Yesterday's Food Is Walking and Talking Today

You are what you eat, give or take a few calories. But how those calories get used depends largely on what you do. In this module, you examine how some daily activities—like walking, talking, or doing the backstroke—affect your dietary needs.

John Carter • Janet Higgins • Paul Swenson • Steve Yockim

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Introduction

By eating, you provide your body with fuel in the form of calories. In this module, you examine the number of calories the body needs to perform some typical activities.

Exploration

A **calorie** is a measure of heat energy. One way to determine the number of calories in a food is to measure the amount of heat released when that food is burned. In the following exploration, you use this technique to estimate the number of calories in a nut.

Note: You must wear eye protection throughout this experiment. Put on a pair of goggles *before* beginning the exploration.

a. Arrange the equipment as shown in Figure **1** below. The bottom of the water can should be no more than 5 cm above the nut.

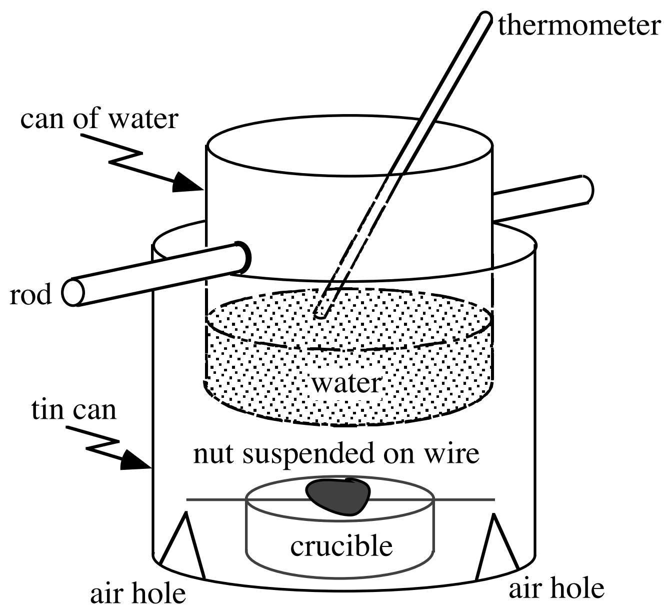


Figure 1: Experiment for estimating calories in a nut

b. Make a table with headings like those in Table **1** and record the types of nuts you plan to test.

Table 1: Data for calorie experiment

Type of Nut	Initial Temperature (˚C)	Maximum Temperature (˚C)	Initial Mass (g)	Final Mass(g)	Volume of Water (mL)

c. Measure and record each of the following:

1. the initial temperature of the water in degrees Celsius (˚C)

2. the volume of the water in milliliters (mL).

3. the total mass of the nut, crucible, and wire in grams (g).

d. **1.** Ignite the nut. As the nut burns, observe the change in water temperature.

2. Allow the nut to burn completely. Record the maximum temperature of the water.

e. Determine the total mass of the ash, crucible, and wire. The difference between this value and the mass measured in Part **c** is the change in mass (the number of grams that burned).

f. Repeat Parts **c–e** for several different types of nuts.

Science Note

A **calorie (cal)** is the amount of energy required to raise the temperature of one milliliter (1 mL) of water one degree Celsius (1˚C).

A **kilocalorie** (**kcal**), 1000 cal, is the amount of energy needed to raise the temperature of one liter (1 L) of water 1˚C.

A **dietary calorie** (typically referred to as a Calorie with a capital *C*) is equal to 1 kcal. The calorie-per-gram rating on most food labels refers to dietary calories.

g. Create a table with headings like those in Table **2** below.

Table 2: Kilocalories in different kinds of nuts

Type of Nut	Change in Temperature (˚C)	Change in Mass (g)	Volume of Water (mL)	Calories per Gram	Kcal per Gram

h. Use the data you recorded in Table **1** and the definitions described in the science note to complete Table **2**. In this case, the number of calories per gram in each type of nut can be calculated using the following formula:

$$\text{calories per gram} = \frac{\text{volume of water (mL)} \bullet \text{change in temperature (˚C)}}{\text{change in mass (g)}}$$

Discussion

a. Do all types of nuts appear to contain about the same number of kilocalories per gram?

b. Compare your results with the Calorie-per-gram rating on a package of nuts.

c. If a nut failed to burn completely, would the experiment produce faulty data? Explain your response.

d. What types of experimental errors might have affected your data?

e. How would using a different quantity of water change the results of the experiment?

Activity 1

The human body uses calories at varying rates, depending on the level of activity. Calorie consumption also depends on a person's size, physical condition, and other factors. Table **3** shows the number of kilocalories used per minute during a variety of activities.

Table 3: Kilocalories used per minute per kilogram of body mass

Activity	$\frac{\text{kcal}}{\text{min} \bullet \text{kg}}$	Activity	$\frac{\text{kcal}}{\text{min} \bullet \text{kg}}$	Activity	$\frac{\text{kcal}}{\text{min} \bullet \text{kg}}$
archery	0.065	drawing	0.036	painting	0.034
badminton	0.097	eating	0.023	racquetball	0.178
basketball	0.138	football	0.132	running	
card playing	0.025	golf	0.085	7.2 min/km	0.135
carpentry	0.052	gymnastics	0.066	5.6 min/km	0.193
circuit training		jumping rope	0.162	5.0 min/km	0.208
Universal	0.116	judo	0.195	3.7 min/km	0.252
Nautilus	0.092	lying at ease	0.022	sitting quietly	0.021
free weights	0.086	music playing		stock clerking	0.054
cycling		cello	0.041	swimming	
slow	0.064	drums	0.066	backstroke	0.169
medium	0.100	flute	0.035	crawl	0.156
fast	0.169	organ	0.053	table tennis	0.068
dancing		piano	0.040	typing	0.027
aerobic	0.135	trumpet	0.031	walking	0.080
normal	0.075	woodwind	0.032	writing	0.029

Sources: McArdle, et al., *Exercise Physiology*; Sharkey, *Physiology of Fitness*. [Reprinted, by permission, from B.J. Sharkey, 1979, *Physiology of Fitness*, 1st ed. (Champaign, IL: Human Kinetics Publishers), 350–353.]

Discussion 1

a. What types of activities burn calories at a high rate?

b. What types of activities burn calories at a low rate?

c. When would your body burn no calories at all?

d. The units for the values in Table **3** are

$$\frac{\text{kcal}}{\text{min} \bullet \text{kg}}$$

Describe how these units can help you determine the number of kilocalories a 60-kg person burns while lying at ease for 30 min.

e. A 60-kg person playing basketball for x minutes burns 500 kcal. This can be represented by the equation $500 = 8.28x$.

1. Both sides of this equation must have the same units: kilocalories. Explain why the expression $8.28x$ represents kilocalories.
2. Describe how to determine x, the number of minutes spent playing basketball.

Mathematics Note

An equation of the form $y = mx$ can be solved for x by dividing each side of the equation by m.

For example, if $10 = 7x$, divide each side of the equation by 7. This results in the equation

$$\frac{10}{7} = x$$

The solution, therefore, is $10/7$.

Exploration

a. Choose one activity from Table **3** that requires a high amount of energy, one that requires a moderate amount of energy, and one that requires a low amount of energy.

b. Create a table with headings like those Table **4** below. Record the names of your chosen activities.

Table 4: Time required to burn kilocalories for three activities

Energy Required	High	Moderate	Low
Activity			
Time to Burn 100 kcal			
Time to Burn 200 kcal			
Time to Burn 300 kcal			
Time to Burn 400 kcal			
Time to Burn 500 kcal			

c. For each activity you chose, calculate the time required for a 60-kg person to burn 100 kcal, 200 kcal, 300 kcal, 400 kcal, and 500 kcal. Record the answers in your table.

d. Graph a scatterplot of the data in Table **4** on a single set of axes. Represent time on the x-axis and energy used on the y-axis.

Mathematics Note

An equation of the form $y = mx$ represents a line that passes through the origin where m is the **slope** of the line. The slope is the ratio of the change in vertical distance to the change in horizontal distance between any two points on the line.

The slope of a line containing two points with coordinates (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

where $y_2 - y_1$ is the change in the vertical distance and $x_2 - x_1$ is the change in the horizontal distance. If $x_1 = x_2$, the line is vertical and has no slope.

For example, the graph in Figure **2** shows the energy used by a 80-kg person playing cards. This graph can be represented by an equation of the form $y = mx$, where y represents energy used in kilocalories and x represents time in minutes.

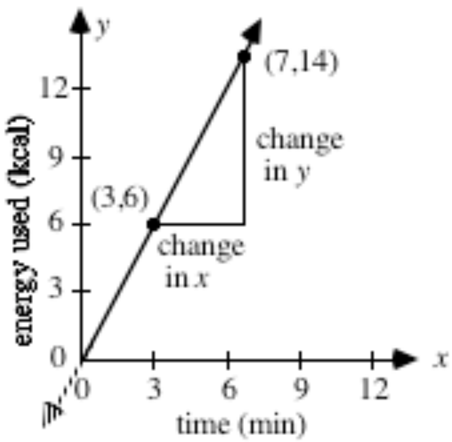


Figure 2: **The slope of a line**

Using the points (7,14) and (3,6), the change in the vertical distance is $14 - 6$; the change in the horizontal distance is $7 - 3$. Therefore, the slope of the line is:

$$m = \frac{14 - 6}{7 - 3} = \frac{8}{4} = 2$$

Since the line passes through the origin and its slope is 2, the equation of the line is $y = 2x$.

e. For each activity in Table **4**, find an equation that expresses y in terms of x, where y represents energy used in kilocalories and x represents time in minutes.

f. Graph your equations from Part **e** on the same set of axes as the scatterplots from Part **d**.

Discussion 2

a. What common point is contained by all the graphs in Part **f** of the exploration?

b. What type of activities have graphs that are closest to vertical?

c. What type of activities have graphs that are closest to horizontal?

d. In Figure **2**, the line $y = 2x$ is used to represent the energy consumed over time by a 80-kg person playing cards.

1. Describe what a vertical change of 6 and a horizontal change of 3 means in terms of playing cards.
2. In this situation, the slope of the line $y = 2x$ is the measure of a rate. Describe this rate.

e.

1. Identify the slope of each equation you wrote in Part **e** of the exploration.
2. Explain how the values in Table **4** can be used to determine the slope of each equation.
3. Describe the rates represented by the slopes of these equations.

Mathematics Note

A **relation** between two variables is a set of ordered pairs of the form (x, y).

The **domain** of a relation is the set of first elements in the ordered pairs (the x-values). The **range** of a relation is the set of second elements in the ordered pairs (the y-values).

A **function** is a relation in which each element of the domain is paired with an element of the range and each element of the domain occurs in only one ordered pair. A function may be described by a rule or equation.

In mathematics, functions usually involve domains and ranges that are sets of real numbers. When a function is written without specifying the domain, you can assume that the domain comes from the set of real numbers. The range can be determined by finding the y-value that corresponds with each number in the domain.

For example, consider the function defined by the equation $y = 2x + 1$. In this case, the domain is the set of all real numbers. Each x-value is paired with only one y-value: a number that is 1 greater than twice the x-value. The range is also the set of all real numbers.

f. **1.** What is the domain of the relation described by each equation you wrote in Part **e** of the exploration?

2. What is the range of each relation?

g. **1.** When the equation $y = 2x$ is used to represent the energy consumed over time by a 80-kg person playing cards, what values of the domain make sense?

2. What values of the range correspond with these values of the domain?

Assignment

1.1 **a.** How many kilocalories does a 57-kg person use playing racquetball for 23 min?

b. Tristin has a mass of 61 kg. If he plays football for 2 hr, how many kilocalories will he burn?

c. Write an equation that describes the number of kilocalories expended by a 58-kg person while playing golf for x minutes.

d. Identify the domain and range of the relation you described in Part **c**.

1.2 **a.** Sigrid has a mass of 58 kg. While practicing judo, she used 300 kcal of energy. How many minutes did she practice?

b. Miguel has a mass of 72 kg. His breakfast contained 320 kcal. How long will it take Miguel to use these kilocalories at his aerobics class?

c. A 60-kg person uses y kilocalories while typing. Write an equation that expresses the amount of time, x, spent typing.

1.3 **a.** While running for 30 min, Alexi burns 500 kcal. Draw and label a graph to represent this situation. Use the graph to estimate how many kilocalories Alexi had burned after 17 min of running.

b. While typing for 30 min, David burns 50 kcal. Draw and label a graph to represent this situation. Use the graph to estimate how many kilocalories David had used after 12 min of typing.

c. While swimming, Alexi burns 150 kcal in 12 min and David burns 300 kcal in 28 min. Draw one graph to represent this situation. Use the graph to determine who would burn more kilocalories in 1 hr of swimming. Explain your reasoning.

d. Which quantity represents the range of the relations described in Parts **a–c**: the number of minutes or the number of kilocalories? Explain your response.

1.4 **a.** Determine the slope for each change in vertical and horizontal distance shown in the table below.

Change in Vertical Distance	Change in Horizontal Distance	Slope
8	4	
4	8	
–8	4	
3	–12	

b. On a single set of axes, sketch the graphs of the lines that pass through the origin with the slopes in Part **a**.

1.5 Find the slope of the line through each of the following pairs of points.

a. $(3, 7)$ and $(12, 3)$

b. $(5, 2)$ and $(6, -4)$

c. $(12, -8)$ and $(10, 4)$

d. $(-7, -2)$ and $(-3, 2)$

e. $\left(\frac{9}{7}, \frac{2}{5}\right)$ and $\left(-\frac{5}{7}, -\frac{3}{5}\right)$

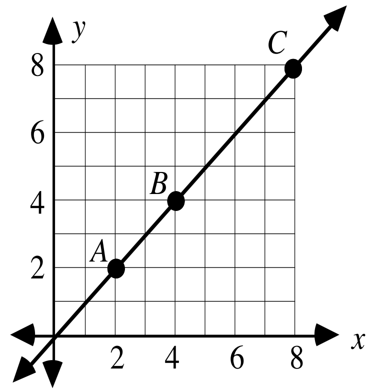
1.6 The slopes of the lines you graphed in the exploration describe rates of energy usage in kilocalories per minute. Give an example of a rate which corresponds with each of the following:

a. a large positive value for slope

b. a small positive value for slope

c. a negative value for slope.

1.7 Use the following graph to complete Parts **a–c**.



a. List the coordinates of points *A*, *B*, and *C*.

b. Find the change in vertical distance, the change in horizontal distance, and the slope (m) between:

1. points *A* and *B*

2. points *B* and *C*

3. points *A* and *C*.

c. Does the pair of points used to calculate the slope of a line affect the value of the slope? Explain your response.

1.8 **a.** Select one activity from Table **3**. Write an equation that describes the energy used over time by a 50-kg person performing this activity, where y represents energy in kilocalories and x represents time in minutes.

b. Graph the equation.

c. Identify the domain and range in this setting.

d. Find the slope of the line and describe what it represents in this situation.

e. Determine the number of kilocalories used when performing this activity for:

1. 30 min

2. 100 min

f. Describe how the slope of the line can be determined from your responses in Part **e**.

1.9 **a.** Draw the graph of a horizontal line and label the coordinates of any two points on the line.

b. Determine the slope of the line.

c. Draw the graph of a vertical line and label the coordinates of any two points on the line.

d. Determine the slope of the line.

Mathematics Note

A horizontal line has a slope of 0 and an equation of the form $y = c$. All the points on a horizontal line have the same y-coordinate, c.

A vertical line has no slope and an equation of the form $x = c$. All the points on a vertical line have the same x-coordinate, c.

e. **1.** Write the equation of the horizontal line in Part **a**.

2. Identify the domain and range for this relation.

f. **1.** Write the equation of the vertical line in Part **c**.

2. Identify the domain and range for this relation.

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1.10 Sam's fast-food lunch contained 865 kcal. If his mass is 75 kg, how long would Sam have to jump rope to burn this amount of energy?

1.11 **a.** A sumo wrestler has a mass of 227 kg. Write an equation that describes the energy he uses over time while lying at ease. Let y represent number of kilocalories and x represent time in minutes.

b. Sam's mass is 75 kg. Write an equation that describes the energy he uses over time while lying at ease. As in Part **a**, let y represent number of kilocalories and x represent time in minutes.

c. Graph the two equations you wrote in Parts **a** and **b** on the same set of axes.

d. Which line appears closer to vertical? What does this observation indicate in this setting?

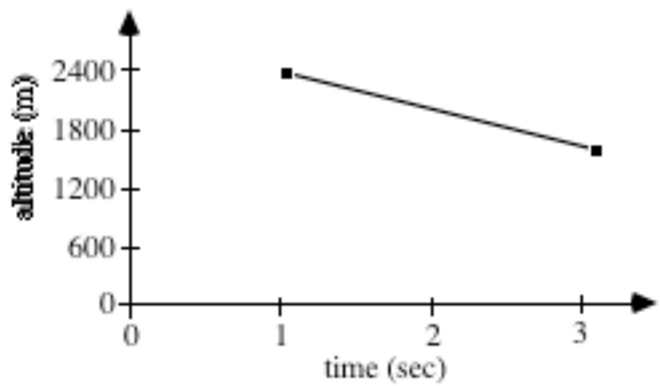
1.12 The slope of a line can be used to describe the average rate of change in one quantity with respect to another. For example, one familiar rate of change is speed, often expressed in kilometers per hour.

a. Rolando is climbing a mountain. At 10:00 A.M., he stood at an elevation of 1000 m. By 2:00 P.M., he had reached an elevation of 1220 m. Determine Rolando's average rate of change in elevation in meters per hour.

b. In 1980, the city of Tucson, Arizona, had a population of 534,000. By 1990, its population had grown to 655,000. Find Tucson's average rate of change in population in people per year.

c. In 1950, the price of a pair of jeans was about $8.00. By 1990, the price had risen to about $32.00. Find the average rate of change in the price of jeans in dollars per year.

d. The graph below shows the change in a parachutist's altitude (in meters) during an interval of 2 sec. Use the graph to estimate the parachutist's average rate of change in altitude in meters per second.



e. Identify the domain and range for the graph in Part **d**.

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Activity 2

Many dietary specialists think of breakfast as the most important meal of the day. In this activity, you use linear equations to help plan an adequate breakfast for an active morning. Table **5** shows the number of kilocalories per serving in some typical breakfast foods.

Table 5: Kilocalories per serving for common breakfast foods

Food	kcal/item	Food	kcal/item
Toast, white	80	Croissant, egg, bacon, cheese	386
Toast, wheat	70	Biscuit, bacon, egg, cheese	483
Doughnut, plain	240	Biscuit with sausage	330
Cereal, with sugar	180	Cherry pie	260
Cereal, plain	120	Egg with muffin	340
Apple	60	English muffin with butter	186
Banana	80	Hotcakes with butter	500
Grapefruit	60	French toast	400
Orange juice	120	Fries	360
Egg, fried	120	Omelet	290
Egg, scrambled	80	Sausage, one patty	200
Egg, substitute	90	Milk, 2%	112
Milk, whole	160	Milk, chocolate	192
Yogurt, plain	120	Peanut butter and jam	500
Coffee	0	Soda pop	144

Sources: Gebhardt and Matthews, *Nutritive Value of Foods*; McArdle, et al., *Exercise Physiology*; Page and Raper, *Food and Your Weight*.

Exploration

a. Use the information in Tables **3** and **5** to design a breakfast that will supply a 62-kg person with the number of kilocalories necessary to play racquetball for 1 hr.

b. Make a table that shows the kilocalories remaining from the meal at the end of each 5-min interval of racquetball.

c. Create a scatterplot of the data from Part **b**. Let the time in minutes be the domain of the relation and the energy remaining be the range.

d. Determine the slope of the data.

e. Determine the coordinates of the point where the data intersects the y-axis.

Mathematics Note

The y-coordinate of the point where a line intersects the y-axis is known as the **y-intercept**.

The equation of a line with slope m and y-intercept b can be written in the form $y = mx + b$. This is the **slope-intercept** form of the equation of a line.

For example, Figure **3** shows a graph of the equation $y = -2x + 4$. This line has a slope of –2 and a y-intercept of 4. It intersects the y-axis at the point (0,4).

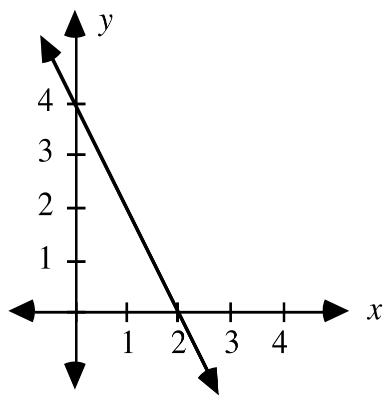


Figure 3: Graph of $y = -2x + 4$

f. On the same set of axes as the scatterplot in Part **c**, graph the equation $y = mx + b$, where m is the slope of data and b is the y-intercept.

g. Repeat Parts **b–f** for a 62-kg person who ate a breakfast of 800 kcal.

Discussion

a. In Part **f** of the exploration, how did the scatterplot compare with the graph of the equation in the form $y = mx + b$?

b. Describe what the variables y and x represent in the exploration.

c. What information besides the slope do you need to identify a specific line?

d. Compare the slopes of the equations in Parts **f** and **g** of the exploration.

Mathematics Note

Two lines that have equal slopes are **parallel**.

As shown in Figure **4**, for example, the graphs of the lines $y = 2x$ and $y = 2x + 3$ are parallel.

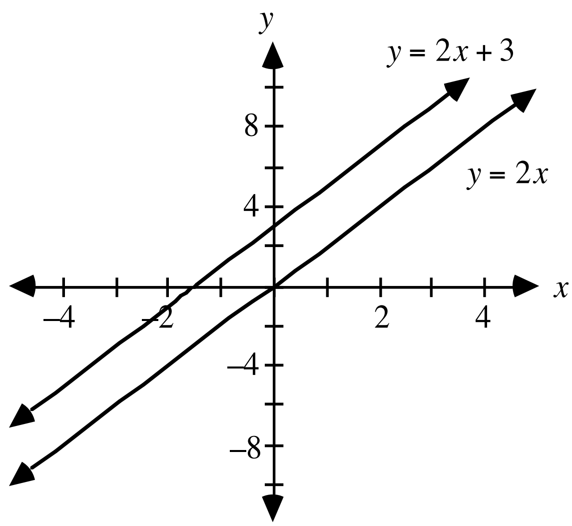


Figure 4: Graphs of $y = 2x$ and $y = 2x + 3$

Although vertical lines have no slope, any two vertical lines are also parallel.

e. Given the equation of a line in the form $y = mx + b$, describe how to determine its slope and y-intercept.

Assignment

2.1 Imagine that a 50-kg person plans to dance aerobically for 1 hr.

a. Use the information in Tables **3** and **5** to design a breakfast that will provide enough energy for this activity.

b. Write an equation in slope-intercept form that describes the number of kilocalories remaining from the meal in Part **a** at the end of each minute of dancing.

c. Write an equation in slope-intercept form that describes the number of kilocalories remaining from a 600-kcal meal at the end of each minute of dancing.

d. Sketch the graphs of both equations on a single set of axes.

2.2 **a.** Identify the y-intercept of a nonvertical line that passes through the origin (0,0).

b. Write an equation for the line with a slope of 3 and a y-intercept of 4.

c. Write an equation for the line that crosses the y-axis at (0,5) and has a slope of –2.

d. Write an equation for the line with a slope of $7/3$ and a y-intercept of $2/5$.

e. Write an equation for the line that crosses the y-axis at (0,–3) and has a slope of $2/5$.

2.3 The following two equations were rewritten in slope-intercept form by solving for y in terms of x.

$$\begin{aligned} y - 5 &= 7x \\ y - 5 + 5 &= 7x + 5 \\ y &= 7x + 5 \end{aligned} \qquad \begin{aligned} y + 5x &= 7x \\ y + 5x + (-5x) &= 7x + (-5x) \\ y &= 7x + (-5x) \\ y &= 2x \end{aligned}$$

Use similar methods to solve each of the following equations for y.

a. $y + 3 = 2x$

b. $y - 5 = 3x + 2$

c. $y + 6x = 2x - 7$

d. $3x + 4y = 7$

e. $2x - 3y = 6$

2.4 The following two equations were rewritten in slope-intercept form by multiplying both sides of each equation by the same quantity.

$$\begin{aligned} \frac{y}{3} &= 4x \\ 3 \bullet \frac{y}{3} &= 3 \bullet 4x \\ y &= 12x \end{aligned} \qquad \begin{aligned} 4y &= 16x \\ \frac{1}{4} \bullet 4y &= \frac{1}{4} \bullet 16x \\ y &= 4x \end{aligned}$$

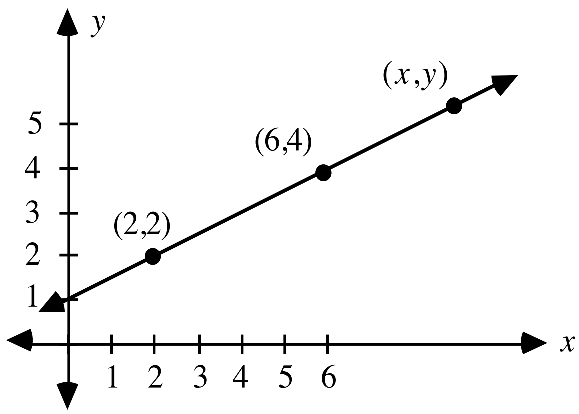
Use a similar method to solve each of the following equations for y.

a. $\dfrac{y}{-5} = 2x$

b. $7y = 4x$

c. $-2y = 8x$

2.5 As shown in the graph below, the points with coordinates (2,2), (6,4), and (x,y) are on the same line. In this case, the coordinates (x,y) represent any point on the line.

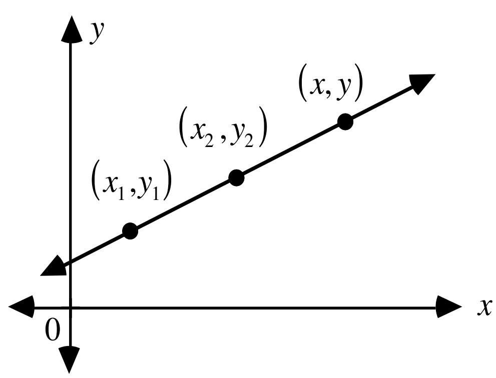


a. Calculate the slope of the line using the points $(2,2)$ and $(6,4)$.

b. Calculate the slope of the line using the points $(2,2)$ and (x,y).

c. Write a mathematical equation that describes the relationship between the two slopes calculated in Parts **a** and **b**.

2.6 As shown in the following graph, the points with coordinates (x,y), (x_1,y_1), and (x_2,y_2) are on the same line. In this case, the coordinates (x,y) represent any point on the line.



a. Write a representation for the slope of the line using the points (x_1,y_1), and (x_2,y_2).

b. Write a representation for the slope of the line using the points (x,y) and (x_1,y_1).

c. What relationship exists between the two representations you wrote in Parts **a** and **b**?

d. Write a mathematical equation that describes this relationship.

Mathematics Note

The equation of a line that passes through the point (x_1, y_1) and has a slope of m can be written in the form: $y - y_1 = m(x - x_1)$. This is the **point-slope form** of the equation of a line.

For example, the point-slope equation of a line that passes through the point $(2, -4)$ and has slope of -7 is $y - (-4) = -7(x - 2)$.

2.7 **a.** Write an equation in point-slope form for each of the following lines.

1. The line that passes through (5,8) and has a slope of –3.
2. The line that passes through (2,10) and (12,5).
3. The line that passes through the origin and has a slope of $2/3$.
4. The line that passes through (–6,4) and (–2,5).

b. Rewrite each equation from Part **a** in slope-intercept form.

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2.8 Some equations do not represent lines. Using appropriate technology, graph the following equations and identify which ones are linear.

a. $y = 2x + 3$

b. $y = x^3 + 2$

c. $y = 3x^2$

d. $y = \dfrac{4}{x}$

e. $y = -\dfrac{1}{3}x - 2$

f. $y = \sqrt{x}$

g. $y = 5x - 2$

2.9 Ordered pairs do not always involve integer values. Even when ordered pairs contain decimal values, however, the slope and equation of a line can still be found using the methods described in this activity.

a. Find the slope of the line through each of the following pairs of points:

1. $(6.4, 8.2)$ and $(1.9, 7.5)$
2. $(9.00, 0.12)$ and $(5.00, -0.46)$

b. Write equations in point-slope form for the lines in Part **a**.

c. Rewrite each equation from Part **b** in slope-intercept form.

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Activity 3

In the previous activities, you examined nutritional information using a number of different methods, including graphs.

Exploration

After breakfast, Maurice and Janet enjoy a leisurely ride on their bicycles. As Janet rides, the number of kilocalories remaining from her meal can be described by the equation $y = -6x + 500$, where x represents time in minutes.

Since Maurice is heavier than Janet, he uses more energy while cycling. The equation that describes the number of kilocalories remaining from his breakfast is $y = -9x + 710$.

a. Describe what the slope and y-intercept of each equation represents in this situation.

b. Graph both equations on the same set of axes.

c. Estimate the coordinates of the point common to both lines.

d. Describe what the coordinates of the common point represent in terms of kilocalories and time.

Mathematics Note

Two lines with different slopes that lie in the same plane always intersect in a single point. The coordinates of the point of intersection satisfy the equations of both lines.

For example, Figure **5** shows the graphs of $y = 3x$ and $y = 0.5x$. The two lines intersect at (0,0). Since $0 = 3(0)$ and $0 = 0.5(0)$, the coordinates of the point of intersection satisfy both equations.

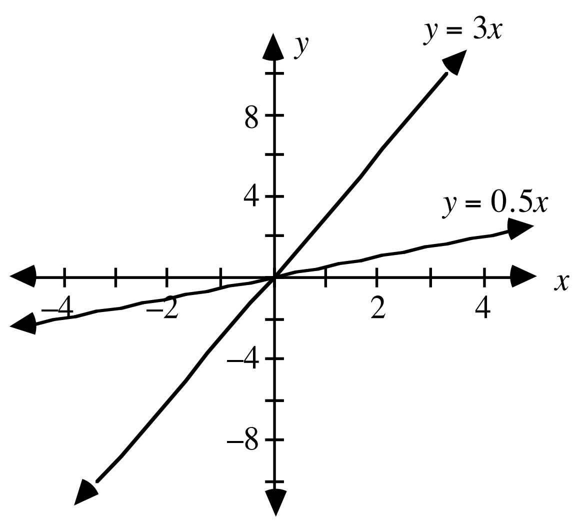


Figure 5: Graphs of $y = 3x$ and $y = 0.5x$

Discussion

a. What information do the two equations in the exploration give about Janet and Maurice?

b. What would have to be true about these two equations if their graphs did not intersect?

c. Describe how to determine when Maurice and Janet had the same number of kilocalories remaining from breakfast.

d. What advantages are there to writing the equation of a line in slope-intercept form?

Mathematics Note

The **distributive property** of multiplication over addition is a key connection between the two operations. It can be described as follows:

$$a(b+c) = ab + ac \text{ or } (b+c)a = ba + ca$$

For example,

$$\begin{aligned} 2(3+4) &= 2 \bullet 3 + 2 \bullet 4 \\ &= 14 \end{aligned} \quad \text{or} \quad \begin{aligned} (3+4)2 &= 3 \bullet 2 + 4 \bullet 2 \\ &= 14 \end{aligned}$$

e. The following two equations were rewritten in slope-intercept form using the distributive property:

$$\begin{aligned} y &= 2(x+3) \\ y &= 2 \bullet x + 2 \bullet 3 \\ y &= 2x + 6 \end{aligned} \qquad \begin{aligned} y &= -5(x-3) \\ y &= -5 \bullet x - (-5) \bullet 3 \\ y &= -5x - (-15) \\ y &= -5x + 15 \end{aligned}$$

Use the distributive property to write each of the following equations in slope-intercept form:

1. $y = 5(x-4)$

2. $y = -3(x+2)$

3. $y = -7(x-5)$

f. Using the distributive property, describe how to change an equation in point-slope form, $y - y_1 = m(x - x_1)$, to an equation in slope-intercept form, $y = mx + b$.

Assignment

3.1 It may require several steps to change some equations to slope-intercept form. For example, consider the following two equations:

$$\begin{aligned} y - 3 &= 2(x - 5) \\ y - 3 &= 2x - 10 \\ y &= 2x - 10 + 3 \\ y &= 2x - 7 \end{aligned} \qquad \begin{aligned} \frac{y}{3} &= x - 1 \\ 3\left(\frac{y}{3}\right) &= 3(x - 1) \\ y &= 3x - 3 \end{aligned}$$

Use similar methods to rewrite each of the following equations in slope-intercept form.

a. $y - 5 = 2(x - 4)$

b. $y + 2 = 2x + 11$

c. $y + 7 = 2(8x - 3)$

d. $\dfrac{y}{5} = x + 2$

e. $2y = 6x + 4$

f. $\dfrac{y}{3} = 2x - 6$

3.2 **a.** The equation $y - 700 = -16(x + 5)$, where x represents time in minutes, describes the number of kilocalories remaining from Ricardo's breakfast during a cross-country race. Write this equation in slope-intercept form and sketch its graph. What does the slope represent in this situation?

b. During the Olympic decathlon competition, Perry burned kilocalories at a rate described by the equation $y - 94 = 2(x + 3)$, where x represents time in minutes. Write this equation in slope-intercept form and sketch its graph. What does the y-intercept represent in this situation?

c. Kelly is spending a rainy summer afternoon reading a mystery novel. While reading, she uses kilocalories at a rate described by the equation $3y - 150 = 5(x - 12)$, where x represents time in minutes. What can you tell about Kelly's energy usage from this equation?

3.3 In Olympic competition, wrestlers must compete in specific classes according to their mass in kilograms. Two athletes, Norjar and Ruiz, plan to wrestle in the same class. On June 12, Norjar had a mass of 78 kg. By June 16, his mass had decreased to 77 kg. Ruiz's mass on June 14 was 62 kg. By June 18, it had risen to 64 kg.

a. Write the dates and masses for Norjar as two ordered pairs. What is the slope of the line containing these points?

b. Write an equation for the line describing Norjar's change in mass in slope-intercept form.

c. Determine an equation that describes Ruiz's change in mass.

d. Graph the two equations from Parts **b** and **c** on the same set of axes.

e. What does the point common to both lines represent in this situation?

f. What are the coordinates of this common point?

g. If the mass of each wrestler continues to change at its previous rate, on what date will their masses be equal?

h. What is the value of their masses on the date the masses are equal?

3.4 **a.** Determine the slope of the line that passes through the points (4,8) and (6,14).

b. Use the point (4,8) and the slope from Part **a** to write an equation of the line in point-slope form.

c. Use the point (6,14) and the slope from Part **a** to write an equation of the line in point-slope form.

d. Are the two equations you wrote in Parts **b** and **c** equivalent? Justify your response.

3.5 Imagine that you are the project director for the next space shuttle flight. For an experiment on energy usage in space, mission specialists Kimberly and Manuel must have the same mass on launch day. To reach this target mass, Manuel has increased his daily activities, while Kimberly has increased her consumption of calories. As project director, you have received the data in the table below.

Astronaut	Day 5	Day 10	Day 15	Day 20
Kimberly	no data	63.0 kg	64.5 kg	66.0 kg
Manuel	74.0 kg	no data	72.0 kg	71.0 kg

If the mass of both astronauts continues to change at the rate for the previous 16 days, on what day can the launch proceed?

3.6 Four other astronauts also have been training for the launch described in Problem **3.5**. They are willing to change their caloric intake as necessary to substitute for Manuel or Kimberly. As project director, you must select one of them as an alternate. The table below shows the mass of each astronaut at some time during the previous 16 days.

Astronaut	Mass
Britte	69.0 kg on day 5
Kwasi	72.5 kg on day 15
Sergei	78.0 kg on day 20
Yukawa	62.5 kg on day 10

a. Using the mass of Manuel and Kimberly and the launch day found in Problem **3.5**, write an equation for each potential alternate that describes the change in mass required to meet the mission restrictions.

b. As project director, which astronaut would you select to serve as an alternate for this launch? Explain your decision.

3.7 Rolf and Tanya meet every morning to exercise. While warming up, Tanya uses 100 kcal of energy. She then burns 4.4 kcal/min during her walk. Rolf uses 60 kcal of energy during his warm-up and burns 15 kcal/min while running.

a. The equation $y = 4.4x + 100$, where x represents time in minutes, describes Tanya's energy usage in kilocalories. Write an equation that describes Rolf's energy usage in kilocalories.

b. Graph both equations in Part **a** on the same set of axes. Estimate the coordinates of the point of intersection and describe what these coordinates represent in this situation.

3.8 **a.** Find the coordinates of the point of intersection for each of the following pairs of equations.

1. $y = 4x - 2$, $y = 2x - 6$
2. $y = 3(x + 5)$, $y = x - 9$
3. $y = \frac{1}{2}(x + 4)$, $y = 2(x - 3)$
4. $(y - 2) = 2(x - 4)$, $(y - 5) = (x - 6)$

b. Check each solution by verifying that the coordinates satisfy both equations.

* * * * *

3.9 Write an equation in slope-intercept form for the line that passes through the given point with the given slope.

a. $(-4,3)$, $m = -1$

b. $(3,-1)$, $m = 0$

c. $(-4,-2)$, $m = 1/2$

3.10 Denali is climbing a cliff 300 m high. After 30 min, she has moved 75 m up the cliff.

a. Assuming that Denali continues to climb at a constant rate, write an equation that describes her distance from the bottom in meters in terms of time in hours.

b. Identify the slope of the equation in Part **a** and describe what it represents in this situation.

c. How long will it take Denali to reach the top of the cliff?

3.11 Find the coordinates of the point common to each pair of lines below.

a. $\begin{cases} y = x - 3 \\ 4x + y = 32 \end{cases}$

b. $\begin{cases} -3x + 6y = 4 \\ 2x + y = 4 \end{cases}$

c. $\begin{cases} 3x + y = 13 \\ 2x - 4y = 18 \end{cases}$

* * * * * * * * * *

Summary Assessment

Rick has a mass of 66 kg. Before sitting down at his desk, he always eats breakfast and completes a morning workout. After eating a large breakfast, Rick exercises by running at the fast rate of 3.7 min/km. After eating a moderate breakfast, he exercises by bicycling at a slow rate.

For Rick, a large breakfast consists of hotcakes with butter, an omelet, a cup of 2% milk, and a glass of orange juice. This provides a total of 1,022 kcal. A moderate breakfast consists of an omelet, a cup of 2% milk, and a glass of orange juice, for a total of 522 kcal.

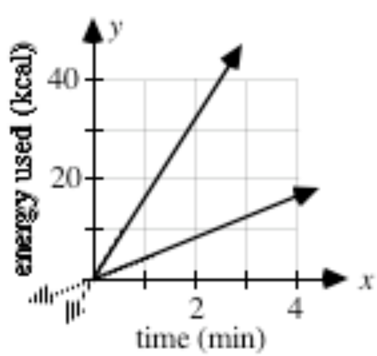
1. On a single set of axes, create a graph of each of the following:

- **a.** the kilocalories remaining from the large breakfast while Rick runs at the fast rate of 3.7 min/km
- **b.** the kilocalories remaining from the moderate breakfast while Rick bicycles at a slow rate

2.
- **a.** Estimate the coordinates of the intersection of the two lines in Problem **1**.
- **b.** Describe what these coordinates represent in this situation.

3. After a certain number of minutes of running or biking, Rick will have the same number of kilocalories remaining from either breakfast. Determine how long Rick can perform his job as a writer using these remaining kilocalories.

4. The graph below shows Rick's energy usage in kilocalories while running fast and cycling slowly.

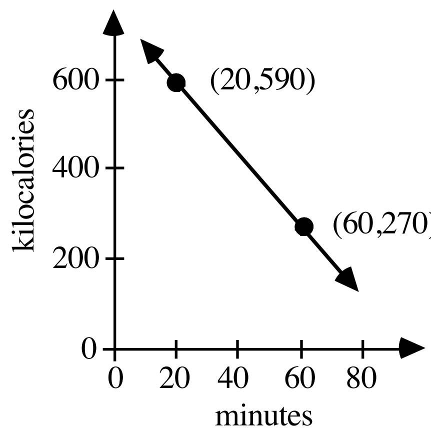


- **a.** Use the concepts of slope and rate to explain which line represents energy used while cycling and which line represents energy used while running.

b. Describe how the graph can be used to approximate the slope of each line.

c. Identify the domain and range of the relations that describe Rick's energy usage during both activities.

5. The following graph shows the number of kilocalories remaining after an entirely different breakfast and exercise routine for Rick.



a. Write an equation in point-slope form that describes the kilocalories remaining from breakfast as Rick exercises.

b. Rewrite the equation in Part **a** so that it shows the number of kilocalories in Rick's breakfast.

c. Identify an exercise that would result in this rate of energy use for Rick.

Module Summary

- A **calorie** is the amount of energy required to raise the temperature of 1 mL of water 1˚C.
- A **kilocalorie (kcal)**, 1000 cal, is the amount of energy needed to raise the temperature of 1 L of water 1˚C.
- A **dietary calorie** (normally referred to as a **Calorie** with a capital *C*) is equal to 1 kcal. The calorie-per-gram rating on most food labels measures dietary calories.
- To solve an equation of the form $y = mx$ for x, divide each side of the equation by m.
- An equation of the form $y = mx$ represents a line that passes through the origin where m is the **slope** of the line. The slope is the ratio of the change in vertical distance to the change in horizontal distance between any two points on the line.
- The slope of a line containing two points with coordinates (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

  where $y_2 - y_1$ is the change in the vertical distance and $x_2 - x_1$ is the change in the horizontal distance. If $x_1 = x_2$, the line is vertical and has no slope.
- A **relation** between two variables is a set of ordered pairs of the form (x, y).

  The **domain** of a relation is the set of first elements in the ordered pairs (the x-values). The **range** of a relation is the set of second elements in the ordered pairs (the y-values).
- A **function** is a relation in which each element of the domain is paired with an element of the range and each element of the domain occurs in only one ordered pair. A function may be described by a rule or equation.
- A horizontal line has a slope of 0 and an equation of the form $y = c$. All the points on a horizontal line have the same y-coordinate, c.
- A vertical line has no slope and an equation of the form $x = c$. All the points on a vertical line have the same x-coordinate, c.
- The y-coordinate of the point where a line intersects the y-axis is known as the ***y*-intercept**.
- The equation of a line with slope m and y-intercept b can be written in the form $y = mx + b$. This is the **slope-intercept** form of the equation of a line.

- Two lines that have equal slopes are **parallel**. Although vertical lines have no slope, any two vertical lines are also parallel.
- The equation of a line that passes through the point (x_1, y_1) and has a slope of m can be written in the form: $y - y_1 = m(x - x_1)$. This is the **point-slope form** of the equation of a line.
- Two lines with different slopes that lie in the same plane always intersect in a single point. The coordinates of the point of intersection satisfy the equations of both lines.
- The **distributive property** of multiplication over addition is a key connection between the two operations. It can be described as follows:

$$a(b + c) = ab + ac \text{ or } (b + c)a = ba + ca$$

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