

# So You Want to Buy a Car



Choosing a new car is never easy. In this module, you'll use mathematics and technology to analyze some common concerns of today's car buyers.

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# So You Want to Buy a Car

## Introduction

In today's world, buying a car is no simple task. Before making a final decision, you must consider numerous manufacturers, dozens of styles, and countless options. Some buyers analyze a car's handling and performance. Others focus more on fuel economy and resale value. Which of these characteristics are most important to you?

## Exploration

- a. Make a list of the 10 factors you consider most important when buying a new car.
- b. Share your list with others in the class. Identify any similarities and differences among the lists.
- c.
  1. As a class, decide on a method that can be used to determine a class list of the "Top 10" factors to consider when buying a car.
  2. Use this method to determine the Top 10 factors.

## Discussion

- a. Do you think that the Top 10 adequately represents the opinions of all members of your class?
- b. What are the benefits or limitations of the strategy used to generate the Top 10?

## *Activity 1*

In the introduction, you determined a class list of the 10 most important factors to consider when buying a new car. Unless you have some way to organize all that information, comparing 10 factors for several different cars can be a difficult task.

## Exploration

In this exploration, you use a spreadsheet to help organize data and create a graph of the results.

- a. Rank the 10 factors from your class list in order of importance to you. Use 1 to indicate the most important and 10 to indicate the least important.

- b. Collect and organize the rank numbers from some other students in the class. Enter the data in a spreadsheet. Table 1 below shows a sample arrangement.

**Table 1: Sample spreadsheet arrangement**

	Student X	Student Y	Student Z
<b>Factor A</b>	6	3	1
<b>Factor B</b>	7	5	4
<b>Factor C</b>	2	2	8
<b>Factor D</b>	9	9	2

### Technology Note

Like a table, a spreadsheet consists of rows and columns. As shown in Figure 1, rows are typically labeled with numbers, while columns are labeled with letters. Each individual section or **cell** in a spreadsheet can be identified by the column and row that contain it.

In Figure 1, for example, the cell containing the number 8.5 is referred to as B2. The cell containing the number 14.3 is referred to as A3.

	A	B	C
1			
2		8.5	
3	14.3		

**Figure 1: Portion of a spreadsheet**

- c. Use the following steps, along with the data you collected in the spreadsheet from Part b, to rank the 10 factors in order of importance.
1. Consider the first factor in the spreadsheet. Write a formula using individual cell notation to find the sum of the rank numbers.
  2. Label a new column for the sum of the rank numbers. Use the formula created in Step 1 to calculate the sum of the rank numbers for the first factor and display it in the first cell of the new column.
  3. Most spreadsheets have a built-in function that allows you to compute the sum of the numbers in a group of cells. Use this function to find the sum of the rank numbers for the second factor and display it in the second cell of the new column.
  4. Find and display the sums of the rank numbers for the remaining factors in the corresponding cells of the new column.
  5. Rank the factors in order of importance according to their sums.

- d. Most spreadsheets also have a built-in function that allows you to find the mean of the numbers in a group of cells.
  - 1. Use this function to find the means of the rank numbers for the 10 factors and display them in another column.
  - 2. Rank the factors in order of importance according to their means. Compare this rank order to the order from Part c.
- e. The information stored in spreadsheets can also be displayed graphically. Use the spreadsheet to create two different bar graphs: one to display the sums of the rank numbers for the 10 factors and another to display the means of the rank numbers.
- f. Spreadsheets are often able to sort entries in increasing or decreasing order, either alphabetically or numerically. When sorting data, it is important to make sure that all the data moves together. Use your spreadsheet to complete Steps **1–3** below.
  - 1. Sort the factors alphabetically.
  - 2. Sort the sums or means of the rank numbers numerically.
  - 3. Sort the factors in the order that you feel is most useful. Create a bar graph to display the sorted information.

### Discussion

- a. Describe some of the benefits of using a spreadsheet to help analyze data.
- b. What problems did you encounter in using the spreadsheet?
- c. Compare the two bar graphs you created in Part e of the exploration. What differences or similarities do you observe?
- d. How can sorting the data in a table or graph help you interpret it?

## Assignment

- 1.1** An automobile dealership has space for two new cars in its showroom. There are four different models available: a convertible, a sports utility vehicle, a luxury car, and a pickup truck. Since the sales staff cannot agree on their two choices, they decide to rank each model in order of preference from 1 to 4, with 1 being the most preferred. The rank numbers for each model are shown in the table below.

Salesperson	Convertible	Utility	Luxury	Truck
A	1	4	3	2
B	3	2	4	1
C	1	3	4	2
D	2	1	3	4

After compiling their rankings, the staff asks you to help them analyze the data and recommend which two models they should order for the showroom.

- Use a spreadsheet to analyze the data using the sums and means of the rank numbers.
  - Display the results graphically.
  - Which two cars would you recommend for the showroom? Write a short summary to support your recommendation.
- 1.2** The data in the following table shows the value of new car exports, in millions of dollars, from the United States to Japan, Canada, and other countries from 1989 to 1992.

Year	Japan	Canada	Other
1989	327	6824	2274
1990	531	6232	2465
1991	496	6195	3070
1992	694	5928	5091

**Source:** U.S. Bureau of the Census.

- Enter the data from the table in a spreadsheet.
- Use the spreadsheet to display the total value of new car exports for the United States per year.
- Create a graph that displays the totals.
- Describe any trends you observe in new car exports.

\* \* \* \* \*

- 1.3** Imagine that you and three other students have been chosen to ride the space shuttle to an international space station. Due to the shuttle’s strict limits on cargo, you can take only a few personal items with you.
- a. To help plan your trip, the space center sends you a list of 10 items that previous astronauts have taken with them into space. This list includes a camera, a medal, a photograph, a stuffed animal, a favorite food, a favorite article of clothing, a compact disc, a book, a hand-held video game, and a portable computer. Enter this information in a spreadsheet.
  - b.
    1. If you could take some of these items with you on the journey, which ones would you choose? Rank each item from 1 to 10, with 1 indicating your first choice.
    2. Collect the rankings from three of your classmates.
    3. Enter the information from Steps 1 and 2 in a spreadsheet.
  - c. Find and display the sums and means of the rank numbers for the 10 items.
  - d. Create two bar graphs: one to display the sums of the rank numbers and the other to display the means of the rank numbers. Compare the two graphs.
  - e. Sort the names of the 10 items alphabetically. How might this information be used?
  - f. Sort the sums or means of the rank numbers numerically. How might this information be used?
  - g. Decide which sorting technique creates the most useful form of the data. Construct a bar graph to display this information.

\* \* \* \* \*

## ***Activity 2***

The year 1992 was an unusual one in automotive history. For the first time, three major publications—*Car and Driver*, *Motor Trend*, and *Automobile Magazine*—chose the same domestic “Car of the Year.”

When making their selections, automotive magazines use both **subjective** and **objective** types of information. For example, a car’s comfort level is based on the passenger’s opinion; therefore, it is a subjective factor. A car’s weight, however, is not based on opinion. Weight is an objective factor.

Each magazine used its own selection process and mix of considerations. The editors of *Motor Trend*, for example, rated eight cars on eight different factors. The six subjective factors were styling and design, quality control, occupant

comfort and convenience, ride and drive, chassis dynamics, and dollar value/market significance. The two objective factors were handling and performance.

A car's handling rating included the results of three tests: braking, maneuverability, and cornering. The braking test measured the distance required to bring a car traveling at 60 mph to a stop. The maneuverability test measured the speed that could be maintained while driving through a slalom course.

The cornering test measured the force required to put a car into a skid. In this case, the unit of measure for force is the *g*, where 1 *g* equals the force of gravity on an object at rest. Cars that receive higher values handle corners better than those that receive lower values. *Motor Trend's* test results for eight cars are shown in Table 2.

**Table 2: Handling**

Model	Braking (ft)	Slalom (mph)	Skid (g)
Cadillac Eldorado Touring Coupe	149	61.7	0.78
Cadillac Seville Touring Sedan	144	65.3	0.80
Ford Taurus SHO	135	63.3	0.80
Mercury Sable LS	142	61.8	0.79
Oldsmobile Achieva SCX	134	67.7	0.86
Oldsmobile 88 Royale LS	137	65.1	0.79
Pontiac Bonneville SSEi	140	65.4	0.80
Pontiac Grand Am GT	141	67.2	0.86

Each car's performance rating combined the results of two fuel economy tests and two acceleration tests. Fuel economy was measured in miles per gallon (mpg) during both city driving and highway driving.

The results of the acceleration tests were reported in terms of the time required to reach 60 mph and to travel a quarter mile. *Motor Trend's* test results for the same eight cars are shown in Table 3.

**Table 3: Performance**

Model	Fuel Economy (mpg)		Acceleration Time (sec)	
	City	Highway	0-60 mph	1/4-mile
Cadillac Eldorado Touring Coupe	16	25	8.3	16.2
Cadillac Seville Touring Sedan	16	25	8.4	16.4
Ford Taurus SHO	18	26	6.8	15.2
Mercury Sable LS	18	28	9.4	17.0
Oldsmobile Achieva SCX	21	31	8.0	16.2
Oldsmobile 88 Royale LS	18	28	9.2	16.8
Pontiac Bonneville SSEi	16	25	8.1	16.1
Pontiac Grand Am GT	21	31	7.5	15.9

## Exploration

In this exploration, you use bar graphs to analyze the data in Tables 2 and 3. Based on your analysis, you then determine your own “Car of the Year.”

- a.
  1. Draw separate bar graphs to display the results of each handling test in Table 2.
  2. For each graph, decide whether a longer bar or a shorter bar indicates a better test result. In the skid test, for example, cars that require more force before skidding handle corners better than cars that require less force. Therefore, a longer bar means a better test result.
- b.
  1. Draw one bar graph to display the test results for both city and highway fuel economy in Table 3.
  2. Draw one bar graph to display the results of the two acceleration tests in Table 3.
  3. For each graph, decide whether a longer bar or a shorter bar indicates a better result. Explain your reasoning.
- c. Use the test results in Table 2 to rank the eight cars on handling.
- d. Use the test results in Table 3 to rank the eight cars on performance.
- e. Rank the eight cars on combined handling and performance where both categories are equally important. Use this information to choose your own 1992 Car of the Year.

## Discussion

- a. Describe the method you used to rank cars in each category.
- b. How did you combine the rankings in the two categories to select a Car of the Year?
- c. Table 2 contains data on three different handling tests, while Table 3 contains the results of four different performance tests. Did you give each of these seven tests equal consideration in making your selection?
- d. *Motor Trend's* 1992 Car of the Year was the Cadillac Seville Touring Sedan. Do your results agree with this selection?
- e. How might the editors of *Motor Trend* justify their choice?



## Assignment

- 2.1**    **a.** Use technology to determine the amount of fuel each car in Table 3 will consume during:
1. 400 miles of city driving
  2. 400 miles of highway driving.
- b.** Create a bar graph that compares the amount of fuel consumed by 400 miles of city driving with the amount consumed by 400 miles of highway driving.
- 2.2**    **a.** Use technology to determine the fuel costs for each car in Table 3 for an 800-mile trip, including 250 miles of city driving. Use the current price of fuel in your area to determine these costs.
- b.** Create a bar graph that compares the fuel costs for the eight cars.
- 2.3**    The slalom course used in the maneuverability test was 600 ft long. Use the information in Table 2 to create a bar graph that compares the number of seconds each car required to complete this course. Assume that each car's speed is constant for the entire length of the course.

\* \* \* \* \*

- 2.4**    Nutritionists generally recommend that you limit your intake of fat, sodium, and cholesterol. The following table contains some nutritional information for hamburgers sold at four fast-food restaurants.

<b>Burger</b>	<b>Calories</b>	<b>Fat (g)</b>	<b>Cholesterol (mg)</b>	<b>Sodium (mg)</b>	<b>Protein (g)</b>
<b>A</b>	500	26	100	890	25
<b>B</b>	584	34	73	733	26
<b>C</b>	630	39	90	850	27
<b>D</b>	730	47	80	1110	28

- a.** Draw a bar graph to display the sodium and cholesterol content of the four hamburgers in the table.
- b.** In your graph from Part **a**, decide whether a longer bar or a shorter bar indicates a better hamburger. Explain your reasoning.
- c.** If separate bar graphs were drawn for each remaining category in the table, would a better hamburger be represented by a longer bar or a shorter bar?
- d.** Use the information in the table to decide which restaurant provides the most nutritious hamburger. Justify your choice.
- e.** Your selection in Part **d** was based on objective data. What subjective information about each hamburger might also help you decide where to eat?

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## Research Project

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The cost of owning a vehicle involves more than just the sticker price. If you finance your purchase, for example, your interest payments increase the cost of the car. Additional costs result from operating and maintaining the vehicle.

In this project, you must determine the cost per mile to operate a vehicle of your choice for the first five years of ownership. Your report should include the purchase price and the criteria you used to choose the vehicle. If applicable, your report also should describe the down payment, the annual interest rate, the number and size of the monthly payments, the total amount of interest paid, and the total cost of the vehicle (including interest).

When determining operating and maintenance expenses, consider each of the following items:

- the number of miles the vehicle is driven annually
  - the cost to insure the vehicle
  - the cost to license the vehicle
  - the cost of replacement tires
  - the cost of regular oil changes, tune-ups, and other scheduled repairs
  - the cost of unscheduled repairs
  - the vehicle's loss in value over the five-year period.
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### *Activity 3*

When designing a new car, engineers must often consider the relationship between two different quantities. For example, the relationship between fuel economy and weight can help determine what materials are used in a vehicle.

#### **Mathematics Note**

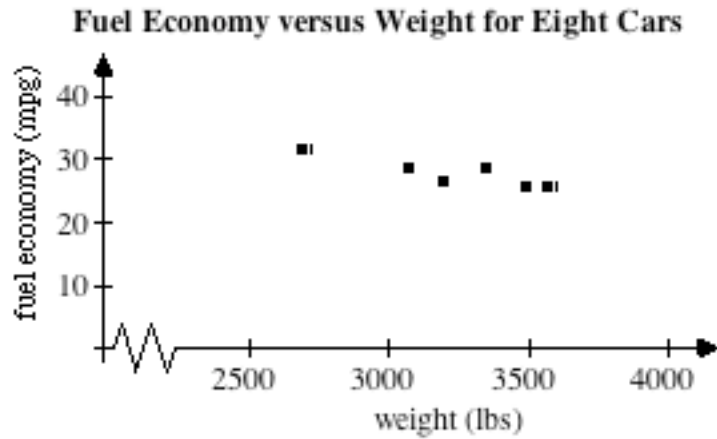
A **scatterplot** is a type of graph in which the corresponding values of two quantities are treated as ordered pairs,  $(x,y)$ . These ordered pairs are plotted as points on a rectangular coordinate system. A scatterplot can be a useful tool for examining the relationship between two quantities.

For example, Table 4 shows the weight and highway fuel economy of eight cars.

**Table 4: Highway fuel economy and weight for eight cars**

Car Model	Fuel Economy (mpg)	Weight (pounds)
Cadillac Eldorado Touring Coupe	25	3612
Cadillac Seville Touring Sedan	25	3707
Ford Taurus SHO	26	3309
Mercury Sable LS	28	3191
Oldsmobile Achieva SCX	31	2816
Oldsmobile 88 Royale LS	28	3468
Pontiac Bonneville SSEi	25	3691
Pontiac Grand Am GT	31	2804

A scatterplot of the data in Table 4 appears in Figure 2.



**Figure 2: Scatterplot of fuel economy versus weight**

The pattern of the points in the scatterplot implies that there is a relationship between weight and fuel economy. As the weight of a vehicle increases, the fuel economy tends to decrease.

## Exploration

In this exploration, you use scatterplots to analyze the relationships between quantities.

- a. As a car is driven through an obstacle course, a scorekeeper records the distance it has traveled at some specified times.
1. Sketch a graph that you believe illustrates the relationship between the time the car has been on the course and the distance it has traveled. Label the axes of the graph.
  2. Table 5 shows the data collected during one car's trip through the obstacle course. The distance traveled is reported in meters (m) and the time is given in seconds (sec). Use technology to create a scatterplot of the data and compare the scatterplot with your sketch.

**Table 5: Distance traveled and time on obstacle course**

Time (sec)	Distance (m)	Time (sec)	Distance (m)
1.0	27	4.0	113
1.5	43	4.5	114
2.0	50	5.0	143
2.5	52	6.0	158
3.0	80	7.0	188
3.5	94	8.0	215

3. Describe the relationship between time and distance traveled.
  4. If possible, use the scatterplot to estimate the distance the car will have traveled in 9 sec.
- b. To collect information on traffic flow, a group of commuters records the number of red lights they encounter as they travel to work.
1. Table 6 shows the data collected by the commuters. The distance traveled is reported in kilometers (km). Use technology to create a scatterplot of the data.

**Table 6: Distance traveled and red lights encountered**

Distance (km)	Red Lights	Distance (km)	Red Lights
1.0	3	4.0	8
1.5	7	4.5	2
2.0	1	5.0	4
2.5	11	6.0	7
3.0	4	7.0	3
3.5	1	8.0	5

2. Describe the relationship between distance traveled and the number of red lights encountered.
3. If possible, use the scatterplot to estimate the number of red lights a commuter will encounter when traveling 9 km.

- c. As you may have observed during the research project, the trade-in value of a vehicle changes as the vehicle ages.
1. Table 7 shows one car's trade-in value in dollars over a period of 9 years. Use technology to create a scatterplot of the data.

**Table 7: Trade-in value and age of car**

Age (years)	Dollar Value	Age (years)	Dollar Value
1.0	12,500	4.0	7600
1.5	11,700	5.0	6500
2.0	11,000	6.0	6000
2.5	10,300	7.0	5200
3.0	9400	8.0	4500
3.5	8100	9.0	4000

2. Describe the relationship between age in years and trade-in value.
3. If possible, use the scatterplot to estimate the car's trade-in value when it is 10 years old.

### Discussion

- a. Describe the three scatterplots you created in the exploration.
- b. Would these graphs show the same relationships if the quantities graphed on the  $x$ -axis and the  $y$ -axis were interchanged?

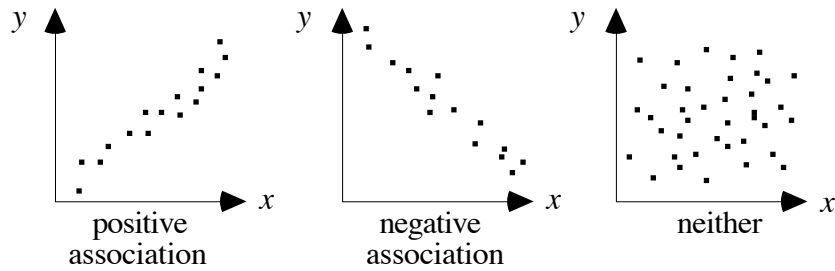
### Mathematics Note

Two quantities are said to have a **positive association** if, as the values of one quantity increase (or decrease), the values of the other quantity also increase (or decrease). When data in a positive association is graphed in a scatterplot, the pattern formed by the data points slants upward from left to right. As the  $x$ -values increase (or decrease), so will the  $y$ -values.

Two quantities are said to have a **negative association** if, as the values of one quantity increase, the values of the other quantity decrease. When data in a negative association is graphed in a scatterplot, the pattern formed by the data points slants downward from left to right. As the  $x$ -values increase, the  $y$ -values decrease.

If two quantities have neither a positive nor a negative association, their scatterplot will show no trend.

These three types of associations are illustrated in Figure 3.



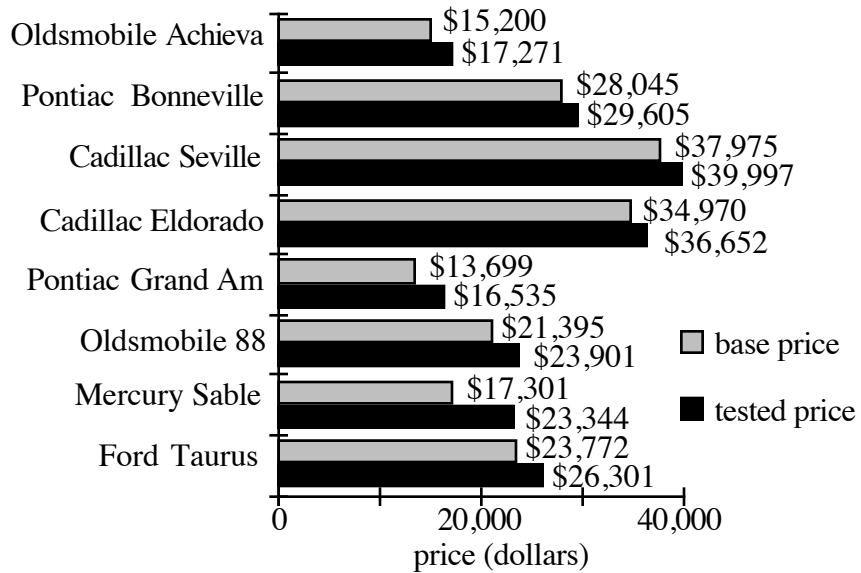
**Figure 3: Three types of associations**

- c. Describe the relationship indicated in each graph from the exploration as a positive association, a negative association, or neither.
- d. When can scatterplots reasonably be used to make predictions?

**Assignment**

- 3.1 The following bar graph shows the base price and tested price of each car nominated for *Motor Trend's* 1992 Car of the Year. (The base price of a car includes no added options. The tested price is the price of the actual model inspected by the magazine, including any options.)

**Base Price and Tested Price of Nominated Cars**



- a. Use a graphing utility to create a scatterplot that compares base price and tested price for these cars.
- b. What type of association exists between base price and tested price? Explain your response.
- c. Why might a manufacturer supply an automotive magazine with a car that features more options than its base model?

- 3.2** The table below shows the weight in pounds of each car nominated for *Motor Trend's* 1992 Car of the Year.

<b>Model</b>	<b>Weight (pounds)</b>
Cadillac Eldorado Touring Coupe	3612
Cadillac Seville Touring Sedan	3707
Ford Taurus SHO	3309
Mercury Sable LS	3191
Oldsmobile Achieva SCX	2816
Oldsmobile 88 Royale LS	3468
Pontiac Bonneville SSEi	3691
Pontiac Grand Am GT	2804

- Do you think that price and weight would have a positive association, a negative association, or neither?
  - To graph the data for price and weight, would you use base price, tested price, or both? Explain your reasoning.
  - Use a graphing utility to create a scatterplot of price versus weight. Does the scatterplot support your response to Part **a**?
  - If possible, use your scatterplot to estimate the cost of a car that weighs 4000 lb. If it is not possible to make a reasonable estimate, explain why.
- 3.3** Use your scatterplot from Problem **3.2** to decide whether each of the following statements is true or false. If the statement is true, justify your decision. If the statement is false, use an example to show why.
- Lighter cars are less expensive than heavier cars.
  - More expensive cars weigh less than less expensive cars.
  - As the weight of a car increases, the price of a car decreases.
  - As the price of a car increases, the weight of a car increases.
- 3.4** The table below shows the weight and the results of two acceleration tests for eight cars. Use a scatterplot of this data to determine if lighter cars accelerate better than heavier cars.

<b>Car Model</b>	<b>Time (sec)</b>		<b>Weight (lb)</b>
	<b>0–60 mph</b>	<b>1/4 mile</b>	
Cadillac Eldorado	8.3	16.2	3612
Cadillac Seville	8.4	16.4	3707
Ford Taurus	6.8	15.2	3309
Mercury Sable	9.4	17.0	3191
Oldsmobile Achieva	8.0	16.2	2816
Oldsmobile 88	9.2	16.8	3468
Pontiac Bonneville	8.1	16.1	3691
Pontiac Grand Am	7.5	15.9	2804

- 3.5** Identify two quantities related to cars (such as speed, weight, or fuel economy) that illustrate each type of association listed below. Use examples that have not been previously described in this module and explain why you believe the two quantities are associated in this way.
- a positive association
  - a negative association
  - neither a positive nor a negative association

\* \* \* \* \*

- 3.6** The following table shows the calories and fat contained in a single serving of some common meats.

Meat Product	Fat (grams)	Calories
chicken breast	2	120
pork tenderloin	5	159
turkey (light meat)	8	176
sirloin steak	8	190
salmon	9	180
hamburger	17	250
beef hot dogs	26	284

**Source:** U.S. Department of Agriculture.

- Create a scatterplot that compares calories in each meat product versus the number of grams of fat.
- Describe the type of association that exists between these two quantities.

\* \* \* \* \*

### *Activity 4*

Car buyers who trade in their old vehicles for newer models often consider resale value when selecting their new cars. A car's resale value often declines as it gets older. This decrease in value over time is called **depreciation**. The rate of depreciation for cars varies from model to model. A car with a low rate of depreciation might rank high with some consumers because they would receive a trade-in allowance closer to the car's original price.



Tables 8–11 show the high and low trade-in values for the 1986 models of four of the cars examined in previous activities. Notice that the high and low values given for 1986 are equal; this was the manufacturer’s suggested retail price for a car with standard equipment.

**Table 8: Cadillac Eldorado**

Year	Low value (dollars)	High value (dollars)
1986	26,756	26,756
1987	17,475	21,925
1988	14,775	18,700
1989	12,200	15,675
1990	10,175	13,375
1991	8,250	11,250
1992	6,300	8,775
1993	5,675	8,125

**Table 9: Cadillac Seville**

Year	Low value (dollars)	High value (dollars)
1986	24,251	24,251
1987	16,275	20,550
1988	13,725	17,475
1989	11,900	15,300
1990	9,550	12,600
1991	7,925	10,775
1992	6,525	9,075
1993	6,125	8,675

**Table 10: Oldsmobile 88**

Year	Low value (dollars)	High value (dollars)
1986	12,760	12,760
1987	8,550	10,925
1988	8,225	10,525
1989	6,975	8,975
1990	5,825	7,875
1991	4,600	6,500
1992	3,475	5,125
1993	2,500	4,025

**Table 11: Pontiac Bonneville**

Year	Low value (dollars)	High value (dollars)
1986	10,249	10,249
1987	7,075	9,100
1988	6,475	8,375
1989	5,425	7,125
1990	4,500	6,325
1991	3,250	4,850
1992	2,100	3,400
1993	1,625	2,900

## Exploration

In this exploration, you use a spreadsheet to compare the rates of depreciation for four vehicles.

- a.
  1. Table 8 contains the trade-in data for a 1986 Cadillac Eldorado. Enter this information in the appropriate columns of a spreadsheet with the headings shown below.

Year	High	Low	Mean

2. Calculate the mean of the low and high values for each year and display it in the appropriate column of the spreadsheet.

- b.
  1. Create a scatterplot of the data that represents the year on the horizontal axis and the mean value on the vertical axis.
  2. Describe the shape of the graph.
  3. Does the mean value of this car appear to increase or decrease over time?
  4. Use the graph to estimate the mean value of the car in 1995.
- c. Repeat Parts **a** and **b** for the other three cars.
- d. Compare the four scatterplots. What similarities or differences do you observe?
- e.
  1. Add another column to each spreadsheet. Use this column to display the decrease in mean value from year to year for each car.
  2. Create a scatterplot that represents a car's age in years on the  $x$ -axis and its decrease in mean value on the  $y$ -axis.
- f. Determine the total depreciation in the mean value of each car over the period from 1986 to 1993.
- g. Imagine that all four cars are driven 70,000 miles over the period from 1986 to 1993. Compare the depreciation per mile for the four cars.

**Note:** Save the spreadsheets you created in this exploration for use in the assignment.

## Discussion

- a.
  1. In Part **e** of the exploration, why are there no spreadsheet entries for decrease in mean value in 1986?
  2. How are the data points for the different cars represented on the scatterplot?
  3. Is the decrease in mean value the same from year to year for any car? How can you tell?
  4. In the scatterplot for the Oldsmobile 88, the  $y$ -value for year 3 is greater than the  $y$ -value for year 2. Does this mean that the car increased in value? Explain your response.
- b. Which car had the greatest decrease in mean value between 1990 and 1991? Did you use your spreadsheets or your graphs to answer this question? Is one easier to use than the other?
- c.
  1. In the exploration, you expressed the amount of depreciation in terms of dollars. How else could you describe a car's loss in value over time?
  2. Why might this be a more reasonable way to compare the depreciation of two different vehicles?

### Mathematics Note

The **percent increase** in a quantity can be found by expressing the increase as a fractional part of the original value, then expressing that fraction as a percentage.

Similarly, the **percent decrease** in a quantity can be found by expressing the decrease as a fractional part of the original value, then expressing that fraction as a percentage.

For example, consider a car whose fuel economy increased from 25 mpg to 28 mpg in its most recent model year. The percent increase in fuel economy can be calculated as shown below:

$$\frac{\text{increase}}{\text{original value}} = \frac{28 - 25}{25} = \frac{3}{25} = 0.12 = 12\%$$

- d.
  1. Which car showed the smallest decrease in mean value over the period from 1986 to 1993?
  2. Did this car also have the least percent decrease in mean value?
- e. If a buyer ranked depreciation as the most important factor in choosing a car, which of the four vehicles would you recommend? Explain your response.

### Assignment

- 4.1 From Table 8, the mean value of a 1986 Cadillac Eldorado in 1987 was \$19,700. Its mean value in 1988 was \$16,738.
- a.
    1. What percentage of the car's 1987 value is represented by its 1988 value?
    2. The Cadillac Eldorado lost \$2,962 of its mean value from 1987 to 1988. What percentage of the 1987 value is represented by this decrease?
  - b. Which of the two percentages calculated in Part a represents the percent decrease in value? Explain your response.
  - c. Add the two percentages you found in Part a. What does this sum represent?

- 4.2**
- Using your spreadsheets from the exploration, calculate and display the percent decrease in the mean value of each car after each year.
  - Create a graph that shows the percent decrease in mean value each year for all four cars.
  - Describe any patterns you observe in the data.
  - Which model(s) had the greatest percent decrease for a single year? Between what years did this decrease occur? Did the same car have the greatest amount of depreciation for that year?
  - Describe a situation in which the car that had the greatest amount of depreciation was not the car that had the greatest percent decrease for that year.
  - Which is the better method for comparing losses in value, amount of depreciation or percent decrease? Explain your reasoning.
- 4.3** In Part **f** of the exploration, you determined the total depreciation in the mean value of each car from 1986 to 1993.
- Find the total percent decrease in mean value for each car.
  - Rank the cars using both the total amount of depreciation and the total percent decrease.
  - Use your rankings to determine which car best holds its value over time.

\* \* \* \* \*

- 4.4** The Shure Shine Company is planning to use the following advertisement to market its shampoo.



- The original bottle contained 200 milliliters (mL) of shampoo. What is the percent increase in the volume of the bottle?
- Do you think that the advertisement should use percent increase to market the shampoo? Explain your response.

- 4.5** The money that an investment earns is referred to as interest. Investments can earn interest in two ways. Using simple interest, interest is paid only on the original amount of money invested.
- a. An investment of \$1000 at an annual interest rate of 10% for 1 year would earn  $\$1000 \cdot 0.10 = \$100$  in simple interest. What is the percent increase in the original investment after 1 year?
  - b. After 5 years, the same investment would earn \$500 in simple interest. What is the percent increase in the original investment after 5 years?

- 4.6** In some types of investments, the interest earned is periodically added to the amount originally invested. In these situations, the interest itself also earns interest. This is known as compound interest.

For example, when \$1,000 is invested at an annual interest rate of 10%, the interest earned after 1 year is  $0.10 \cdot \$1,000 = \$100$ . If this amount is added to the original investment, the total value of the investment during the second year is  $\$100 + \$1,000$  or \$1,100.

The interest earned on this amount after the second year is  $0.10 \cdot \$1,100 = \$110$ . The value of the investment during the third year is  $\$110 + \$1,100$  or \$1,210.

- a. Use a spreadsheet to find the value of this investment at the end of each year for 15 years.
- b. Determine the percent increase in the investment's value after 15 years.
- c. Explain why the percent increase after 15 years is not  $15 \cdot 10\% = 150\%$ .

\* \* \* \* \*

## ***Summary Assessment***

1. Not all cars lose value over time. After decreasing in value for the first decade of use, some cars become collector's items. The value of these vintage cars then starts to increase. For example, consider the following three vintage cars.
  - A 1967 Jaguar XE-6 worth \$22,000 is expected to increase in value by 5% each year.
  - A 1948 Mercedes sedan worth \$31,000 is expected to increase in value by 4% each year.
  - A 1924 Ford Model A worth \$10,000 is expected to increase in value by 8% each year.

Imagine that an investor plans to buy one of these three cars and keep it for either 10 or 20 years. After 10 or 20 years, the investor would then sell the car for a profit.

  - a. To help this investor decide which car to purchase, determine the value of each car after 10 years and after 20 years.
  - b. For each time period, compare each car's increase in value to its percent increase.
  - c. In either time period, does the car that shows the greatest increase in value also show the greatest percent increase?
2. Would you recommend the same car for both a 10-year investment and a 20-year investment? Explain your response.
3. Write a report to the investor that explains which car you would recommend for each investment period. Use appropriate data and at least one graph to support your selections.

## *Module Summary*

- **Subjective** information is based on opinion.
- **Objective** information is based on measurable results.
- A **scatterplot** is a type of graph in which the corresponding values of two quantities are treated as ordered pairs,  $(x,y)$ . These ordered pairs are plotted as points on a rectangular coordinate system.
- Two quantities are said to have a **positive association** if, as the values of one quantity increase (or decrease), the values of the other quantity also increase (or decrease). When data in a positive association is graphed in a scatterplot, the pattern formed by the data points slants upward from left to right. As the  $x$ -values increase (or decrease), so will the  $y$ -values.
- Two quantities are said to have a **negative association** if, as the values of one quantity increase, the values of the other quantity decrease. When data in a negative association is graphed in a scatterplot, the pattern formed by the data points slants downward from left to right. As the  $x$ -values increase, the  $y$ -values decrease.
- If two quantities have neither a positive nor a negative association, their scatterplot will show no trend.
- The decrease in value of a car or other property over time is **depreciation**.
- The **percent decrease** in a quantity can be found by expressing the decrease as a fractional part of the original value, then expressing that fraction as a percentage.
- The **percent increase** in a quantity can be found by expressing the increase as a fractional part of the original value, then expressing that fraction as a percentage.

## **Selected References**

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Other useful resources include: *Automobile Magazine*, *Car and Driver*, *Consumer Reports*, *Fodor's Used Car Guide*, and *Road and Track*.