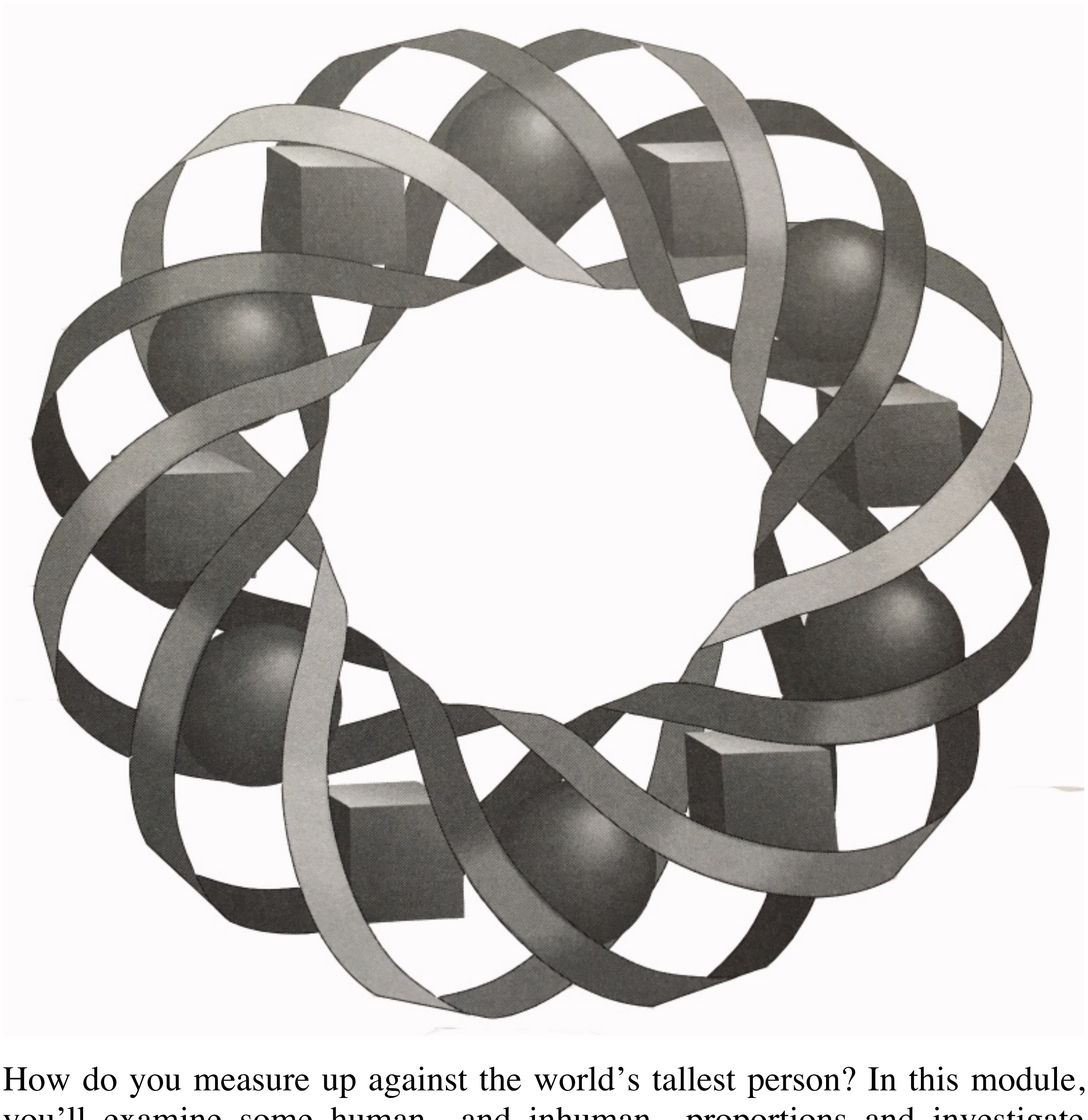
Are You Just A Small Giant?



How do you measure up against the world's tallest person? In this module, you'll examine some human—and inhuman—proportions and investigate the concept of similarity.

Randy Carspecken • Bonnie Eichenberger • Sandy Johnson • Terry Souhrada

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Introduction

According to the 1990 *Guinness Book of World Records*, Robert Wadlow (1918–1940) was the world's tallest human. Wadlow's measurements at the time of his death are listed in Table **1**.

Table 1: Robert Wadlow's measurements at death

Height	272 cm
Weight	1950 N
Mass	199 kg
Shoe Length	47 cm
Hand Length	32 cm
Arm Span	289 cm
Ring Size	25

How does your size and shape compare with Wadlow's? If Robert Wadlow were alive today, would the two of you look similar? The answer to this question depends on the definition of *similar*.

Mathematics Note

Two ratios, a/b $(b \neq 0)$ and c/d $(d \neq 0)$, are **proportional**, or **in proportion**, if:

$$\frac{a}{b} = \frac{c}{d}$$

When two such ratios are proportional, it is also true that a/c and b/d are proportional, where $c \neq 0$ and $d \neq 0$.

In mathematics, two objects are **similar** if they have the same shape and the ratios of corresponding lengths are proportional. The ratio of corresponding sides is the **scale factor**.

For example, Figure **1** shows two similar triangles, *ABC* and *DEF*.

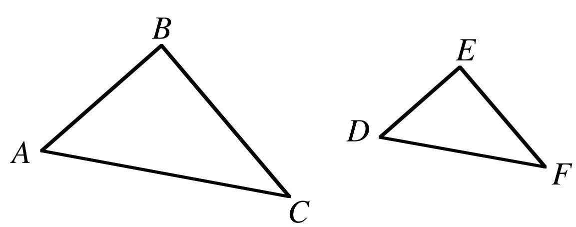


Figure 1: Two similar triangles

Since ABC and DEF are similar, the ratios of corresponding sides are proportional and equal to the scale factor. In this case, the scale factor is 1.5.

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = 1.5$$

When two triangles are similar, the measures of the corresponding angles also are equal.

Discussion

a. When a photograph is enlarged, is the new image similar to the original? Explain your response.

b. Explain why a photographic image of a person is not similar to the actual person.

c. **1.** How are scale factors used in scale drawings?

2. Do scale drawings always produce similar figures?

d. Describe the equation that results when the following proportion is solved for y.

$$\frac{y}{x} = \frac{a}{b}$$

e. In the module "Oil: Black Gold," you learned that a direct proportion can be described by a linear equation of the form $y = mx$, where m is the constant of proportionality.

Does the equation you described in Part **d** represent a direct proportion? If so, identify the constant of proportionality. If not, explain why not.

Activity 1

Are Robert Wadlow's dimensions proportional to those of other people? In this activity, you compare Wadlow's measurements at the time of his death with the measurements of other humans.

Exploration

In this exploration, you use proportions to investigate similarity in scale drawings. You also use proportions to determine if two people are similar.

Figure **2** shows a scale drawing of Robert Wadlow and his father.

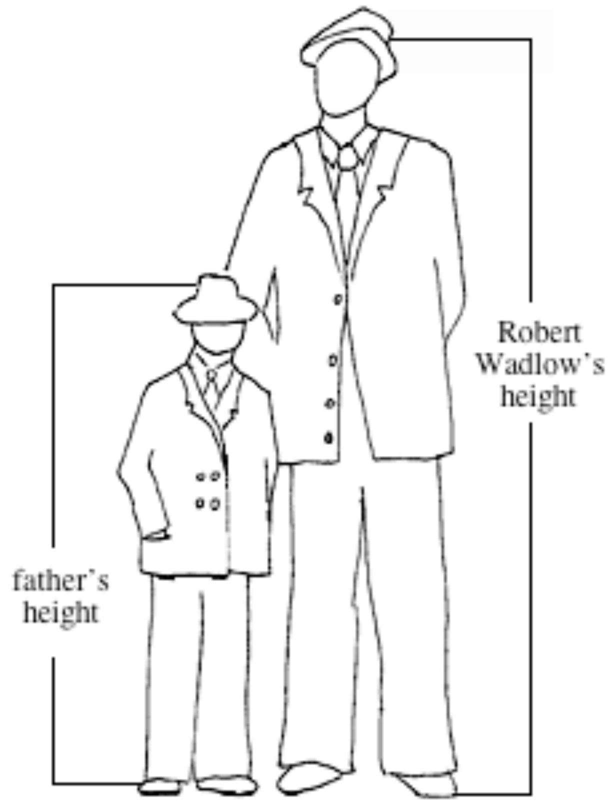


Figure 2: Scale drawing of Robert Wadlow and his father

a. Obtain a second scale drawing of Robert Wadlow and his father from your teacher.

   1. Measure Robert Wadlow's height, in centimeters, in both drawings.

   2. Determine the ratio of these two measurements.

b. **1.** Measure the height, in centimeters, of Wadlow's father in both drawings.

   2. Determine the ratio of these two measurements.

c. **1.** Do your results in Parts **a** and **b** indicate that the measurements in the two drawings are proportional?

   2. Are the two drawings similar? Explain your response.

d. Provide additional evidence for (or against) your response to Part **c1** by taking a third measurement on each drawing and determining the ratio of the measurements.

e.
1. Write the ratio of Wadlow's actual height to the height of his image in Figure **2**.
2. Use this ratio and the height of Wadlow's father in Figure **2** to write a proportion that can be used to determine the father's actual height.
3. Determine the father's actual height.

f. Repeat Part **e** using the scale drawing on the template.

g. Compare the height you determined in Part **f** with the height you determined in Part **e3**.

h.
1. Measure your height and shoe length in centimeters.
2. Use these measurements to determine if you are similar to Robert Wadlow at the time of his death.

i.
1. Use your height and shoe length and the shoe length of a classmate to predict the height of that classmate.
2. Compare the predicted height with your classmate's actual height.
3. What do your results indicate about the two of you?

Discussion

a. Can you use your measurements from Part **h** of the exploration to make predictions about the measurements of other people? Explain your response.

b. If two polygons are similar, their corresponding sides are proportional. What is the relationship between their corresponding angles?

c. In the equation below, why can't b and d be equal to 0?

$$\frac{a}{b} = \frac{c}{d}$$

d. Describe how a photocopier that enlarges or reduces preserves similarity using the following terms: *proportional, similar*, and *scale factor*.

Assignment

1.1 Use Robert Wadlow's shoe length and height to estimate each of the following:

a. the height of a similar person with a shoe length of 40 cm

b. the shoe length of a similar person with a height of 165 cm.

1.2 **a.** A family of five all have proportional shoe lengths and heights. The shoe length and height, in centimeters, of one member of the family can be written as the ordered pair (21,126). Use these dimensions to complete the ordered pairs for the rest of the family.

1. brother: (12, __)
2. mother: (24, __)
3. sister: (__, 108)
4. father: (__, 216)

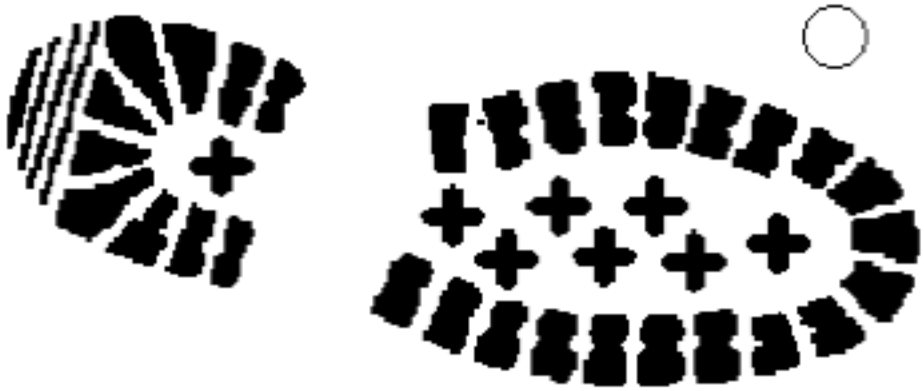
b. In the following table, the ratios of corresponding lengths are expressed in the form "row:column." Use the ordered pairs from Part **a** to complete a copy of the table.

Family Member	father	mother	brother	sister
father	1:1	3:2		
mother				
brother			1:1	2:3
sister				

1.3 A person similar to you has a thumb length of 6.5 cm. Use your thumb length and height to estimate this person's height.

1.4 A newborn baby is 46 cm long, with a head circumference of 33 cm. Are you similar to this baby? Explain your response.

1.5 During a criminal investigation, a detective photographed the print of a suspect's shoe next to the outline of a penny, as shown below.



a. Use this photograph to determine the length of the footprint.

b. Predict the suspect's height if the suspect is similar to Robert Wadlow.

c. What other useful predictions might the detective make from this footprint?

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1.6 A basketball player's height is 2.5 m. Assuming that the player's shape is similar to yours and that height is proportional to wrist circumference, determine the circumference of the player's wrist.

1.7 **a.** Using a word-processing program, Theo can keyboard 2000 words in 50 min. He has 90 min available to enter a 3500-word essay in the computer. Will he be able to finish in time?

b. To check the spelling in a document, the word processor requires 3 min per 1000 words. How much time will it take to check the spelling in Theo's essay?

c. Will Theo need more than 90 min to finish his essay if he also plans to check the spelling? Explain your response.

1.8 When an object is caught in a whirlpool, its speed is inversely proportional to its distance from the whirlpool's center. If an object's speed is 7.8 cm/sec at a distance of 900 cm from the center, what is its speed at a distance of 10 cm from the center?

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Activity 2

In Activity **1**, you used scale factors to predict lengths in similar figures. In this activity, scale factors will be used to predict areas of similar figures.

Exploration

All squares are similar. How is the scale factor for two squares related to their areas? To answer this question, complete Parts **a–e**.

a. Table **2** lists the side lengths of nine different squares. Draw each of these squares on a sheet of centimeter graph paper.

Table 2: Side lengths of squares

Square	Side Length (cm)
A	1.0
B	2.0
C	3.0
D	4.0
E	5.0
F	6.0
G	1.5
H	2.5
I	1.2

b. How do you think scale factors might be used to predict the areas of these squares?

c. Complete Table **3** using the formula for the area of a square.

Table 3: Side lengths of squares

Square	Side Length (cm)	Area (cm^2)
A	1	
B	2	
C	3	
D	4	
E	5	
F	6	
G	1.5	
H	2.5	
I	1.2	
J	z	
K	y	

d. Use the information in Table **3** to complete Table **4**.

Table 4: Scale factors and ratios of areas of squares

Squares	**Ratio of Side Lengths (Scale Factor)**	**Ratio of Areas**
A to B		
E to A		
B to D		
E to C		
I to A		
G to A		
J to F		
J to K		

Mathematics Note

To raise a fraction to a power n, where n is a non-negative integer, both the numerator and the denominator may be raised to the indicated power. In general,

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

where $b \neq 0$.

For example, the fraction $3/4$ can be raised to the third power as follows:

$$\left(\frac{3}{4}\right)^3 = \frac{3^3}{4^3} = \frac{27}{64}$$

e. Use the information in Table **4** to describe the relationship between the scale factor for two squares and the ratio of their areas.

f. Predict whether the relationship between scale factor and area for squares is also true for similar triangles.

g. To test your prediction in Part **f**, complete Steps **1–4** below.

1. Using a geometry utility, construct a triangle *ABC*.
2. Connect the midpoints of the sides of triangle *ABC* to form triangle *IGH*. Your construction should now resemble the one shown in Figure **3**.

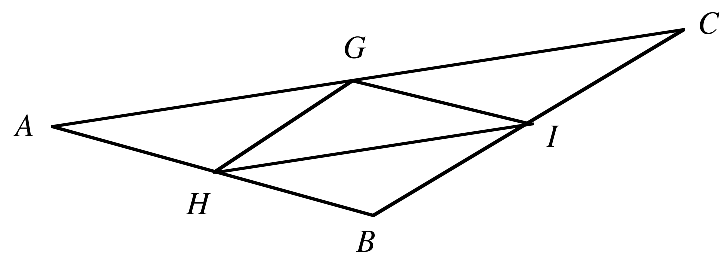


Figure 3: Triangles *ABC* and *IGH*

3. Using the lengths of corresponding sides and the measures of corresponding angles, prove that triangles *ABC* and *IGH* are similar.
4. Determine the relationship between the scale factor of the two triangles and the ratio of their areas.

Discussion

a. Why are all squares similar?

b. When the side length of a square is doubled, what happens to the area of the square?

c. When the side lengths of a triangle are tripled, what happens to the area of the triangle?

Mathematics Note

A **square root** of a non-negative number a is a number s such that $s^2 = a$.

For example, since $5^2 = 25$, the number 5 is a square root of 25. Because $(-5)^2 = 25$, the number -5 is also a square root of 25.

The positive square root of a number is its **principal square root**. The principal square root of a is usually denoted by $\sqrt{a}$, although it may also be written as $\sqrt[2]{a}$. For example, $\sqrt[2]{25} = \sqrt{25} = 5$.

In general, an nth root of a non-negative number a is a number s such that $s^n = a$. The non-negative nth root of a is denoted as $\sqrt[n]{a}$.

The nth root of a fraction can be found by taking the nth root of the numerator and dividing it by the nth root of the denominator. In general,

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

where $b \neq 0$.

For example, the principal square root of $9/16$ can be found as follows:

$$\sqrt[2]{\frac{9}{16}} = \sqrt{\frac{9}{16}} = \frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}$$

d. If the area of a square is $a \text{ cm}^2$, what is the side length of the square?

e. Consider two squares, one with an area of 49 cm^2 and another with an area of 1 cm^2. What is the scale factor for these squares?

f. In Table **4**, you recorded the ratio of the area of square J to the area of square K. What is the square root of this ratio?

g. Given the area of square A and the ratio of the side length of square A to the side length of square B, how could you determine the area of square B?

h. Do you think that the relationship between scale factor and the ratio of areas discovered in the exploration is true for all similar figures? Explain your response.

Mathematics Note

The ratio of the areas of two similar objects is the square of the ratio of the lengths of corresponding sides (the scale factor).

For example, if the scale factor for two similar objects is $4/5$, the ratio of their areas is:

$$\left(\frac{4}{5}\right)^2 = \frac{4^2}{5^2} = \frac{16}{25}$$

Assignment

2.1 A figure is enlarged until its area is 225 times the area of the original figure. By what number was each length in the original figure multiplied?

2.2 A circle with a radius of 3 cm is enlarged by a scale factor of 10. How many times the area of the original circle is the area of the larger circle?

2.3 A rectangle with an area of 80 m^2 is enlarged by a scale factor of 3. Find the area of the larger rectangle.

2.4 Zino's Pizzeria charges $11.30 for a pizza with a diameter of 30 cm and $18.95 for a pizza with a diameter of 41 cm.

a. The two pizzas are similar. Determine the scale factor when comparing the larger pizza to the smaller one.

b. Find the ratio of their areas.

c. Use the ratio of the areas and the price of the smaller pizza to determine a corresponding price for the larger pizza.

d. Decide which pizza is the better buy and explain your reasoning.

2.5 A professional basketball player 215 cm tall has a footprint with an area of 468 cm^2. This basketball player and Nelson have similar bodies. If the area of Nelson's footprint is 325 cm^2, how tall is Nelson?

2.6 The size of a television screen is typically reported in terms of the length of its diagonal. For example, if a store advertises a 50-cm screen, this means that the length of the screen's diagonal is 50 cm.

a. What is the ratio of the area of a 63-cm screen to the area of a 33-cm screen?

b. What is the length of the diagonal for a screen with twice the area of a 33-cm screen?

2.7 **a.** Determine the area of a right triangle with sides that measure 3 cm, 4 cm, and 5 cm.

b. Use your answer to Part **a** to predict the area of a right triangle with sides that measure 18 cm, 24 cm, and 30 cm.

c. Verify your prediction using the formula for the area of a triangle.

* * * * *

2.8 Andreas and Jonalynn installed new carpeting in two of their bedrooms. The floors in both rooms are rectangles. The smaller of the two rooms is 3.2 m wide and 4.1 m long. The ratio of the widths of the two rooms is 1.2, while the ratio of the lengths is 1.6.

a. Find the length and width of the larger room.

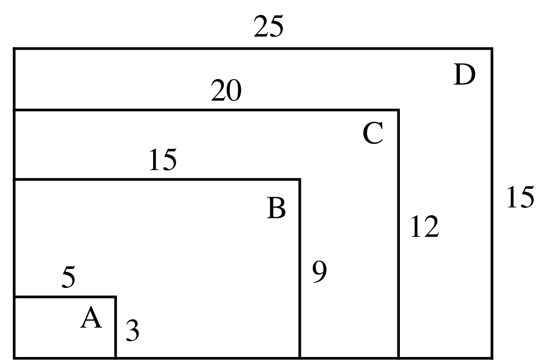
b. Can the ratios of the widths and the lengths of the two rooms be used to determine the ratio of the areas? Explain your response.

c. Find the areas of both rooms.

d. What is the ratio of the areas of the two rooms? Does this ratio verify your response to Part **b**? Why or why not?

2.9 Damion's Turf Service charges $50.00 to clean a section of artificial turf 10 yd wide and 20 yd long. Excluding the end zones, a football field is 50 yd wide and 100 yd long. How much should the company charge to clean a football field? Explain your response.

2.10 The figure below shows four rectangles and their dimensions in centimeters.



a. Compare the perimeters and areas of these rectangles.

b. Identify the rectangles that are similar and explain how you determined your response.

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Activity 3

In previous activities, you used scale factors to predict lengths and areas for similar figures. In this activity, scale factors are used to predict volumes of similar objects.

Exploration

a. **1.** Using a set of unit cubes, create a cube with an edge length of 2 units. Determine the volume of this cube and record it in the appropriate cell of Table **5**.

Table 5: Edge length and volume of cubes

Cube	Edge Length	Volume
A	1	1
B	2	
C		27
N	n	

2. Determine the edge length of a cube with a volume of 27 units^3 and record it in the appropriate cell of Table **5**.

3. Determine the volume of a cube with an edge length of n and record it in the appropriate cell of Table **5**.

b. **1.** Use the information in Table **5** to complete Table **6**.

Table 6: Ratios of edge lengths and volumes

Cubes	Ratio of Edge Lengths (scale factor)	Ratio of Volumes
B to A	2/1	
B to C	2/3	
C to A		27/1
	a/b	

2. Describe the relationship between the ratio of the edge lengths (scale factor) for two cubes and the ratio of their volumes.

Mathematics Note

A **cube root** of a number a is a number b such that $b^3 = a$. Since $2^3 = 8$, for example, 2 is the cube root of 8.

The cube root of a is denoted by $\sqrt[3]{a}$. For example, $\sqrt[3]{-8} = -2$ and

$$\sqrt[3]{\frac{64}{125}} = \frac{\sqrt[3]{64}}{\sqrt[3]{125}} = \frac{4}{5}$$

Discussion

a. **1.** What is the ratio of volumes for two cubes with volumes of 125 cm^3 and 8 cm^3?

2. What is the ratio of the edge lengths for these cubes?

b. In general, what is the relationship between the scale factor and the ratio of volumes for two cubes?

c. If a cube is enlarged until its volume is 64 times the original volume, by what scale factor has each edge length been multiplied?

d. If the volume of a cube is $d \text{ cm}^3$, what is the length of one edge?

e. Do you think that the relationship you described in Part **b** is true for all similar figures? Explain your response.

Mathematics Note

If the scale factor of two similar figures is a/b, then the ratio of their volumes is:

$$\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$$

For example, Figure **4** shows two cubes with edge lengths of 2 cm and 3 cm, respectively.

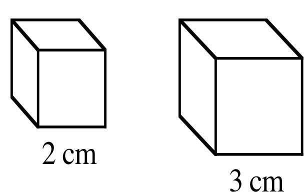

Figure 4: Two cubes

Since the ratio of the edge lengths (scale factor) is $2/3$, the ratio of the volumes is:

$$\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$$

Assignment

3.1 Consider two spheres: the smaller has a volume of 1 m^3, the larger has a volume of 343 m^3. What is the scale factor for these spheres?

3.2 Two eggs are similar in shape. One egg is twice as long as the other. What is the ratio of their volumes?

> ### Science Note
> The **density** of a substance is the ratio of its mass to its volume. For example, the density of water is 1 g/cm^3.
>
> Since the density of any pure substance is a constant, the mass of the substance is directly proportional to its volume. In other words, as the volume increases, so does the mass.

3.3 A steel sphere with a diameter of 4 cm has a volume of 33.5 cm^3. Its mass is 261 g.

a. What is the density of the sphere?

b. What is the density of a steel sphere with a diameter of 6 cm?

c. What is the scale factor between the 6-cm sphere and the 4-cm sphere?

d. What is the ratio of the volume of the 6-cm sphere to the volume of the 4-cm sphere?

e. How does the ratio of their volumes compare to:

1. the ratio of their masses?

2. the scale factor?

3.4 The two similar fish shown below are drawn to scale. The smaller fish is 25 cm long and has a mass of 0.75 kg. Assuming that the densities of the two fish are the same, use scale factor to estimate the length and mass of the larger fish.

* * * * *

3.5 A baker's favorite pie recipe calls for 6 cups of apples to make a 9-inch pie. How many cups of apples should he use to make a similar 8-inch pie?

3.6 The circumference of one egg is three times the circumference of another egg. If the two eggs are similar, what is the ratio of the mass of the larger egg to the mass of the smaller egg?

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Activity 4

In Activity **2**, you discovered that when a square's side length is doubled, its area is quadrupled. In this activity, you examine the relationship between length and area from a graphical point of view.

Exploration

The area of a square is related to the length of its sides. The area of a shoe print is related to the length of the shoe. How are these two relationships similar? How are they different?

a. Figure **5** shows a scale drawing of a child's shoe print. Each square in the drawing represents an area of 1 cm^2. Use this drawing to estimate the actual length and area of the shoe print.

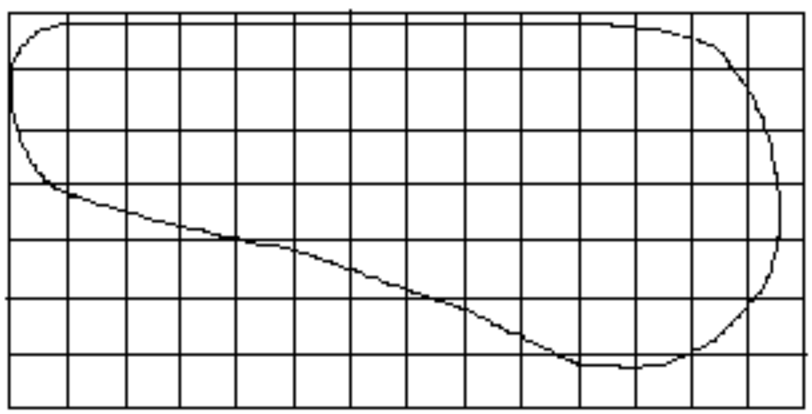


Figure 5: A child's shoe print

b. Draw your own shoe print on centimeter graph paper. Measure its length and estimate its area.

c.
1. Collect and organize this information for the entire class. To this data set, add the child's data from Part **a** as well as a shoe length of 0 cm and the corresponding area.
2. Create a scatterplot of the class data. Let *y* represent area and *x* represent shoe length.

d.
1. Graph the formula for the area of a square on the same coordinate system as the scatterplot in Part **c**. Let *y* represent area and *x* represent side length.
2. Describe any similarities or differences you observe in the two graphs.

Mathematics Note

An equation of the form $y = ax^b$ is a **power equation**.

For example, the formula for the area of a square, $y = x^2$, is a power equation in which $a = 1$ and $b = 2$.

e. Find an equation of the form $y = ax^2$ that models the scatterplot in Part **c** by varying the value of a until the graph of the equation reasonably approximates the data points.

f. One way to determine if another power equation fits the data better than your equation from Part **e** is to compare the residuals.

1. Enter the class data in a spreadsheet with the following headings:

Length (x)	Area (y)	Predicted Area	Absolute Value of Residual

2. Use your model from Part **e** to determine the predicted area for each value of x.
3. Determine the absolute value of each residual.
4. Find the sum of the absolute values of the residuals.

g. Vary the value of a in your model of the form $y = ax^2$ to determine the equation that minimizes the sum of the absolute values of the residuals. Record this equation.

h.
1. Graph your equation from Part **g** on the same coordinate system as the scatterplot from Part **c** and print a copy of the resulting graph.
2. Mark the point on the scatterplot that represents your shoe length and area.
3. Mark the point on the curve that represents the ordered pair (x,y), where x is your shoe length and y is the predicted area. Connect this point to the point in Step **2**.

Discussion

a. Describe the general relationship between shoe length and the area of a shoe print.

b. Compare your equation in Part **g**, along with its corresponding sum of the absolute values of the residuals, with those obtained by others in the class.

c. How well does your equation model the data for your own shoe length and area?

d. How does the value of a affect the graph of the equation $y = ax^2$?

e. If the lengths of two similar shoe prints have a scale factor of 3, what is the ratio of their areas?

Assignment

4.1 In the exploration, you graphed equations of the form $y = ax^b$ where $b = 2$ and both the domain and range were limited to positive numbers. To observe how different values of a and b affect the graph of $y = ax^b$, use a graphing utility to complete Parts **a** and **b** below.

a. Graph at least three examples of each of the following. Let the domain be the set of real numbers from –10 to 10.

1. $y = ax^2$ where a is positive

2. $y = ax^2$ where a is negative

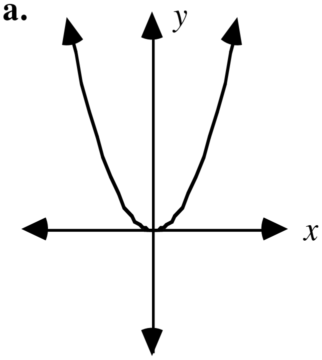
b. Repeat Part **a** for $y = ax^3$.

c. Repeat Part **a** for $y = ax^b$ where b is greater than 3.

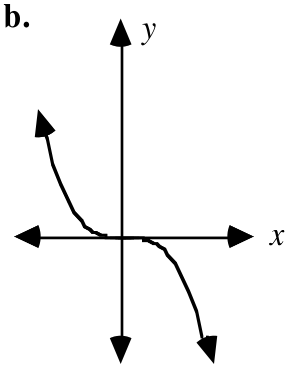
d. Describe how the values of a and b appear to affect the graph of $y = ax^b$.

4.2 Each of the following graphs was generated by an equation of the form $y = ax^b$. For each one, determine two equations with different values of b that produce graphs with roughly the same general shape.

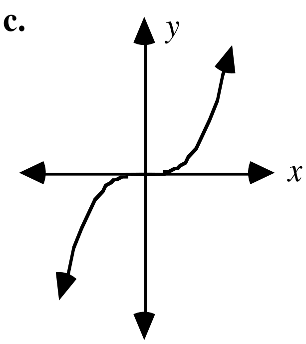
a.



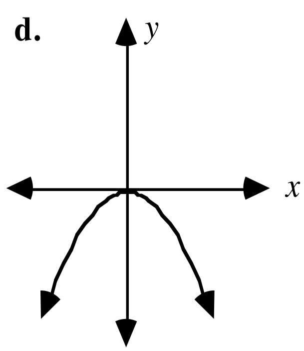
b.



c.



d.



4.3 Assuming that Robert Wadlow is similar to you, estimate the area of his shoe print.

4.4 Neva is 170 cm tall. Estimate her shoe length and the area of her shoe print if she is similar to you.

4.5 The table below shows some data on the length and mass of bird eggs.

Type of Egg	Length (cm)	Mass (g)
hummingbird	1.3	0.5
black swift	2.5	3.5
dove	3.18	6.41
partridge	3.0	8.7
Arctic tern	4.2	18
grebe	4.3	19.7
Louisiana egret	4.5	27.5
very small chicken	5.2	44.8
mallard duck	6.17	80
very large chicken	6.5	85
great black-backed gull	7.62	111
Canada goose	8.9	197
condor	11.0	270
ostrich	17.0	1400

a. Make a scatterplot of this data. Let y represent mass in grams and x represent length in centimeters.

b. Find an equation of the form $y = ax^3$ that models the data.

c. Which type of egg fits the curve least well? Explain your response.

d. The egg of a bald eagle is 7.3 cm long. Use your equation to predict the mass of this egg.

e. Write a paragraph summarizing the relationship between the length and mass of bird eggs.

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4.6 The table below shows the data collected as an object fell through the air.

Time (sec)	Distance (m)	Time (sec)	Distance (m)
0.00	0.000	0.35	0.539
0.05	0.000	0.40	0.717
0.10	0.017	0.45	0.920
0.15	0.072	0.50	1.144
0.20	0.152	0.55	1.393
0.25	0.256	0.60	1.668
0.30	0.387	0.65	1.889

a. Create a scatterplot of this data. Let y represent distance in meters and x represent time in seconds.

b. Use the process described in Parts **e–g** of the exploration to find an equation that models the data.

c. Graph this equation on the same coordinate system as in Part **a**.

d. Use your model to predict how far the object will fall in 2 sec.

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Activity 5

When you stand upright, the soles of your feet place a certain amount of pressure on the ground. In this activity, you explore the differences in pressure created by wearing shoes with flat soles, shoes with small heels, or no shoes at all.

Science Note

Although the kilogram is often referred to as a metric unit of weight, it is actually a unit of mass. Weight is a force determined by gravity. One metric unit of force is the **newton** (**N**). On the surface of the earth, the weight of an object in newtons is its mass in kilograms multiplied by 9.8 m/sec^2 (the acceleration due to gravity).

For example, the weight in newtons of a 70-kg person can be calculated as follows:

$$70 \text{ kg} \bullet \frac{9.8 \text{ m}}{\text{sec}^2} = \frac{686 \text{ kg} \bullet \text{m}}{\text{sec}^2} = 686 \text{ N}$$

Exploration

a.
1. Place your shoeless foot on a sheet of centimeter graph paper and trace around it. Use the tracing to estimate the area of your footprint without shoes.
2. Estimate the area of your shoe print when wearing shoes with flat soles. Assume that the bottom of your shoe makes complete contact with the ground.
3. Estimate the area of your shoe print when wearing a shoe with small heels.

b. Compare the three areas determined in Part **a**.

c. Determine your weight in newtons.

d. Find the pressure, in newtons per square centimeter, that your feet place on the ground in each of the following situations. Hint: Since your weight is distributed over both feet, divide this weight by the area of two footprints.

1. while not wearing shoes

2. while wearing shoes with flat soles

3. while wearing shoes with small heels

e. Compare the three pressures determined in Part **d**.

Discussion

a. When designing shoes for specific purposes, manufacturers often consider the amount of pressure that the foot places on the ground. For example, the sole on a running shoe typically has a greater area than the sole on a casual shoe.

Describe the purposes of some different types of shoes and the approximate area of the sole for each type.

b. Why do you think that high heels (and other shoes with small heel areas) are banned in some buildings?

c. Why does the frame of a bed with a mattress and box spring differ from the frame of a waterbed?

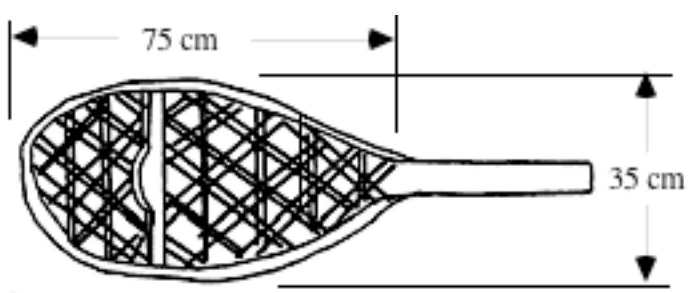
Assignment

5.1 Walking involves the smooth transfer of weight from one foot to the other. At the beginning of each step, about half your weight rests on the heel of the forward shoe.

a. Arlis wears shoes with a heel area of 0.75 cm^2 each. Sketch the print of one heel.

b. Arlis weighs 500 N. Estimate the pressure that the heel of her forward shoe places on the ground at the beginning of each step.

5.2 The diagram below shows the shape and dimensions of a typical snowshoe. Use this diagram to explain why snowshoes make it easier to stand or walk on snow.



5.3 Assume that you and Robert Wadlow are similar. Determine the pressure he places on the ground when standing upright. How does this amount differ from the pressure you place on the ground?

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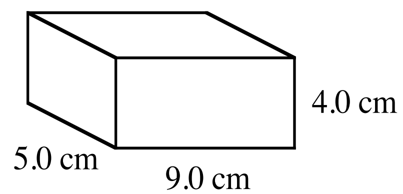
5.4 The diagram below shows a block of lead in the shape of a rectangular prism. The density of lead is 11.34 g/cm^3.



a. Calculate the pressure, in newtons per square centimeter, that the block exerts on the floor.

b. Suppose that the lead block is balanced on top of a wooden cube with an edge length of 2.5 cm. Assuming that the cube adds no significant mass to this situation, determine the pressure that the combination of block and cube exerts on the floor.

c. Explain any differences you observe in the pressures calculated in Parts **a** and **b**.

5.5 When placed upright on its base, a cylinder exerts a pressure of 0.50 N/cm^2 on the floor. The radius of the base is 35 cm. What is the mass of the cylinder?

5.6 Gravity on the planet Mars is about $1/3$ the gravity on Earth. How much pressure would you place on the Martian surface when standing upright?

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Activity 6

The **femur**, or thigh bone, extends from the hip to the knee. In humans, two femurs (one in each leg) support the weight of the body. A horse, however, has four related bones to bear its weight. Figure **6** shows scale drawings of both a human femur and the femur of a horse.

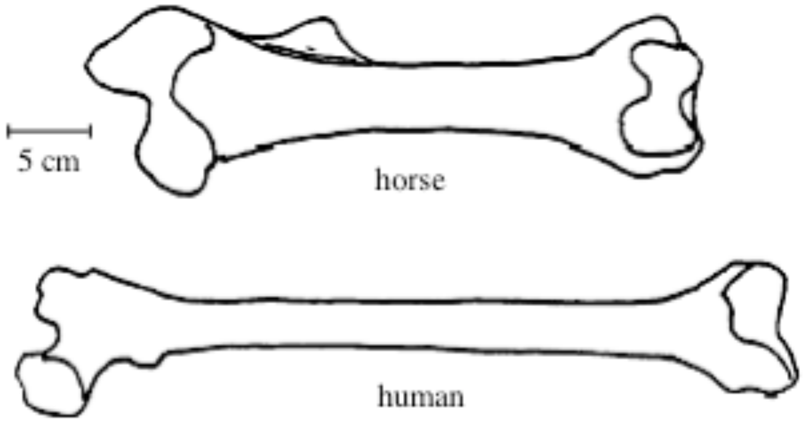

Figure 6: Two femurs

How much weight can these bones support without breaking? In this activity, you build two model femurs and use them to investigate one limit to biological growth.

Exploration

a. Use paper cylinders to model each femur in Figure **6**. The diameter of each cylinder should equal the smallest diameter of each bone.

Write the corresponding length, diameter, and circumference on each model. Compare the two models for similarity and record your observations.

b. As shown in Figure **7**, a **cross section** is the surface found by slicing an object perpendicular to its length.

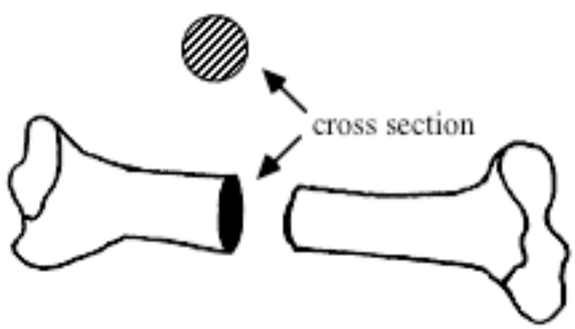

Figure 7: A cross section of bone

1. Using centimeter graph paper, draw an accurate cross section of each femur. Estimate the area of each cross section in square centimeters.
2. Calculate the cross-sectional area of each femur and compare these values with your estimates.
3. Write the calculated cross-sectional area on each model.
4. The cross-sectional area of a bone is a good indicator of relative

strength. Compare the strength of the femurs shown in Figure **6**.

c. The person whose femur appears in Figure **6** was 180 cm tall and weighed 600 N. Imagine a similar person who is twice as tall (a giant).

1. Use a strip of paper to make a cross-sectional model of the giant's femur, as shown in Figure **8**.

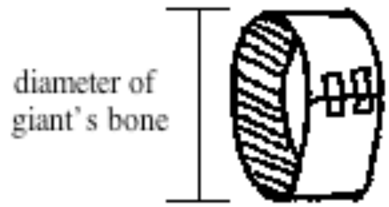

Figure 8: Paper model of cross section of bone

2. Write the diameter, circumference, and cross-sectional area on your model.

3. Would the giant's bone have to support the same weight per square unit as the bone of the person who weighed 600 N? Explain your response.

4. Write a paragraph describing the femurs of the 600-N human and the giant, including the dimensions of the bones and the pressures on the cross-sectional areas.

5. The relationship between volume and area creates a tension on bone structure that affects the biological growth of animals. How might this tension affect the giant?

d. A vertical compression test measures how much pressure a bone can withstand before crushing. In one test, a human femur withstood a pressure of approximately 1200 N/cm^2.

Considering the results of this test, do you think that there is a limit on the size of humans? Explain your response.

e. To investigate this question further, create a spreadsheet with columns as in Table **7**. Complete the spreadsheet using the data from Parts **a** and **b** of this exploration.

Table 7: Scale factor and human body structure

Scale Factor	Body Height (cm)	Femur Diameter (cm)	Cross-sectional Area of Femur (cm^2)	Body Weight (N)	Pressure on Femur (N/cm^2)
1	180			600	
2					
3					
⋮					
30					

f. Considering the information in your spreadsheet, do you think that there is a limit on the size of humans? Explain your response.

g. Use the data in your spreadsheet to create the following:

1. a scatterplot of femur diameter versus scale factor

2. a scatterplot of the cross-sectional area of a femur versus scale factor

3. a scatterplot of body weight versus scale factor.

h. **1.** Predict the shape of a scatterplot of pressure on the femur versus scale factor.

2. Create a scatterplot of pressure on the femur versus scale factor. How does the shape of the graph compare with your prediction?

Discussion

a. What did you discover about the femurs of humans and horses?

b. Is it possible for a human to grow taller than Robert Wadlow? Explain your response.

c. Compare the three graphs you created in Part **g** of the exploration.

d. Describe the relationship between pressure on the femur and scale factor.

Assignment

6.1 The person whose femur is shown in Figure **6** weighed 600 N and was 180 cm tall. Assuming that your body is similar, use your height to estimate the pressure on your femur in newtons per square centimeter.

6.2 Assume that a human femur can withstand a pressure of approximately 1200 N/cm^2. Use your response to Problem **6.1** to determine the maximum weight that your femur can support.

6.3 Describe some other factors that might limit the maximum height of humans.

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6.4 The mean height of male gorillas is approximately 1.8 m. Their mean mass is about 200 kg.

a. Determine the mean weight, in newtons, of male gorillas.

b. A gorilla's femur can support a maximum of 10 times the mean body weight. What is this maximum weight?

c. In one version of the story, the gorilla King Kong was supposed to be about 9.8 m tall. Is it possible for a gorilla like King Kong to exist? Explain your response.

6.5 The femur of a Tyrannosaurus in a Montana museum is 103 cm long and has a diameter of 25 cm. Scientists estimate that this dinosaur weighed between 35,000 N and 62,000 N. Which bone is subject to greater pressure: the femur of this Tyrannosaurus or the femur of a 600-N person? Explain your response.

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Summary Assessment

Do you think that human siblings are similar? Use the data in the following table (or collect data from your own family) to support your response.

Sibling	Length of Foot (cm)	Circumference of Foot (cm)	Area of Footprint (cm^2)	Body Weight (N)
A	15.0	35	61	93
B	19.5	45	101	208
C	23.0	54	149	340
D	30.0	70	251	756

In justifying your position, include examples of each of the following:

- proportionality
- scale factors
- linear equations of the form $y = ax$
- power equations of the forms $y = ax^2$ and $y = ax^3$
- square roots
- cube roots

Your report should also include predictions made using scale factors, graphs, and equations and use residuals to determine how well an equation models a data set.

Module Summary

- Two ratios, a/b $(b \neq 0)$ and c/d $d \neq 0$, are **proportional**, or **in proportion**, if

  $$\frac{a}{b} = \frac{c}{d}$$

  When two such ratios are proportional, it is also true that

  $$\frac{a}{c} = \frac{b}{d}$$

  where $c \neq 0$ and $d \neq 0$.

- Two objects are **similar** if they have the same shape and the ratios of corresponding lengths are proportional. The ratio of corresponding sides is the **scale factor**.

- To raise a fraction to a power n, where n is a non-negative integer, both the numerator and the denominator may be raised to the indicated power. In general,

  $$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

  where $b \neq 0$.

- A **square root** of a non-negative number a is a number s such that $s^2 = a$.

- The positive square root of a number is its **principal square root**. The principal square root of a is usually denoted by $\sqrt{a}$, although it may also be written as $\sqrt[2]{a}$.

- A **cube root** of a number a is a number b such that $b^3 = a$. The cube root of a is denoted by $\sqrt[3]{a}$.

- In general, the nth root of a non-negative number a is a number s such that $s^n = a$. The non-negative nth root of a is denoted as $\sqrt[n]{a}$.

- The nth root of a fraction can be found by taking the nth root of the numerator and dividing it by the nth root of the denominator. In general,

  $$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

  where $b \neq 0$.

- An equation of the form $y = ax^b$ is a **power equation**.

- The ratio of the areas of two similar objects is the square of the ratio of the lengths of corresponding sides (the scale factor).
- If the scale factor of two similar figures is a/b, then the ratio of their areas is:

  $$\left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}$$

  while the ratio of their volumes is:

  $$\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$$

- The **density** of a substance is the ratio of its mass to its volume.
- One metric unit of force is the **newton** (**N**). The weight of an object in newtons is its mass in kilograms multiplied by $9.8\ \text{m}/\text{sec}^2$ (the acceleration due to gravity).

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