

One Step Beyond



How much did you earn last week? How much was that long-distance phone call? And what did it cost to send that package to Pawtucket? Every time you cash a paycheck, dial a telephone, or mail a postcard, you're stepping off into the unknown.

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One Step Beyond

Introduction

Shelly wants to open a new business: “Shelly’s Shirt Shop by the Sea.” As part of her business plan, Shelly must consider the cost of paying employees, the cost of telephone service, and the cost of shipping her products. Each of these costs is an example of a **step function**. In this module, you examine the step functions which Shelly encounters as she analyzes the cost of doing business.

Activity 1

Before hiring any employees, Shelly must determine exactly how she will pay them for their work. She plans to offer a wage of \$6.00 per hour, but has not decided how to pay workers for parts of hours. She is considering one of three different methods, all based on multiples of 15 minutes—rounding down, rounding to the nearest, or rounding up. These three rounding methods are described in Table 1.

Table 1: Rounding methods for employee hours

Rounding Method	Rounding Down	Rounding to the Nearest	Rounding Up
Description	rounds to previous multiple of 15	rounds to nearest multiple of 15	rounds to next multiple of 15
For 17 min of Work	paid for 15 min	paid for 15 min	paid for 30 min
For 27 min of Work	paid for 15 min	paid for 30 min	paid for 30 min

Exploration

In this exploration, you create two different graphs of an employee’s pay when time is rounded down to the previous multiple of 15 minutes.

- a.
 1. Obtain a copy of the employee’s pay template from your teacher. Each ordered pair on the template represents a number of minutes worked and the corresponding amount earned. Complete the ordered pairs for each minute in the period from 0 min to 79 min.

- Cut out each square on the template, creating a set of 80 squares each labeled with a different ordered pair. Figure 1 shows the ordered pairs for 17 min and 53 min.



Figure 1: Two paper squares

- On a sheet of freezer paper, create a two-dimensional coordinate system with the origin located near the lower left-hand corner of the paper. Represent an employee's pay in dollars on the y -axis and time in minutes on the x -axis.
- As shown in Figure 2, use the paper squares to plot the corresponding ordered pairs on the coordinate system from Step 3.

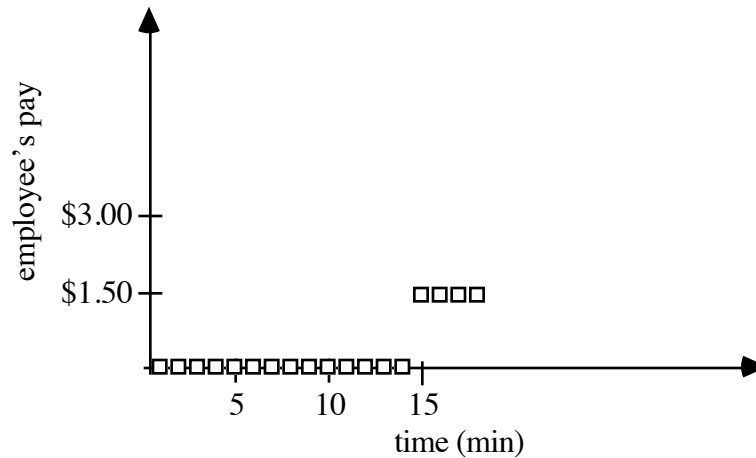


Figure 2: Points plotted on coordinate axes

- Another way of representing an employee's pay when rounding down is to graph the 15-min intervals in the period from 0 min to 79 min.

Mathematics Note

An **interval** is the set of all real numbers between two fixed endpoints. Each of these endpoints may or may not be included in the interval.

When an interval is graphed on a number line, a closed circle indicates that the endpoint is included in the interval. An open circle indicates that the endpoint is not included in the interval.

For example, the graph in Figure 3 represents all real numbers between 15 and 30, including 15 but not including 30.



Figure 3: Graph of an interval

One way to describe an interval is with an **inequality**. For example, the interval in Figure 3 can be described by the inequality $15 \leq x < 30$.

A real-number interval also can be described using **interval notation**. In interval notation, a square bracket,] or [, indicates that the endpoint is included in the interval. A parenthesis,) or (, indicates that the endpoint is not included in the interval. For example, the interval shown in Figure 3 can be written as $[15, 30)$.

1. On a sheet of graph paper, create another two-dimensional coordinate system, representing pay in dollars on the y -axis and time in minutes on the x -axis.
2. Graph as many 15-min intervals as necessary to show the pay earned for working from 0 min to 79 min. Use open or closed circles to indicate whether or not each endpoint is included in an interval.

Discussion

- a. Describe the general shapes of the graphs created in the exploration.
- b. If you consider each paper square as representing a pixel on a screen, the graph you created in Part a of the exploration resembles the graphs created by some types of technology. Compare this graph to the graph created in Part b of the exploration.
- c. Does either graph show the exact time when an employee's pay jumps from one step to another? Explain your response.

Mathematics Note

A **relation** between two variables is a set of ordered pairs of the form (x,y) .

The **domain** of a relation is the set of first elements in the ordered pairs (the x -values). The **range** of a relation is the set of second elements in the ordered pairs (the y -values).

A **function** is a relation in which each element of the domain is paired with an element of the range and each element of the domain occurs in only one ordered pair. A function may be described by a rule or equation.

For example, consider a function that pairs each number of hours with a number of minutes. This function can be written as $m = 60h$, where h represents hours and m represents minutes. Figure 4 shows a graph of this function. In this case, both the domain and the range are limited to the set of non-negative real numbers.

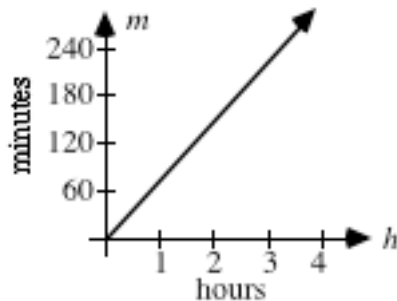


Figure 4: Graph of the function $m = 60h$

- d. Is your graph from Part a of the exploration a function? Explain your response.
- e. Given a graph on a coordinate plane with the domain represented on the horizontal axis and the range on the vertical axis, a **vertical line test** can sometimes be used to determine when the graph is not a function. If a vertical line can be drawn that intersects the graph in more than one point, this indicates that an element of the domain is paired with more than one element of the range. If this occurs, the graph does not represent a function.

Does your graph from Part a of the exploration pass the vertical line test? Explain your response.

- f.
 1. How is exactly 15 min represented on your graph from Part b of the exploration?
 2. What does the length of each step represent?
 3. What does the vertical distance between successive steps represent?
- g. To determine pay for parts of hours, Shelly could round down using multiples of 10 minutes instead of multiples of 15. How would this affect employee earnings?

- h.** The symbol $+\infty$ can be used to indicate that an interval contains all the numbers greater than a particular value. Similarly, the symbol $-\infty$ can be used to indicate that an interval contains all the numbers less than a particular value. For example, the set of all numbers less than or equal to -2 could be written as $(-\infty, -2]$.


Using these symbols, how could you represent the domain of the function $m = 60h$ in Figure 4 as an interval?

Assignment

- 1.1** Shelly plans to pay her employees \$6.00 per hour. Assuming that she rounds parts of hours down to the previous multiple of 15 minutes, complete the following table.

Time	Wages Earned
14 min	
14 min, 20 sec	
14 min, 57 sec	
15 min	
15 min 2 sec	
16 min	

- 1.2** If Shelly rounds down to the previous multiple of 15 minutes, there is an interval for which an employee will earn \$0.
- Represent this period of time as an inequality.
 - Describe this period of time in a sentence.
 - Graph this period of time on a number line.
 - Express this interval using interval notation.
- 1.3** Some of Shelly's employees begin work at 7:00 A.M. The following table includes a column for the wages earned in each 15-min interval, beginning at 7:00 A.M., when parts of hours are rounded down to the previous multiple of 15.

Time Interval (min)	Interval Notation	Number-line Graph	Wages Earned
$0 \leq t < 15$			
	$[15, 30)$		\$1.50
$30 \leq t < 45$			
\vdots	\vdots	\vdots	\vdots

- Use your responses from Problem 1.2 to complete the first row.
- Continue to complete each row until the table includes 8:19 A.M.

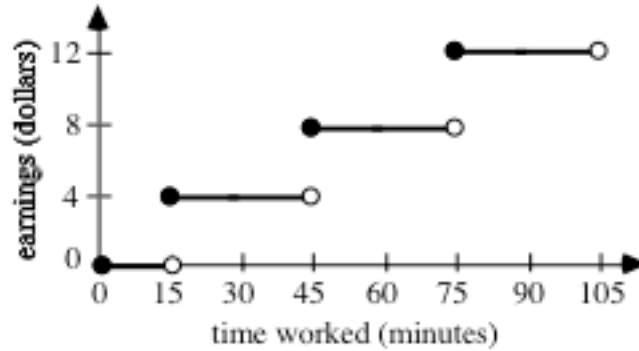
- 1.4** As shown in Table 1, rounding to the nearest multiple of 15 minutes and rounding up to the next multiple of 15 are two different methods for rounding an employee's time. Complete Parts **a** and **b** below using a wage of \$6.00 per hour.
- On a coordinate plane, create a graph of an employee's earnings from 0 min to 79 min when time is rounded to the nearest multiple of 15.
 - Create a graph as in Part **a** for an employee's earnings from 0 min to 79 min when time is rounded up to the next multiple of 15.

- 1.5** Assume that Shelly considers time in 15-min intervals. One of her employees has worked x min where x is in the interval $[0, 60]$.
- List the five different amounts this employee could earn for working x min at a wage of \$6.00 per hour.
 - If time is rounded to the nearest multiple of 15 minutes, determine the time interval that would pay \$3.00. Write this interval both as an inequality and using interval notation.
 - If time is rounded up to the next multiple of 15 minutes, determine the time interval that would pay \$3.00. Write the interval both as an inequality and using interval notation.
 - Complete the following table, listing the time intervals for which the two rounding methods pay each dollar amount identified in Part **a**.

Money Earned	Time Worked in Minutes	
	Rounding Up	Rounding to the Nearest
\$0.00	$x = 0$	$0 \leq x < 7.5$
	$0 < x \leq 15$	
⋮	⋮	⋮
\$6.00		

- Identify all the time intervals in the first hour of work for which the two rounding methods result in the same amount of pay.
- 1.6** Use your graphs for the three different rounding methods to help you answer the following questions.
- Using each rounding method, what is the minimum amount of time an employee can work to earn \$1.50?
 - Which of the three rounding methods favors employees? Explain your response.
 - Which of the three rounding methods favors the employer? Explain your response.
 - Which method might serve as a compromise for employees and employers? Explain your response.

- 1.7 Identify all the time intervals in the first hour of work for which all three rounding methods in Table 1 result in the same amount of pay.
- 1.8 The following graph shows another method which Shelly could use to pay her employees.



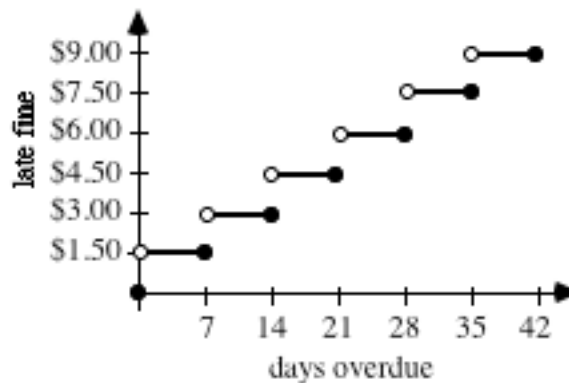
- a. Using this method, how much would Shelly pay her employees per hour?
- b. Describe this new payment method.

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- 1.9 A salesperson in a clothing store receives a commission of \$135 for each \$1000 in sales. The store “rounds down” when determining commissions. For example, if the amount of merchandise sold is less than \$1000, the salesperson gets no commission. If \$1200 worth of merchandise is sold, the salesperson gets \$135.

- a. How much commission would a salesperson receive for \$4800 in sales?
- b. Identify the intervals of sales for which a salesperson would earn from \$0 to \$540 in commissions. List the commission earned for each interval.
- c. Create a graph of commission earned versus merchandise sold for the intervals identified in Part b.

- 1.10 A library uses the schedule shown below to determine fines for overdue books.



- a. Describe the method the library uses for rounding the number of days overdue.
- b. Explain why the graph represents a function.
- c. Identify the domain and range of the function.
- d. Given a fine of \$4.50, determine the interval of days for which the book might have been overdue. Write this interval both as an inequality and using interval notation.

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Activity 2

Shelly’s employees park their cars at Jeff’s Parking Lot, which is located next to their store. To accommodate shoppers and delivery trucks, parking is free at Jeff’s for the first hour. After that, the lot charges \$1.00 per hour. Parts of hours are rounded up to the next whole hour.

Exploration 1

In this exploration, you examine an equation Jeff can use to calculate parking charges.

- a. Table 2 shows the cost of parking in Jeff’s lot. Use the patterns you observe in the table to complete the bottom row.

Table 2: Cost of parking in Jeff’s lot

Hours Parked (interval notation)	Number of Hours Charged	Parking Cost
[0, 1)	0	\$0
[1, 2)	1	$1 \cdot \$1.00 = \1.00
[2, 3)	2	$2 \cdot \$1.00 = \2.00
⋮	⋮	⋮
[x, (x + 1))		

- b. Create a graph of parking cost versus hours parked using the domain [0, 8).
- c. Determine the parking costs for each of the following times.
 1. 1.5 hr
 2. 5.1 hr
 3. 7.9 hr

Mathematics Note

The **greatest integer function** assigns every element x in the domain to the greatest integer less than or equal to x . This function can be written as $y = [x]$.

Using the greatest integer function, the number 2.7, for example, would be assigned the value of 2. This can be written as $[2.7] = 2$. Similarly, $[1.9] = 1$, $[0.4] = 0$, and $[-3.4] = -4$.

- d. Evaluate $[x]$ for each of the following values of x :
1. 2.2
 2. 2.9
 3. -2.2
 4. -2.9
- e. Determine how the technology you use handles the greatest integer function.
- f. When y represents hours parked and x represents parking costs, the greatest integer function can be used to determine the charges for Jeff's Parking Lot.
1. Use technology to graph $y = [x]$.
 2. How does the graph of $y = [x]$ differ from your graph in Part **b**?

Discussion 1

- a. Describe how to use the equation $y = [x]$ to determine the cost of parking in Jeff's Lot for each of the following:
1. 7.5 hr
 2. 0.9 hr
- b. After the first hour, parking in Jeff's Lot is no longer free. How is this represented in your graphs from Parts **b** and **f** of Exploration 1?
- c. For what values of the domain does $y = [x]$ model the cost of parking at Jeff's Lot?

Exploration 2

As you observed in Exploration 1, the function $y = [x]$ can be used to model the parking costs at Jeff's Lot. In this exploration, you discover how greatest integer functions can be used to model other situations.

- a. 1. In the equation $y = a[x]$, the greatest integer function is multiplied by a constant a . Complete Table 3 for $a = 1$, $a = 3$, and $a = -2$.

Table 3: Multiplying $[x]$ by a constant

x	$y = 1[x]$	$y = 3[x]$	$y = -2[x]$
0			
0.4			
1.6			
2.4			

2. Graph $y = 1[x]$, $y = 3[x]$, and $y = -2[x]$ on the same set of axes.
3. Compare the three graphs.
- b. 1. In the equation $y = [x] + d$, a constant d is added to the greatest integer function. Complete Table 4 for $d = 0$, $d = 3$, and $d = -5$.

Table 4: Adding a constant to $[x]$

x	$y = [x] + 0$	$y = [x] + 3$	$y = [x] - 5$
0			
0.4			
1.6			
2.4			

2. Graph $y = [x] + 0$, $y = [x] + 3$, and $y = [x] - 5$ on the same set of axes.
3. Compare the three graphs.

- c. 1. In the equation $y = [bx]$, the variable x is multiplied by a constant b before the greatest integer function is applied. Complete Table 5 for $b = 1$ and $b = 0.5$.

Table 5: Multiplying x by a constant b in $[x]$

x	$y = [1x]$	$y = [0.5x]$
0		
0.4		
1.6		
2.4		
3.6		
4.2		

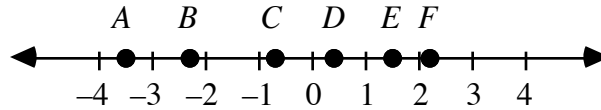
2. Graph $y = [1x]$ and $y = [0.5x]$ on the same set of axes.
 3. Compare the two graphs.

Discussion 2

- a. If $y = 3[x]$ models the cost to park x hours, what effect does the constant 3 have on the graph of $y = [x]$.
- b. Describe how the value of a affects the graph of $y = a[x]$.
- c. If $y = [x] + 3$ models the cost to park x hours, what effect does the constant 3 have on the graph of $y = [x]$.
- d. Describe how the value of d affects the graph of $y = [x] + d$.
- e. If $y = [0.5x]$ models the cost to park x hours, what effect does the constant 0.5 have on the graph of $y = [x]$.
- f. Describe how the value of b affects the graph of $y = [bx]$.
- g. The function $y = [x]$ describes the cost of parking x hours in Jeff's Parking Lot. The function $y = [0.5x]$ describes the cost of parking for x hours in a competitor's lot.
- Which lot would you prefer to park in?
 - How could you use the graphs of the functions to choose the better parking lot?

Assignment

- 2.1** Determine the value of the greatest integer less than or equal to each of the six points on the following graph.



- 2.2** Determine the value of y in each of the following equations:

- $y = [-2.3]$
- $y = [0]$
- $y = [1.5] + 0.5$
- $y = 3[(0.45) \cdot 10] + 7$

- 2.3** **a. 1.** Explain why the solution set of $[x] = 9$ can be represented as shown below.



- 2.** Express the solution set of $[x] = 9$ as an inequality.
- b.** Represent the solution set of each of the following equations as a graph on a number line and as an inequality.
- $[x] = 5$
 - $[x] = -3$
 - $[x] = n$
- 2.4** Jeff is considering three different ways to change the parking fees at his lot. Write an equation that could be used to model each of the following situations. Use technology to verify that each equation correctly describes the fees.
- Jeff would charge \$1.00 per hour or any part of an hour. Using this method, it would cost \$1.00 to park for 20 min and \$3.00 to park for 2 hr 20 min.
 - Jeff would charge \$1.50 per hour or any part of an hour. Using this method, it would cost \$1.50 to park for 20 min and \$4.50 to park for 2 hr 20 min.
 - Jeff would charge \$0.75 for each full 30-min period. Using this method, there would be no cost to park 20 min. It would cost \$0.75 to park 45 min and \$6.75 to park 4 hr 35 min.

- 2.5**
- On a coordinate plane, draw a graph that shows values of x in the interval $[-4, 5.5)$ rounded to the nearest integer. Using this method of rounding, 2.4 is rounded to 2, while 2.6 is rounded to 3. Since 2.5 is exactly between 2 and 3, it is rounded to 3.
 - Describe the shape of the graph. Determine whether or not it represents a function and justify your response.
 - Write an equation of the form $y = [x + c]$ to represent the graph. Use technology to check your equation for several values of x .
 - Graph your equation from Part **c** for values of x in the interval $[-4, 5.5)$. Does your equation work for $x = -1.5$ and $x = -2.5$? Explain your response.
 - Determine how the rounding function on the technology you use rounds negative numbers such as -1.5 and -2.5 . Does your equation from Part **c** produce the same results?

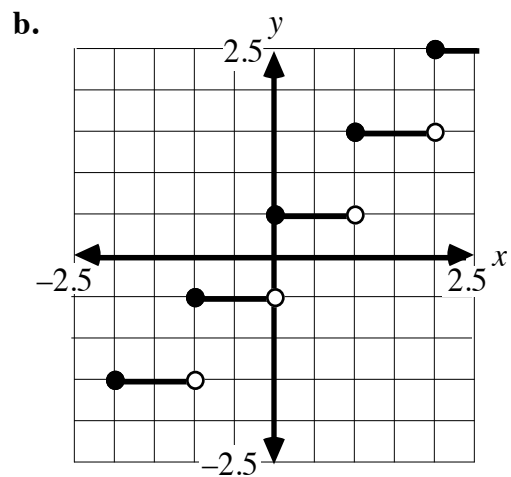
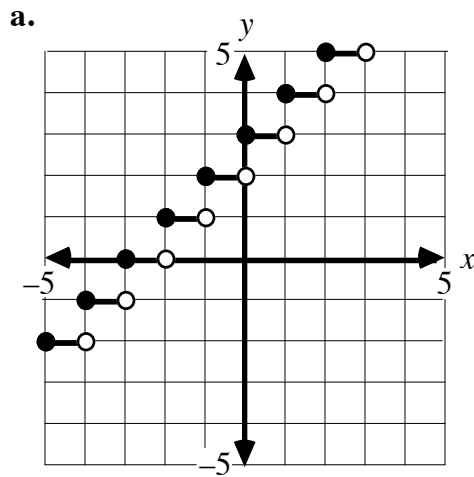
- 2.6**
- On a coordinate plane, draw a graph that shows values of x in the interval $(-5, 5]$ rounded up to the next integer. Using this method of rounding, 2.4, 2.5, and 2.6 are all rounded up to 3, while -2.4 , -2.5 , and -2.6 are all rounded up to -2 .
 - Graph $y = -[-x]$. Compare this graph to your graph in Part **a**.
 - What might you call the function given in Part **b**?

- 2.7**
- The table below shows four different functions. Complete the table using $-4, -3, -2, -1, 0, 1, 2, 3,$ and 4 for x .

x	$y = [x + 2]$	$y = [x] + 2$	$y = [x + 2.5]$	$y = [x] + 2.5$
-4				
-3				
-2				
-1				
0				
1				
2				
3				
4				

- Using the domain $[-10, 10]$, graph $y = [x + 2]$ and $y = [x] + 2$ on the same set of axes. Compare the two graphs.
- Do you think that $[x] + c = [x + c]$? Explain your response.

2.8 Write an equation for each step function graphed in Parts a and b.

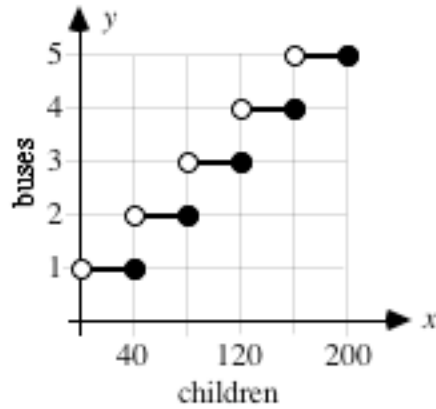


2.9 One teacher rounds student scores to the nearest integer, then applies the grading scale shown in the table below.

Grade	Score
A	90-100
B	80-89
C	70-79
D	60-69
F	0-59

- Why does this teacher round scores before assigning letter grades?
- Write an inequality that describes all the scores that correspond to a grade of A.
- Draw a graph to represent this teacher's grading system.
- Does your graph in Part c represent a function? Explain your response.
- Do you think that this method of rounding scores is fair? Explain your response.

- 2.10** a. While planning for the summer season, a camp director drew the following graph. This graph shows the number of buses needed to transport up to 200 children to camp.



1. Describe the domain for this graph.
 2. How many children can each bus carry?
 3. Write an equation that models the graph. Use technology to test your equation.
- b.**
1. The director of another camp plans to hire buses that can carry up to 79 children each. Write a step function that models this situation.
 2. Use this function to determine the number of buses required to transport 1543 students to camp.
- 2.11** Dana runs the concession stand at a concert hall. She is paid \$175 per week plus an additional \$45 for each \$300 in sales.
- a. Determine how much Dana earns for a week in which sales reach \$1000.
 - b. Draw a graph to represent Dana's earnings for weekly sales from \$0 to \$1000.
 - c. Write an equation to represent Dana's weekly earnings.

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Activity 3

Shelly's budget for her new store includes telephone costs. Before selecting a long-distance company, she wants to compare the discounts they offer. In this activity, you use the greatest integer function to graph these discounts.

Exploration

Company A offers an initial evening rate of \$0.10 per min. After the first 5 min, the company cuts the rate to \$0.09 per min. After the next 5 min, the rate falls another penny to \$0.08 per min. This pattern continues for the next three multiples of 5 minutes, until a minimum rate of \$0.05 per min is reached.

When calculating the cost of a call, company A always rounds parts of minutes up to the next whole minute.

- a. Table 6 shows an incomplete list of the company's rates.

Table 6: Long-distance telephone rates

Interval of Minutes	Decrease in Initial Rate	Discounted Rate (Cost per Minute)	Cost of Call
(0, 1)	\$0.00	\$0.10	\$0.10
[1, 2)	\$0.00	\$0.10	\$0.20
[2, 3)	\$0.00	\$0.10	\$0.30
⋮	⋮	⋮	⋮
[7, 8)	\$0.01	\$0.09	\$0.77
⋮	⋮	⋮	⋮
		\$0.05	

Create a spreadsheet with headings like those in the table, then extend the spreadsheet until the discounted rate falls to \$0.05 per min.

- b.
1. Create a graph using the number of minutes as the domain and the discounted rate as the range. (Hint: Select at least one value from each interval of minutes in Table 6. Plot these values with the corresponding discounted rates.)
 2. Make a sketch of your graph. Make sure that your sketch clearly indicates the points where the rate changes. **Note:** Save your graph for use in Problem 3.2.
 3. After a certain number of minutes, the rate reaches the minimum of \$0.05 per minute. Extend your sketch to show what happens to the graph after this point is reached.

Discussion

- a. What type of association exists between the number of minutes and the discounted rate? Explain your response.
- b. Compare the general shape of the graph in Part **b** of the exploration with the graphs you created in previous activities.
- c. Explain why the graph of the phone company's rates is a function.
- d. How did you use the spreadsheet to determine the cost of a call that lasts a given number of minutes?

Assignment

- 3.1 Company B offers a different discount than Company A. The initial evening rate is \$0.10 per minute. After the first 10 min, the company cuts the rate to \$0.07 per min. After the next 10 min, the rate falls to \$0.04 per min. This is the minimum rate for company B.
 - a. Create a table like Table 6 to show company B's rates for evening calls up to 30 min long.
 - b. Using company B's rates, determine the total cost of a call for each minute from 1 min to 30 min.
 - c. Using company A's rates, determine the total cost of a call for each minute from 1 min to 30 min.
 - d. Compare the costs of calls made using each company.
- 3.2
 - a. Create a graph of the discounted rate versus the number of minutes for company B. Use a domain of [0 min, 30 min].
 - b. Compare this graph with the graph of company A's rates you created in the exploration.
 1. How does the vertical distance between steps compare?
 2. When do the rates for each company change?
 3. When are the companies' rates equal?
 - c. Why are these graphs meaningful only in the first quadrant? Explain your response.

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- 3.3** Shelly's Shirt Shop charges by the number of shirts ordered. The following table shows the cost per shirt for orders from 1–47 shirts.

Number of Shirts	Cost per Shirt
[1, 11]	\$16.00
[12, 23]	\$14.50
[24, 35]	\$13.00
[36, 47]	\$11.50

- Describe the domain for the relationship in the table.
- Create a graph of the cost per shirt versus the number of shirts.
- Write an equation that models the cost per shirt for any number of shirts in the interval [1, 47].

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Summary Assessment

1. The U.S. Postal Service uses the following chart to determine the cost of mailing letters first class (based on rates effective January 1, 1995). Using this chart, a letter that weighs 0.5 oz costs \$0.32 to mail, while one that weighs 3.3 oz costs \$1.01.

Weight up to and including (oz)	Cost
1	\$0.32
2	\$0.55
3	\$0.78
4	\$1.01
5	\$1.24
6	\$1.47
7	\$1.70
8	\$1.93
9	\$2.16
10	\$2.39
11	\$2.62

When a parcel's weight exceeds 11 oz, the post office uses a different chart to determine mailing costs, as shown below.

Weight up to and including (lb)	Cost
2	\$3.00
3	\$4.00
4	\$5.00
5	\$6.00

When a parcel's weight exceeds 5 lb, the cost depends on both weight and destination.

- a. In a paragraph, describe the cost of mailing letters that weigh up to 11 oz.
- b. Use inequalities to express the weights in each row of the chart for parcels that weigh more than 11 oz.
- c. Draw a graph that represents the cost of mailing letters or parcels that weigh up to 5 lb.
- d. Describe the advantages of each of the following ways of representing postal rates: charts, explanations, inequalities, and graphs.

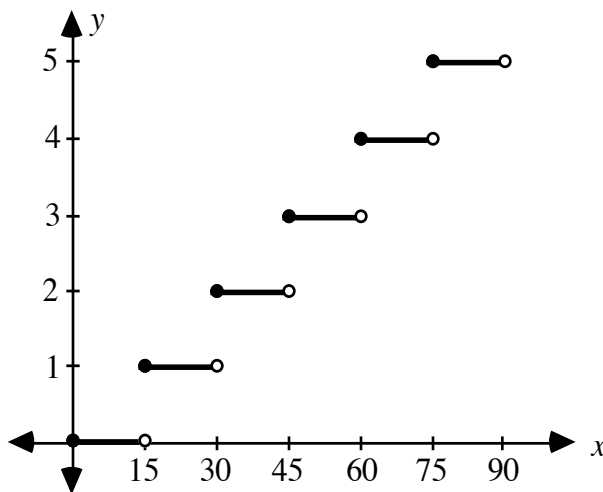
2. Another function that rounds numbers is the **integer part function**. The integer part function removes the decimal or fractional part of a number, leaving only the integer. This process is often referred to as **truncation**.

When $x = 7.2$, for example, the integer part function removes the decimal part, leaving the integer 7. When $x = 7.9$, the result is also 7.

- a. Describe how the integer part function is related to the functions $y = [x]$ and $y = -[-x]$.
- b. Find two intervals, one including positive numbers and one including negative numbers, for which the following functions give the same result:
 1. “integer part of x ” and “ x rounded to the nearest integer”
 2. $[x]$, “ x rounded to the nearest integer,” and “integer part of x ”

Module Summary

- An **interval** is the set of all real numbers between two fixed endpoints. Each of these endpoints may or may not be included in the interval.
- When an interval is graphed on a number line, a closed circle indicates that the endpoint is included in the interval. An open circle indicates that the endpoint is not included in the interval.
- A real-number interval can be described with an **inequality** or by using **interval notation**. In interval notation, a square bracket,] or [, indicates that the endpoint is included in the interval. A parenthesis,) or (, indicates that the endpoint is not included in the interval.
- A **relation** between two variables is a set of ordered pairs of the form (x,y) .
- The **domain** of a relation is the set of first elements in the ordered pairs (the x -values). The **range** of a relation is the set of second elements in the ordered pairs (the y -values).
- A **function** is a relation in which each element of the domain is paired with an element of the range and each element of the domain occurs in only one ordered pair. A function may be described by a rule or equation.
- The graph below shows one example of a **step function**.



- The **greatest integer function** is a step function that assigns to the number x the greatest integer less than or equal to x . The function may be symbolized as $y = [x]$.

Selected References

Lott, J. W., and A. W. Wilson. "Applications of the Step Function." *The Illinois Mathematics Teacher* 30 (January 1979): 2–7.

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Usiskin, Zalman. "The Greatest Integer Symbol—An Applications Approach." *The Mathematics Teacher* 70 (December 1977): 739–743.