What Are My Child's Chances?

Did you ever wonder why your eyes match your mother's? Or why you have your father's chin? In this module, you study the genetics of several inherited traits.

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Introduction

Genetic diseases are passed on from generation to generation. Many of them can be crippling, even fatal. While effective therapies are available for some of these disorders, others are currently untreatable.

Scientists are developing tests to identify genetic diseases in unborn children and to detect adult carriers. Through prenatal screening, and by determining the probability that a particular disease will appear in certain families, many genetic diseases can be prevented. A genetics counselor is an important part of this effort.

The mathematical science of probability, combined with a knowledge of biology and medicine, can help a genetics counselor determine the chances that a child will inherit a genetic disease.

Activity 1

Genetics is the study of **heredity**, the process by which characteristics are passed from one generation to the next. The biological structures that control heredity are **genes**. Your particular combination of genes determines your eye color and hair color, for example, as well as many of the other characteristics that distinguish you from siblings, parents, and friends.

Most physical traits are determined by several sets of genes. Some, however, such as the presence of free earlobes, are controlled by a single set of genes. Single sets of genes also control blood type and determine the availability of certain hormones and enzymes in the body. How can you calculate a child's chances of having free earlobes or type A blood? In the following activities, you discover how mathematics can help you predict the likelihood of receiving a particular trait.

Exploration

Table **1** describes the characteristics associated with five physical traits. Read the descriptions and determine whether or not you have each trait.

Table 1: Five traits determined by single sets of genes

Trait	Description
Mid-digit hair	The middle section of each finger has hair on it.
Widow's peak	When the hair on the head is pulled back, a distinct point in the hairline can be seen in the center of the forehead.
Free earlobes	The bottom parts of the earlobes are not attached to the side of the head.
Rolled tongue	The tongue can be rolled to form a "U" shape.
Folded hands	When the hands are folded so that fingers interlace, the left thumb falls naturally on top.

To explore your own inherited characteristics, you can use a wheel of traits. Figure **1** shows a wheel for the five traits described above.

a. Place your finger at the trait in the center of the wheel, "mid-digit hair." If you have this trait, move your finger to the "yes" portion. If you do not have it, move your finger to the "no" portion.

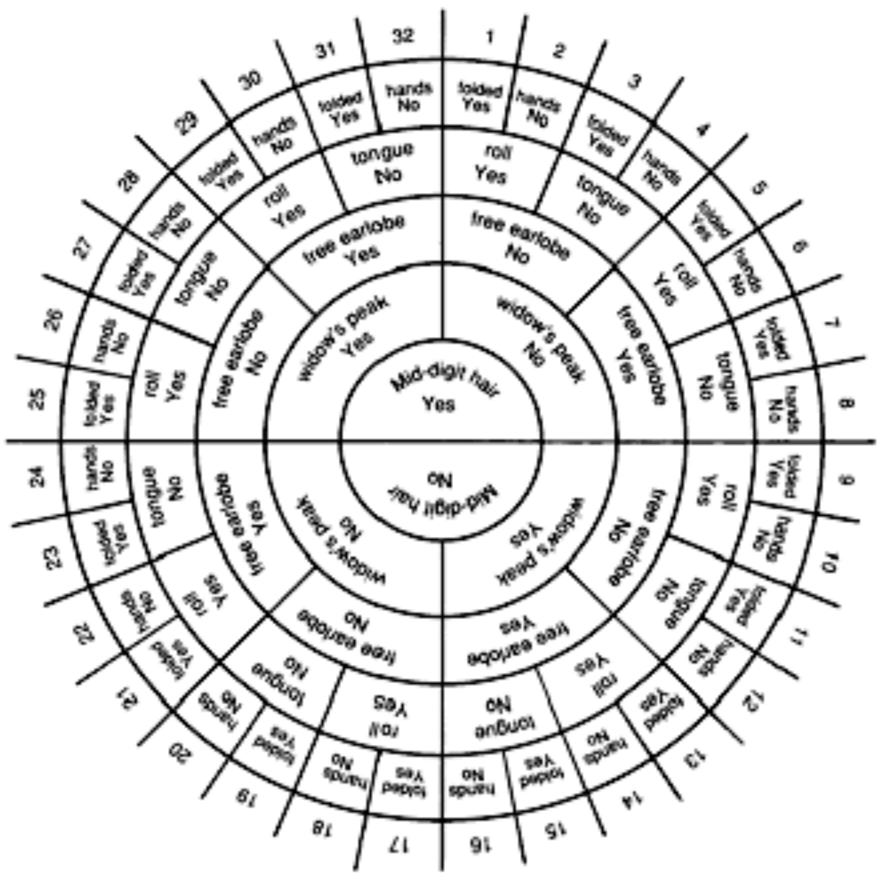

Figure 1: Wheel of traits

b. From the portion of the ring where you placed your finger, move outward to the next ring and determine whether or not you have the trait "widow's peak." Again, move your finger to the appropriate portion of the ring.

c. Continue this process for the next three traits, working your way outward through each ring of the wheel.

d. The last portion of the ring where you placed your finger determines your personal number. Record this number.

e. Organize the class results from Parts **a–c** in Table **2**. **Note:** Save this table for use in Problem **1.1**.

Table 2: Class results from wheel of traits

Trait	No. with Trait	No. without Trait
mid-digit hair		
widow's peak		
free earlobes		
rolled tongue		
folded hands		

f. The set of all possible outcomes for an experiment is the **sample space**. An **event** is a subset of the sample space.

One method of predicting the likelihood of an event is to perform many trials under controlled conditions. The results of these trials provide the **experimental** (or **empirical**) **probability** of the event occurring. The experimental probability of an event is the following ratio:

$$\frac{\text{number of times event occurs}}{\text{total number of trials}}$$

Using the class data, calculate the experimental probability of having each of the five traits on the wheel.

Discussion

a. Is each trait on the wheel equally likely to occur in your class? Explain your response.

b. Why does the wheel in Figure **1** have 32 different personal numbers?

c. Given the five traits in the wheel in Figure **1**, how many different pairs of traits exist?

Mathematics Note

Given any event E, the event that E does not occur is its **complement**. The complement of E can be represented by the symbol E', read "E prime" or "E complement." The sum of the probabilities of two complementary events is 1.

For example, consider an experiment in which a die is tossed 5 times and the number of sixes is counted. If S represents the event that a six occurs, the event that a six does not occur can be represented by S'. If 3 sixes are obtained, the experimental probability of S is $3/5$, while the experimental probability of S' is $2/5$. The sum of the probabilities for these two complementary events is 1.

d. Describe how you could use a Venn diagram to show the relationship between two complementary events E and E'.

e. If you know the experimental probability of having a specific trait, how can you determine the experimental probability of not having that trait?

f. How could you use the experimental probabilities found in the exploration to estimate the number of students in your school with each trait?

Assignment

1.1 **a.** Use the class data to determine the experimental probability of not having each trait on the wheel. Organize these probabilities in a table like the one below.

Trait	Experimental Probability of Having Trait	Experimental Probability of Not Having Trait
mid-digit hair		
widow's peak		
free earlobes		
rolled tongue		
folded hands		

b. Using the class data, predict how many students in your school have free earlobes.

c. What is the sum of the experimental probabilities of having and not having each trait? Why does this occur?

1.2 Suppose that you used the wheel of traits in Figure **1** to collect data from across the nation. Do you think the combination of five traits that occurred most often in your class would occur most often in the national data? Why or why not?

1.3 In one class of students, the experimental probability of having a widow's peak was 0.7. What was the experimental probability of not having a widow's peak in this class? Explain how you determined your response.

1.4 Describe the event that is complementary to having the trait "folded hands."

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1.5 In 1991, approximately 2,041,000 fires were reported in the United States. The table below shows how these fires were distributed by type.

Type of Fire	Number Reported
Structure—accidental	542,000
Structure—incendiary device used	62,000
Structure—suspicious origin	36,000
Outside of structure (includes timber, crops, and outside storage)	54,000
Brush and rubbish	806,000
Vehicle	428,000
Other	113,000

Source: U.S. Bureau of the Census.

a. What is the probability that a fire reported in 1991 was a structure fire?

b. What percentage of all reported fires in 1991 involved incendiary devices or suspicious causes?

c. What is the probability that a fire reported in 1991 was not a vehicle fire?

1.6 In the United States, heart disease is the most common cause of death for adults. The table below shows the number of deaths due to heart disease for men of various age groups in 1990.

Age (years)	No. in Age Group	No. of Deaths
25–34	21,564,000	3278
35–44	18,009,000	18,585
45–54	12,232,000	46,078
55–64	9,955,000	948,276
65–74	7,907,000	1,716,056
75–84	3,745,000	169,828
over 85	841,000	70,864

Source: U.S. Bureau of the Census.

a. For men aged between 45 and 54 in 1990, what was the probability of dying from heart disease?

b. What percentage of the men who died from heart disease in 1990 were between 65 and 74 years old?

c. For men aged 25 and over in 1990, what was the probability of dying from heart disease?

d. For men aged between 25 and 34 in 1990, what was the probability of not dying from heart disease?

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Activity 2

The genes you inherited from your parents are carried on 23 pairs of **chromosomes**. Figure **2** shows one pair of chromosomes, magnified many thousands of times.

Figure 2: A pair of human chromosomes

Like chromosomes, genes also come in pairs. Each member of a pair is called an **allele** (pronounced "uh leel"). During fertilization, two alleles combine—one from each parent. These two alleles may be alike or different. However, only one trait appears for each pair.

The pair of alleles that determines the presence or absence of a particular characteristic is the **genotype**. The trait that actually occurs is referred to as the **phenotype**. When the alleles in a pair are different, the trait that appears is the **dominant** trait. The other is referred to as the **recessive** trait.

Exploration

In the case of earlobes, there are two possible phenotypes: free earlobes or attached earlobes. Having free earlobes is the dominant trait, while having attached earlobes is the recessive trait.

In genetics, the dominant trait is typically represented by a capital letter, while the recessive trait is represented by the corresponding lowercase letter. If F represents free earlobes and f represents attached earlobes, then the four possible combinations of these two alleles can be written as follows: FF, Ff, fF, and ff.

However, the combinations Ff and fF, which each contain one dominant and one recessive allele, are genetically the same. This means there are only three possible genotypes: FF, Ff, and ff. When the two alleles in a pair are different, the dominant trait masks the recessive one. Since having free earlobes is the dominant trait, the genotypes FF and Ff both result in the phenotype of free earlobes. Only the genotype ff results in the phenotype of attached earlobes.

a. A parent can have any one of the three possible genotypes: FF, Ff, or ff.

  1. Select two genotypes, one to represent each parent.

  2. List all the genotypes that are possible in their children.

b. Describe the method you used in Part **a** to ensure that all possible combinations were identified.

Science Note

The act of combining genes from two parents is called a **cross**. In a cross, each allele in a parent's genotype is assumed to have an equal chance of being passed on to the offspring.

Given the genotypes of both parents for a particular trait, you can use a **Punnett square** to help identify the possible combinations from a cross. For example, Figure **3** shows a Punnett square for the cross of two parents with the same genotype, Ff. The possible genotypes from this cross are FF, Ff, and ff.

		Mother	
		F	**f**
Father	**F**	FF	Ff
	f	Ff	ff

Figure 3: Punnett square for the cross Ff ∞ Ff

c. Draw a Punnett square for the cross you chose in Part **a**. Identify all the possible genotypes and all the possible phenotypes.

Mathematics Note

If each outcome in a sample space has the same chance of occurring, then the **theoretical probability** of an event can be calculated using the following ratio:

$$\frac{\text{number of outcomes in the event}}{\text{total number of outcomes in the sample space}}$$

The theoretical probability of an event E can be written as $P(\mathrm{E})$.

For example, consider the cross of two parents with the same genotype, Ff. As shown in Figure **3**, the sample space contains 4 equally likely outcomes: FF, Ff, Ff, and ff. Since the FF genotype occurs 1 time in the sample space, its theoretical probability can be expressed as $1/4$. In other words, $P(\mathrm{FF}) = 1/4$. Similarly, $P(\mathrm{ff}) = 1/4$. Since the genotype Ff occurs 2 times in the sample space, $P(\mathrm{Ff}) = 2/4$, or $1/2$. The sum of the probabilities for all the possible outcomes in the sample space is 1.

Because the allele for free earlobes (F) is dominant, the genotypes FF and Ff both result in the phenotype of free earlobes. For the cross shown in Figure **3**, the probability that a child has free earlobes is $3/4$, while the probability that a child has attached earlobes is $1/4$.

d. For each genotype identified in Part **c**, determine the probability that a child of these parents will possess that genotype.

e. For each possible phenotype identified in Part **c**, determine the probability that a child of these parents will have that phenotype.

Discussion

a.
1. In Part **c** of the exploration, how many different genotypes did you identify?
2. How many different phenotypes did you identify?

b. Why is it possible for your responses to Part **a** of the discussion to differ from those of your classmates?

c. How do the parents' genotypes affect the probability of the child's phenotype?

Assignment

2.1 Consider the cross FF ∞ ff, where F represents free earlobes and f represents attached earlobes.

a. Draw a Punnett square for this cross.

b. What is the probability of each possible genotype?

c. Describe the phenotype that results for each possible genotype.

d. What is the probability of each possible phenotype?

2.2 Repeat Problem **2.1** using the cross Ff ∞ ff.

2.3 Consider the cross Rr ∞ RR, where R represents the ability to roll the tongue, a dominant trait.

a. Determine the probability of the genotype RR. Justify your response using a Punnett square.

b. Determine the probability of the genotype rr. Defend your response using a tree diagram.

c. What is the probability that a child of these parents has the ability to roll the tongue? Explain your response.

2.4 Having a widow's peak is a dominant trait, represented by the capital letter W. Consider the cross Ww ∞ ww, where the father has a widow's peak.

a. Which pair of alleles belongs to the father? Explain your response.

b. Using a Punnett square, identify the genotypes that can result from this cross.

c. Determine the probability that a child of this cross does not have a widow's peak.

2.5 Consider the cross Ww ∞ Ww, where W represents the trait widow's peak.

a. What is the probability that a child of this cross receives one dominant allele (W) and one recessive allele (w)?

b. Why do the two outcomes Ww and WW result in the same phenotype?

c. For a child of this cross, what is the ratio of the probability of having a widow's peak to the probability of not having one?

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2.6 Both of Deirdre's parents have free earlobes, a dominant gene. Deirdre does not.

a. Use Punnett squares to find all the possible genotypes for a child of Deirdre's parents.

b. Determine the probability that a child of Deirdre's parents does not have free earlobes.

2.7 Cystic fibrosis is a genetic disease. Its victims lack certain chemicals in the body and typically have severe respiratory and digestive problems. For children in the United States, it is the leading cause of death due to disease.

A baby can be born with cystic fibrosis if both its parents are **carriers** of the disease. Carriers are people who show no signs of the disease but can pass it on to their children. Carriers have the genotype Cc, while people with the disease have the genotype cc.

a. Draw a Punnett square that represents a cross between two parents that are carriers of cystic fibrosis.

b. What is the probability that a child with cystic fibrosis is born to parents who are both carriers of the disease?

c. One out of every 25 people in the United States is a carrier of cystic fibrosis. What are the chances that a healthy person chosen at random in the United States is not a carrier of cystic fibrosis?

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Activity 3

In the previous activities, you examined traits one at a time. When genetic counselors advise their clients, however, they must consider the parents' traits as groups. Some characteristics, for example, are associated only with males, while some are associated only with females. A few characteristics are linked to certain other traits regardless of sex. Grouping traits affects the probabilities in each situation. It also affects the way in which those probabilities are calculated.

Discussion 1

a. Consider a father with the genotype Rr and a mother with the genotype RR, where R represents a dominant trait.

1. Assuming that each allele in a parent's genotype has an equal chance of being passed on to any offspring, what is the probability that a child of this couple will receive the dominant allele from the father?
2. What is the probability that a child of this couple will receive the dominant allele from the mother?
3. What is the probability that a child of this couple will have the genotype RR?

b. If the mother's genotype were Rr, would this change your response to Part **a.1**?

Mathematics Note

Two events A and B are **independent** if $P(\text{A and B}) = P(\text{A}) \bullet P(\text{B})$. It follows that for independent events A, B, and C, $P(\text{A and B and C}) = P(\text{A}) \bullet P(\text{B}) \bullet P(\text{C})$. This definition can be extended to any number of independent events.

Given two independent events, the occurrence of one has no effect on the likelihood of the occurrence of the other. Two events that are not independent are said to be **dependent**.

For example, consider tossing a coin heads up and rolling a 4 on a die. These two events are independent if $P(\text{head and } 4) = P(\text{head}) \bullet P(4)$. The sample space for tossing a coin and rolling a die consists of 12 equally likely outcomes: a head with each of the six faces on the die and a tail with each of the six faces. Only one of these outcomes includes a head and a 4. Therefore, $P(\text{head and } 4) = 1/12$.

The theoretical probability of tossing a head is $1/2$, while the theoretical probability of rolling a 4 is $1/6$. Therefore,

$$P(\text{head}) \bullet P(4) = \frac{1}{2} \bullet \frac{1}{6} = \frac{1}{12}$$

Since $P(\text{head and } 4) = P(\text{head}) \bullet P(4)$, the two events are independent.

c. In the cross Rr ∞ Rr, the probability of the genotypes RR and rr is $1/4$, while the probability of Rr is $1/2$. How could you find the probability of each genotype using the probabilities of receiving one allele from the father and one from the mother?

Exploration

a. Select two of the following five traits: mid-digit hair, widow's peak, free earlobes, rolled tongue, and folded hands.

b.
1. Determine the number of students in your class who have only the first trait (A), only the second trait (B), and both the first and second traits (A and B). Record this data in a table with headings like those in Table **3** below.

   Table 3: Experimental probabilities of two traits

Trait A:		
Trait B:		
Total Number in Class:		
Trait	**Frequency**	**Experimental Probability**
A		
B		
A and B		

2. Find the experimental probabilities of having trait A, having trait B, and having both A and B. Record these probabilities in Table **3**.

c. Choose another pair of traits from the five traits listed in Part **a**. Repeat Part **b** using this pair of traits. Continue this process until you have completed a table for each of the 10 possible pairs of traits.

d. Experimental probabilities are often used to approximate theoretical probabilities. Judging from the experimental probabilities in your tables from Parts **b** and **c**, which pairs of traits appear to represent independent events?

Discussion 2

a.
1. Using the information in your tables, do any of the pairs of traits appear to represent independent events? Explain your response.
2. Do any of the pairs of traits appear to represent dependent events? Explain your response.

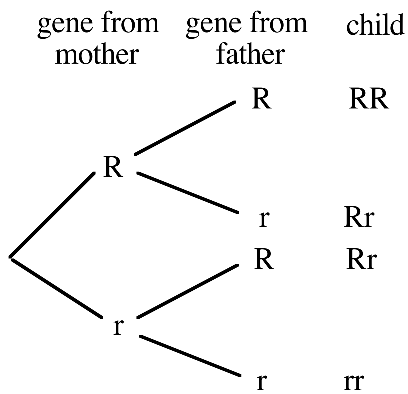
b.
1. Identify two events that appear to be independent. Describe how you could demonstrate that they are independent.
2. Identify two events that appear to be dependent. Describe how you could demonstrate that they are dependent.

c. Given the theoretical probability of having a particular trait, how would you predict the number of students in your class with that trait?

Assignment

3.1 Assume that each of the following five traits is independent of the others: mid-digit hair, widow’s peak, free earlobes, rolled tongue, and folded hands. Using the class data from the exploration, estimate the probability of having free earlobes and the ability to roll your tongue and no widow’s peak.

3.2 The tree diagram below shows the possible genotypes for the cross Rr ∞ Rr.



a. Make a copy of this diagram. On each branch of the tree, write the probability of a child receiving that allele.

b. Use the tree diagram to find the probability of each genotype. Describe the method you used.

c. Create a Punnett square for the cross Rr ∞ Rr. Use it to calculate the probability of each genotype.

d. How do the probabilities you found using the tree diagram compare with the probabilities found using a Punnett square?

3.3 Two potential parents both have the genotype Mm for mid-digit hair. The father has the genotype WW for widow’s peak, while the mother has the genotype Ww. Assuming that mid-digit hair and widow’s peak are independent traits, determine the probability that a child of these parents has both mid-digit hair and a widow’s peak.

3.4 **Antigens** are proteins that can activate the immune system. An individual's blood type is determined by the presence or absence of two antigens, A and B, on red blood cells. Type A and type B blood each have one antigen on red blood cells, type O blood contains neither antigen, while type AB contains both.

Blood type is also affected by another antigen, the Rh factor. Blood that contains this factor is referred to as Rh positive (Rh+), while blood that does not contain this factor is referred to as Rh negative (Rh–). The presence or absence of the Rh factor, along with the four basic blood types, results in eight different kinds of blood: A+, B+, AB+, O+, A–, B–, AB–, and O–.

A biology class has collected data on the blood of 26 students. The following table summarizes this data.

Blood Type	No. of Students	Rh Factor	No. of Students
A	10	Rh+	20
B	4	Rh–	6
AB	1		
O	11		

a. Determine the experimental probability of each of the following blood types: A, B, AB, and O.

b. Determine the experimental probability each of the following: Rh+ and Rh–.

c. Assuming that blood type and Rh factor are independent traits, draw a tree diagram that shows the estimated probability of each of the eight possible kinds of blood.

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3.5 The presence of curly hair (H) is dominant over straight hair (h). If the parents' genotypes are hh and Hh, determine the probability that their two children will both have curly hair. Assume that the traits of one child are independent of the traits of the other.

3.6 As mentioned in Problem **2.7**, 1 out of every 25 people in the United States is a carrier of cystic fibrosis. What is the probability that two people chosen at random are both carriers?

3.7 A marketing firm surveyed 5000 people about their favorite sodas and snack foods. The results of the survey are shown in the table below. Each value in the "percentage" column represents the percentage of the survey group who preferred a particular soda or snack food.

Soda	Percentage	Snack Food	Percentage
cola A	30	corn chips	30
cola B	25	potato chips	40
cherry cola	10	pretzels	10
orange	5	variety mix	15
root beer	15	other	5
other	15		

a. What is the probability that a person chosen at random from this group prefers cola?

b. What is the probability that a person chosen at random from this group prefers cola and pretzels?

c. What is the probability that a person chosen at random from this group does not prefer root beer?

3.8 Huntington's disease is a rare and deadly genetic disorder. All carriers eventually develop the disease. Since Huntington's disease may lie dormant until a person is between 35 and 45 years old, many carriers have children before they develop symptoms.

a. Draw a Punnett square of a cross between a carrier of Huntington's disease (Hh) and a person who is not a carrier (hh).

b. What is the probability that a child from this cross does not carry the disease?

c. If the parents from Part **a** have one child who is not a carrier, what is the probability that a second child will carry the disease?

d. What is the probability that the parents from Part **a** have two children who are not carriers?

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Research Project

Research one of the following genetic diseases: cystic fibrosis, Tay-Sach's disease, Huntington's disease, sickle-cell anemia, or phenylketonuria (PKU). Determine its symptoms, the frequency of its occurrence, and the probability that parents pass the disease to their children.

Activity 4

Your individual combination of traits makes you different from every other person on the planet. Some of these inherited traits, however, can automatically prevent you from having some others. In this activity, you discover how the probability of having one trait or another can be affected by the relationship between them.

Exploration

a. To complete Steps **1** and **2** below, let trait A represent a preference for writing with the left hand and trait B represent a preference for writing with the right hand.

1. Determine the number of students in your class with trait A only, trait B only, both trait A and trait B, and either trait A or trait B. Record this data in a table with headings like those in Table **4**.

Table 4: Experimental probabilities of two traits

Trait A:		
Trait B:		
Total Number in Class:		
Trait	**Frequency**	**Experimental Probability**
A		
B		
A and B		
A or B		

2. Find the experimental probabilities of having trait A only, having trait B only, having both trait A and trait B, and having either trait A or trait B. Record these probabilities in your table.

b. Repeat Part **a** for each of the following pairs of traits.

1. blue eyes and dark hair

2. dark hair and folded hands

Mathematics Note

Given two events A and B, the theoretical probability of either A or B occurring can be found as follows:

$$P(\text{A or B}) = P(\text{A}) + P(\text{B}) - P(\text{A and B})$$

For example, suppose event A is drawing a black card from a standard deck of 52 playing cards, and event B is drawing a king from a standard deck of playing cards. Since there are 26 black cards in a standard deck, $P(\text{black}) = 26/52$. Similarly, because there are 4 kings in the deck, $P(\text{king}) = 4/52$. The probability of either A or B occurring can be calculated as shown below:

$$\begin{aligned} P(\text{black or king}) &= P(\text{black}) + P(\text{king}) - P(\text{black and king}) \\ &= \frac{26}{52} + \frac{4}{52} - \left(\frac{26}{52} \bullet \frac{4}{52}\right) \\ &= \frac{26}{52} + \frac{4}{52} - \frac{2}{52} \\ &= \frac{28}{52} = \frac{7}{13} \end{aligned}$$

c. Use the formula in the mathematics note to determine $P(\text{A or B})$ for each of the following pairs of traits.

1. left-handedness and right-handedness
2. blue eyes and dark hair
3. dark hair and folded hands

d. Compare your results from Part **c** to the experimental probability of A or B for each pair of traits.

Discussion

a. Did your results in Part **c** of the exploration support the formula for determining $P(\text{A or B})$? Explain your response.

b. For which pair(s) of traits in the exploration was the experimental probability of A and B occurring at the same time 0?

Mathematics Note

Two events are **mutually exclusive** if they cannot occur at the same time in a single trial. For two mutually exclusive events A and B, $P(\text{A and B}) = 0$.

For example, consider the toss of a single coin. In this case, heads and tails are mutually exclusive, since both cannot occur at the same time on a single toss. However, because one person could be both right-handed and have the ability to roll the tongue, these two events are not mutually exclusive.

c. **1.** In your class data, which pairs of traits were mutually exclusive?

2. Do you think that these traits are always mutually exclusive? Explain your response.

d. For two mutually exclusive events A and B, the theoretical probability of A or B can be calculated using either the formula given in the mathematics note or the one shown below:

$$P(\text{A or B}) = P(\text{A}) + P(\text{B})$$

Why are these two formulas equivalent for mutually exclusive events?

e. Are complementary events always mutually exclusive? Explain your response.

f. Are mutually exclusive events always complementary? Use an example to support your response.

g. Draw a Venn diagram that shows the relationship between two mutually exclusive events. How does this diagram show that $P(\text{A and B}) = 0$?

Assignment

4.1 Decide whether or not each of the following pairs of traits represents two mutually exclusive events. Justify your response for each pair.

a. having the ability to roll the tongue and not having this ability

b. having a widow's peak and having curly hair

c. having two blue eyes and having two brown eyes

4.2 Familial hypercholesterolemia causes high levels of cholesterol in the blood and can lead to clogging of the arteries at a young age. This genetic disorder is caused by the dominant allele (D). People with the genotype DD are severely affected, those with the genotype Dd are mildly affected, and those with the genotype dd are not affected at all.

Hank and Erma both have the genotype Dd. Use two different methods to find the probability that a child of this couple is either mildly affected or not affected at all.

4.3 The Norgaard family has five children. Both parents have the genotype Mm for mid-digit hair. The father's genotype for widow's peak is WW, while the mother's is Ww.

a. What is the probability that a child of these parents has either mid-digit hair or a widow's peak?

b. How many of the family's five children would you expect to have either mid-digit hair or a widow's peak? Justify your response.

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4.4 A survey of 20 students reported the following hair colors: 10 brown, 6 black, 3 blond, and 1 red. If one student is randomly selected from this group, what is the probability that the student has blond hair or red hair? (Assume that each student has only one hair color.)

4.5 In 1990, the Internal Revenue Service audited 0.8% of all tax returns. In that same year, Carla's Accounting Service prepared tax returns for 250 clients. Assume that each taxpayer had the same chance of receiving an audit.

a. What was the probability of receiving an audit in 1990?

b. Estimate the number of Carla's clients who received an audit.

c. Eli and Marc both used Carla's Accounting Service in 1990. Determine the probability of each of the following.

1. both Eli and Marc received audits

2. either Eli or Marc received an audit

4.6 Umberto plans to buy a chain with a tumbler lock to secure his bicycle. The first lock he examines has three tumblers, each of which can be set to any digit from 0 to 9. Only one combination of three numbers will open the lock.

a. What is the probability of guessing the correct combination on the first attempt?

b. The second lock Umberto examines has five tumblers. Is a five-tumbler lock more secure than a three-tumbler lock? Explain your response.

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Activity 5

Like hair color and free earlobes, your gender is an inherited trait. As shown in Figure **4**, the sex chromosomes for a female are indicated by XX, while those for a male are indicated by XY. Each parent contributes one chromosome to the child's pair. The father can contribute either an X or a Y chromosome; the mother can contribute only an X chromosome.

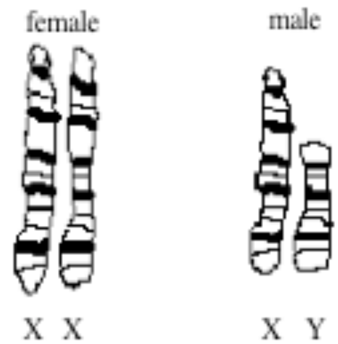

Figure 4: Human sex chromosomes

Exploration

In this exploration, you use a simulation to investigate the probability that a combination of sex chromosomes results in a female child. You then use another simulation to examine the probability that a family with two children will include one male child and one female child.

a. Assume that the probability of receiving either allele from either parent is the same. Predict the probability that a combination of sex chromosomes results in a female child.

b.
1. To model the mother's chromosomes, label both sides of a coin "X."
2. To model the father's chromosomes, label a second coin "X" on one side and "Y" on the other.

c.
1. Simulate the combination of male and female chromosomes by tossing the two coins. Determine and record the gender of the resulting combination.
2. Repeat Step **1** 19 more times.
3. Determine the experimental probability that a combination of male and female chromosomes results in a female child.

d. Use a Punnett square or tree diagram to determine the theoretical probability that a combination of male and female chromosomes results in a female child.

e. Predict the probability that two independent combinations of male and female chromosomes result in one male and one female child.

f.

1. To simulate two independent combinations of sex chromosomes, toss the two coins twice. Determine and record the genders of the resulting pair of children.
2. Repeat Step **1** 19 more times.
3. Determine the experimental probability that two independent combinations of male and female chromosomes result in one male and one female child.

g. Determine the theoretical probability that two independent combinations of male and female chromosomes result in one male and one female child.

h. Compile the class results from Parts **c** and **f**. Using the class results, determine the experimental probability of each of the following.

1. A combination of male and female chromosomes results in a female child.
2. Two independent combinations of male and female chromosomes results in one male and one female child.

Discussion

a. When tossing two coins, why is the outcome for each coin independent of the other?

b. In Part **a** of the exploration, did you predict that a male child and a female child are equally likely? Why or why not?

c.

1. How did the experimental probabilities you calculated using the results of 20 trials compare with the experimental probabilities calculated using the class results?
2. Which probabilities were closer to the theoretical probabilities?

d. In Part **f** of the exploration, you simulated two combinations of sex chromosomes by tossing the coins twice.

1. How many outcomes are there in the sample space for this experiment?
2. How would the size of the sample space change if the experiment simulated three combinations of sex chromosomes?
3. How would the size of the sample space change if the experiment simulated *n* combinations of sex chromosomes?

Assignment

5.1 **a.** Consider a couple with three children. Draw a tree diagram that shows all the possible outcomes for the genders of the children.

b. Assume that having a boy and having a girl are equally likely outcomes. Determine the probability that the couple has a girl, a girl, and a boy, in that order.

c. Determine the probability that exactly two of the children are girls.

d. Are your responses to Parts **b** and **c** different? Why or why not?

5.2 A family has four boys. Assuming that the birth of each child is an independent event, is their fifth child more likely to be a girl or a boy? Explain your reasoning.

5.3 To find the probability that a family with five children will have five girls, a student used the following formula:

$$P(5 \text{ girls}) = \frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2} = \frac{1}{32}$$

Do you agree with this student's method? Why or why not?

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5.4 After analyzing its customer data, a rental car agency found that approximately 2% of its customers were involved in accidents.

a. What is the probability that a customer chosen at random is involved in an accident?

b. If 5 different customers are chosen at random, what is the probability that all 5 are involved in accidents? Explain your response.

c. If the agency rents 250 cars in one day, how many should they expect to be involved in accidents?

5.5 In one lottery game, players may choose any sequence of six digits. A computer generates the winning number by randomly selecting one digit at a time, in order from first to sixth.

a. What is the probability that the fourth digit of the winning number is 7?

b. What is the probability that the winning number is 682399?

c. What is the probability that the winning number is 123456?

d. What is the probability that the winning number is 000000?

5.6 In the game of craps, a player rolls two dice. A roll of 7 or 11 results in a win.

a. What is the probability of winning on one roll? Explain your response.

b. What is the probability of winning on three rolls in a row? Explain your response.

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Summary Assessment

In order to ensure a safe and steady supply of blood for medical use, many hospitals maintain banks of donated blood. The human body normally contains about 5 L of blood. Each volunteer donates about 0.5 L. In healthy people, that 0.5 L is readily replenished.

Imagine that you work for a hospital blood bank. The blood bank likes to have at least 5 L of each blood type available—enough for one complete transfusion, if necessary. The hospital is planning to make a public service announcement requesting more donors. Your job is to determine which blood types are most needed.

1. An individual's blood type—O, A, B, or AB—is determined by a combination of the parents' genes, one allele from each parent. The allele for type O blood is always recessive, while the alleles for types A and B are always dominant. The combination of alleles A and B produces the blood type AB.

   a. List all the possible genotypes for each blood type.

   b. Assuming that each possible genotype is equally likely to occur in the population, determine the probability of having each blood type.

2. The blood bank currently has a total of 75 L of blood. How many liters of each blood type would you expect them to have?

3. The presence or absence of the Rh factor also affects blood type. In the United States, about 85% of the population is Rh positive (Rh+); the rest is Rh negative (Rh–).

   The presence or absence of the Rh factor, along with the four basic blood types, results in eight different kinds of blood: A+, B+, AB+, O+, A–, B–, AB–, and O–. How many liters of each would you expect the blood bank to have?

4. In its public service announcement, which types of blood should the hospital say are most needed? Explain your response.

Module Summary

- The set of all possible outcomes for an experiment is the **sample space**. An **event** is a subset of the sample space.
- One method of predicting the likelihood of an event is to perform many trials under controlled conditions. The results of these trials provide the **experimental** (or **empirical**) **probability** of the event occurring. The experimental probability of an event is the following ratio:

$$\frac{\text{number of times event occurs}}{\text{total number of trials}}$$

- Given any event E, the event that E does not occur is its **complement**. The complement of E can be represented by the symbol E', read "E prime" or "E complement." The sum of the probabilities of two complementary events is 1.
- If each outcome in a sample space has the same chance of occurring, then the **theoretical probability** of an event can be calculated using the following ratio:

$$\frac{\text{number of outcomes in the event}}{\text{total number of outcomes in the sample space}}$$

  The theoretical probability of an event E can be written as $P(\text{E})$.
- Two events A and B are **independent** if $P(\text{A and B}) = P(\text{A}) \bullet P(\text{B})$.

  It follows that for independent events A, B, and C, $P(\text{A and B and C}) = P(\text{A}) \bullet P(\text{B}) \bullet P(\text{C})$. This definition can be extended to any number of independent events.
- Two events that are not independent are said to be **dependent**.
- Given two events A and B, the theoretical probability of either A or B occurring can be found as follows:

$$P(\text{A or B}) = P(\text{A}) + P(\text{B}) - P(\text{A and B})$$

- Two events are **mutually exclusive** if they cannot occur at the same time in a single trial. For two mutually exclusive events A and B, $P(\text{A and B}) = 0$.
- **Genetics** is the study of **heredity**, the process by which characteristics are passed from one generation to the next. The biological structures that control the heredity of traits are **genes**.
- Human genes are carried on 23 pairs of **chromosomes**. Genes also come in pairs; each member of a pair is an **allele**.

- When the alleles in a pair are different, the trait that appears is called the **dominant** trait. The other is called the **recessive** trait.
- The pair of alleles that determines the presence or absence of a particular characteristic is the **genotype**. The trait that actually occurs is referred to as the **phenotype**.
- The act of combining genes from two parents is called a **cross**.
- Given the genotypes of both parents for a particular trait, you can use a **Punnett square** to help identify the possible combinations from a cross.

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