Who Gets What and Why?

Why do some U.S. states have more senators than House representatives? In this module, you examine the process—and the politics—of apportionment.

Patricia Bean • Dean Preble • Teri Willard

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Introduction

In the not-too-distant future, a world of unstable countries and changing borders is a real possibility. In the early 1990s, for example, the Soviet Union splintered into 15 different countries, each with its own government. As the next century approaches, maps will continue to be redrawn. The following scenario describes one possible version of the future.

> The year is 2096. Antarctica, the continent that lies about the South Pole, is the last remaining land mass not yet exploited for its natural resources. First discovered by whalers in the early 1800s, it remained unexplored until the 20th century. However, repeated expeditions and the eventual establishment of several research stations led to conflicting territorial claims. To head off future disagreements, 12 nations signed an Antarctic treaty in 1959. The treaty went into effect in 1961. Although it has been modified over the years, three fundamental principles remain: the suspension of territorial claims, the freedom of scientific research, and a ban on all military activities on or around the Antarctic continent.
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> In order to maintain peace in the late 21st century and beyond, the countries of the seven major continents have agreed to form seven distinct global regions. To replace the United Nations, they have created the Global Congress for Peace (GCP). The most pressing issues facing the GCP are fair representation for the people of the new global regions, and the threat of war over rights to the resources of Antarctica.

The problem of ownership in Antarctica raises many of the same difficult issues at the core of other international disputes. Who owns Antarctica? Is there some reasonable method for dividing the region, or parts of it, among individual nations or groups of nations?

Discussion

a. Name two countries whose governments are currently involved in a border dispute.

b. Identify some of the causes for the dispute.

c. What factors must be considered when determining boundaries between two neighboring countries?

Activity 1

Figure **1** shows a map of the continent of Antarctica. With more than 8.3 million square kilometers (km^2) of land and ice, Antarctica is a vast potential source of minerals and fresh water. The continent may contain coal, iron, copper, nickel, lead, silver, cobalt, manganese, and titanium. To whom do these resources belong? How should the Global Congress distribute the mining rights?

Figure 1: Antarctica

Exploration

In this exploration, you devise methods for distributing Antarctica's resources and assigning delegates to the Global Congress.

a. Table **1** contains population estimates and land areas for the seven global regions in 2096. Use this information to find the population density (in people per square kilometer) for each region.

Table 1: Populations and land areas of global regions

Region	Population	Area (km^2)
Asia	$6.32 \cdot 10^9$	$1.77 \cdot 10^7$
Africa	$1.35 \cdot 10^9$	$1.95 \cdot 10^7$
North America	$8.60 \cdot 10^8$	$1.56 \cdot 10^7$
South America	$6.04 \cdot 10^8$	$1.15 \cdot 10^7$
Europe	$1.04 \cdot 10^9$	$3.18 \cdot 10^6$
Oceania	$5.40 \cdot 10^7$	$5.47 \cdot 10^6$
Sino-Soviet	$5.84 \cdot 10^8$	$1.44 \cdot 10^7$

b. Assume that Antarctica's natural resources are distributed evenly about its land area. Using a map of the continent, divide it among the seven global regions listed in Table **1** so that each receives a fair share.

c. In 2096, the Global Congress decides to limit itself to a total of 49 delegates. Determine a method for assigning a specific number of delegates to represent each region based on population.

Discussion

a. How did you determine the amount of land to allot to each global region?

b. What problems did you encounter when dividing 49 delegates among the seven regions?

c. In the United States, the distribution of representatives is referred to as **apportionment**. How does your method of apportionment compare to those suggested by others in your class?

Assignment

1.1 In 1996, the United Nations allowed only one representative from each member country.

a. Do you agree with this form of representation? Explain your response.

b. Suggest an alternative form of representation and describe why you believe it might be more fair.

1.2 Since the establishment of the U.S. Constitution, the United States has had two chambers of Congress: the Senate and the House of Representatives. Each state receives two senators, regardless of land area or population. In 1996, the House of Representatives consisted of 435 members, divided according to population. Why do you think the country's founders included both forms of representation?

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1.3 **a.** Express the values in Table **1** as values with one significant digit.

b. Use the values from Part **a** to find both the total population and the total area of the seven regions.

c. Express the population and area of each region as a fraction of the total. Record these fractions in a table similar to Table **1**.

d. Find the sums of the population fractions and area fractions in Part **c**.

e. **1.** Using the information from Part **c**, divide the 49 representatives to the Global Congress among the seven regions.

2. Compare this apportionment to the one you made in Part **c** of the exploration.

1.4 Juanita wants to give her collection of 957 rare books to four former students: Elizabeth, Bill, Isabella, and Kareem. She plans to divide the collection according to the number of years of education each student has completed after high school.

Elizabeth attended college and graduate school for a total of 10 years, Bill for 4 years, Isabella for 6 years, and Kareem for 7 years. Determine how many books each former student should receive and describe the method you used.

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Activity 2

For advice on potential methods of apportionment, the Global Congress consulted an American historian. The historian reported that in 1787, the U.S. Constitution (Article 1, Section 2) set forth some basic rules for the apportionment of delegates to the House of Representatives. These rules included the following:

- representatives will be apportioned based on the population of the state
- populations will be determined by a census every 10 years
- the method to be used will be determined by congressional law every 10 years
- each state will receive at least one representative.

The historian also reported that several methods of apportionment were used during the nation's first 200 years. For example, a method devised by Thomas Jefferson was used from 1790 to 1830.

Exploration

In this exploration, you consider a hypothetical country with four regions. This country must fairly apportion a total of 10 representatives. **Note:** Throughout this module, assume that every region must receive at least one representative.

Historical Note

The **standard divisor** is the ratio of the total population to the total number of representatives.

For example, if the total population is P and the total number of representatives is d, the standard divisor is P/d. Notice that this value also describes the mean number of people per representative.

a. Table **2** shows the population of each of the four regions. Use this information to find the standard divisor.

Table 2: Populations of four regions

Region	Population
A	813
B	652
C	1385
D	95
Total	2945

b.

1. To determine the approximate number of representatives to assign to a region, divide its population by the standard divisor. This value is known as the **standard quota** for the region.
2. Round each standard quota to the nearest hundredth and record it in a table with headings like those in Table **3**.

Table 3: Populations of four regions

Region	Population	Standard Quota
A	813	2.76
B	652	
C	1385	
D	95	
Total	2945	

3. Find the sum of the standard quotas.

c.

1. Use the standard quotas to assign an appropriate number of representatives to each region. (Remember that each region must receive at least one representative.)
2. Determine the total number of representatives assigned. Did you allot all 10 representatives?

d. Each of the historical methods of apportionment deals with the decimal portions of the standard quotas in a different way. The Jefferson method of apportionment rounds each standard quota down to the previous whole number.

1. Use the Jefferson method to round each standard quota in Table **3**.
2. Use the rounded quotas to assign an appropriate number of representatives to each region. (Remember that each region must receive at least one representative.)
3. Determine the total number of representatives assigned. Did you allot all 10 representatives?

e. When the total number of representatives assigned is different than the number available, the Jefferson Method states that the standard divisor must be changed.

Raise or lower the value of the standard divisor until the total number of representatives assigned is 10.

Discussion

a. The standard quota is the basis for many methods of apportionment. Describe a mathematical expression for the standard quota in terms of the total population (P), the total number of representatives (d), and the population of the region (R).

b. What problems arise when using the standard quota to assign a number of representatives to each region?

c. Does the method you used in Parts **a–c** of the exploration assign representatives according to the guidelines set forth in the U.S. Constitution?

d. Describe how technology could be used to round standard quotas according to the Jefferson method.

e. Describe how technology could be used to modify the standard divisor according to the Jefferson method.

Assignment

2.1 Suppose that the country described in the exploration decides to add two more representatives to its governing body. This results in a total of 12 representatives. Use the Jefferson method to assign the appropriate number of representatives to each region described in Table **2**.

2.2 In 1842, Congress replaced the Jefferson method of apportionment with one proposed by Daniel Webster. The Webster method was used for the remainder of the 1840s and again from 1900 to 1940. Although the two methods have some similarities, the Webster method rounds standard quotas to the nearest integer.

a. Round the standard quotas in the table below using the Webster method.

Region	Population	Standard Quota
A	813	2.76
B	652	2.21
C	1385	4.70
D	95	0.32

b. Use the rounded quotas to assign a number of representatives to each region. (Remember that each region must receive at least one representative.)

c. When the total number of representatives assigned does not equal the number available, the Webster method—like the Jefferson method—modifies the standard divisor. Adjust the standard divisor until the total number of representatives assigned is 10. Describe the process you used to find a new divisor.

2.3 After the 1850 census, Congress adopted a method of apportionment based on the work of Alexander Hamilton. The Hamilton method determines the standard divisor, calculates the standard quota for each state, then rounds each quota down to the previous whole number.

If the total number of representatives assigned is less than the number available, the decimal parts of the standard quotas are ranked from greatest to least. This ranking is used to assign the remaining seats in the House of Representatives. The state with the greatest decimal part receives the first extra seat, the state with the second greatest receives the second extra seat, and so on, until all extra seats are assigned.

a. Use the Hamilton method to apportion 10 representatives among the four regions in Table **2**.

b. For each region, determine the number of people represented by each delegate.

2.4 **a.** Write a paragraph comparing the Jefferson, Hamilton, and Webster methods of apportionment.

b. Compare the number of representatives received by each region in Table **2** using the three methods.

2.5 As described in Activity **1**, the Global Congress for Peace has a total of 49 delegates. Use each of the following methods of apportionment to assign delegates to the seven regions in Table **1**.

a. the Jefferson method

b. the Webster method

c. the Hamilton method

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2.6 Suppose that the country described in the exploration adds 10 more representatives to its governing body, for a total of 20. Use each of the following methods of apportionment to assign representatives to the four regions in Table **2**.

a. the Jefferson method

b. the Webster method

c. the Hamilton method

2.7 In 2096, scientists estimate that the total amount of fresh water available from the Antarctic region is $1.58 \cdot 10^{10}$ kL (kiloliters). The Global Congress wants to apportion this resource among the seven regions in Table **1**.

a. The water will be allotted in units of 1 million kiloliters ($1 \cdot 10^6$ kL). Determine the number of units of this size in the total amount of water available.

b. The total population of the seven regions is approximately $1.08 \cdot 10^{10}$. To determine the standard divisor in this situation, divide the total population by the total number of $1 \cdot 10^6$ kL units. Round the divisor to the nearest whole number.

c. Using the result from Part **b**, find the standard quota for each region.

d. Determine the allotment of fresh water for each region using either the Jefferson or the Webster method of apportionment.

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Research Project

To learn more about the history of apportionment in the United States, complete either Part **a** or Part **b** below.

a. The following table shows the apportionment methods used in the United States from 1790 to 1990. (In 1941, the size of the U.S. House of Representatives was fixed at 435.) Select one of the entries in the table and write a report describing its historical circumstances.

Census Years	Apportionment Method Used
1790–1830	Jefferson
1840	Webster
1850	Hamilton
1860–1870	Hamilton (modified)
1880–1890	Hamilton
1900–1910	Webster
1920	No new apportionment
1930	Webster
1940–1990	Huntington

b. Identify an event in U.S. history that was affected by apportionment. Describe the event and explain how apportionment affected its occurrence or outcome.

Activity 3

The method of apportionment for the U.S. House of Representatives has been the subject of frequent debate. In 1929, Congress mandated that this apportionment be done in one of three ways: by applying the method used for the previous apportionment, by applying the Webster method, or by applying a method of equal proportions.

The method of equal proportions was suggested by Joseph Hill in 1911 and presented to Congress by Edward Huntington. When Hill designed this method, he decided that the number of people represented by each delegate was important.

Discussion 1

a. In the apportionment of 1990, 434 of the 435 representatives to the U.S. House were assigned with relative ease. The lone remaining seat, however, was coveted by several states. Massachusetts, with a population of 6,029,050, had already been assigned 10 representatives. The state of Washington, with a population of 4,887,945, had been assigned 8 representatives.

Given that M is the population of Massachusetts in 1990 and m its number of seats in the U.S. House, the number of people represented by each delegate can be described by the ratio M/m.

How could you use similar notation to describe the number of people represented by each delegate for the state of Washington?

b. Using the ratios described in Part **a**, which state—Massachusetts or Washington—was better represented in the U.S. House in 1990? Justify your response.

c. To determine which state should receive an unassigned delegate, Hill used what he referred to as **relative difference**. When Huntington presented Hill's method of apportionment to Congress, he defined relative difference using an expression like the one below, where W/w is greater than M/m:

$$\frac{\frac{W}{w} - \frac{M}{m}}{\frac{M}{m}}$$

Use this expression to find the relative difference between Massachusetts and Washington in 1990.

d. What would happen to the relative difference if M/m were greater than W/w?

Exploration

In the method of apportionment proposed by Huntington and Hill, relative difference is used to decide which state has priority for receiving another delegate. This priority is determined by minimizing the relative difference. In this exploration, you examine the relative differences among several states.

a. In the 1990 apportionment, Massachusetts initially received 10 representatives, while Washington initially received 8 representatives. One unassigned seat remained.

1. Determine the relative difference that occurs when Massachusetts receives the additional seat.

2. Determine the relative difference that occurs when Washington receives the additional seat.

b. In order to minimize the effect of adding another representative, would you give the additional seat to Massachusetts or to Washington?

c. In 1988, New Jersey had 14 seats in the U.S. House. During the 1990 reapportionment, the state was initially assigned only 13 representatives. Naturally, many of New Jersey's 7,748,634 citizens felt that they should receive the remaining seat.

1. Determine the relative difference between New Jersey and Washington if New Jersey receives the additional seat.

2. Determine the relative difference between New Jersey and Washington if Washington receives the additional seat.

3. Determine the relative difference between New Jersey and Massachusetts if New Jersey receives the additional seat.

4. Determine the relative difference between New Jersey and Massachusetts if Massachusetts receives the additional seat.

d. In order to minimize the effect of adding another representative, would you give the additional seat to Massachusetts, Washington, or New Jersey?

e. When determining which of three states should receive an additional representative, you calculated a relative difference six times.

How many times would you have to calculate relative difference to determine priority among each of the following:

1. four states?

2. five states?

Discussion 2

a. In the following expression, M represents the population of Massachusetts in 1990 and m its number of seats in the U.S. House, while W represents the population of Washington and w its number of seats in the house. What does the entire expression represent?

$$\frac{\frac{W}{w} - \frac{M}{m+1}}{\frac{M}{m+1}}$$

b. Using M, m, W, and w, describe an expression for the relative difference when Washington receives an additional House seat.

c. In Part **a** of the exploration, you determined two relative differences: one when Massachusetts received the additional seat, and one when Washington received it.

1. Use an inequality to describe the relationship between these two relative differences.

2. How could you express this inequality using M, m, W, and w?

Assignment

3.1 Expressions like the one described in Part **a** of Discussion **2** are sometimes called **complex fractions**. Simplify each of the following complex fractions to a simple fraction by performing the indicated operations.

a. $\dfrac{\frac{3}{5}}{\frac{4}{7}}$

b. $\dfrac{\frac{2}{5}+\frac{1}{4}}{\frac{4}{3}}$

c. $\dfrac{\frac{6}{5}+\frac{9}{4}}{\frac{2}{3}}$

d. $\dfrac{\frac{4}{3}-\frac{2}{x+1}}{\frac{2}{x+1}}$

3.2 In Part **c** of Discussion **2**, you used the variables M, m, W, and w to write the following inequality.

$$\frac{\frac{W}{w} - \frac{M}{m+1}}{\frac{M}{m+1}} > \frac{\frac{M}{m} - \frac{W}{w+1}}{\frac{W}{w+1}}$$

a. **1.** Explain why the fraction on the left-hand side of the inequality can be rewritten as shown below.

$$\frac{\frac{W}{w}}{\frac{M}{m+1}} - \frac{\frac{M}{m+1}}{\frac{M}{m+1}}$$

2. Explain why this expression simplifies to the following:

$$\frac{\frac{W}{w}}{\frac{M}{m+1}} - 1$$

b. Use the method described in Part **a** to simplify the right-hand side of the inequality.

c. The entire inequality can be simplified further by adding 1 to both sides. Write the simplified inequality.

d. The inequality can be simplified again by multiplying both sides by the common denominator of the remaining two complex fractions. However, if the common denominator is negative, this multiplication would change the direction of the inequality.

1. Is the common denominator shown below positive or negative? Explain your response.

$$\left(\frac{M}{m+1}\right)\left(\frac{W}{w+1}\right)$$

2. Multiply both sides of the inequality by this common denominator and write the simplified inequality.

e. The simplified inequality you wrote in Part **d** should now contain two fractions with numerators W^2 and M^2, respectively.

1. Determine whether each of these fractions is positive or negative.

2. Describe how the inequality can be rewritten so that the numerators are W and M.

3. Rewrite the inequality so that the numerators are W and M. The values on each side of this inequality are referred to as **priority numbers**.

4. Why was it important to determine whether the fractions were positive or negative before completing Step **3**?

f. In Parts **a–e**, you demonstrated that these inequalities are equivalent:

$$\frac{\frac{W}{w}-\frac{M}{m+1}}{\frac{M}{m+1}} > \frac{\frac{M}{m}-\frac{W}{w+1}}{\frac{W}{w+1}} \qquad \frac{W}{\sqrt{w(w+1)}} > \frac{M}{\sqrt{m(m+1)}}$$

1. Verify that each inequality is true for the 1990 values of M, m, W, and w. (Massachusetts had a population of 6,029,050 and 10 representatives. Washington had a population of 4,887,945 and 8 representatives.)

2. Which inequality do you think is easier to use? Explain your response.

Mathematics Note

The **geometric mean** of two positive numbers a and b is $\sqrt{ab}$. For example, the geometric mean of 4 and 5 is $\sqrt{4 \cdot 5} = \sqrt{20}$ or about 4.47.

3.3 **a.** Consider the arithmetic sequence 0, 2, 4, 6, 8. The arithmetic mean of the terms 0 and 8 is 4, while the arithmetic mean of 2 and 8 is 4. Describe how the arithmetic mean is calculated.

b. Consider the geometric sequence 2, 4, 8, 16, 32. The geometric mean of the terms 2 and 8 is 4, while the geometric mean of 2 and 16 is $\sqrt{32}$. Determine the geometric means of each of the following:

1. 8 and 32

2. 4 and 32

3.4 **a.** Where did the geometric mean occur in your comparison of relative differences for Massachusetts and Washington?

b. How would you use priority numbers to determine which state should receive the additional delegate?

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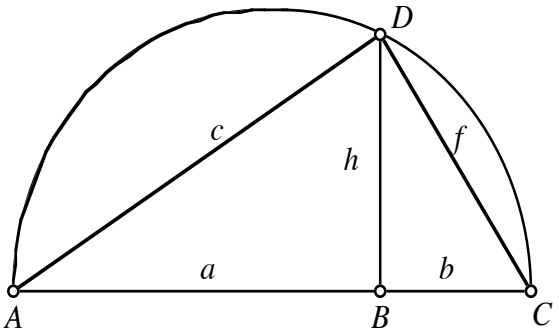
3.5 The geometric mean of a and b also can be described as the height h of the triangle circumscribed in the semicircle below.



a. In the diagram, the measures of $\angle ADC$, $\angle ABD$, and $\angle CBD$ are all 90°. Use similar triangles to show that $h = \sqrt{ab}$.

b. **1.** If $a = 5.1$ cm and $b = 2.1$ cm, what is the value of h?

2. If $a = 7.4$ cm and $h = 4.8$ cm, what is the value of b?

3. If $b = 4.0$ cm and $h = 5.9$ cm, what is the value of a?

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Activity 4

As you observed in Activity **3**, when the number of states vying for an additional representative increases, the number of comparisons required also increases. The method of equal proportions reduces the number of calculations necessary by assigning priority numbers based on relative differences.

Since 1929, the method of equal proportions has become known as the Huntington method. Using this method, each state receives one representative. Priority numbers are then used to apportion the remaining House seats.

Exploration 1

Table **4** shows the populations of six regions in a hypothetical country. These six regions share a total of 45 representatives to the country's governing body.

Table 4: Populations and representatives for six regions

Region	Population	Rounded Quota	(Rounded Quota) + 2
A	264		
B	481		
C	330		
D	1652		
E	1442		
F	477		
Total	4646		

In this exploration, you use the Huntington method to assign an appropriate number of representatives to each region.

a. In order to determine how many priority numbers to calculate, you must estimate the number of representatives each region will receive. To make this estimate, complete Steps **1** and **2** below.

1. Determine the standard quota for each region and round each one up to the next integer. Record the rounded quotas in Table **4**.

2. Since the rounded quota may be less than the number of representatives assigned, add 2 to each rounded quota and record these values in Table **4**. This will ensure that enough priority numbers are calculated for each region.

Historical Note

The **method of equal proportions** is based on a comparison of each state or region's priority numbers. These **priority numbers** are determined according to the following formula:

$$\text{priority no.} = \text{population} \bullet \frac{1}{\sqrt{(\text{current no. of reps.})(\text{possible no. of reps.})}}$$

Using this method, states or regions may have several priority numbers, depending on whether one or more representatives could be added. For example, if a region can be considered for nine representatives, nine priority numbers must be calculated (one for each representative).

In the apportionment of 1990, the lone remaining seat in the U.S. House was to be assigned to either Massachusetts, with 10 representatives and a population of 6,029,050, or the state of Washington, with 8 representatives and a population of 4,887,945. Using the method of equal proportions, the priority number for Washington's 9th representative was found as follows:

$$\begin{aligned}\text{priority no.} &= 4{,}887{,}945 \bullet \frac{1}{\sqrt{8(9)}} \\ &= 4{,}887{,}945 \bullet 0.11785113 \\ &= 576{,}049.843\end{aligned}$$

In a similar manner, the priority number for Massachusetts' 11th representative was found as shown below:

$$\begin{aligned}\text{priority no.} &= 6{,}029{,}050 \bullet \frac{1}{\sqrt{10(11)}} \\ &= 6{,}029{,}050 \bullet 0.095346259 \\ &= 574{,}847.362\end{aligned}$$

Since the priority number for Washington was greater than the priority number for Massachusetts, Washington received the additional seat in the U.S. House.

b. In Activity **3**, you assigned representatives by comparing relative differences. To assign 45 representatives among 6 regions by this method, you would have to make 410 comparisons.

How many priority numbers do you need to calculate to assign these 45 representatives using the method of equal proportions?

Discussion 1

a. The partial spreadsheet in Table **5** below shows one way to calculate priority numbers for the six regions. Why are the values in each row of column C one more than the corresponding values in column B?

Table 5: Calculating priority numbers

	A	B	C	D	E
1	**Region**	**Current No. of Reps.**	**Possible No. of Reps.**	**Pop.**	**Priority Number**
2	A	1	2	264	186.68
3	A	2	3	264	107.78
4	A	3	4	264	76.21
5	A	4	5	264	59.03
6	B	1	2	481	340.12
7	B	2	3	481	196.37
8	B	3	4	481	138.85
9	B	4	5	481	107.55
10	B	5	6	481	87.82
11	B	6	7	481	74.22
12	C	1	2	330	233.35
13	C	2	3	330	134.72
⋮	⋮	⋮	⋮	⋮	⋮

b. The estimated number of priority numbers needed for each region was based on the standard quota.

1. Why have four priority numbers been calculated for region A?

2. How many priority numbers should be calculated for region C?

c. Describe a spreadsheet formula that could be used to determine the priority numbers in column E.

d. Which region shown in the partial spreadsheet has the highest priority for receiving the next representative? Do you think this will be the highest priority number among all six regions?

Exploration 2

a. Complete the partial spreadsheet given in Table **5**.

b. According to the Huntington method, each of the six regions automatically receives 1 representative. Use priority numbers to assign the remaining 39 representatives.

Discussion 2

a. Describe how the priority numbers for Massachusetts and Washington could be expressed using the variables M, m, W and w.

b. In Problem **3.2**, you used the following inequality to compare relative differences for Massachusetts and Washington:

$$\frac{W}{\sqrt{w(w+1)}} > \frac{M}{\sqrt{m(m+1)}}$$

Compare this inequality to the expressions you described in Part **a**.

c. By comparing the results of an apportionment with the corresponding standard quotas, some critics have found fault with the Huntington method. Compare the number of representatives assigned by the Huntington method with the standard quota for each region in Table **6**. What potential drawbacks do you observe?

Table 6: Standard quotas for six regions

Region	Population	Standard Quota
A	264	2.557
B	481	4.659
C	330	3.196
D	1652	16.001
E	1442	13.967
F	477	4.620
Total	4646	

Assignment

4.1 As described in Activity **2**, you can assign representatives using the Hamilton method of apportionment by completing the following steps:

- finding the standard divisor
- calculating the standard quota
- rounding each quota down
- assigning any remaining representatives by ranking the decimal parts of the standard quotas from greatest to least.

a. Use the Hamilton method to apportion 45 representatives among the six regions in Table **4**.

b. Compare your results in Part **a** with the apportionment made using the Huntington method.

4.2 Table **4** in Exploration **1** shows the populations of six regions. Suppose that the legislature for these regions decides to increase the total number of representatives from 45 to 50. Use the Huntington method of apportionment to assign the additional seats.

4.3 As described in Activity **1**, the Global Congress consists of 49 representatives from the seven regions described in the table below. Use the Huntington method to assign an appropriate number of representatives to each region.

Region	**Population**	**Area (km^2)**
Asia	$6.32 \cdot 10^9$	$1.77 \cdot 10^7$
Africa	$1.35 \cdot 10^9$	$1.95 \cdot 10^7$
North America	$8.60 \cdot 10^8$	$1.56 \cdot 10^7$
South America	$6.04 \cdot 10^8$	$1.15 \cdot 10^7$
Europe	$1.04 \cdot 10^9$	$3.18 \cdot 10^6$
Oceania	$5.40 \cdot 10^7$	$5.47 \cdot 10^6$
Sino-Soviet	$5.84 \cdot 10^8$	$1.44 \cdot 10^7$

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4.4 As shown in Table **4**, the population of region D is 1652, while the population of region E is 1442. Suppose that one person from region E moves to region D. Using the Huntington method of apportionment, describe how this change will affect the number of representatives assigned to each region. (Assume that the total number of representatives for the six regions remains 45.)

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Summary Assessment

Like the U.S. House of Representatives, the Canadian House of Commons is apportioned according to population. A total of 295 members of the House of Commons are divided among 10 provinces and 2 territories. The 1991 population of each of these regions is shown below.

Province	Population
Alberta	2,545,553
British Columbia	3,282,061
Manitoba	1,091,942
New Brunswick	723,900
Newfoundland	568,474
Nova Scotia	899,942
Ontario	10,084,885
Prince Edward Island	129,765
Quebec	6,895,963
Saskatchewan	988,928
Northwest Territories	57,649
Yukon Territory	27,797

Source: Statistics Canada.

1. Use each of the following methods of apportionment to assign representatives to the provinces and territories of Canada.

- **a.** the Jefferson method
- **b.** the Webster method
- **c.** the Hamilton method
- **d.** the Huntington method

2. In this situation, which of the four methods of apportionment do you prefer? Justify your selection.

Module Summary

- In the United States, the division and sharing of representatives is referred to as **apportionment**.
- The **standard divisor** is the ratio of the total population to the total number of representatives.
- The **standard quota** for each region is calculated by dividing its population by the standard divisor.
- The **Jefferson method** rounds each standard quota down to the previous integer. Using this method, the standard divisor is adjusted until the sum of the rounded quotas equals the number of available representatives.
- The **Webster method** rounds standard quotas to the nearest integer. Like the Jefferson method, it modifies the standard divisor until the sum of the rounded quotas equals the number of available representatives.
- The **Hamilton method** determines the standard divisor, calculates standard quotas, then rounds each quota down to the previous whole number. If the total number of representatives assigned is less than the number available, the decimal parts of the standard quotas are ranked from greatest to least. This ranking is used to assign any remaining seats. The region with the greatest decimal part receives the first extra seat, the region with the second greatest receives the second extra seat, and so on, until all remaining seats are assigned.
- The **geometric mean** of two positive numbers a and b is $\sqrt{a \cdot b}$.
- The **method of equal proportions**, or **Huntington method**, is based on a comparison of each state or region's priority numbers. These **priority numbers** are determined according to the following formula:

$$\text{priority no.} = \text{population} \cdot \frac{1}{\sqrt{(\text{current no. of reps.})(\text{possible no. of reps.})}}$$

In this formula, a region's population is multiplied by the reciprocal of the geometric mean of two consecutive numbers. Using this method, a region may have several priority numbers, depending on whether one or more representatives could be added.

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