Under the Big Top But Above the Floor

It's carnival time at Dantzig High School. Before you play "Guess My Number" or "Roll-a-rama," however, you'll need some practice. In this module, you'll develop the skills necessary to model and solve problems involving linear inequalities.

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Under the Big Top but Above the Floor

Introduction

Every year at Dantzig High School, the mathematics club sponsors a school carnival. The club designs game booths to earn money. As booths compete to attract more customers, the atmosphere grows festive. Many students dress in crazy costumes. Balloons, tinsel, and streamers are everywhere, and the aromas of pizza, popcorn, and hot dogs waft through the gym.

The first game booth that you visit is "Guess My Number." Inside the booth, Yvette stands next to a tumbling basket of numbers. Yvette picks a number from the basket. You must now guess the number by asking questions which can be answered by "yes" or "no." If you guess correctly using seven or fewer questions, you win a prize.

Exploration

a. To play Guess My Number with a classmate, each of you should select an integer in the interval [–50, 50]. Do not reveal your number to your classmate. Take turns asking yes-or-no questions until you discover each other's number. Record the questions asked, the answers given, and the number of questions.

Play the game at least twice and consider some strategies you might use to guess correctly with the fewest questions.

b. Play Guess My Number at least two more times, using only questions that include inequalities (for example, "Is your number less that 5?"). Record each question using inequality symbols. Record the responses on a number line.

c. Play Guess My Number at least three more times, using questions that include inequalities.

Discussion

a. If your opponent must choose an integer in the interval [–50, 50], can you develop a strategy that guarantees you will know the number in seven questions or less?

b. What do you think the minimum number of questions would be to determine an integer in the interval [–75, 75]?

Activity 1

The next booth you visit is "Guess My Location." The object of this game is to determine the exact location of your opponent's point on a grid. Pablo and Lisa are ahead of you at the booth. Watch them play while you wait for your turn. Figure **1** shows the beginning of their game.

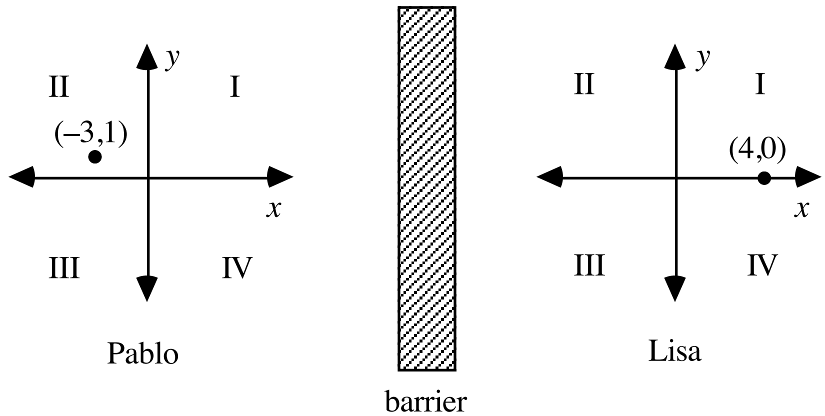


Figure 1: Pablo and Lisa play Guess My Location

Pablo asks, "Is $x > 0$?"

Lisa answers, "Yes."

Pablo knows that Lisa's point must be in quadrant I, quadrant IV, or on the positive x-axis.

Lisa asks, "Is $y \leq 0$?"

"No," answers Pablo.

Lisa knows that Pablo's point is above the x-axis.

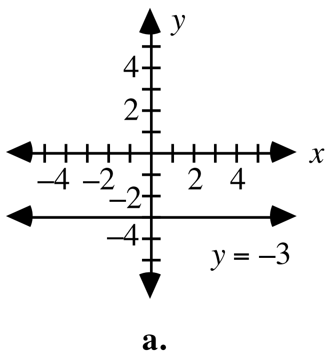
It's Pablo's turn again. "Is $y \geq 0$?" he asks.

"Yes," answers Lisa.

Pablo knows that Lisa's point can only be in the first quadrant or on the positive x-axis.

Discussion 1

a. The graph of the equation $y = -3$ is shown in Figure **2a**. As shown by the shaded region in Figure **2b**, the line separates the plane into two parts.



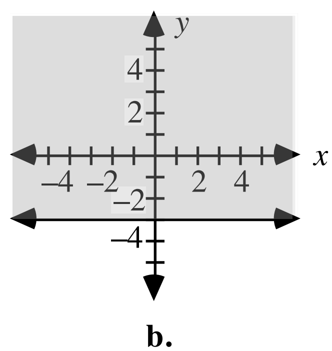


Figure 2: Graphs of $y = -3$ and the half-plane above $y = -3$

1. What must be true about all the points in the shaded region in Figure **2b**?
2. How can you use inequality notation to describe the points either on or above the line $y = -3$?

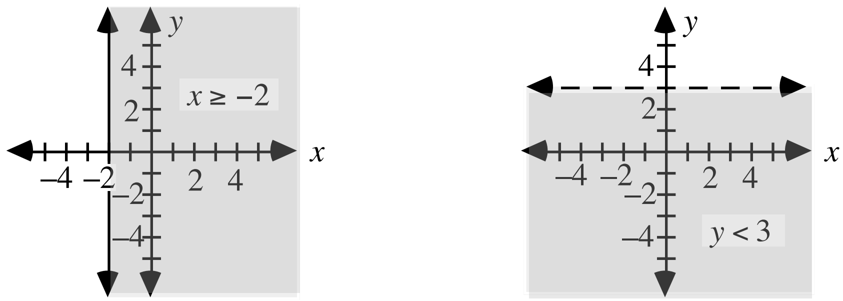
b. The line $x = 5$ defines the boundary for the inequality $x < 5$.

1. Would a graph of $x < 5$ be shaded to the right or to the left of $x = 5$? Explain your response.
2. Should the boundary be a part of the graph of $x < 5$? Explain your response.

Mathematics Note

The graph of a linear inequality is a shaded region that represents the **solution set** of the inequality. The solution set contains all the points, or solutions, that make the inequality true. The graph of a linear equation forms the **boundary line** for the region. A solid boundary indicates that the points on the line are part of the solution set. A dashed boundary indicates that the points on the line are not part of the solution set.

For example, Figure **3a** shows the graph of $x \geq -2$ and Figure **3b** shows the graph of $y < 3$.



a. **b.**

Figure 3: Graphs of $x \geq -2$ and $y < 3$

c. Explain why the point (2,–7) is in the solution set for $x \geq 2$ but not for $y < -7$.

Exploration 1

In this exploration, you play Guess My Location with a classmate. Read Parts **a–d** before beginning play.

a. Each player makes a game sheet by drawing a coordinate system on a sheet of graph paper. Both game sheets should have the same scales on the *x*- and *y*-axes.

A book should be placed upright between the two game sheets so that neither player can see the other's graph.

b. Both players choose and mark a point on their game sheets, without revealing the point's location to the opponent. The coordinates of the point must be integers.

c. To gain information about the location of the opponent's point, players take turns asking yes-or-no questions involving inequalities.

Record each question you ask as an inequality. Record each answer you receive as the graph of an inequality on your coordinate system.

d. The first player to identify the coordinates of the opponent's point wins the game. If a player guesses incorrectly, then that player loses the next turn.

Discussion 2

a. Describe the strategies you used to play Guess My Location.

b. Assume that your opponent's point is located in the third quadrant. What two mathematical statements would describe this region?

Mathematics Note

A **conjunction** combines two mathematical statements with the word *and*. A conjunction can be represented as the intersection of two sets.

As shown in Figure **4**, for example, the conjunction $x > 0$ and $y > 0$ can be represented by the intersection of their solution sets.

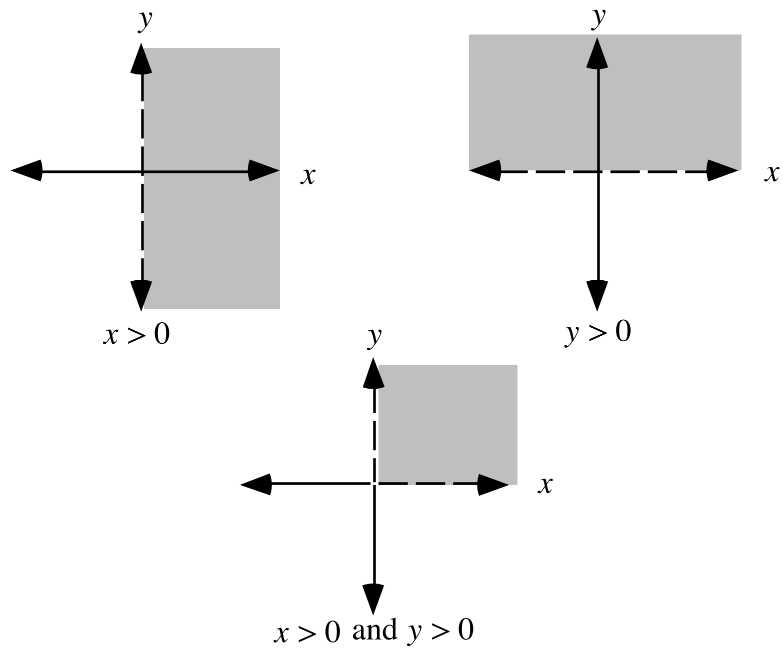


Figure 4: Intersection of two linear inequalities

c. For all points in the first quadrant, the *x*- and *y*-coordinates are greater than 0. Does the conjunction shown in Figure **4** represent these points? Explain your response.

d. Imagine that Pablo asked Lisa "Is $x > 0$ and $y < 0$?"

1. If Lisa answers "No," in what quadrant(s) could her point be located? Explain your response.
2. If Lisa answers "Yes," in what quadrant(s) could her point be located? Explain your response.

e. What conjunction describes the intersection of the two inequalities shown in Figure **5**?

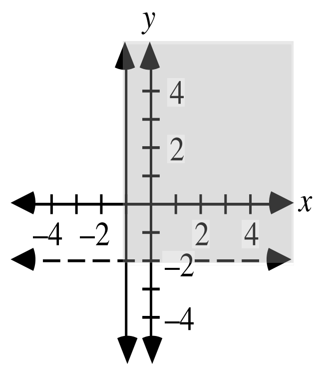


Figure 5: Two inequalities and their intersection

Exploration 2

Imagine that Pablo and Lisa replay the original game of Guess My Location shown in Figure **1**. Pablo has chosen the point (–3,1) and Lisa has chosen (4,0). This time around, their questions use conjunctions.

"Is $x > 0$ and $y < 0$?" asks Pablo.

"No," Lisa responds.

Pablo knows that the point is not in Quadrant IV.

Now it's Lisa's turn. She asks, "Is $x \leq 0$ and $y \geq 0$?"

Pablo answers, "Yes."

Lisa knows that the point is not in Quadrant II.

Now Pablo asks, "Is $x \geq 0$ and $y \geq 0$?"

"Yes," answers Lisa.

Following Pablo and Lisa's example, play another game of Guess My Location. For this game, use the two additional rules described in Parts **a** and **b**.

a. Each question must be a conjunction.

b. One mathematical statement in the conjunction must refer only to *x*; the other statement must refer only to *y*.

Discussion 3

a. What advantages or disadvantages did you notice when using conjunctions to play Guess My Location?

b. Consider Pablo's first conjunction, "Is $x > 0$ and $y < 0$?" The intersection of the two boundary lines for these inequalities, $x = 0$ and $y = 0$, is referred to as a **corner point**.

Is this corner point part of the solution set for the conjunction? Explain your response.

c. How could you verify that a point in the first quadrant is a solution to $x > 0$ and $y > 0$?

d. Consider the points in the solution set of $x > 0$ and $y > 0$. The *x*-coordinates of these points must be positive real numbers. This condition on the *x*-coordinates is called a **constraint**.

Describe the constraints on the *y*-coordinates of the points in this solution set.

e. How could you use conjunctions to describe the region shown in Figure **6**?

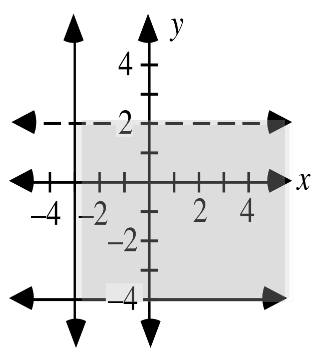


Figure 6: A region defined by conjunctions

Assignment

1.1 **a.** Sketch a graph of the line $y = 4$.

b. Write an inequality to describe all the points located on and above the line.

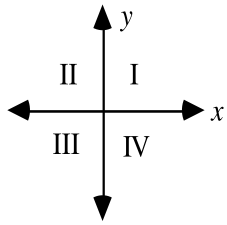
c. Shade the region that corresponds to the solution set of the inequality.

d. Write the coordinates of at least three points included in the solution set.

e. Describe the constraints on the *x*-coordinates of the points in the solution set.

f. Describe the constraints on the *y*-coordinates of the points in the solution set.

1.2 The following graph shows the four quadrants of the coordinate plane. Use conjunctions of linear inequalities to describe each quadrant. **Note:** The x- and y-axes are not included in any quadrant.



1.3 **a.** On a coordinate plane, graph the region defined by the intersection of the linear inequalities $x \geq -4$ and $y \leq 2$.

b. Select a point in the intersection of the two inequalities that is not on a boundary. Verify that the point is a solution to both inequalities.

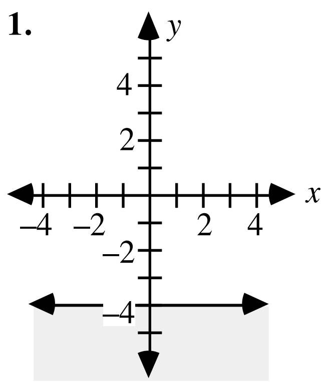
c. Select a point outside the shaded region. Verify that the point is not a solution for at least one of the inequalities.

d. Select a point on a boundary. Is this point a solution to both inequalities? Explain your response.

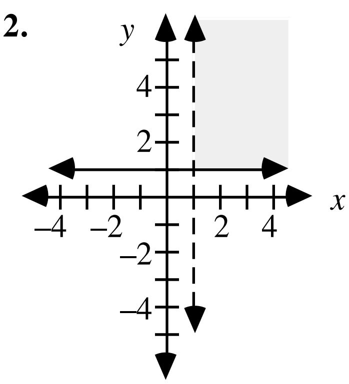
e. Identify the corner point of the conjunction.

1.4 **a.** Use inequalities to describe the shaded region in each of the following graphs.

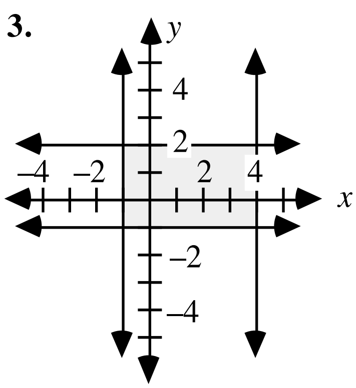
1.



2.



3.



b. Identify the coordinates of any corner points for each shaded region in Part **a**.

1.5 In a game of Guess My Location, your opponent chooses the point with coordinates (0,–2). Using conjunctions, write two different questions about this point that will receive a "yes" answer.

1.6 In another game of Guess My Location, you ask the question, "Is $x = 5$ and $y > -2$?" Your opponent answers, "Yes."

a. On a coordinate plane, graph the set of all points that fit this information.

b. Describe the graph formed.

1.7 The tables in Parts **a** and **b** show some questions and answers from two different games of Guess My Location. For each game, graph the "smallest" region that contains all possible solutions and use linear inequalities to describe the region.

a.

Question	**Answer**
Is $x \geq 0$ and $y \geq 0$?	Yes
Is $x \leq 2$ and $y \geq 0$?	No
Is $x \geq 3$ and $y \geq 1$?	Yes
Is $x \geq 4$ and $y \leq 5$?	Yes

b.

Question	**Answer**
Is $x \geq 0$ and $y \leq 0$?	Yes
Is $x \geq 5$ and $y \geq -3$?	No
Is $x \leq 4$ and $y \geq -3$?	Yes

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1.8 A cash register contains bills and coins. Using the horizontal axis to represent the value of the bills and the vertical axis to represent the value of the coins, draw a graph to describe each of the following statements:

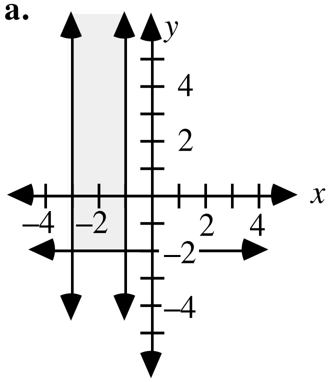
a. The cash register contains at least \$50.00 in bills and no more than \$25.00 in coins.

b. The cash register contains less than \$100.00 in bills and at least \$10.00 in coins.

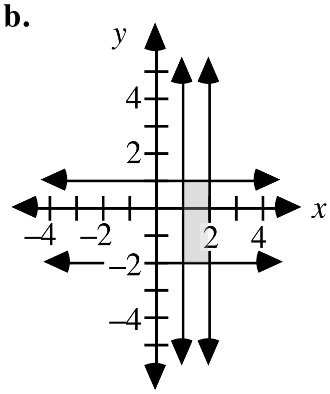
c. The cash register contains more than \$75.00 in bills and no more than \$50.00 in coins.

1.9 Use linear inequalities to describe each shaded region in Parts **a–c**.

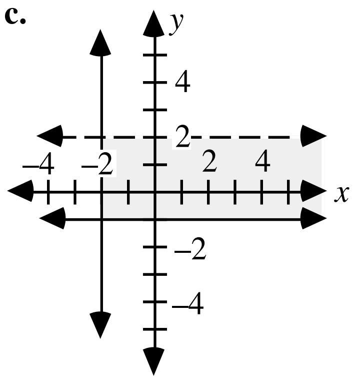
a.



b.



c.



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Activity 2

Anton and Sharline are the school champions at Guess My Location. In this activity, you investigate some of the tools they use to analyze the game.

Exploration 1

Anton and Sharline are playing Guess My Location. Both players have selected points with *x*- and *y*-coordinates between –10 and 10. After receiving an answer to his first question, Anton shades his game sheet as shown in Figure **7**.

The points *A*, *B*, *C*, and *D* in Figure **7** lie at the intersections of the dashed line, the solid line, and the lines that define the right-hand and lower limits of the grid. The solid line through points *A* and *C* also passes through (0,–3) and (1,–1). The horizontal dashed line has a *y*-intercept of 7.

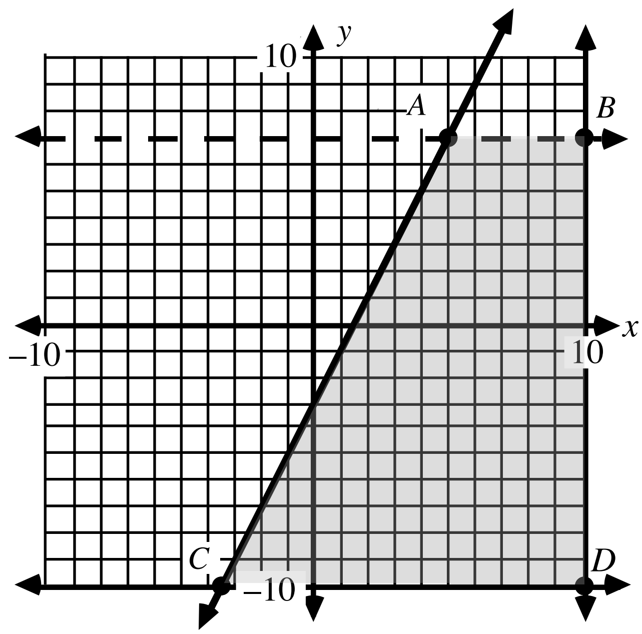


Figure 7: Anton's graph

a. Determine the equation of each line that borders the shaded region in Anton's graph.

Mathematics Note

A **system of linear equations** is a set of two or more equations whose graphs are lines. A **solution** to a system of linear equations is a point where all the lines intersect. The coordinates of this point satisfy all the equations in the system.

For example, consider the following system of linear equations:

$$\begin{cases} y = 2x + 3 \\ y = x + 1 \end{cases}$$

As shown in Figure **8**, the graphs of the two lines intersect at the point with coordinates (–2,–1). By substitution, this point is a solution to the system since $-1 = 2(-2) + 3$ and $-1 = -2 + 1$.

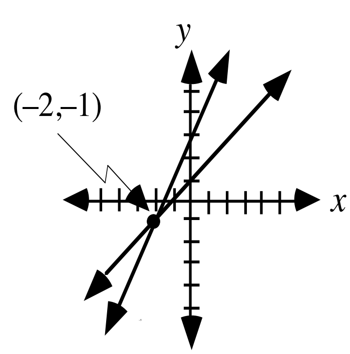


Figure 8: Solution to a system of linear equations

b. In order to shade the region shown in Figure **7**, what question might Anton have asked? What answer must Sharline have given?

c. In Figure **7**, point C is one of the corner points of the shaded region. Estimate the coordinates of point C as accurately as possible.

Discussion 1

a. The shaded region in Figure **7** contains the location of Sharline's point. Anton used the limits of the grid, one solid line, and one dashed line to define the boundaries of the region.

1. If Anton had wanted to define the same region using two dashed boundary lines, how could he have modified his question?
2. If Anton had wanted to use two solid boundary lines, how could he have modified his question?

b. What points are eliminated from a solution set when a dashed boundary line replaces a solid boundary line?

c. The shaded region in Figure **7** shows the solution set for a system of linear inequalities. Which of the corner points are included in this solution set?

d. The corner point C appears to have at least one coordinate that is not an integer. Did you find the exact coordinates of this point? Explain your response.

e. The exact coordinates of a point where two lines intersect is the solution to the corresponding system of equations. To find these exact coordinates, you can use a method involving substitution.

For example, Table **1** lists the steps necessary to find the solution to one system of equations by substitution.

Table 1: Substituting to solve a system of equations

Consider the given system of equations.	$\begin{cases} y = 2x - 3 \\ y = 7 \end{cases}$
Since $y = 7$ in this case, substitute 7 for y.	$7 = 2x - 3$
Solve the resulting equation for x.	$5 = x$
Write the solution to the system in the form (x, y).	$(5, 7)$

To check the solution, you can substitute the coordinates of the point in each of the original equations in the system. If $x = 5$ and $y = 7$, then $y = 2x - 3 = 2(5) - 3 = 7$. Therefore, (5,7) is a solution to the equation $y = 2x - 3$. Since (5,7) is also a solution to the equation $y = 7$, then it is a solution to the system of equations.

1. Why are the coordinates of point *C* a solution to a system of linear equations?

2. Use the substitution method to check your estimate for the coordinates of point *C*.

Exploration 2

Figure **9** shows a graph of three linear equations. Point *B* is the intersection of $5x - 5y = 4$ and $3x + y = 8$.

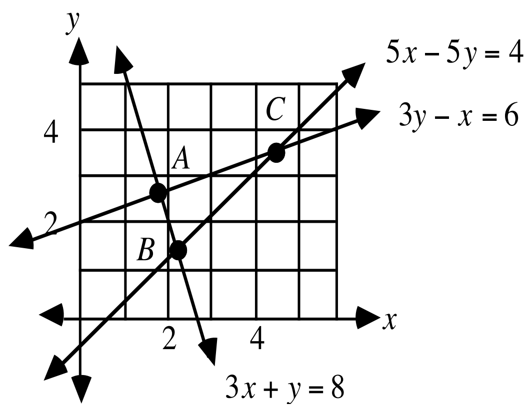


Figure 9: Three linear equations

a. Estimate the coordinates of point *B* in Figure **9**.

b. Table **2** lists the steps necessary to find the solution to another system of equations by substitution.

Table 2: Substituting to solve a system of linear equations

Consider the given system of equations.	$\begin{cases} 2x + 3y = 6 \\ 9y - 4x = 13 \end{cases}$
Solve one of the equations for y.	$y = 2 - \frac{2}{3}x$
Substitute for y in the other equation.	$9\left(2 - \frac{2}{3}x\right) - 4x = 13$
Solve for x.	$x = \frac{1}{2}$
Substitute for x in one of the original equations.	$2\left(\frac{1}{2}\right) + 3y = 6$
Solve the resulting equation for y.	$y = \frac{5}{3}$
Write the solution to the system in the form (x,y).	$\left(\frac{1}{2}, \frac{5}{3}\right)$

To check the solution, you can substitute the coordinates of the point in each of the original equations.

$$2x + 3y = 6 \qquad\qquad 9y - 4x = 13$$

$$2\left(\frac{1}{2}\right) + 3\left(\frac{5}{3}\right) \stackrel{?}{=} 6 \quad \text{and} \quad 9\left(\frac{5}{3}\right) - 4\left(\frac{1}{2}\right) \stackrel{?}{=} 13$$

$$1 + 5 = 6 \qquad\qquad 15 - 2 = 13$$

1. Use the method described in Table **2** to find the exact coordinates of point B in Figure **9**.
2. Compare your solution with the estimate you made in Part **a**.
3. Check your solution by substituting the coordinates of the point in each of the original equations.

c. Repeat Parts **a** and **b** for points A and C in Figure **9**.

Discussion 2

a. Did your estimated coordinates for points A, B, and C support the solutions you found by solving systems of equations?

b. Describe how to confirm the solution to a system of linear equations.

c. What system of linear inequalities could be used to describe the region in Figure **9** with corner points A, B, and C? Assume that the region includes the lines defining its boundaries.

Assignment

2.1 In another game of Guess My Location, Sharline asks, "Does $x = y + 4$ and $y = 1$?" Anton answers, "Yes." Use the substitution method to find the coordinates of Anton's point. Check your results by graphing.

2.2 Anton asks, "Does $y = 3x - 5$ and $6x = -2y - 6$?" Sharline answers, "Yes." What are the coordinates of the point that satisfies this system of equations?

2.3 Graph each of the following systems of equations and use the graphs to estimate the solutions. Determine the solution to each system by substitution. Use your estimates to check your solutions.

a. $y = x + 1$ and $y = 3x - 3$

b. $x + y = 10$ and $2x - 4y = 2$

c. $y = x + 7$ and $y = -2x + 6$

d. $3x + y = 10$ and $4x + 1 = y$

2.4 **a.** Graph the line $y + 3 = 2x$.

b. Write a linear inequality that describes the region below the line and including the line.

c. List the coordinates of two points that satisfy the inequality.

2.5 **a.** Graph the line $y + 2x = 1$ on the same set of axes you used in Problem **2.4**.

b. Write a linear inequality that describes the region above the line and including the line.

c. List the coordinates of two points that satisfy both this inequality and the one you wrote in Problem **2.4b**.

2.6 **a.** Identify the point of intersection of the lines $y - 2x + 3 = 0$ and $y + 2x = 1$.

b. Use substitution to verify your response to Part **a**.

2.7 **a.** Draw the region defined by the following system of inequalities: $x \geq 0$, $y \geq 0$, $x \leq 8$, and $y \leq 8$.

b. Describe the shape of this region.

c. Identify the corner points, or vertices, of the region.

d. Rewrite the system of inequalities in Part **a** so that the boundaries are not included in the region.

e. Write the equations of three different lines that divide the region in Part **a** into two parts with equal area.

2.8 Graph the region defined by each of the following inequalities:

a. $y > 4x - 3$

b. $y \leq -2x + 7$

c. $-3x + 2y \geq 6$

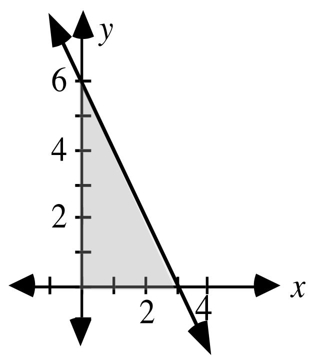
2.9 **a.** Graph the following system of inequalities:

$$\begin{cases} x + y \leq 6 \\ y \geq -2 \\ x \geq -2 \\ x \leq 4 \end{cases}$$

b. Describe the shape of the region.

c. List the coordinates of its vertices.

2.10 Use a system of linear inequalities to describe the shaded region in the following graph.



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2.11 Find the solution to each of the following systems of equations. Verify your solutions by substitution or graphing.

a. $y = -4$ and $y = 2x + 10$

b. $y = x - 2$ and $y = -x - 6$

2.12 Graph the region defined by each of the following systems of inequalities:

a. $y > 4x - 3$ and $y \leq -2x + 7$

b. $y \leq -2x + 7$ and $-3x + 2y \geq 6$

2.13 Jasmine has \$290.00 and saves \$5.00 per week. Alan has \$200.00 and saves \$8.00 per week.

a. Write equations that describe the amount of money each person has after any week w.

b. In how many weeks will the two have the same amount of money?

2.14 In a basketball game, a player made at least five baskets (2 points each), at least four free throws (1 point each), and scored a total of no more than 20 points. (Assume no three-point baskets were made.)

a. Write a system of three inequalities to describe this situation.

b. Graph this system of inequalities. Use the horizontal axis to represent the number of baskets and the vertical axis to represent the number of free throws.

c. List the set of ordered pairs that satisfy this system of inequalities.

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Activity 3

Another booth at the carnival features a game called "Roll-a-rama." This game offers many winning outcomes. In this activity, you examine the chances of winning at Roll-a-rama.

Exploration

In Roll-a-rama, players roll one white die and one red die. One roll of each die is considered a game. To win, the following must be true:

- the number on the red die must be at least 2 but no more than 5
- the sum of the two dice must be less than or equal to 7.

a. Play Roll-a-rama 10 times and record your results.

b. Determine the experimental probability of winning a game.

c. List all the possible rolls in Roll-a-rama.

d. Determine the theoretical probability of winning a game.

Discussion

a. Are you more likely to win or lose when playing Roll-a-rama?

b. Do you think it is possible to find a strategy that increases the probability of winning? Explain your response.

c. If all the possible rolls were graphed on a coordinate plane where the roll on one die is represented on the x-axis and the roll of the other die is represented on the y-axis, what would the graph look like?

Mathematics Note

A **feasible region** is the graph of the solution set of a system of linear inequalities.

For example, consider the following system of linear inequalities: $0 \le x \le 6$, $0 \le y \le 4$, and $x + y \le 8$. The shaded region in Figure **10** is the feasible region for this system.

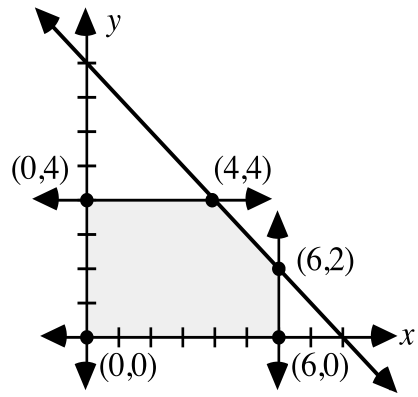


Figure 10: A feasible region

d. The winning combinations for Roll-a-rama could be graphed as a feasible region.

1. Why might it be misleading to shade the feasible region for Roll-a-rama?
2. How might you represent the feasible region for Roll-a-rama?

Assignment

3.1 You can identify the winning combinations in Roll-a-rama by graphing a system of linear inequalities on a coordinate plane. The inequalities that define the constraints on a winning roll can be written as follows:

$$\begin{cases} x \ge 1 \\ y \ge 2 \\ y \le 5 \\ x + y \le 7 \end{cases}$$

a. What do x and y represent in these inequalities?

b. Describe the rule of the game that corresponds with each constraint.

c. Graph this system of inequalities on a set of coordinate axes and determine the feasible region.

d. Identify the number of points in the feasible region that represent winning rolls. **Note**: Save this graph for use in Activity **4**.

3.2 While playing Roll-a-rama, Sharline recorded each roll as an ordered pair in the form (white die, red die). In 10 games, she obtained the following rolls: (1,4), (3,2), (6,2), (4,4), (2,1), (4,3), (5,1), (3,3), (1,6), and (5,3). How many games did she win? Justify your response using your graph from Problem **3.1c**.

3.3 **a.** How many different losing combinations are there in Roll-a-rama?

b. List at least three of these losing combinations. Use your graph from Problem **3.1c** to explain why they do not represent winning rolls.

3.4 Is the point where the line $x + y = 7$ intersects the y-axis in the feasible region for Roll-a-rama? Explain your response.

3.5 **a.** What are the constraints on the region that includes all possible rolls in Roll-a-rama?

b. Write inequalities to describe these constraints.

3.6 Rewrite the rules for "Roll-a-rama" so that the probability of winning is the same as the probability of losing. To check your rules, play the new game several times and record your results.

* * * * *

3.7 Graph the feasible region for each of the following systems of inequalities and label the vertices.

a.

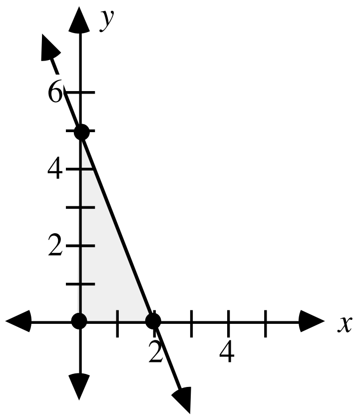
$$\begin{cases} w \geq 0 \\ t \geq 0 \\ 2w + 3t \leq 10 \end{cases}$$

b.

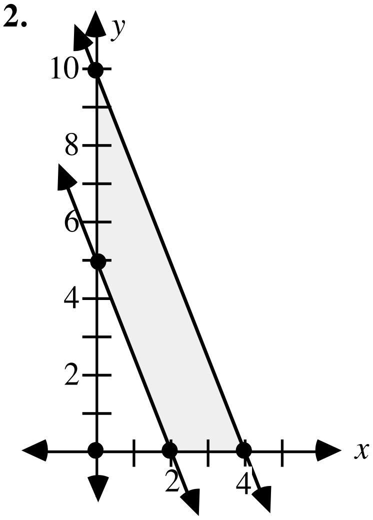
$$\begin{cases} x \geq -1 \\ y \leq 2 \\ y \geq x - 1 \end{cases}$$

3.8 **a.** Describe the constraints on the feasible region shown in each of the following graphs:

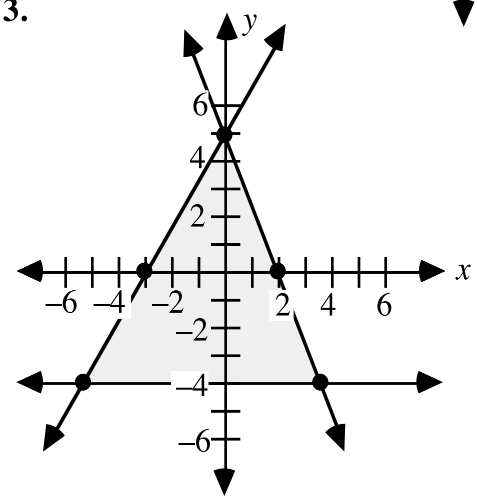
1.



2.



3.



b. List the coordinates of the corner points for each feasible region in Part **a**.

3.9 A furniture manufacturer makes upholstered chairs and sofas. On average, each chair requires 4 hr to assemble and each sofa requires 8 hr to assemble. The company's carpenters can provide a total of no more than 124 hr of assembly work per day. Each chair also requires 2 hr to upholster, while each sofa requires 6 hr to upholster. The company's upholsterers can provide a total of no more than 72 hr of work per day.

a. Write a system of inequalities that describes the numbers of chairs and sofas which the company can build and upholster in one day.

b. Graph the feasible region described by the system you wrote in Part **a** and identify the coordinates of the corner points. Use the horizontal axis to represent the number of chairs and the vertical axis to represent the number of sofas. **Note:** Save your graph for use in Problem **4.3**.

3.10 A group of friends would like to order two large pizzas with extra toppings and some soft drinks. Each pizza costs \$10.00. Each extra topping costs \$0.50, while each soft drink costs \$1.00. The group has \$30.00 to spend.

a. Graph the feasible region that shows the possible numbers of toppings and soft drinks which the group can order.

b. Identify the coordinates of the corner points.

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Activity 4

In Roll-a-rama, each winning player receives a second roll of the dice. If the player also wins on this second roll, a score is determined based on the number showing on each die. The score is calculated using the following rules:

- the number on the white die is multiplied by 2
- the number on the red die is multiplied by 3
- the two resulting numbers are added.

The object of the second roll is to obtain as high a score as possible. This score (S) can be determined by the equation $S = 2x + 3y$, where x represents the number on the white die and y represents the number on the red die. Since finding the best value of S is the objective, this is called an **objective equation**.

Exploration

a. As noted in the previous activity, players roll one red die and one white die in Roll-a-Rama. To win,

- the number on the red die must be at least 2 but no more than 5
- the sum of the two dice must be less than or equal to 7.

The winning combinations can be described by a feasible region for a system of inequalities. On a copy of the Roll-a-Rama template, graph the lines that define this feasible region.

b. Use the objective equation to find the value of S for each roll in the feasible region. Record this information in a table with headings like those in Table **3**.

Table 3: Scores for each winning roll

Winning Roll (x,y)	Score (S)
(1,2)	8

c.
1. Select two points in the feasible region with the same value for S. Draw the line that passes through these two points.
2. Write the equation of this line in the form $y = mx + b$.
3. Repeat Steps **1** and **2** for the remaining points in the feasible region.

d.
1. Use the information in Table **3** to identify the maximum value for S.
2. Substitute this value for S in the objective equation $S = 2x + 3y$. Solve the equation for y.
3. Graph this equation on the template.

e. In Roll-a-rama, you want to find the greatest value of the objective equation. In other situations, you may want to find the least value of an objective equation. Repeat Part **d** for the minimum value for S.

Discussion

a. Where are the maximum and minimum values for S located on your graph from the exploration?

Mathematics Note

In problems which involve a system of linear inequalities (and where the corner points are part of the feasible region), the **corner principle** provides a method of finding the maximum or minimum values for an objective equation. According to this principle, the maximum and minimum values of an objective equation occur at corner points of the feasible region.

b.
1. What similarities did you observe among the lines graphed in Parts **c–e** of the exploration?
2. How are the equations of the lines related to the equation for S?

c. What must a player roll to obtain the greatest possible score?

Assignment

4.1 Sharline and Anton are designing a new game for the Dantzig High carnival. They have already selected two different prizes from a catalog, and need only to decide how many of each to buy. The small prizes cost \$0.25 cents each; the large ones cost \$0.75 each.

According to carnival guidelines, the game may offer no more than 100 prizes. To make the game attractive to players, Sharline and Anton want to buy at least 10 of the large prizes and no more than 60 of the small prizes.

a. Write a linear inequality for each constraint in this situation. Let x represent the number of small prizes and y represent the number of large prizes.

b. Graph your system of linear inequalities on a coordinate plane. Identify the coordinates of the vertices.

c. List the coordinates of three points in the feasible region. What does each point mean in terms of buying prizes?

d. Sharline and Anton have decided to charge \$1.00 to play the game and give each player a prize. Since they have no other expenses, their **profit** is the money they take in minus the cost of the prizes.

1. What is their profit on each small prize?
2. What is their profit on each large prize?

e. Since Sharline and Anton wish to maximize profit, write an equation that describes the amount of profit (P) for each point (x,y) in the feasible region.

f. Use the corner principle to find the coordinates of the point that maximizes P. Assuming that 100 people play the game, how many of each prize should they buy?

4.2 Imagine that you are responsible for managing the concession stand at the carnival. Your budget for purchasing pizza and canned soft drinks is \$115.00. A six-pack of soft drinks costs \$3.00. A 12-serving pizza costs \$10.00. The number of soft drinks you order should not exceed the number of slices of pizza.

a. The concession stand sells pizza for \$1.50 per slice, while soft drinks are sold for \$1.00 each. Assuming that the concession stand sells everything you purchase, write an equation that describes the profit from pizza and soft-drink sales.

b. How many six-packs and how many pizzas should you order? Explain your response.

* * * * *

4.3 The furniture company described in Problem **3.9** earns a profit of \$50.00 on each chair and \$80.00 on each sofa.

a. Write an equation that describes the company's profit from sales of sofas and chairs.

b. Using your graph from Problem **3.9**, determine the number of sofas and chairs the company should make each day to maximize profit.

4.4 Imagine that you are a production manager at an electronics company. Your company makes two types of calculators: a scientific calculator and a graphing calculator.

a. Each model uses the same plastic case and the same circuits. However, the graphing calculator requires 20 circuits while the scientific calculator requires only 10. The company has 240 plastic cases and 3200 circuits in stock. Graph the system of inequalities that represents these constraints.

b. The profit on a scientific calculator is \$9.00, while the profit on a graphing calculator is \$15.00. Write an equation that describes the company's profit from calculator sales.

c. How many of each type of calculator should the company produce to maximize profit using the stock on hand?

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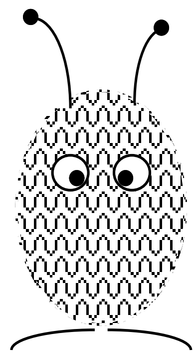
Summary Assessment

Instead of having a game booth, the Dantzig High Pep Club has decided to sell "Spirit Animals" at the carnival. The following illustration shows a Spirit Animal.



Spirit Animals are made from pompons and come in two sizes: small and large. Small animals require 1 pompon, while large animals require 3 pompons. The club has 270 pompons.

Seven members of the club have agreed to work for 30 min each making animals. It takes 1 min to make a small animal and 1.5 min to make a large animal.

The club plans to sell large animals for \$0.90 each and small animals for \$0.65 each. The club paid \$0.05 for each pompon. Your job is to help the Pep Club maximize their profit.

1. Choose variables to represent the numbers of each size of Spirit Animal the club can make.

2. The project is limited by the number of pompons available. Consider how many pompons are needed for each size of animal. Write the linear inequalities for these constraints.

3. The number of minutes that members can work on the project is also limited. Consider how much time is needed to make each size of animal. Write the linear inequalities for these constraints.

4. Graph the linear inequalities from Problems **2** and **3** on a coordinate plane and shade the feasible region.

5. What is the profit on each size of animal?

6. Assuming that club members sell every animal they make, write an equation to describe the club's profit.

7. How many of each size of Spirit Animal should the Pep Club make to maximize profit?

8. Describe the steps you used to find the maximum profit.

Module Summary

- The graph of a linear inequality is a shaded region that represents the **solution set** of the inequality. The solution set contains all the points, or solutions, that make the inequality true. The graph of a linear equation forms the **boundary line** for the region. A solid boundary indicates that the points on the line are part of the solution set. A dashed boundary indicates that the points on the line are not part of the solution set.
- A **conjunction** combines two mathematical statements with the word *and*. A conjunction can be represented as the intersection of two sets.
- A **system of linear equations** is a set of two or more equations whose graphs are lines. A **solution** to a system of linear equations is a point where all the lines intersect. The coordinates of this point satisfy all the equations in the system.
- The **constraints** on a problem are the conditions that limit the number of possible solutions.
- A **feasible region** is the graph of the solution set of a system of linear inequalities.
- The intersection of two or more boundary lines forms a **corner point** (or vertex) of the feasible region.
- In problems which involve a system of linear equalities (and where the corner points are part of the feasible region), the **corner principle** provides a method of finding the maximum or minimum values for an objective equation. According to this principle, the maximum and minimum values of an objective equation occur at corner points of the feasible region.

Selected References

Farlow, S. J., and G. Haggard. *Finite Mathematics and Its Applications*. New York: Random House, 1988.