Traditional Design

In this module, you examine the geometric properties of some traditional American Indian art forms.

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Introduction

The traditional artwork of many American Indian peoples is prized for its beauty, meaning, and symmetry. Its forms are as diverse as the tribes themselves—varying by region, by culture, and by individual artist. In this module, you examine the star quilts of the Assiniboine and Sioux, the medicine wheels of the Plains Indians (particularly the Northern Cheyenne), and the sandpaintings of the Navajo.

Exploration

American Indian art often emphasizes nature and its forces through pictures and symbols. The symbol shown in Figure **1**, for example, represents both a mountain and abundance. It may have been painted on a prayer bowl or woven into a blanket.

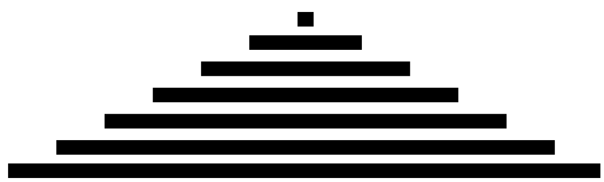


Figure 1: Representation of a mountain (abundance)

The geometric properties of designs like this one can be examined through paper folding. In this exploration, you will practice some basic paper-folding techniques. One such technique allows you to investigate the line symmetry of symbols like the one in Figure **1**.

Mathematics Note

An object has **line symmetry** if the object is its own image in a reflection in a line. This line is the **line of symmetry**.

For example, Figure **2** shows three lines that each divide a rectangle into two congruent parts. This rectangle has line symmetry with respect to lines l and m. The reflection of the rectangle in line n, however, is not itself. Therefore, line n is not a line of symmetry for this figure.

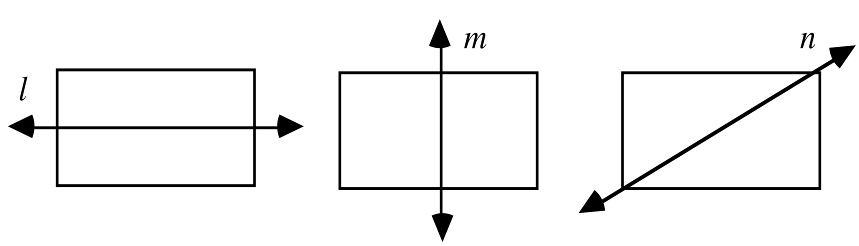


Figure 2: Lines dividing a rectangle into congruent parts

Complete Parts **a–c** using a reproduction of the symbol in Figure **1**.

a. **1.** Fold your paper along the symbol's line of symmetry. Unfold the paper and label the resulting crease line *a*.

2. Measure the length of the bars on each side of crease *a*.

3. Measure the angles formed by line *a* and the bars in the symbol.

4. Make a conjecture about the relationship of the line of symmetry to the bars in the symbol.

b. **1.** Fold the paper to form a line parallel to the bars in the symbol. Unfold the paper and label this crease line *b*.

2. Identify an angle formed by crease *a* and crease *b*. As shown in Figure **3** below, fold the paper so that one side of the angle coincides with its other side. Unfold the paper and label this crease line *c*.

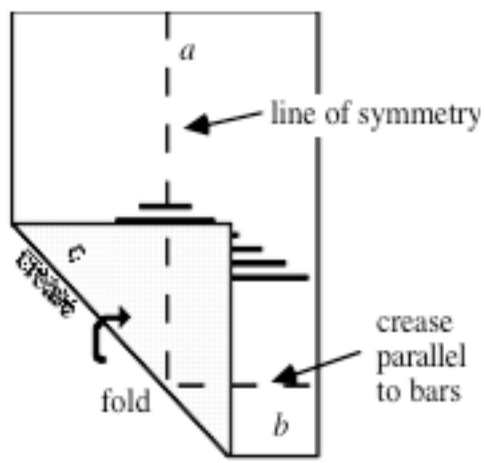


Figure 3: Folding one side of an angle onto the other

Mathematics Note

Perpendicular distance is the distance from a point to a line or a plane. It is equal to the length of the segment, perpendicular to the line or plane, that connects the point to that line or plane.

For example, Figure **4** shows a point *A* and a line *l*. Since $\overline{AX}$ is perpendicular to line *l*, the perpendicular distance from *A* to *l* is 5 units.

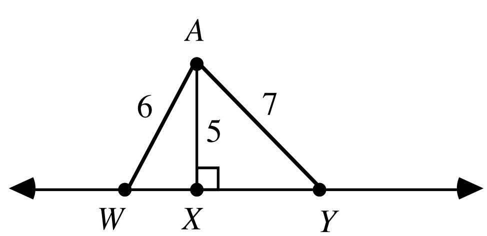


Figure 4: Perpendicular distance

c.

1. Select a point on crease *c* and measure the perpendicular distance from this point to each side of the angle formed by creases *a* and *b*.
2. Measure the angles formed by creases *c* and *b* and the angles formed by creases *c* and *a*.
3. Make a conjecture about the relationship of crease *c* to the angle formed by creases *a* and *b*.

Discussion

a. Describe a paper-folding method that could be used to create each of the following:

1. an object's reflection in a line
2. a perpendicular bisector of a line segment
3. a bisector of an angle
4. a midpoint of a line segment.

b. Describe a paper-folding method that could be used to form a square from a rectangular sheet of paper.

Activity 1

For hundreds of years, some American Indian families marked important events in the lives of their children by giving gifts. These gifts were not intended for the individual child, however, but for some other admired person. In this tradition, gift giving demonstrated the deep pride a family felt for its children.

Today, the Assiniboine and Sioux tribes of northeastern Montana preserve this tradition through the star quilt ceremony. Each star quilt represents many hours of painstaking labor. It is a great honor both to give away a star quilt and to receive one. Star quilt ceremonies are usually held at community gatherings, particularly athletic events. For example, the family of a basketball player or cheerleader might make a quilt to present between games at a state tournament. The person to receive the quilt might be a coach, an admired member of the community, or a player from another team.

The family often wraps the quilt around the shoulders of the chosen recipient while introductions are made and the audience stands respectfully. This ceremony is a powerful medium for reflecting the values, attitudes, pride, and identity of the presenting family.

Although there are many kinds of star quilts, all of them have the lone star as their primary focus. Figure **5** shows one example of a lone star pattern.

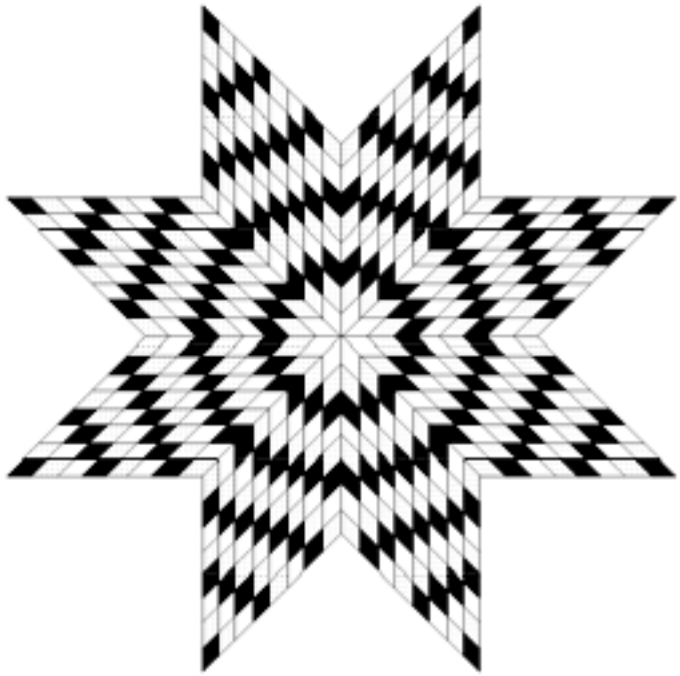


Figure 5: Lone star quilt pattern

Discussion 1

a. Identify some of the geometric shapes you recognize in the lone star pattern in Figure **5**.

b. **1.** What basic shape do you think was used to construct the star?

2. Describe the properties of this shape.

Exploration

a. Fold and cut a standard sheet of paper to form a square. If no crease has been formed along a diagonal of the square, use another fold to create one.

b. As shown in Figure **6a**, use paper folding to bisect an angle formed by a side of the square and the creased diagonal. Using the same vertex—and without unfolding the paper—repeat this step for the adjacent angle, as shown in Figure **6b**. Do not unfold this shape.

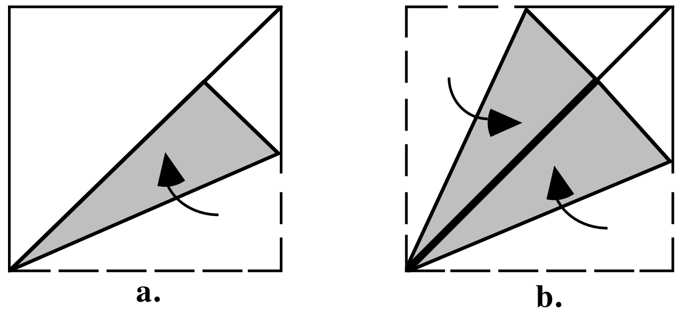


Figure 6: Folding in the sides of a square

c. Repeat Parts **a** and **b** using the opposite vertex of the original square, as shown in Figure **7**. The resulting shape is a **rhombus**.

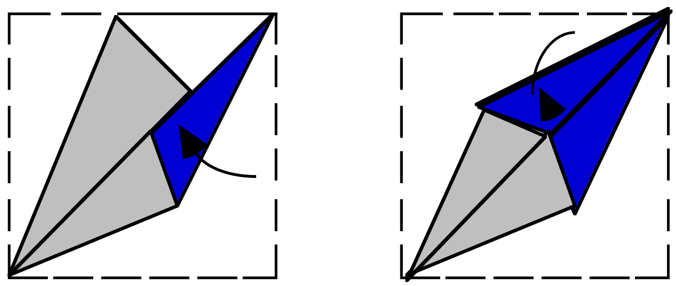


Figure 7: Folding at the opposite vertex

d.
1. By carefully examining the folds made in creating this rhombus, determine and record the measures of its interior angles.
2. Predict the relationship that occurs between opposite angles of a rhombus.

e. Turn the rhombus over so that the previous folds are not visible. Fold it along the shorter diagonal. Unfold it, fold along the longer diagonal, and then unfold again. Make a conjecture concerning the diagonals of this rhombus.

f.
1. Find the midpoint of each side of the rhombus.
2. Using a straightedge, draw the line that contains the midpoints of two opposite sides. Identify the shapes formed by these lines and the sides of the rhombus.
3. Draw the line that contains the midpoints of the other two sides.

g. Identify the four shapes formed by the lines in Part **f** and the sides of the rhombus.

h. Record the measure of each angle of the shapes you identified in Part **g**. **Note:** Save the folded rhombus for use in Discussion **2** and in the assignment.

Mathematics Note

Two polygons are **similar** if there is a one-to-one correspondence between their vertices so that corresponding sides are proportional and corresponding angles are congruent.

For example, Figure **8** shows two triangles, the approximate lengths of their sides, and the approximate measures of their angles.

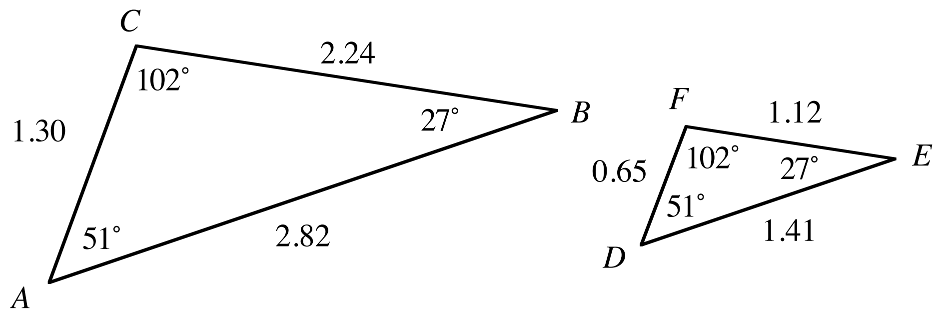


Figure 8: Two similar triangles

In this example, point *A* corresponds to point *D*, *B* to *E*, and *C* to *F* so that:

$$\frac{BC}{EF} = \frac{AB}{DE} = \frac{AC}{DF} = 2$$

and $\angle F \cong \angle C$, $\angle A \cong \angle D$, and $\angle B \cong \angle E$. Therefore, triangle *ABC* is similar to triangle *DEF*. This can be denoted as follows: $\Delta ABC \sim \Delta DEF$.

Discussion 2

a. Are the new shapes you identified in Part **g** of the exploration similar to the original rhombus created in Part **c**?

b. Describe the relationship between the two parallelograms formed in Part **f** of the exploration.

c. Compare the conjecture you made in Part **e** of the exploration concerning the diagonals of the rhombus with others in your class.

d.
1. Describe the relationships that exist among the interior angles of a rhombus.
2. What methods could you use to demonstrate these relationships?

e. After connecting the midpoints of opposite sides in Part **f** of the exploration, your paper rhombus should have resembled the diagram in Figure **9**.

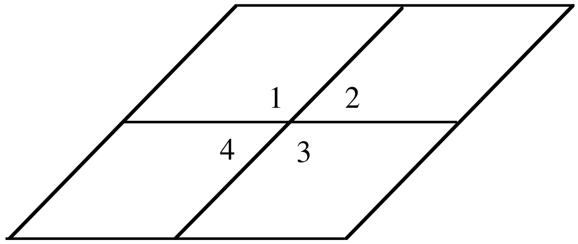


Figure 9: A rhombus

1. Angles 1 and 3 are **vertical angles**. What is the relationship between the measures of these angles?
2. Does another pair of angles in Figure **9** have this same relationship?
3. What characteristics do you think might define a pair of vertical angles?
4. Using the rhombus you created in the exploration, identify several pairs of vertical angles formed by crease lines.

f. Connecting the midpoints of opposite sides in Part **f** of the exploration created new, smaller shapes similar to the original rhombus. If you repeat this process on the smaller rhombi, what is the result?

Mathematics Note

A **rep tile** is a shape that can be partitioned into congruent shapes, each one similar to the original. For example, ΔABC in Figure **10** is a rep tile.

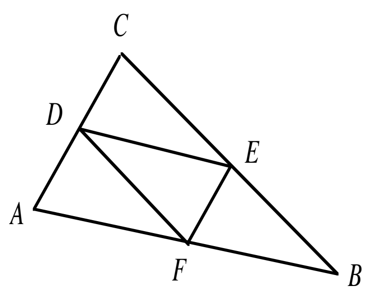


Figure 10: A rep tile

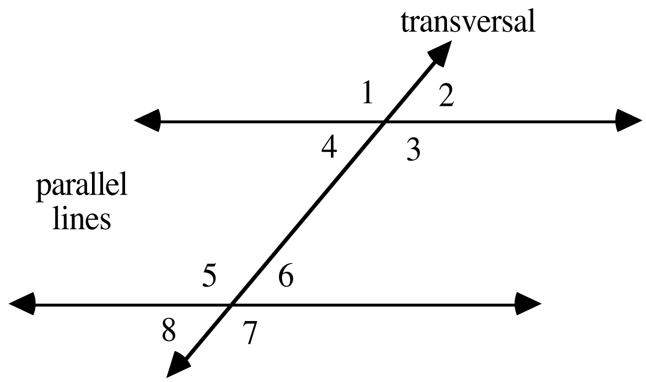
A rep tile can also be used to create a larger similar figure. For example, ΔDEF is a rep tile creating ΔABC. **Note:** The name *rep tile* is derived from the phrase "repetition of a tile."

g. Based on your response to Part **f** above, is a rhombus a rep tile?

Assignment

1.1 **a.** On a sheet of paper, extend the pattern shown in Figure **9** to show that it could tessellate the plane.

b. **1.** On your paper, find two parallel lines and a transversal similar to the ones shown in the following diagram.



Describe the relationship between the measures of $\angle 2$ and $\angle 6$, and between the measures of $\angle 4$ and $\angle 8$. These are pairs of **corresponding angles**.

2. Using the diagram above, identify as many sets of corresponding angles as possible.

3. Suggest a general definition for a pair of corresponding angles.

c. Identify all pairs of vertical angles in the diagram in Part **b**.

d. In the diagram in Part **b**, $\angle 4$ and $\angle 6$ are **alternate interior angles**, while $\angle 2$ and $\angle 8$ are **alternate exterior angles**.

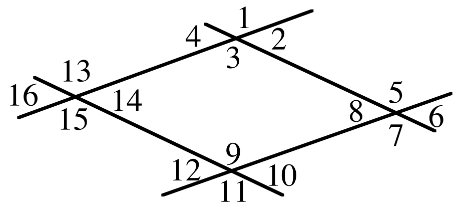
1. Describe the relationship between the measures of $\angle 4$ and $\angle 6$.

2. Describe the relationship between the measures of $\angle 2$ and $\angle 8$.

3. Using the diagram in Part **b**, identify as many pairs of alternate interior and alternate exterior angles as possible.

4. Suggest how these pairs of angles might have received their names.

1.2 Consider the following rhombus with its sides extended.



a. Determine the relationship that exists between $\angle 1$ and each of the remaining angles at that vertex.

b. Explain why the measure of $\angle 1$ is equal to the measure of $\angle 5$, $\angle 13$, and $\angle 9$.

c. What is the minimum number of angles you must measure in order to determine the measures of all 16 angles in the diagram?

d. Suppose none of the sides of the figure were parallel. How would this change your response to Part **c**?

Mathematics Note

An object has **rotational symmetry** about a point if, when rotated through an angle about that point, each point in the image coincides with a point in the preimage.

For example, Figure **11** shows a regular pentagon *ABCDE* and its center *O*. This object has rotational symmetry about *O*. When pentagon *ABCDE* is rotated 72° about *O*, each point in the image coincides with a point in the preimage.

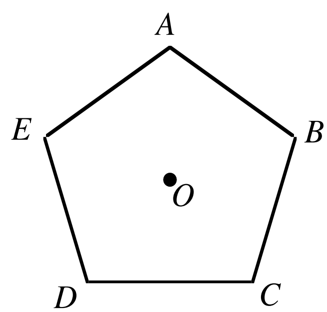


Figure 11: A regular pentagon and its center

1.3 Obtain a copy of the lone star design from your teacher.

a. Draw a line that passes through the center of the star. If you cut the star along this line, how would the two pieces be related?

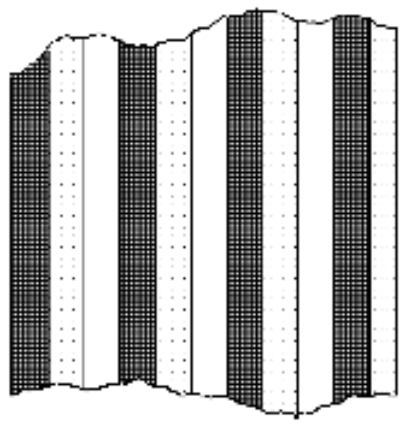
b. Describe any symmetries that would exist between the two pieces.

c. Would your observations in Parts **a** and **b** hold true for any line that passes through the center of the star? Explain your response.

d. Summarize your responses to Parts **a–c** in a paragraph.

1.4 **a.** Because making a star quilt requires many hours of work, quilters often use certain cutting and sewing techniques to simplify the task. To make the lone star in Figure **5**, for example, quilters generally start by sewing long strips of material together as shown below.

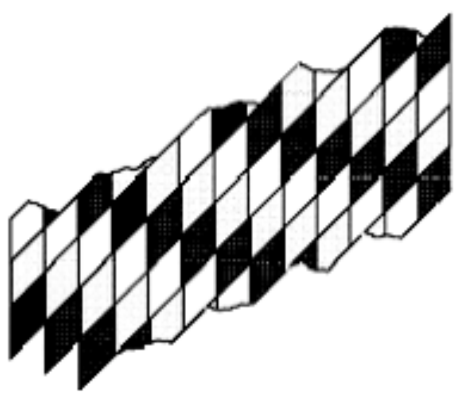
Suppose that you drew a set of parallel lines on this material so that each line was perpendicular to the strips. Describe the geometric figures that would be formed by the lines and strips.



b. On a copy of the diagram above, draw a set of parallel lines that are not perpendicular to the strips of material. Describe what kind of geometric figures are obtained.

c. The lone star pattern in Figure **5** has eight evenly spaced "points." What is the measure of the central angle for each of these points?

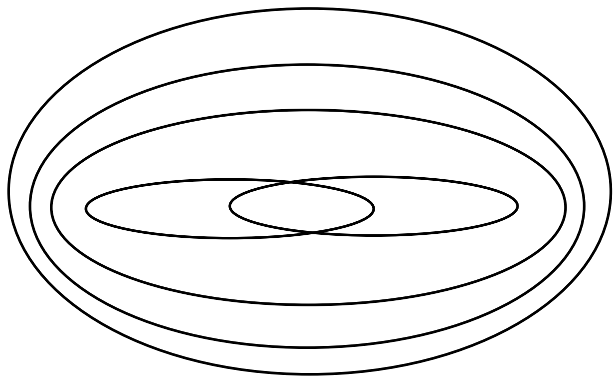
d. By cutting the quilt material along a set of parallel lines such as those in Part **b**, then shifting each new strip by one color, a quilter can create the following arrangement of rhombi. Describe how to draw this set of parallel lines. Verify your method using a copy of the diagram in Part **b**.



e. Imagine that you are stitching the strips created in Part **d** to make a lone star quilt. As you sew, the quilt starts to curl and buckle. Describe what might be causing this problem, and what you might do to correct it.

1.5 One way of classifying quadrilaterals is by the number of parallel sides. When characterized in this manner, a trapezoid can be defined as a quadrilateral with at least one pair of parallel sides. In the same fashion, a parallelogram can be defined as a trapezoid with two pairs of parallel sides.

Create a copy of the following Venn diagram. Using the definitions described above, label each region in the diagram with one of the following classifications: quadrilaterals, parallelograms, trapezoids, rectangles, squares, rhombi.



1.6 Use a geometry utility to examine the properties of the diagonals of each of the following polygons. Describe your observations.

a. a trapezoid with exactly one pair of parallel sides

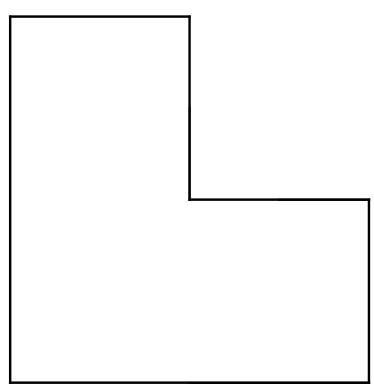
b. a parallelogram that is neither a rhombus nor a rectangle

c. a rectangle that is not a square

d. a rhombus that is not a square

e. a square.

1.7 **a.** Make a copy of the figure below. Draw line segments on the figure to create a rep tile. Hint: Begin by examining the midpoints of the edges.



b. Design a rep tile that could be used in a quilt pattern. Describe the roles of angles and parallel lines, if any, in your design.

* * * * *

1.8 Describe four ways in which you could determine if a quadrilateral is a parallelogram.

1.9 Many buildings have rectangular floors. Describe how a carpenter can verify that a floor is a rectangle using only a tape measure.

1.10 List the quadrilaterals (trapezoids, parallelograms, rhombi, rectangles, or squares) that have each of the following properties:

a. a diagonal forms two congruent triangles

b. a diagonal bisects opposite angles

c. the two diagonals form four congruent triangles.

* * * * * * * * * *

Activity 2

The medicine wheels of the Northern Cheyenne and other Plains peoples are both beautiful and symmetrical. Almost every aspect of each wheel—its symbols, color, and design—holds a special significance for the person who creates it.

In the past, simple wheels were built by placing small stones or pebbles on the ground. One pattern for such a wheel is shown in Figure **12**. Each stone represented one of the many things in the universe. For example, an individual stone might have represented an animal, an individual person, or a nation.

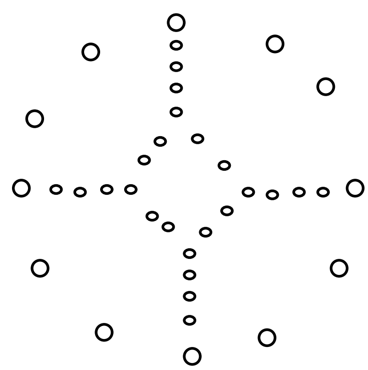


Figure 12: Medicine wheel formed with pebbles

Medicine wheels were also painted on personal shields. These shields were made from a variety of animal hides—including buffalo, bear, deer, coyote, weasel, and mouse—and often decorated with eagle plumes or tassels of animal fur. The designs and symbols on each wheel held deep personal meaning for the bearer.

Figure **13** shows a medicine wheel with two **concentric circles** (circles that have the same center), a set of evenly spaced quadrilaterals, and a Cheyenne design in the middle.

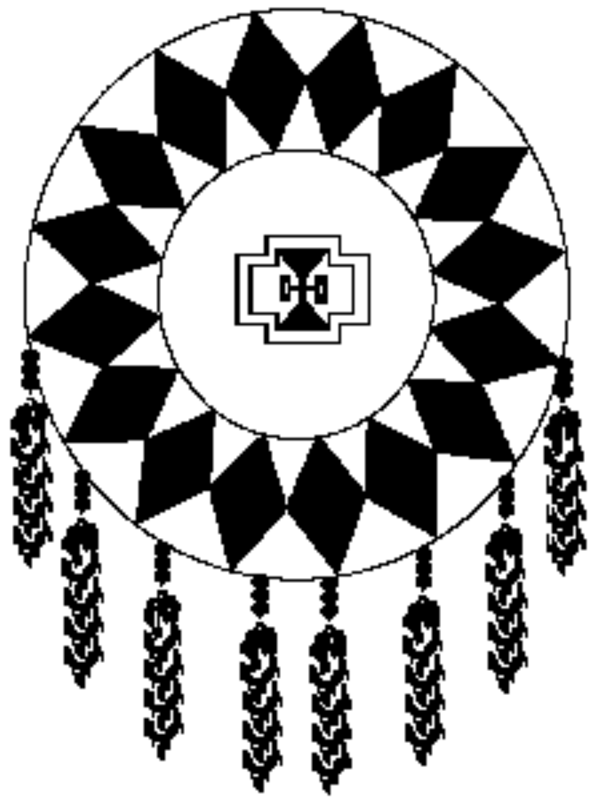


Figure 13: A Cheyenne medicine wheel

In this activity, you use some geometric properties of a circle to create your own medicine wheel.

Discussion 1

a. Disregarding the tassels and beads, what types of symmetry do you observe in the medicine wheel in Figure **13**?

b. In the Level 2 module "Crazy Cartoons," you explored four types of transformations: reflections, rotations, dilations, and translations. Explain how you could use transformations to reconstruct this medicine wheel—including tassels and beads—on a geometry utility.

Exploration

a. Using a geometry utility, draw a circle whose diameter is approximately two-thirds the width of the screen.

b.
1. A **chord** is a line segment joining any two points on a circle. Draw two chords of your circle.
2. Predict where the perpendicular bisectors of the two chords will intersect. Draw a point at that location.
3. Construct the perpendicular bisectors of the chords. Mark the intersection point (if different from the location predicted in Step **2**).
4. Drag the endpoints of the chords to change their sizes and locations. Record your observations.
5. Change the size of the circle. Record your observations.

c.
1. Draw a new circle and construct a diameter.
2. Identify the intersection points of the diameter and the circle.
3. Construct a line through one of the points of intersection.
4. Measure an angle at the intersection of the diameter and the line constructed in Step **3**.
5. Adjust the angle constructed in Step **3** until its measure is 90˚.
6. Record your observations.

Mathematics Note

A **secant** of a circle is a line that intersects a circle in two points.

A **tangent** of a circle is a line, segment, or ray in the plane of the circle that intersects the circle in exactly one point and is perpendicular to a radius at that point. This intersection is the **point of tangency**.

For example, Figure **14** shows circle O with secant $\overleftrightarrow{AB}$, tangent $\overleftrightarrow{BC}$, and B, a point of tangency.

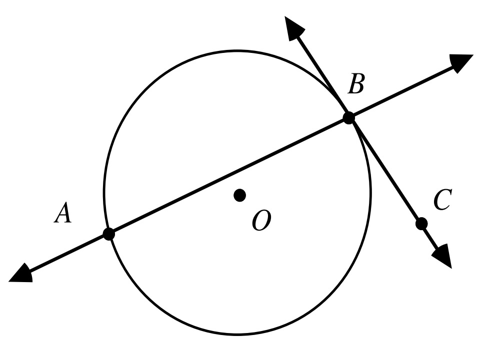


Figure 14: Circle with secant and tangent lines

Discussion 2

a. **1.** What is the measure of the angle between a diameter of a circle and a tangent to that circle whose point of tangency is the endpoint of the diameter?

2. Suppose that a secant does not contain the center of a circle. If a tangent to the circle is then drawn as in Figure **14**, what can be said about the measure of an angle formed by this secant and tangent?

b. Describe how to use paper folding to find a line tangent to a circle.

c. What is true about two tangents whose points of tangency are opposite endpoints of a diameter? Explain your response.

d. Figure **15** shows a fragment of American Indian pottery. Before beginning reconstruction of the circular plate, a museum curator might first make a sketch of the original artifact, including its center. Using your observations from the exploration, describe how to find the center of this circular plate.

Figure 15: Pottery fragment

e. Describe how to use paper folding to find the center of a circle.

f. The perpendicular bisector of a chord is the set of points in the plane equidistant from the ends of the chord. This means that for any point C in Figure **16** below, $AC = BC$.

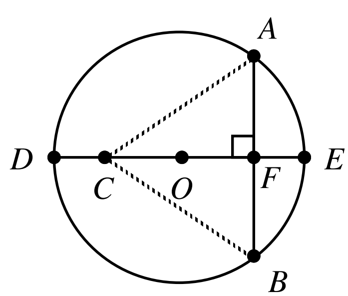


Figure 16: Chord *AB* and its perpendicular bisector

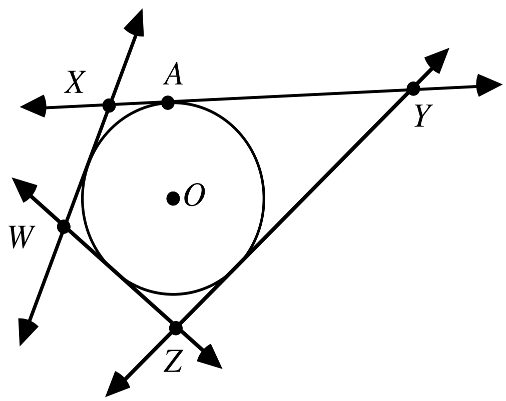
How does this verify that the intersection of the perpendicular bisectors of two chords is the center of the circle?

g. In Figure **16**, ΔABC is isosceles and $\overline{CF}$ is an altitude. What is the relationship between the altitude of an isosceles triangle and its base?

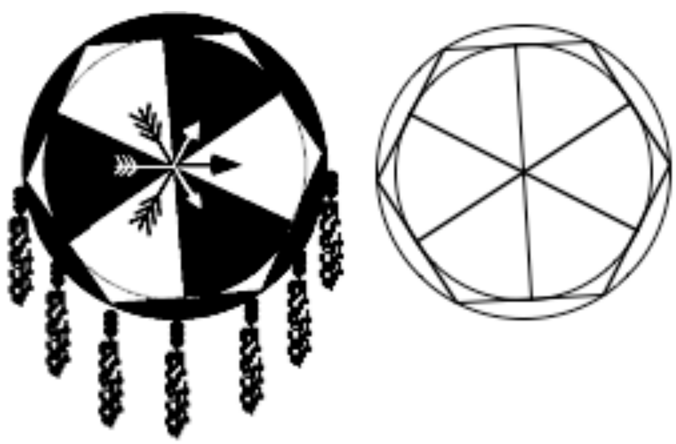
Assignment

2.1 **a.** Using appropriate technology, construct a point *A* on a circle and a tangent to the circle at that point.

b. Continue the construction from Part **a** to create a circle inscribed in quadrilateral *XYZW*, as shown in the diagram below. Describe the process you used to complete the drawing.



2.2 The diagram below shows two drawings of a medicine wheel. The drawing on the right represents the design of the wheel without the feathers, arrows, and shading of the drawing on the left.



a. Recreate the drawing on the right using a geometry utility. Describe the techniques you used.

b. **1.** Does the drawing you created in Part **a** have line symmetry? Explain your response.

2. Does it have rotational symmetry? Explain your response.

2.3 **a.** The diagram below shows the outline of a medicine wheel.

Using a copy of this diagram, find the center of the circle. Use the center to construct two smaller, concentric circles on the wheel.

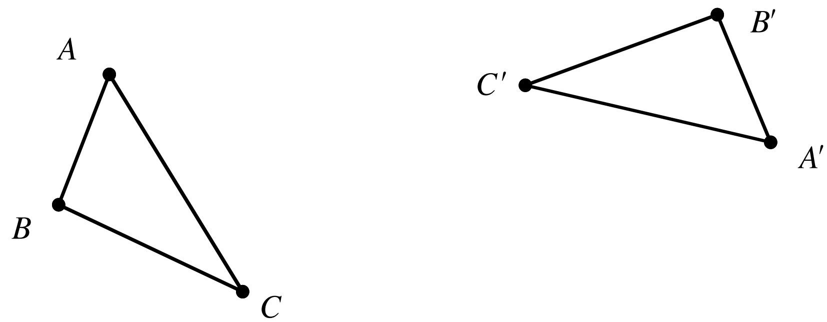
b. Describe how you found the center of the circle.

c. Personal medicine wheels often display the symbols of an individual or clan. What symbols could you use to represent yourself, your class, or your school?

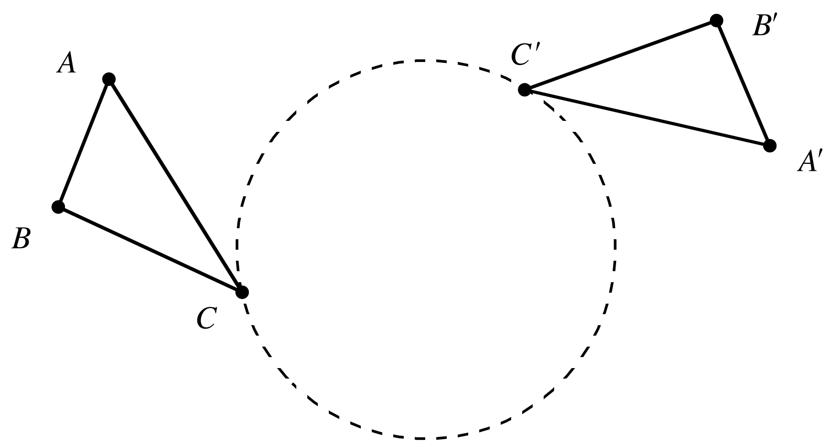
d. Using the three circles drawn in Part **a**, create a medicine wheel that combines chords, tangents, and at least one traditional American Indian symbol in a design that radiates from the center.

e. Describe how you used each required component in your design.

2.4 The following illustration shows ΔABC and its image under a rotation, $\Delta A'B'C'$.



Any two corresponding points on the image and the preimage, such as C' and C, are contained on a circle whose center is also the center of rotation. The center of the circle is not shown in the illustration below.



a. Explain why the center of the circle must be contained on the perpendicular bisector of $\overline{CC'}$.

b. Using a copy of the diagram, determine the center of rotation. Describe the method you used.

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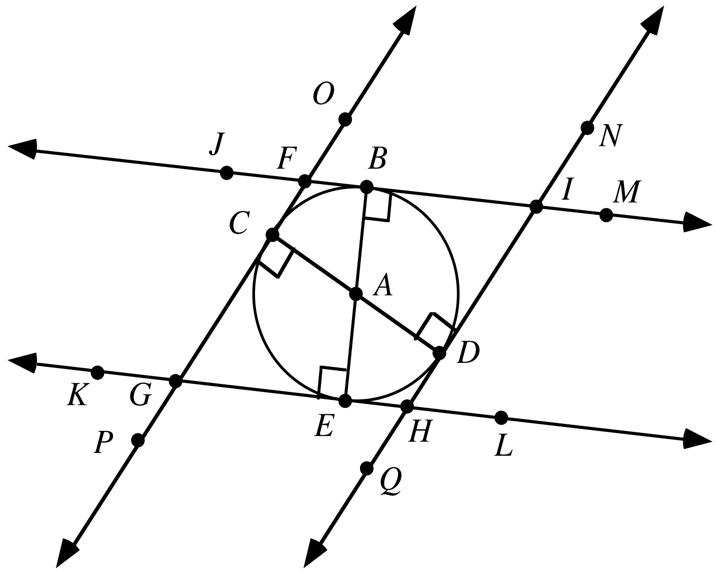
2.5 **a.** In the following diagram, $\overline{CD}$ and $\overline{BE}$ both contain A, the center of the circle. What other geometric term describes these chords?



b. In the diagram above, $\overleftrightarrow{FI}$ and $\overleftrightarrow{GH}$ are tangent to the circle at B and E, respectively, while $\overleftrightarrow{FG}$ and $\overleftrightarrow{IH}$ are tangent to the circle at C and D, respectively.

1. Explain why $\overleftrightarrow{FI}$ and $\overleftrightarrow{GH}$ are parallel.
2. Is there another set of parallel lines in the diagram? Explain your response.

c. **1.** Recreate the diagram on a geometric utility.

2. Measure $\angle KGP$, $\angle CGE$, $\angle KGC$ and $\angle CFB$.

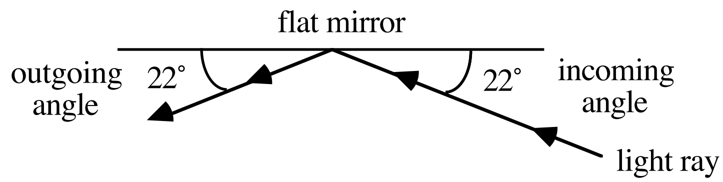
3. Any two of the angles listed in Step **2** can be used to illustrate a special angle relationship. Identify several of these relationships.

d. Identify all pairs of congruent angles formed by $\overleftrightarrow{IH}$, $\overleftrightarrow{FI}$ and $\overleftrightarrow{GH}$. Describe the relationship that exists between the angles in each pair.

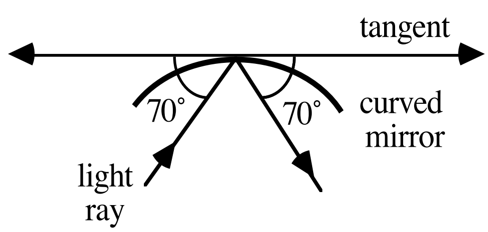
e. Make a conjecture about the lengths of $\overline{FI}$, $\overline{IH}$, $\overline{HG}$, and $\overline{GF}$. Confirm your conjecture by measuring these lengths.

f. Do you believe $GFIH$ is a rhombus? Explain your response.

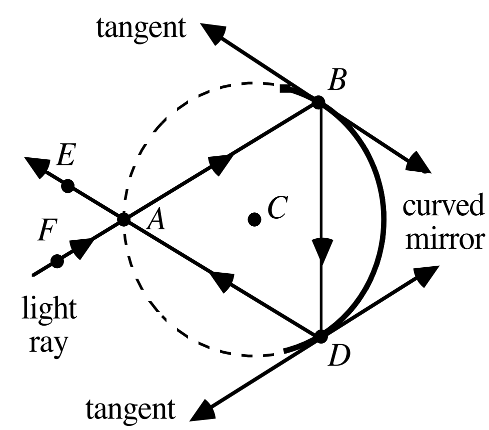
2.6 As you learned in the Level 1 module "Reflect on This," when a light ray reflects off a flat mirror, the incoming angle is congruent to the outgoing angle. For example, the diagram below shows a light ray reflecting off a flat mirror at an angle of 22°.



When light reflects off a curved mirror, the incoming angle also is congruent to the outgoing angle. In this case, the angles are measured with respect to the tangent at the point of reflection. For example, the following diagram shows a light ray reflecting off a curved mirror at an angle of 70°.



Consider a mirror whose cross section is a portion of a circle with center at point *C*. As shown in the following illustration, points *A*, *B*, and *D* are on circle *C*. A light ray travels from *A* to *B* with an incoming angle of 60°, reflects off the mirror at *B*, reflects again off the mirror at *D*, then passes back through *A*.



a. If segments are drawn from *C* to each vertex of ΔABD in the diagram above, three triangles are formed: ΔACB, ΔACD, and ΔBCD. What type of triangles are these?

b. 1. Each segment described in Part **a** cuts an interior angle of ΔABD into two smaller angles. What is the relationship between these two smaller angles? Explain your response.

2. What term can be used to identify each of these segments?

c. Why must the outgoing light ray at point *D* be parallel to the tangent to the circle at point *B*?

* * * * * * * * * *

Activity 3

The art of the Navajo people of the southwestern United States is rich in symbolism and meaning. Navajo sandpaintings, for example, are an important component of traditional healing rituals. The sandpaintings themselves are done with five sacred, symbolic colors and usually represent a particular character in tribal legend.

Figure **17** shows one sandpainting design recreated on a geometry utility.

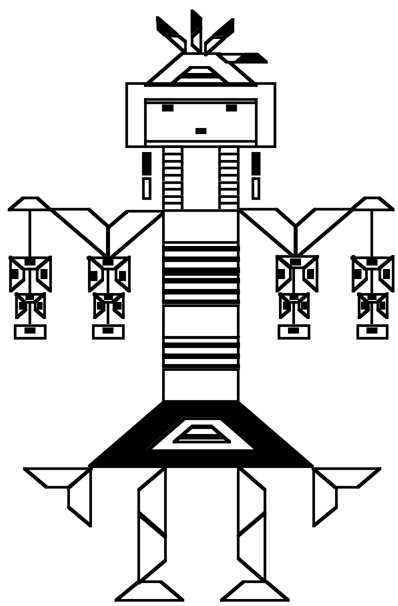


Figure 17: Design resembling a Navajo sandpainting

Discussion 1

a. Where does symmetry appear to play a part in the design in Figure **17**?

b. Describe the two basic geometric figures used in the design.

c. Where does similarity appear to play a part in the design?

Exploration

In this exploration, you examine one shape from the design in Figure **17** and investigate how that shape can be transformed to create other parts of the design.

a. **1.** On a geometry utility, construct a rectangle to represent the head of the figure in the design. Label the vertices of the rectangle *HEAD*, as shown in Figure **18** below.

2. Create a point *P* outside the rectangle to serve as a center of dilation.

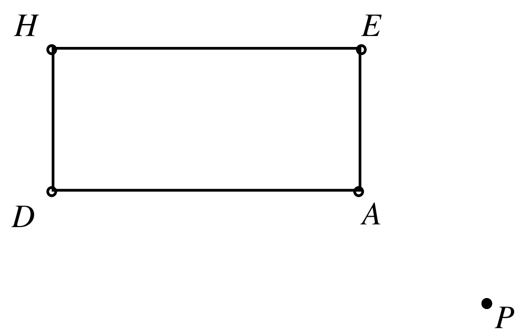


Figure 18: A rectangle with center of dilation

3. Construct ΔPEA.

4. Find the midpoints of $\overline{PE}$ and $\overline{PA}$. Label these points E' and A'.

5. Construct $\overline{E'A'}$.

b. Verify that $\Delta PE'A'$ is similar to ΔPEA.

c. Using the same techniques as in Part **a**, construct $\overline{D'A'}$, $\overline{H'D'}$, and $\overline{E'H'}$ to create rectangle $H'E'A'D'$.

d. Rectangle $H'E'A'D'$ is a **dilation** of rectangle *HEAD* with center at point *P* and a scale factor of $H'E'/HE$. Describe the properties of the preimage that are preserved in a dilation.

e. Move point *P* to several other locations on your screen. Record your observations.

f. In Parts **a** and **c**, you used the midpoints of segments to construct rectangle $H'E'A'D'$.

Predict what would happen if you repeated this process using points $1/4$ the distance from *P* to each vertex of rectangle *HEAD*.

Discussion 2

a.
1. What is the scale factor for the dilation in Part **d** of the exploration?
2. What would be the scale factor for the dilation suggested in Part **f** of the exploration?

b. Describe how changing the location of the center of dilation in Part **e** affected the resulting image.

c. Describe how you could use a dilation to create an image larger than the preimage.

d. A transformation that produces an image congruent to the preimage is an **isometry**. Which of the following transformations—reflections, rotations, dilations, and translations—can be classified as isometries? Explain your responses.

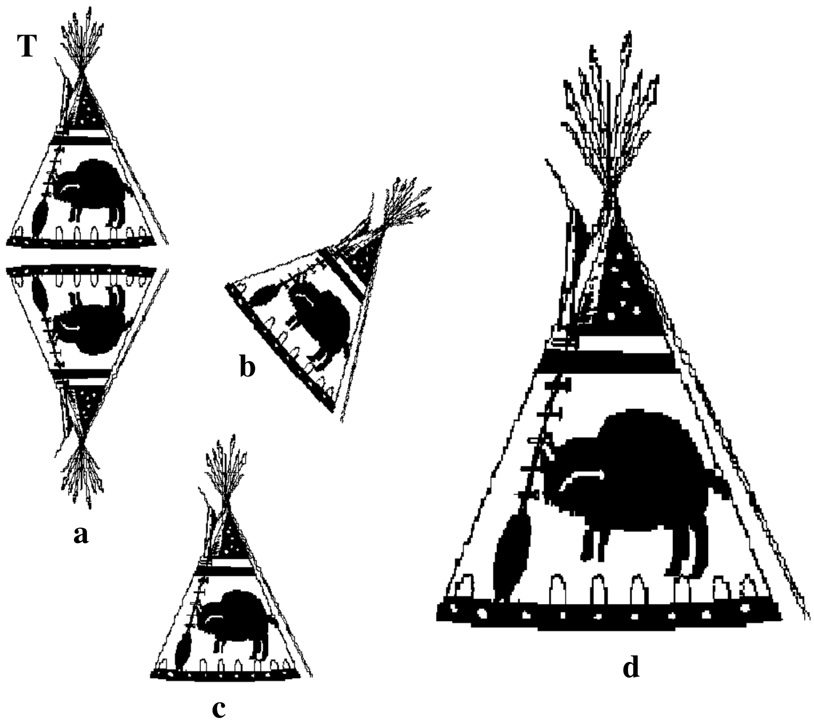
Assignment

3.1 **a.** Using the template supplied by your teacher, find the images of several points of the following figure under the transformations described below.

1. a dilation with center at point P and a scale factor of $1/2$
2. a translation by the vector $\overrightarrow{PP'}$ of the image in Step **1**
3. a reflection in $\overleftrightarrow{PP'}$ of the image found in Step **2**
4. a 90° clockwise rotation, with center at point P, of the image found in Step **3**

b. Sketch the remainder of each image in Part **a**.

3.2 The Blackfeet tribe of the American Northwest sometimes painted symbols and designs on their tipis. In the diagram below, the tipi in the upper left-hand corner, labeled **T**, is the preimage. Use a copy of the diagram to complete Parts **a–e**.



a. Image **a** is a reflection of **T**. Draw the line of reflection.

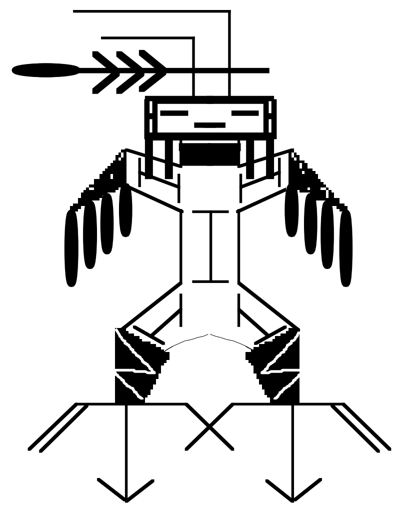
b. Image **b** is a transformation of **T** using a rotation. Find the center of rotation.

c. Image **c** is a translation of **T**. Draw a vector to represent the translation.

d. Image **d** is a dilation of **T**. Find the center of dilation and identify the scale factor.

e. The preimage **T** can also be transformed to the image **c** by a composite transformation involving two reflections. Draw two lines of reflection that could be used to perform this transformation.

3.3 Navajo sandpaintings frequently represent legendary characters. The following design is associated with Big Thunder.



a. Create your own design for a sandpainting using transformational geometry. (You may want to use a geometry utility.)

b. Write a paragraph describing how you used transformations to create your design.

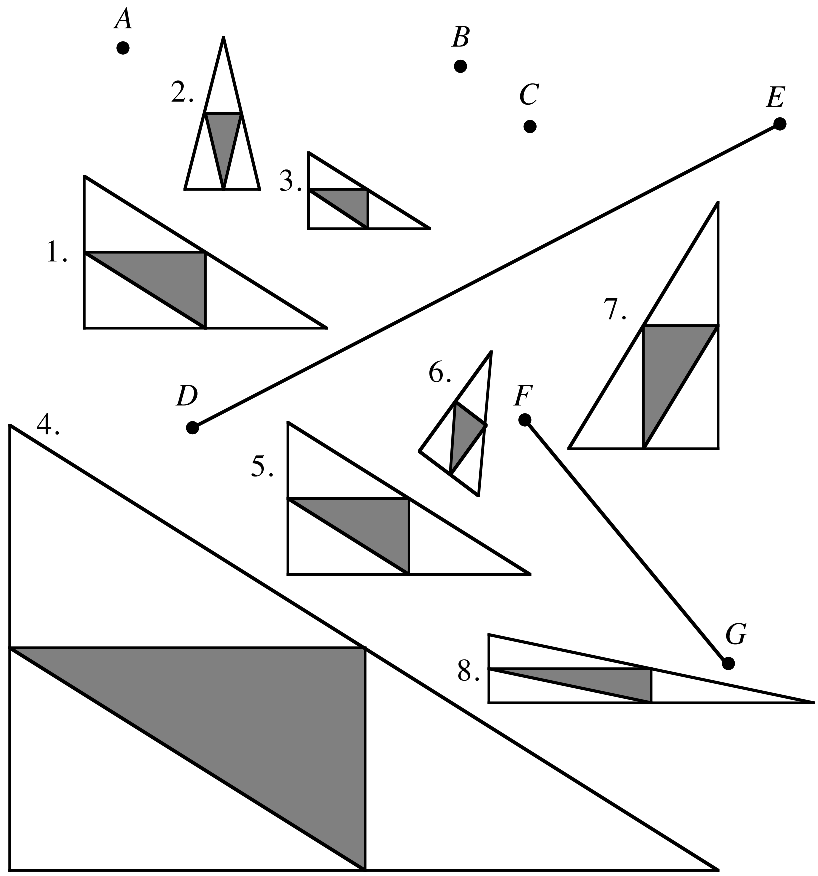
* * * * *

Research Project

Geometric designs appear in the art of many cultures and historical periods. Byzantine mosaics, for example, contain many polygonal designs and symmetries. Celtic knot designs are two-dimensional representations of three-dimensional patterns. And many well-known Renaissance painters used similarity and line perspective to portray landscapes and the human form.

Choose a culture and historical period. Research the use of geometric shape and design in the art of that time and place. Write a paper discussing your findings.

3.4 The following diagram shows eight numbered figures. Some of these figures are the images of figure 1 under transformations involving the labeled points and segments.



If possible, describe the transformation used to produce each numbered image, with figure 1 as its preimage. Each description should respond to the questions listed below:

a. Is the transformation an isometry?

b. Is the image similar to the preimage?

c. Which labeled points or lines were used in the transformation?

d. If the transformation includes a dilation, what is the approximate scale factor?

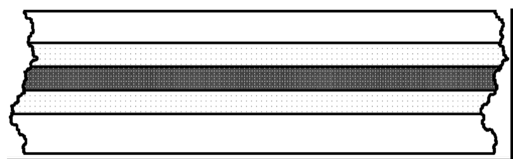
e. If the transformation includes a rotation, what is the approximate angle of rotation?

f. If the transformation includes a translation, what is the approximate magnitude and direction of the translation vector?

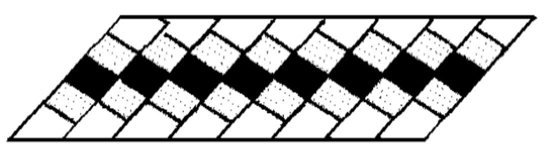
* * * * * * * * * *

Summary Assessment

1. The people of Florida's Seminole tribe are known for their patchwork designs. As early as the 1880s, they used hand-cranked sewing machines to stitch their patterns. Although there are many variations in Seminole patchwork, each one is started by sewing strips of fabric together as shown in the diagram below.

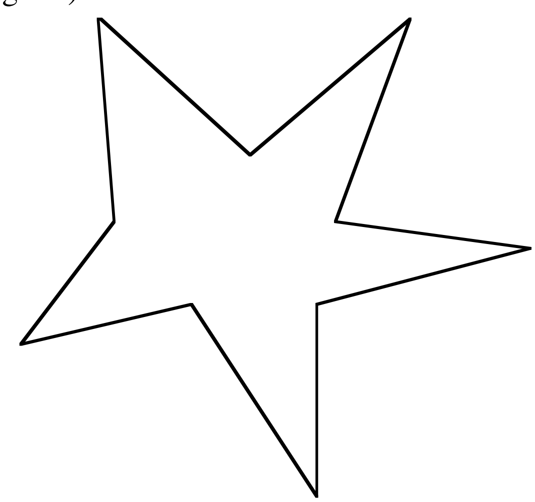


a. How could these strips of fabric be cut and sewn to create the following pattern?



b. Describe the method you developed in Part **a** in a paragraph. Include specific directions to complete the task and use geometric terms to explain why your method works.

2. A symbol often used by the Hopi people resembles the five-pointed star (or pentagram) shown below.



a. Use appropriate technology to create a pentagram like the one above.

b. Dilate the pentagram by a scale factor of your choice.

c. Explain why the corresponding sides of the image and preimage are parallel.

3. The Navajo symbol for a medicine man's eye represents wisdom. The eye symbol on the right-hand side of the diagram below is the image of the one on the left under a rotation.

a. Using a copy of this diagram, find the center of rotation. Describe the procedure you used to locate the center.

b. Using the center found in Part **a**, dilate the preimage by a scale factor of $1/2$.

Module Summary

- An object has **line symmetry** if the object is its own image in a reflection in a line. This line is the **line of symmetry**.
- **Perpendicular distance** is the distance from a point to a line or a plane. It is equal to the length of the segment, perpendicular to the line or plane, that connects the point to that line or plane.
- Two polygons are **similar** if there is a one-to-one correspondence between their vertices so that corresponding sides are proportional and corresponding angles are congruent.
- **Vertical angles** are two non-straight, non-adjacent angles formed by two intersecting lines. Vertical angles are congruent.
- A **rep tile** is a shape that can be partitioned into congruent shapes, each one similar to the original.
- If two parallel lines are cut by a transversal, then the **corresponding angles** formed are congruent.
- If two parallel lines are cut by a transversal, then the **alternate interior angles** formed are congruent.
- If two parallel lines are cut by a transversal, the **alternate exterior angles** formed are congruent.
- A **rhombus** is a parallelogram with two consecutive congruent sides.
- An object has **rotational symmetry** about a point if, when rotated through an angle about that point, each point in the image coincides with a point in the preimage.
- A **chord** is a line segment joining any two points on a circle.
- A **secant** of a circle is a line that intersects the circle in two points.
- A **tangent** of a circle is a line, segment, or ray in the plane of the circle that intersects the circle in exactly one point and is perpendicular to a radius at that point. This intersection is the **point of tangency**.
- The perpendicular bisector of a chord passes through the center of the circle.
- **Isometries** are transformations in which each preimage is congruent to its image.

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