, the data can be modeled by the exponential equation $y = 250(1.8)^x$, where $y$ is the population at the end of each decade, $a$ is the initial population, and $x$ is the number of decades after 1940.

**a.** **1.** Determine an exponential equation that models the data for the simulation in Table **1**.

**2.** Graph this equation on the same set of axes as the scatterplot from Part **f** of Exploration **1**.

**3.** Determine the sum of the absolute values of the residuals for this exponential model.

**b.** **1.** Determine an exponential equation that models the number of AIDS cases in Table **2**.

**2.** Graph this equation on the same set of axes as the scatterplot from Part **g** of Exploration **1**.

**3.** Determine the sum of the absolute values of the residuals for this exponential model.

## Discussion 2

**a.** **1.** In Part **a** of Exploration **2**, you found an exponential model for the data from the simulation. Describe how you determined this model.

**2.** Does your equation appear to provide a good model for the data? Explain your response.

**b.** **1.** In Part **a** of Exploration **2**, you found an exponential model for the number of AIDS cases. Describe how you determined this model.

**2.** Does your equation appear to provide a good model for this data? Explain your response.

## Assignment

**1.1** The growth of a population can be modeled by the equation $y = a(1.5)^x$, where $x$ represents time in years. When $x = 1$, the population is 210. What is the size of the initial population?

**1.2** The following table lists the total number of AIDS cases reported in the United States from 1980 to 1985.

| Year | No. of Cases |
|---|---|
| 1980 | 94 |
| 1981 | 417 |
| 1982 | 1601 |
| 1983 | 4720 |
| 1984 | 10,996 |
| 1985 | 22,890 |

**Source:** Centers for Disease Control, 1995.

**a.** Create a scatterplot of the data. Let $y$ represent the number of AIDS cases; $x$ represent the number of years after 1980 (for 1980, $x = 0$).

**b.** Find an exponential equation that models the data.

**c.** Does your equation appear to be a good model of the data? Explain your response.

**d.** Using your equation from Part **b**, predict the total number of AIDS cases reported in each year from 1986 to 1991.

**e.** Do your predictions seem reasonable? Explain your response.

**1.3** The following table lists the total number of AIDS cases reported in the United States from 1986 to 1991.

| Year | No. of Cases |
|---|---|
| 1986 | 42,085 |
| 1987 | 70,918 |
| 1988 | 106,580 |
| 1989 | 149,223 |
| 1990 | 197,315 |
| 1991 | 255,864 |

**Source:** Centers for Disease Control, 1995.

**a.** Compare this data with the predictions you made in Problem **1.2d**.

**b.** Why do you think the actual numbers of reported cases differ from your predictions?

**c.** Create a scatterplot of the AIDS data from 1980 to 1991. Let $y$ represent the number of AIDS cases and $x$ represent the number of years after 1980 (for 1980, $x = 0$).

**d.** Find an exponential equation that models the data.

**e.** Does your equation appear to be a good model of the data? Explain your response.

**1.4** The following table shows estimates for the total number of U.S. children and adolescents orphaned by AIDS from 1983 to 1994.

| Year | No. of Orphans | Year | No. of Orphans |
|---|---|---|---|
| 1983 | 300 | 1989 | 10,000 |
| 1984 | 700 | 1990 | 14,400 |
| 1985 | 1500 | 1991 | 18,500 |
| 1986 | 2900 | 1992 | 23,900 |
| 1987 | 4500 | 1993 | 31,100 |
| 1988 | 6800 | 1994 | 38,400 |

**Source:** D. Michaels and C. Levine, "Estimates of the Number of Motherless Youth Orphaned by AIDS in the United States."

**a.** Make a scatterplot of this data. Let *y* represent the number of children orphaned by AIDS and *x* represent the number of years after 1983 (for 1983, $x = 0$).

**b.** Find an exponential equation that models the data.

**c.** Determine the sum of the absolute values of the residuals for your exponential model.

**d.** Do you think that your equation is a good model for the data? Justify your response.

**1.5** The following table shows a dramatic decrease in the population of the Blackfeet tribe due to an epidemic of smallpox.

| Year | Population |
|---|---|
| 1822 | 2800 |
| 1838 | 600 |
| 1841 | 200 |

**Source:** J. F. Decker, "Depopulation of the Northern Plains Natives."

**a.** Find an exponential equation that models the data.

**b.** Do you think that three data points are enough to determine an accurate model? Explain your response.

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**1.6** The table below shows the amount of money in an investment account from 1988 to 1994.

| Year | Amount |
|---|---|
| 1988 | $500 |
| 1989 | $550 |
| 1990 | $605 |
| 1991 | $666 |
| 1992 | $732 |
| 1993 | $805 |
| 1994 | $886 |

**a.** Make a scatterplot of the data. Let *y* represent the amount of money and *x* represent the number of years after 1988 (for 1988, $x = 0$).

**b.** Find an exponential equation that models the data.

**c.** Use your model to predict the amount of money in the account in each year from 1995 to 2000.

**d.** Do you think that your model is a good one? Justify your response.

**1.7** The table below shows the number of deaths due to AIDS in the United States per 10 million people for selected years from 1981 to 1985.

| Year | Deaths per 10 million |
|---|---|
| 1981 | 6 |
| 1983 | 64 |
| 1984 | 148 |
| 1985 | 292 |

**Source:** U.S. Bureau of the Census.

**a.** Make a scatterplot of the data. Let $y$ represent the number of deaths per 10 million people and $x$ represent the number of years after 1981 (for 1981, $x = 0$).

**b.** Find an exponential equation that models the data.

**c.** Do you think that your equation is a good model for the data? Justify your response.

**d.** Use your model to estimate the number of deaths due to AIDS per 10 million people in each of the following years:

**1.** 1982

**2.** 1987

**e.** In 1982, there were approximately 20 deaths due to AIDS per 10 million people. In 1987, there were approximately 672 deaths due to AIDS per 10 million people. Did your model provide reasonable estimates for either one of these statistics?

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## *Activity 2*

There are several ways in which a person can be exposed to HIV. The Centers for Disease Control (CDC) records the source of exposure for each AIDS-related death or illness in categories similar to those shown in Table **4** below.

**Table 4: Categories of Exposure**

| Category of Exposure | No. of Cases Reported through June 1995 |
|---|---|
| **heterosexual contact**—any kind of sexual intercourse between members of the opposite sex | 64,676 |
| **intravenous drug use**—drug use involving needles | 149,718 |
| **receipt of blood**—receiving blood transfusions for transplants and surgical procedures or for hemophilia and coagulation disorder (two diseases requiring blood transfusions) | 18,451 |
| **male homosexual contact** (there are no known cases of women infecting other women with HIV through homosexual contact) | 275,259 |
| **other**—cases in which the type of exposure does not belong to any of the above categories or has not been determined | 52 |

When a person is diagnosed as HIV positive, the patient may report one or more potential sources of exposure. For example, a person may have had a blood transfusion and also used intravenous drugs. This person would be listed under two categories of exposure: receipt of blood and intravenous drug use.

Organizing this data for thousands of cases is a very complicated task. In this activity, you explore ways to organize large amounts of information.

## Exploration

Data like that collected by the CDC about AIDS is done by conducting surveys. In this activity, you organize the data you collected in the survey of childhood diseases.

**a.** Combine the data you collected in the childhood disease survey with the information collected by your classmates.

Use the combined data to complete Table **5** below, placing an X in the appropriate cell for each individual who has had a particular disease.

**Table 5: Results of childhood disease survey**

| **Individual** | **Disease** | | | | |
|---|---|---|---|---|---|
| | Mumps | Measles | Strep Throat | Chicken Pox | Whooping Cough |
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| ⋮ | | | | | |
| 50 | | | | | |

**b.** Use the information in Table **5** to answer the following questions.

1. How many people in the survey have had strep throat?
2. How many people have had chicken pox?
3. How many people have had strep throat and chicken pox?
4. How many people have had mumps or chicken pox?
5. How many people have had strep throat, mumps, and chicken pox?
6. How many people have had whooping cough, but did not have strep throat?
7. How many people have had whooping cough and measles?

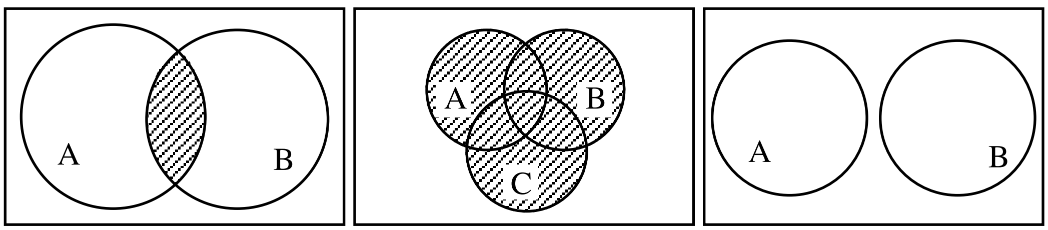
## Mathematics Note

**Venn diagrams** are mathematical models that show relationships among different sets of data. Each circle in a Venn diagram represents a set of data.

The shaded region in Figure **1a** shows the **intersection** of set A with set B, or the set of elements common to both A and B. The intersection of A and B can be written as $A \cap B$.

The shaded region in Figure **1b** shows the **union** of sets A, B, and C. This union contains all the elements in sets A, B, or C. It can be written as $(A \cup B) \cup C$ or $A \cup B \cup C$. Parentheses are not necessary to show order.

Figure **1c** shows two **disjoint sets**, or sets that have no elements in common. The intersection of disjoint sets is the **empty set**, $\varnothing$. This can be written as $A \cap B = \varnothing$.




**a.** intersection of sets **b.** union of sets **c.** disjoint sets

**Figure 1: Venn diagrams**

Venn diagrams can provide a useful tool for displaying relationships among groups. For example, the Venn diagram in Figure **2** shows some data on the incidence of two diseases in a school of 100 students. Fifty of the students have had chicken pox and 18 have had mumps. Eight of the them have had both; while 40 have had neither disease.

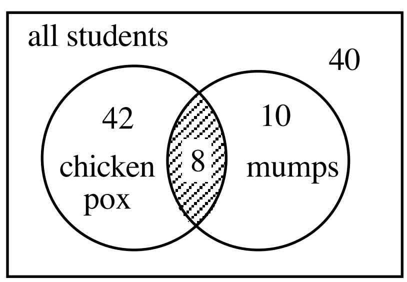



**Figure 2: Incidence of chicken pox and mumps**

Venn diagrams also can be helpful when you must determine the probability of a particular event. For example, suppose you choose one student at random from the group who has had either chicken pox or mumps. The probability that this student has had only the mumps can be determined as follows: From Figure **2**, the number of students who have had either disease is $42 + 8 + 10 = 60$, while the number who have had mumps only is 10. Therefore, the probability that a randomly selected student has had mumps only is $10/60 \approx 0.17 = 17\%$.

**c.** Draw a Venn diagram to represent each of the following groups in your survey results. Shade the appropriate region in the diagram and identify each shaded region as either an intersection of sets, a union of sets, or an empty set.

1. the number of people who have had strep throat and chicken pox
2. the number of people who have had mumps or chicken pox
3. the number of people who have had strep throat, mumps, and chicken pox
4. the number of people who have had whooping cough, but did not have strep throat
5. the number of people who have had whooping cough and measles

**d.**
1. Determine the probability that a person chosen at random from the survey group has had both strep throat and chicken pox.
2. Determine the probability that a person chosen at random from the survey group has had both whooping cough and measles.

## Discussion

**a.** How are the childhood diseases named in the survey similar to AIDS? How are they different?

**b.** Which provides a better way to organize the information in the exploration—a table or a Venn diagram?

**c.** If you had to organize the disease data for everyone in the United States, would you use a table or a Venn diagram? Explain your response.

**d.** How could Venn diagrams have helped you to answer the questions in Part **b** of the exploration?

**e.** How can Venn diagrams help you determine the probabilities of events?

## Assignment

**2.1** The table below contains data for two of the five categories of HIV exposure listed in Table **4**, along with the number of cases who fall into both categories.

| Category of Exposure | No. of Cases Reported through June 1995 |
|---|---|
| intravenous drug use | 149,718 |
| receipt of blood | 18,451 |
| intravenous drug use and receipt of blood | 2969 |

**Source:** Centers for Disease Control, 1995.

**a.** Use a Venn diagram to organize this data.

**b.** If an AIDS patient is selected at random from the two exposure categories in the table, what is the probability that this patient was exposed to HIV only by intravenous drug use?

**2.2** The following table contains data for three of the five categories of HIV exposure listed in Table **4**, along with the number of cases who fall into more than one of the categories.

| Category of Exposure | No. of Cases Reported through June 1995 |
|---|---|
| intravenous drug use (set A) | 149,718 |
| male homosexual contact (set B) | 275,259 |
| receipt of blood (set C) | 18,451 |
| male homosexual contact and intravenous drug use | 31,024 |
| male homosexual contact and receipt of blood | 4086 |
| intravenous drug use and receipt of blood | 2969 |
| all three of these categories | 690 |

**Source:** Centers for Disease Control, 1995.

**a.** Use a Venn diagram to organize this data.

**b.** What is the total number of AIDS cases represented in the table?

**c.** If an AIDS patient is selected at random from the three exposure categories in the table, what is the probability that this patient was infected by both intravenous drug use and homosexual contact?

**2.3** The table below contains data for three of the five categories of HIV exposure listed in Table **4**, along with the number of cases who fall into more than one of the categories.

| Category of Exposure | No. of Cases Reported through June 1995 |
|---|---|
| heterosexual contact | 64,676 |
| intravenous drug use | 149,718 |
| male homosexual contact | 275,259 |
| male homosexual contact and intravenous drug use | 31,024 |
| male homosexual contact and heterosexual contact | 9345 |
| intravenous drug use and heterosexual contact | 22,856 |
| all three of these categories | 3304 |

**Source:** Centers for Disease Control, 1995.

**a.** Use a Venn diagram to organize this data.

**b.** If an AIDS patient is selected at random from the three exposure categories in the table, what is the probability that this patient was exposed to the virus both by intravenous drug use and heterosexual contact?

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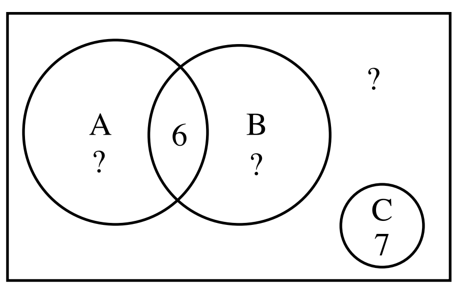
**2.4** In a group of 458 students, 228 are taking mathematics, 166 are taking business, and 7 are taking consumer decision-making. In the incomplete Venn diagram below, students enrolled in mathematics are represented by set A, those in business are represented by set B, and those in consumer decision-making are represented by set C.




**a.** Draw a complete copy of this Venn diagram.

**b.** What is the probability that a student chosen at random from this group:

**1.** is taking both mathematics and business?

**2.** is not taking mathematics, business, or consumer decision-making?

**3.** is taking all three classes?

**2.5** Consider the following six definitions of polygons.

- A quadrilateral is any four-sided figure.
- A trapezoid is a quadrilateral in which at least one pair of opposite sides is parallel.
- A parallelogram is a quadrilateral in which both pairs of opposite sides are parallel.
- A rhombus is a parallelogram with four congruent sides.
- A rectangle is a parallelogram with four right angles.
- A square is a parallelogram that is both a rhombus and a rectangle.

**a.** Sketch a trapezoid.

**b.** Sketch a parallelogram that is neither a rhombus nor a rectangle.

**c.** Sketch a rhombus that is not a rectangle.

**d.** Draw a Venn diagram that shows the relationships among these six types of polygons.

**2.6** The following table shows the distribution of students among three clubs at Canterville High School.

| **Name of Club** | **No. of Students** |
|---|---|
| Pep Club | 150 |
| Spanish Club | 58 |
| Science Club | 29 |
| Spanish and Pep Clubs | 23 |
| Spanish and Science Clubs | 15 |
| Pep and Science Clubs | 11 |
| all three clubs | 5 |

**a.** Organize this data using a Venn diagram.

**b.** The total student population at Canterville High is 230. Modify your Venn diagram to reflect this total.

**c.** If a student is selected at random from the group who participates in at least one of the three clubs, what is the probability that the student is a member of both the Spanish Club and the Science Club?

**d.** What is the probability that a student chosen at random from the high school is in none of the three clubs?

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## Research Project

Obtain the most recent data for U.S. AIDS cases and exposure categories. You may use the National AIDS Hotline (1-800-342-AIDS), a physician, medical clinic, or local health department as sources. You may also obtain information from the Centers for Disease Control's National Aids Clearinghouse at the following electronic address:

http://cdcnac.aspensys.com:86/aidsinfo.html

**a.** Compare the most recent data with the data presented in this module.

1. What changes do you notice?
2. Which category seems to be growing the fastest?

**b.** Write at least two problems involving this data that can be solved with the help of a Venn diagram.

# *Activity 3*

The study of disease and epidemics often involves probability. Most laboratory tests, for example, are not 100% accurate. Similarly, few diseases infect all members of a population and medical treatments are not effective for every patient. In this activity, you learn more about probability and investigate another way to help organize information.

## Exploration

In this exploration, you play a game called "Epidemic."

**a.** The rules for Epidemic are listed below. Read Steps **1-5** before beginning the exploration.

1. Select three students as players. Identify these students as player 1, player 2, and player 3.
2. Without revealing their choices to the class, each of these players randomly chooses one outcome from the appropriate list.
   - Player 1: whooping cough, measles, mumps, healthy
   - Player 2: measles, healthy
   - Player 3: strep throat, chicken pox, healthy
3. Each remaining member of the class predicts the outcomes chosen by the three players. For example, one student might make the following prediction: "Player 1 has mumps, player 2 is healthy, and player 3 has chicken pox."

4. The three players then reveal their choices.
5. Record the numbers of correct and incorrect predictions for the entire class.

**b.** Play Epidemic 10 times.

## Mathematics Note

A **tree diagram** is a mathematical model that shows all the possible outcomes for a series of events or decisions. Each line segment in a tree diagram is a **branch**. Each branch may be assigned a probability.

For example, Figure **3** shows a tree diagram for Epidemic. Notice how the number of branches at a given location in the diagram corresponds with the number of choices available to that player.

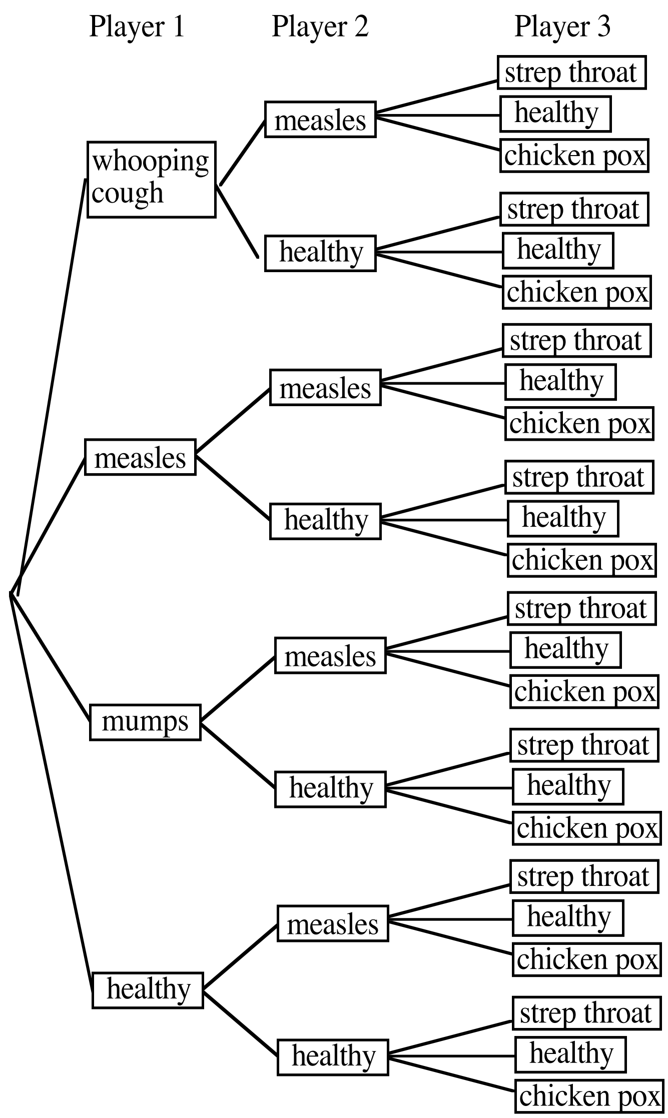


**Figure 3: A tree diagram for Epidemic**

**c.** **1.** List all the possible outcomes for one round of Epidemic.

**2.** Determine the size of the sample space for one round of Epidemic.

**3.** Describe how your strategy in Step **1** ensures that you have listed all the possibilities.

**d.** In one round of Epidemic, what is the probability that:

**1.** player 1 has the measles?

**2.** at least one player is healthy?

## Mathematics Note

The **fundamental counting principle** provides a method for determining the total number of outcomes for a series of events or decisions. If an event that can occur in $m$ ways is followed by an event that can occur in $n$ ways, then the total number of ways in which the two events can occur is $m \bullet n$.

In Epidemic, for example, player 1 has three choices and player 2 has two choices. Considering these players, the number of possible outcomes is $3 \bullet 2 = 6$.

**e.** Use the fundamental counting principle to verify the size of the sample space you determined in Part **c** of the exploration.

**f.** **1.** How many outcomes are possible in a game like Epidemic in which three players each have three choices?

**2.** What is the probability of each of these outcomes?

**g.** **1.** How many outcomes are possible in a game like Epidemic in which player 1 has four choices, player 2 has two choices, player 3 has three choices, and player 4 has five choices?

**2.** The selection "healthy" is a possible choice for players 1, 2, and 3, but not for player 4. In one round of this game, what is the probability that at least one player chooses "healthy"?

## Discussion

**a.** What is the probability of making a correct prediction in Epidemic?

**b**. If a class of 30 students plays Epidemic, how many would you expect to make a correct prediction in one round of the game?

**c.** In one round of Epidemic, is it possible for all three players to choose "healthy"? Is it very likely? Explain your responses.

**d.** In one round of Epidemic, is it possible for all three players to choose a disease? Is it very likely? Explain your responses.

**e.** How can a tree diagram help you calculate the probability of a particular outcome?

**f.** How can the fundamental counting principle help you calculate the probability of a particular outcome?

## Assignment

**3.1** One version of Epidemic involves only two players. Player 1 chooses randomly from either AIDS or hepatitis, while player 2 chooses randomly from AIDS, hepatitis, polio, or healthy.

**a.** Draw a tree diagram that shows all the possible outcomes for one round of this game.

**b.** List all the possible outcomes for one round.

**c.** Determine the size of the sample space for this game.

**d.** In one round of this game, what is the probability that:

**1.** both players choose "AIDS"?

**2.** one player chooses "healthy"?

**3.2** Consider a game like Epidemic in which each of three players has two choices. What is the size of the sample space for this game? Describe how you determined your response.

**3.3** In another game like Epidemic, player 1 has five choices, player 2 has $k$ choices, and player 3 has $n$ choices. Write a formula that can be used to determine the number of possible outcomes for this game.

**3.4** Statistics show that condom use is not 100% effective in preventing the exchange of body fluids**.** When used correctly, approximately 2 out of 100 condoms are ineffective. However, since condoms are not always used correctly, about 10 out of 100 are actually ineffective.

**a.** If a condom is used correctly, what is the probability that it is effective?

**b.** What is the actual probability that a condom is effective?

**c.** Is there a significant difference between these two probabilities? Explain your reasoning.

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**3.5** Although Shane plans to take art during third period, he has not yet selected the classes for the rest of his schedule. He has a choice of 2 classes each for first and second periods, a choice of 5 classes each for fourth and fifth periods, and a choice of 3 classes for sixth period. How many different six-period schedules are possible for him?

**3.6** When Cheyenne arrived at the university for a three-week agricultural camp, she received three posters: one featured cattle, one showed a wheat field, and one displayed a barnyard scene. After meeting her two neighbors in the dormitory, she agreed to the following plan: the three students would display a different poster on each door so that, at any one time, all three posters would be displayed. They agreed to exchange posters after the first week, then again after the second week. In addition, they decided that no one of them would display a particular poster more than once.

**a.** Draw a tree diagram to represent all the possible poster displays during the three-week camp.

**b.** What is the probability that Cheyenne will display the posters in the following order: cattle, wheat field, barnyard?

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## *Summary Assessment*

**1.** The following table lists the total number of U.S. AIDS cases for all persons aged 13 or older for selected years between 1983 and 1987.

| Year | No. of Cases |
|---|---|
| 1983 | 4589 |
| 1984 | 10,750 |
| 1985 | 22,399 |
| 1987 | 69,592 |

**Source:** Centers for Disease Control, 1995.

**a.** Make a scatterplot of this data. Let $y$ represent the number of AIDS cases and $x$ represent the number of years after 1983.

**b.** Determine an exponential equation that models the data.

**c. 1.** Use your model to estimate the number of AIDS cases in 1986.

**2.** The actual number of AIDS cases in 1986 was 41,256. Does your estimate seem reasonable? Explain your response.

**d. 1.** Use your model to predict when the number of AIDS cases in the United States will exceed 500,000.

**2.** Does this prediction seem reasonable? Explain your response.

**2.** The ninth-grade class at Piedmont High School has 472 students. To evaluate class participation in extracurricular activities, the principal requests rosters from the athletics, music, and drama departments.

The athletics roster lists 217 students, while the music roster lists 312 students. There were 92 students named on both the athletics and music rosters, 45 on both the athletics and drama rosters, and 87 on both the music and drama rosters. Only 15 members of the class were listed on all three rosters, while 30 students did not appear on any of the rosters.

**a.** Draw a Venn diagram to illustrate the involvement of this ninth-grade class in extracurricular activities.

**b.** How many members of the class participate in drama?

**c.** What is the probability that a student chosen at random from this class is involved only in athletics?

**d.** How many members of the class are involved in athletics or music? Describe how you determined your response.

**3.** A benefit carnival features a game in which a stack of three cards is turned face down. The cards are numbered 1, 2, and 3. To play the game, you shuffle the cards, then turn them over one at a time. To win the game, the first card turned over must be greater than 1, the second card turned over also must be greater than 1, and the third card turned over must be 1.

Using what you know about tree diagrams and probability, decide whether or not this is a good game for the carnival and explain the reasons for your decision.

## *Module Summary*

- The equation $y = a \bullet b^x$ describes a pattern of **exponential growth**. When describing population growth, $a$ is the initial population and $x$ is the number of time periods. The value of $b$ is the sum of two percentages: 100 (representing the initial population) and $r$, the growth rate from one time period to the next.
- **Venn diagrams** are mathematical models that show relationships among different sets of data. Each circle in a Venn diagram represents a set of data.
- The **intersection** of set A with set B is the set of elements common to both A and B. This can be written as $\text{A} \cap \text{B}$.
- The **union** of sets A, B, and C is the set of all elements in sets A, B, or C. This can be written as $(\text{A} \cup \text{B}) \cup \text{C}$ or $\text{A} \cup \text{B} \cup \text{C}$. Parentheses are not necessary to show order.
- Two **disjoint sets** have no elements in common. The intersection of disjoint sets is called the **empty set**, $\varnothing$. This can be written as $\text{A} \cap \text{B} = \varnothing$.
- A **tree diagram** is a mathematical model that shows all the possible outcomes for a series of events or decisions. Each line segment in a tree diagram is a **branch**. Each branch may be assigned a probability.
- The **fundamental counting principle** provides a method for determining the total number of outcomes for a series of events or decisions. If an event that can occur in $m$ ways is followed by an event that can occur in $n$ ways, then the total number of ways in which the two events can occur is $m \bullet n$.

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For more information on AIDS and HIV, call the Centers for Disease Control (CDC) National AIDS Hotline: 1-800-342-2437.