## What Will We Do

## When the Well Runs Dry?



The availability of fresh, clean water affects us personally, locally, and globally. In this module, you'll use volume, rates of change, and linear models to assess individual water use.

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## What Will We Do When the Well Runs Dry?

## Introduction

What is a drink of fresh, clean water worth to you? Imagine a time when citizens pay $\$ 1.00$ for a glass of water and $\$ 10.00$ for a bath. At those prices, public fountains will disappear and green lawns will become rare luxuries. If this sounds far-fetched, think about the millions of people who already buy bottled water. Although humans can only survive a few days without water, its availability is often taken for granted.

## Discussion

a. Do you think that water will become as expensive as the introduction suggests? Why or why not?
b. Is there a water shortage in your area?
c. 1. What causes water shortages?
2. Can these causes be avoided?
d. 1. Estimate the amount of water you use each day.
2. How could you measure your daily water use?

## Activity 1

How can you estimate the amount of water you use in one day? Before confronting any of the issues involved with water use and conservation, you must first understand volume.

## Mathematics Note

A prism is a three-dimensional figure determined by two congruent polygons in parallel planes whose corresponding vertices are connected by segments. The two congruent and parallel faces are the prism's bases. The parallelograms formed by joining the corresponding vertices of the bases are the prism's lateral faces.

Prisms are named by the polygonal shape of the two bases. The height of a prism is the perpendicular distance between the bases.

For example, Figure 1 shows a trapezoidal prism and a triangular prism.


Figure 1: Two prisms
The amount of space occupied by an object is its volume. Volume is measured in units such as cubic centimeters $\left(\mathrm{cm}^{3}\right)$ or liters (L).

The volume ( $V$ ) of a prism may be found by multiplying the area of the polygonal base ( $B$ ) by the height $(h): V=B h$.

For example, if the area of the base of a triangular prism is $120 \mathrm{~cm}^{2}$ and its height is 30 cm , then its volume is $V=B h=(120)(30)=3600 \mathrm{~cm}^{3}$.

## Exploration 1

In this exploration, you determine the volume of a cube.
a. On a sheet of cardboard, draw a net for a cube with $10 \mathrm{~cm} \infty 10 \mathrm{~cm}$ faces.
b. Cut out, fold, and tape the net to form a cube.
c. Calculate the volume of the cube in each of the following units:

1. cubic centimeters $\left(\mathrm{cm}^{3}\right)$
2. cubic decimeters $\left(\mathrm{dm}^{3}\right)$.
d. Determine a relationship between cubic centimeters and cubic decimeters.
e. Estimate the number of liters (L) of water which the cube will hold.
f. Since it would not be practical to pour water into your cardboard cube, check your estimate by completing Steps 1-3 below.
3. Open one face of the cube.
4. Fill the cube with rice. Note: Make sure that the edges of the cube are securely taped. To prevent spills, you may wish to place the cube inside a bucket while pouring rice.
5. Use a 1-L container to measure the amount of rice in the cube.
g. Determine a relationship between liters and each of the following units:
6. cubic centimeters
7. cubic decimeters.

## Discussion 1

a. 1. Is the cube you created in Exploration 1 a prism? Why or why not?
2. How can you distinguish between the bases of a cube and its lateral faces?
b. 1. How many bases does a prism have?
2. How many lateral faces does a prism have?
3. How many lateral edges are there in a prism?
4. What is the total number of edges in a prism?
c. Describe the relationship between each of the following:

1. a cubic centimeter and a cubic decimeter
2. a liter and a cubic centimeter
3. a liter and a cubic decimeter.

## Exploration 2

In Exploration 1, you estimated then calculated the volume of a prism. In many communities, however, water supplies are stored in reservoirs that do not have polygonal bases. In this exploration, you examine a method for estimating the volume of objects that are not prisms.
a. On a $10 \mathrm{~cm} \infty 10 \mathrm{~cm}$ sheet of graph paper, draw a closed geometric figure that is not a polygon, such as the one shown in Figure $\mathbf{2}$ below.


Figure 2: A closed figure
b. To estimate the area of the figure, complete Steps 1-4 below.

1. Determine the area of each square on the graph paper.
2. Count the number of whole squares in the figure.
3. Count the number of partial squares in the figure and divide this number by 2 .
4. Find the sum of your answers to Steps 2 and 3. Multiply this sum by the area of one square determined in Step 1.
c. Using bases shaped like your geometric figure from Part a, make a three-dimensional solid 10 cm high, such as the one shown in Figure 3. Tape the edges of the solid as securely as possible.


Figure 3: A three-dimensional solid
d. Use the estimated area of the figure found in Part b to estimate the volume of the solid in each of the following units:

1. cubic centimeters
2. liters.
e. Check your estimate from Part d by opening your solid, filling it with rice, then measuring the amount of rice in the solid. Note: To prevent spills, you may wish to place the solid inside a bucket while pouring rice.

## Discussion 2

a. In Part bof Exploration 2, you estimated the area of a figure by counting squares. What other methods could you use to determine the area of a figure that is not a polygon?
b. 1. How did you determine the volume of the solid in Part $\mathbf{d}$ of Exploration 2?
2. How did this value compare to the amount of rice that filled the solid?
c. Describe how you could calculate the volume of a prism with bases shaped like each of the following:

1. triangles, rectangles, or trapezoids
2. polygons other than triangles, rectangles, or trapezoids.

## Assignment

1.1 José has a bathtub that holds 250 L of water. Thaddeus has a tub that has inside measurements of $5 \mathrm{dm} \infty 3 \mathrm{dm} \infty 15 \mathrm{dm}$. Which tub holds more water? Justify your response.
1.2 A forced-air heating duct has dimensions $2 \mathrm{~m} \infty 6 \mathrm{dm} \infty 50 \mathrm{~cm}$.
a. Make a scale drawing of this duct in centimeters.
b. Determine its volume in cubic centimeters.
1.3 Determine the volume of each of the following objects:
a. a water trough

b. a toilet tank

1.4 A Canadian town of 50,000 draws its water from the lake pictured below. The average depth of the lake is 15 m .

a. Estimate the volume of water in the lake and describe how you determined your estimate.
b. The average person in this town uses 380 L of water per day. If the lake is not replenished, how long will the water last?

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1.5 The municipal reservoir for a U.S. city of 6 million people lies in a V -shaped valley. One end of the reservoir is dammed, while the other is faced by a steep rock wall. The body of water itself is 6 km long, 2 km wide, and has an average depth of 400 m .
a. Make a scale drawing of this reservoir in kilometers.
b. Determine its volume in liters.
c. The average person in this city uses 420 L of water per day.

Assuming that the reservoir is not refilled, how long can it supply the city with water?
1.6 Although cubic centimeters are generally used to measure the volume of solid materials, health professionals also use these units to measure liquid volumes. In this setting, $1 \mathrm{~cm}^{3}$ is referred to as 1 cc . What is the relationship between cubic centimeters and milliliters?
1.7 The following diagram shows the dimensions of a swimming pool with rectangular ends. How many liters of water does this pool hold?

1.8 Construction companies often measure volume using cubic yards. How many cubic yards of concrete are needed to pave a driveway with the following dimensions: $20 \mathrm{ft} \infty 15 \mathrm{ft} \propto 6$ in?
1.9 The diagram below shows a small fish pond.


Determine the volume of the pond in each of the following units:
a. cubic meters
b. liters.

## Activity 2

How much water does a leaky faucet waste? How long does it take to fill a bathtub? How long would it take a broken water main to flood a basement? The answers to these questions depend partly on the water's rate of flow. In this activity, you use graphs to relate the rate of flow to the slope of a line. This activity extends the notions of lines found in "Yesterday's Food is Walking and Talking Today" to model a rate of flow.

## Exploration

In this exploration, you measure the rate at which water flows through a funnel. Note: To prevent damage to papers, books, and electronic equipment, rice is used to simulate water.
a. Use cardboard to make a sturdy funnel with a volume of at least 2 L and an opening at the narrow end approximately 2 cm in diameter.
b. 1. Hold the funnel over a bucket and place one hand under the narrow end to block the flow of rice. Pour 2.0 L of rice into the funnel.
2. Simultaneously remove your hand from the narrow end of the funnel and start a timer. Determine the time (to the nearest 0.1 sec ) required for the funnel to empty completely. Record the time in a table with headings like those in Table $\mathbf{1}$ below.

Table 1: Time data

| Funnel <br> Opening (cm) | Time for <br> Trial 1 | Time for <br> Trial 2 | Time for <br> Trial 3 |
| :---: | :---: | :---: | :---: |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |

c. Repeat Part b for two more trials.
d. To complete Table 1, repeat Parts a-c using funnels with openings of 3 cm and 4 cm .

## Mathematics Note

A rate compares the change in one quantity to the change in another quantity.
For example, the rate of flowing water compares a change in volume to a change in time. A rate of flow of 20 liters per minute may be written as $20 \mathrm{~L} / \mathrm{min}$
e. Calculate the average time (to the nearest 0.1 sec ) required for each size of funnel to empty.
f. Use these average times to determine the rate at which rice flowed from each funnel in the following units.

1. liters per second
2. liters per minute
3. liters per hour
g. Use the rates from Part $\mathbf{f}$ to predict the volume (to the nearest 0.1 L ) that could flow through each funnel in $1 \mathrm{~min}, 2 \mathrm{~min}, 5 \mathrm{~min}$, and 10 min .
Record your predictions in a table with headings like those in Table 2.
Table 2: Time vs. volume data

| Time <br> $(\min )$ | Volume (L) |  |  |
| :---: | :---: | :---: | :---: |
|  | 2-cm Funnel | 3-cm Funnel | 4-cm Funnel |
| 1 |  |  |  |
| 2 |  |  |  |
| 5 |  |  |  |
| 10 |  |  |  |

h. 1. Make a scatterplot of the predictions in Table 2 for the $2-\mathrm{cm}$ funnel. Let $x$ represent the time in minutes and $y$ represent the volume in liters.
2. Find a line that fits the points as closely as possible. Draw the line on the same set of axes as the scatterplot.
3. Determine the slope of the line.
i. Use your line to estimate the volume that could flow through the funnel in 7 min and in 12 min . Note: Save your work in this exploration for use in the assignment.

## Discussion

a. Compare the data you collected in Table $\mathbf{1}$ with that of others in the class. What may have caused the differences you observe?
b. What relationship did you observe between the size of the funnel opening and the rate of flow?
c. Do you think that the rates determined in Part $\mathbf{f}$ of the exploration are reliable? Explain your response.
d. Describe the relationship between the rate of flow and the slope of the line you found in Part $\mathbf{h}$ of the exploration.
e. If a leaky faucet drips once every second and each drip has a volume of 0.50 mL , what volume of water will leak from the faucet in 1 day?

## Assignment

2.1 Use your line from Part $\mathbf{h}$ of the exploration to complete the following.
a. 1. Identify the $y$-intercept of the line.
2. What does the $y$-intercept represent in terms of the exploration?
b. Write an equation of the line in the form $y=m x+b$, where $y$ represents volume in liters and $x$ represents time in minutes.
c. Use your equation to predict the volume that could flow through the funnel in 7 min and in 12 min .
d. Compare these predictions with the ones you made in Part $\mathbf{i}$ of the exploration.
2.2 a. 1. Make a scatterplot of the data in Table $\mathbf{2}$ for the $3-\mathrm{cm}$ funnel.
2. Find a line that fits the data as closely as possible. Draw the line on the same set of axes as the scatterplot.
3. Write an equation of the line in the form $y=m x+b$, where $y$ represents volume in liters and $x$ represents time in minutes.
b. Repeat Part a for the data for the $4-\mathrm{cm}$ funnel.
2.3 Graph the equations found in Problem 2.1b and Problem 2.2a and b on the same coordinate system. Describe any similarities or differences you observe and explain why they occur in terms of the exploration.
2.4 The table below shows the average time required to empty a $2.0-\mathrm{L}$ water bottle using openings of different sizes.

| Bottle Opening (cm) | Time to Empty (sec) |
| :---: | :---: |
| 0.6 | 78.0 |
| 1.3 | 18.0 |
| 2.5 | 5.0 |

a. Use this data to determine the rate at which water flowed through each opening in the following units.

1. liters per second
2. liters per minute
3. liters per hour
b. Write an equation of the form $y=m x+b$ to describe the flow of water through each opening, where $y$ represents volume in liters and $x$ represents time in minutes.
2.5 The All School Club is a service organization at Larry's school. When Larry became president of the club, he decided to start a membership drive. The campaign hopes to sign 4 new members per week. After 10 weeks, this should bring the club's total membership to 375 students. The following table shows the membership during the first five weeks of the campaign.

| Week | Total Membership |
| :---: | :---: |
| 1 | 335 |
| 2 | 339 |
| 3 | 343 |
| 4 | 347 |
| 5 | 351 |

a. 1. Draw a scatterplot of this data. Let $x$ represent the week and $y$ represent the total membership.
2. Draw a line that fits the data as closely as possible.
b. 1. Is the slope of the line positive or negative?
2. What does the slope indicate in terms of the membership drive?
c. 1. Identify the $y$-intercept of the line.
2. What does the $y$-intercept represent in terms of the membership drive?
2.6 Crickets make chirping sounds by rubbing their wings together. For some crickets, the relationship between the number of chirps per minute and the air temperature is very close to a line. When the air temperature is $20^{\circ} \mathrm{C}$, these crickets chirp 124 times per minute. When the temperature is $26.6^{\circ} \mathrm{C}$, they chirp 172 times per minute.
a. Graph this information on a scatterplot. Let $x$ represent the temperature in degrees Celsius and $y$ represent the number of chirps per minute.
b. Draw a line that fits the data as closely as possible.
c. Identify the $y$-intercept of this line.
d. Determine the slope of the line.
e. Write an equation of the line in the form $y=m x+b$.
f. Predict the temperature at which these crickets make 150 chirps per minute.
2.7 Lumber is typically sold in units called board feet. The table below shows the number of board feet contained in lengths of three common dimensions of lumber: $1 \times 4,2 \times 4$, and $2 \times 12$. (A $2 \times 4$ is approximately 2 in . thick and 4 in . wide.)

|  | Length |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Dimensions | $\mathbf{5} \mathbf{f t}$ | $\mathbf{1 0} \mathbf{f t}$ | $\mathbf{2 0} \mathbf{f t}$ | $\mathbf{3 0} \mathbf{f t}$ | $\mathbf{4 0} \mathbf{f t}$ |
| $1 \times 4$ | $1 \frac{2}{3}$ | $3 \frac{1}{3}$ | $6 \frac{2}{3}$ | 10 | $13 \frac{1}{3}$ |
| $2 \times 4$ | $3 \frac{1}{3}$ | $6 \frac{2}{3}$ | $13 \frac{1}{3}$ | 20 | $26 \frac{2}{3}$ |
| $2 \times 12$ | 10 | 20 | 40 | 60 | 80 |

a. On a single coordinate system, create a scatterplot that shows the number of board feet in a piece of lumber versus its length for each of the following dimensions:

1. $1 \times 4$
2. $2 \times 4$
3. $2 \times 12$.
b. Describe the relationship between the length of a piece of lumber and the number of board feet it contains.
c. Draw a line that fits each scatterplot as closely as possible.
d. Write an equation for each line from Part $\mathbf{c}$ in the form $y=m x+b$
e. Find the dimensions, including the length, of a piece of lumber that contains 1 board foot. Describe how you determined your response.

## Activity 3

An aquifer is a water-filled layer of sand or gravel-a sort of underground deposit of water. The High Plains Aquifer, which underlies parts of eight states, is one of the largest known. It contains as much water as Lake Huron: about 4.24 quadrillion liters. For this reason, some geologists call it the "sixth Great Lake."

The geographical boundaries of the aquifer are shown in Figure 4.


Figure 4: The High Plains Aquifer

## Source: Dugan and Schild, Water-Level Changes in the High Plains Aquifer

This region accounts for nearly $15 \%$ of the grain, $25 \%$ of the cotton, and almost $40 \%$ of the beef produced in the United States. Much of this production is due at least in part to the availability of water from the High Plains Aquifer.

Although rain and snow help to replenish the aquifer, scientists predict that, in some areas, it may be depleted in less than 100 years. To prevent this, many farmers are planting crops that require less water, while others are irrigating in ways that conserve the resource. Without irrigation, however, acres of productive farms would return to the original prairie.

## Exploration

To monitor changes in the aquifer, researchers drilled wells in Chase County, Nebraska. They measured the distance from the surface of the ground to the water. As shown in Figure 5, high water readings were taken in June, while low water readings were taken in September.


Figure 5: High and low water lines

Table 3 shows the distances from the surface to the high and low water lines from 1964 to 1978. The information in Table $\mathbf{3}$ seems to indicate that some sort of change is occurring in the aquifer. In this exploration, you determine whether or not this data can be reasonably modeled by a line.
Table 3: Distance from surface to water line (in meters)

| Year | High | Low | Year | High | Low |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 9 6 4}$ | 17.1 | 18.1 | $\mathbf{1 9 7 2}$ | 19.7 | 21.3 |
| $\mathbf{1 9 6 5}$ | 17.2 | 18.1 | $\mathbf{1 9 7 3}$ | 19.8 | 22.3 |
| $\mathbf{1 9 6 6}$ | 17.1 | 18.2 | $\mathbf{1 9 7 4}$ | 20.1 | 22.9 |
| $\mathbf{1 9 6 7}$ | 17.4 | 18.2 | $\mathbf{1 9 7 5}$ | 20.6 | 23.8 |
| $\mathbf{1 9 6 8}$ | 17.7 | 19.4 | $\mathbf{1 9 7 6}$ | 21.3 | 26.0 |
| $\mathbf{1 9 6 9}$ | 18.1 | 19.8 | $\mathbf{1 9 7 7}$ | 22.3 | 26.5 |
| $\mathbf{1 9 7 0}$ | 18.9 | 20.7 | $\mathbf{1 9 7 8}$ | 22.7 | 27.6 |
| $\mathbf{1 9 7 1}$ | 19.4 | 21.3 |  |  |  |

Source: Dugan and Schild, Water-Level Changes in the High Plains Aquifer.
a. Enter the distances to the high water level in a spreadsheet with headings like those in Table 4. Note that the first column is headed "Years after 1964." This means that 1964 corresponds with 0,1965 corresponds with 1 , and so on.

Table 4: Distances to high water level after 1964

| Years after 1964 | Distance (m) |
| :---: | :---: |
| 0 | 17.1 |
| 1 | 17.2 |
| $\vdots$ | $\vdots$ |
| 14 | 22.7 |

b. Create a scatterplot of the data from Part a. Let $y$ represent the distance and $x$ represent the number of years after 1964.
c. Mathematical models often use graphs or equations to describe relationships that arise in real-world situations. A mathematical model that consists of a line or its equation is a linear model.

1. Draw a linear model that fits the data from Part a as closely as possible.
2. Find the equation of the line.
d. 1. Add a third column to your spreadsheet with the heading "Predicted Distance."
3. Use the equation from Part $\mathbf{c}$ to predict the distance to the high water level for each year in Table 3.
4. Enter these predicted values in the appropriate column of the spreadsheet.

## Mathematics Note

As shown in Figure 6, a linear model may not exactly fit every data point. However, even when a line does not fit every point, it can still provide a reasonable model of the data.

A residual is the difference between an observed value and the predicted value. In Figure 6, the residual for each data point is the difference between the $y$-coordinate of the data point and the corresponding $y$-value of the model. Since data points may be located above or below the line, the values of residuals may be positive or negative.


Figure 6: A linear model
The absolute value of a residual is a measure of the distance from the data point to the linear model. In general, the smaller the sum of the absolute values of the residuals, the more closely a line approximates the data.
e. 1. Add a fourth column to your spreadsheet with the heading "Residuals."
2. Using your predicted distances from Part d, find and enter the residual for each data point.
3. Determine the sum of the residuals.
f. 1. Add a fifth column to your spreadsheet with the heading "Absolute Value of the Residuals."
2. Calculate and record the absolute value of each residual.
3. Determine the sum of the absolute values of the residuals.
g. Divide the sum of the absolute values of the residuals by the number of data points. This is the average distance from each data point to the model.

## Discussion

a. Does the line you drew in Part $\mathbf{c}$ of the exploration fit the data exactly? Explain your response.
b. How could you use the average distance from each data point to the model to determine if a model fits reasonably well?
c. Does your line appear to be a good model for the data?
d. If the sum of the absolute values of the residuals is 0 , then a linear model fits the data perfectly. Is this also true for the sum of the residuals? Use an example to justify your response.
e. Describe the slope of your linear model. What might this slope indicate about the aquifer in Chase County?
f. Do you think that water-level data has been recorded for enough years to support your response to Part $\mathbf{e}$ of the discussion? Why or why not?
g. What practices might be implemented to ensure that future generations can continue to draw water from the High Plains Aquifer?
h. Is there an aquifer in your area? If so, where would you go to find more information about it?

## Assignment

3.1 a. Create a scatterplot like the one in the exploration for the distances to the low water levels in Table 3.
b. Draw a line that closely approximates the data.
c. Write an equation of the line in the form $y=m x+b$. Describe the method you used to find this equation.
d. Use your equation to predict the distances to the low water levels of the aquifer from 1964 to 1978.
e. 1. Find the residual of each point in the scatterplot.
2. Calculate the sum of the absolute values of the residuals.
f. Explain whether or not your line is a good model for the data.
3.2 Imagine that you have a leaky pipe under the kitchen sink. To catch the water, you place a coffee can under the leak.

The following table shows the time that the can has been in place and the total mass of the can and water.

| Time (min) | Total Mass (g) |
| :---: | :---: |
| 5 | 160 |
| 10 | 350 |
| 15 | 470 |
| 20 | 570 |
| 25 | 790 |

If $y$ represents the total mass in grams and $x$ represents the time in minutes, which of the following equations more closely approximates this data? Support your choice by determining the sum of the absolute values of the residuals for each model.

$$
\begin{aligned}
& y=30 x+16 \\
& y=33 x+15
\end{aligned}
$$

3.3 When an inflated balloon is placed in a freezer, its volume decreases as the air inside it grows colder. When the balloon is removed from the freezer, its volume increases as it warms.

The following table shows some data comparing the temperature of the air in the balloon to its volume.

| Temperature $\left({ }^{\circ} \mathbf{C}\right)$ | Volume (mL) |
| :---: | :---: |
| 10 | 500 |
| 20 | 520 |
| 30 | 531 |
| 40 | 558 |

a. Make a scatterplot of this data. Let $y$ represent the volume in milliliters and $x$ represent the temperature in degrees Celsius.
b. Draw a line that closely approximates the data.
c. Write an equation of the line in Part $\mathbf{b}$ in slope-intercept form.
d. 1. Find the sum of the absolute values of the residuals.
2. Describe what the calculation from Part d1 means in relationship to the linear model.
e. Predict the volume of the balloon at an air temperature of $100^{\circ} \mathrm{C}$.
3.4 Jason is raising hamsters to sell to pet stores. As shown in the following table, he has 64 hamsters after 5 months.

| Month (x) | Hamsters (y) |
| :---: | :---: |
| 1 | 2 |
| 2 | 6 |
| 3 | 18 |
| 4 | 35 |
| 5 | 64 |

The local pet supplier buys hamsters only in lots of 200. In order to predict when he will have enough hamsters to sell, Jason decides to create a graph of the data in the table.
a. Make a scatterplot of Jason's data.
b. Draw a line that approximates the data.
c. Write an equation of the line in slope-intercept form.
d. Find the sum of the absolute values of the residuals.
e. Do you think Jason should use a linear model to predict when he will have 200 hamsters? If so, predict the time. If not, explain why not.
3.5 Over the past 70 years, Olympic swimmers have lowered the winning time in the women's 400 -meter freestyle by more than 2 min . The table below shows the winning times in each race from 1924 to 1988, rounded to the nearest 0.01 min .

| Year | Time | Year | Time | Year | Time |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1924 | 6.04 | 1952 | 5.20 | 1972 | 4.32 |
| 1928 | 5.71 | 1956 | 4.91 | 1976 | 4.17 |
| 1932 | 5.48 | 1960 | 4.84 | 1980 | 4.15 |
| 1936 | 5.44 | 1964 | 4.72 | 1984 | 4.12 |
| 1948 | 5.30 | 1968 | 4.53 | 1988 | 4.06 |

a. 1. Graph this information on a scatterplot. Let $x$ represent the number of years after 1920 and $y$ represent time in minutes.
2. Draw a line that fits the data as closely as possible.
b. Write an equation of the line in the form $y=m x+b$.
c. Determine how well your model fits the data.
d. Use your model to predict the winning time in the 400 -meter freestyle in each of the following years:

1. 2000
2. 2080
e. Do you think that the predictions you made in Part $\mathbf{e}$ are realistic? Explain your response.
3.6 Another method for determining how well a linear model fits a specific data point uses percent error. The percent error is the absolute value of the difference between the estimated value and the measured value, divided by the measured value, and expressed as a percentage:

$$
\text { percent error }=\left|\frac{\text { estimated }- \text { measured }}{\text { measured }}\right| \bullet 100
$$

For example, if a data point has coordinates $(10,24)$ and the corresponding point on the linear model has coordinates $(10,21)$, then the percent error can be calculated as follows:

$$
\left|\frac{21-24}{24}\right| \cdot 100=12.5 \%
$$

If the measured value is 0 , then percent error cannot be calculated.
a. Using the predicted distances to the high water levels from Part d of the exploration, find the percent error for your model for each of the following years:

1. 1964
2. 1970
3. 1975 .
b. The equation $y=0.7 x+17$ is one possible model for the distances to the low water levels in Table 3. Find the percent error for this model for each of the following years:
4. 1965
5. 1970
6. 1975. 

c. Do you think that percent error would be a good measure of fit for a set of data? Why or why not?

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## Research Project

On average, each resident of the United States uses about 420 L of water per day. How does your daily water use compare to this value? In the following research project, you analyze your own personal water use.

Create a table with headings like the one below. To complete the table, you will need to develop some innovative ways to measure water use. For example, how can you determine the volume of water used in a shower or bath? And how much water does it take to flush a toilet?

| Use | Rate of Use | No. of Uses or Time <br> Used | Daily Volume |
| :---: | :---: | :---: | :---: |
| washing machine |  |  |  |
| dishwasher |  |  |  |
| bathroom sink |  |  |  |
| kitchen sink |  |  |  |
| toilet |  |  |  |
| shower or bath |  |  |  |
| other |  |  |  |
| other |  | Total |  |
|  |  |  |  |

Your report should include a description of the methods you used to determine each measurement, a comparison of your daily water usage to the national average, and a discussion of any differences you observe.

## Summary Assessment

1. As shown in the diagram below, four farms share irrigation water from the same reservoir. The average depth of the reservoir is 8.0 m . Each farmer has 2 center-pivot irrigation sprinklers. Each sprinkler can pump more than 4000 L of water per minute.

scale: $1 \mathrm{~cm}=200 \mathrm{~m}$
a. During the 3-month growing season, the farmers plan to irrigate their fields for 16 hours a day, every day. Assuming that the reservoir is not refilled during the growing season, do they have enough water for their plan? Justify your response, showing all calculations.
b. If the farmers do not modify their plan, for how many days can the reservoir supply them with water?
c. What is the maximum number of hours per day the farmers can operate the sprinklers and still irrigate for the full 3 months?
2. The table below shows the volume of water flowing from a garden hose over time.

| Time (min) | Volume (L) |
| :---: | :---: |
| 0.00 | 0.0 |
| 1.00 | 15.0 |
| 2.00 | 29.0 |
| 3.00 | 48.0 |
| 4.00 | 65.0 |
| 5.00 | 73.0 |
| 6.00 | 94.0 |

a. Draw a scatterplot of the data. Let $y$ represent volume in liters and $x$ represent time in minutes.
b. Draw a line on the scatterplot that closely models the data points.
c. Write an equation for the line in the form $y=m x+b$.
d. Determine the average rate of flow in each of the following units:

1. liters per minute
2. liters per second
3. liters per hour
e. How is the rate of flow in liters per minute related to the graph in Part b? Include mathematical terms and concepts in your response.
f. Assuming that the rate of flow remains constant, determine the volume of water which will flow from the hose in:
4. 15 min
5. 2 hr .
g. 1. Find the absolute value of the residual for each data point.
6. Determine the sum of the absolute values of the residuals.
7. Describe what this sum tells you about your model.
h. Another possible model for this data is the line $y=14 x+2$. Compare the sum of the absolute values of the residuals found using this equation to the sum you calculated in Part $\mathbf{g}$. Use the comparison to determine which equation is the better model.

## Module

## Summary

- A prism is a three-dimensional figure determined by two congruent polygons in parallel planes whose corresponding vertices are connected by segments. The two congruent and parallel faces are the prism's bases. The parallelograms formed by joining the corresponding vertices of the bases are the prism's lateral faces.
- Prisms are named by the polygonal shape of the two bases. The height of a prism is the perpendicular distance between the bases.
- The amount of space occupied by an object is its volume. Volume is measured in units such as cubic centimeters $\left(\mathrm{cm}^{3}\right)$ or liters (L).
- The volume ( $V$ ) of a prism may be found by multiplying the area of the polygonal base $(B)$ by the height $(h): V=B h$.
- A rate compares the change in one quantity to the change in another quantity.
- Mathematical models often use graphs or equations to describe relationships that arise in real-world situations. A mathematical model that consists of a line or its equation is a linear model.
- The difference between the $y$-coordinate of a data point and the corresponding $y$-value of a linear model is a residual. Since data points may be located above or below the line, the values of residuals may be positive or negative.
- The absolute value of a residual is a measure of the distance from the data point to the linear model. In general, the smaller the sum of the absolute values of the residuals, the more closely a line approximates the data.


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