There’s No Place Like Home

What geometric shape does your home resemble? What do you think the houses of the future will look like? In this module, you explore some traditional American Indian housing designs—from the ground up.

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Introduction

Traditional American Indian dwellings vary from tribe to tribe and from region to region. Those who lived in what is now South Dakota used tipis, the tribes of the American Southwest chose pueblos, while the peoples of the Far North built igloos (also called *illuliaqs*). Differences in climate, terrain, building materials, and cultures were all important factors in the development of appropriate housing designs.

The design of a house affects such practical considerations as the amount of building material needed, the amount of space enclosed, and the relative heating or cooling efficiency of the dwelling. As the end 21st century begins, both building materials and heating fuels are growing scarce. Houses of the future must make efficient use of our resources.

Examining some traditional American Indian housing designs may help suggest some possible designs for efficient homes in the next century. In this module, you examine several types of structures, the geometric shapes that represent them, and the properties that help make buildings efficient.

Activity 1

The amount of floor space in a home is important to the comfort of its owners. Many colonial houses had rectangular floors. In this case, the area of the floor is relatively easy to determine. The floor of a large tipi, however, could be a regular polygon with as many as 18 sides.

Exploration

How could you find the area of a floor shaped like an 18-sided regular polygon? The area of a geometric figure refers to the region enclosed by it. The area of a polygon may be found by estimating, by using a formula, or by other means.

In this exploration, you calculate area by inscribing a regular polygon inside a circle. An **inscribed polygon** is one in which each vertex lies on a circle. The **radius** of an inscribed regular polygon is equal to the radius of the circle.

For example, Figure **1** shows a square inscribed in a circle, with the radius of the square drawn from the square's center to one of the vertices.

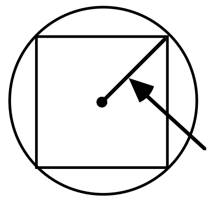


Figure 1: Inscribed square with a radius of 2 cm

a.
  1. Construct a circle on a geometry utility. Record its radius, area, and perimeter.
  2. Select three points on the circle to represent the vertices of an inscribed triangle.
  3. Connect the center of the circle to each of the three points to form three central angles. A **central angle** is an angle with its vertex at the center of a circle.
  4. Move the points along the circle, one at a time, until the central angles all have equal measures.
  5. Connect the three points on the circle to form a regular triangle.

b.
  1. Use the geometry utility to calculate the triangle's area and perimeter.
  2. Record this information in a table.

c. Using a process similar to that described in Part **a**, inscribe a regular quadrilateral, pentagon, hexagon, octagon, and 18-sided polygon in circles of the same radius.

d. In your table from Part **b**, record the area and perimeter of each regular polygon, as well as the area and perimeter of the circle in which they were inscribed. **Note:** Save your data for use in the discussion.

Discussion

a. How did you determine the locations of the vertices for your inscribed polygons?

b. In Parts **a** and **c** of the exploration, why does creating congruent central angles guarantee that a regular polygon will be formed?

c.
  1. What methods do you know for determining the area of regular polygons?
  2. What are the advantages and disadvantages of each one?

d. Consider a set of regular polygons inscribed in the same circle. As the number of sides in a polygon increases, describe what happens to each of the following characteristics:

1. the shape of the polygon
2. the area of the polygon
3. the perimeter of the polygon.

e. A **tangent** to a circle is a line, segment, or ray that intersects a circle in one point and is perpendicular to a radius at that point. Consider a pentagon whose sides are each tangent to a circle, as shown in Figure **2** below. In this case, the polygon **circumscribes** the circle.

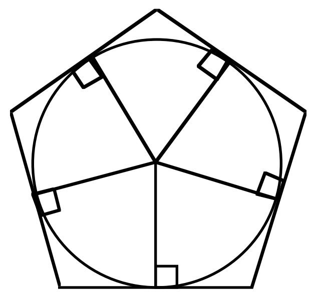


Figure 2: A circle circumscribed by a regular pentagon

Consider a set of regular polygons that circumscribe the same circle. As the number of sides in a polygon increases, describe what happens to each of the following characteristics:

1. the shape of the polygon
2. the area of the polygon
3. the perimeter of the polygon.

f. Figure **3** below shows a regular pentagon inscribed in a circle. In this case, $\overline{AG}$ is an **apothem**, a segment whose measure is the perpendicular distance from the center of a regular polygon to one of its sides.

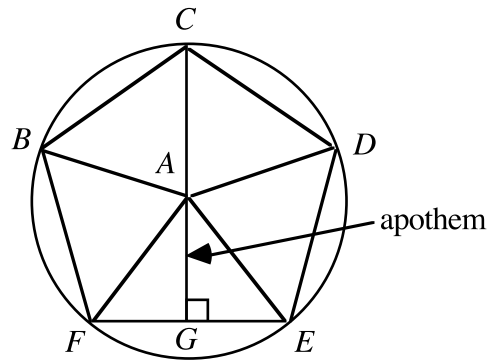


Figure 3: A regular pentagon

One formula for the area of a regular polygon is

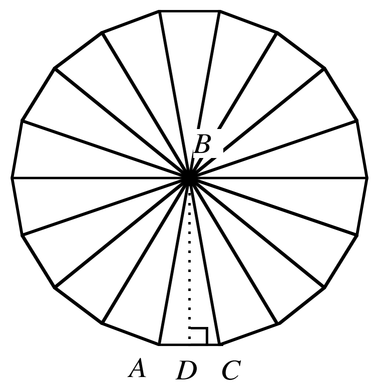
$$A = \frac{1}{2}ap$$

where a is the length of the apothem and p is the perimeter of the polygon. (See the Level 1 module, "A New Look at Boxing.")

1. As the number of sides of an inscribed regular polygon increases, what do the values of a and p approach in relation to the circle?
2. How does this affect the formula for the area of a regular polygon given above?

Assignment

1.1 The diagram below shows a view of a tipi from above. The floor of the tipi is an 18-sided regular polygon with a radius of 5 m.



Use this diagram to determine each of the following:

a. the measures of $\angle ABC$ and $\angle ABD$

b. the length of $\overline{AD}$ (using trigonometry)

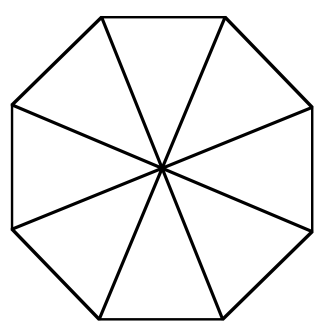
c. the length of $\overline{AC}$

d. the length of $\overline{BD}$

e. the area of ΔABC

f. the area of the tipi's floor.

1.2 The Powder House in Williamsburg, Pennsylvania, was built in 1714 and used for storing gunpowder. As shown in the diagram below, the structure has a regular octagonal base. Given that the longest diagonals measure 16 m, determine the area of the floor.



1.3 Igloos are built from blocks of snow. The floor of an igloo is a circle. If an igloo has a diameter of 3.5 m, what is the area of its floor?

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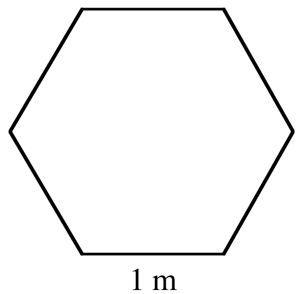
1.4 The diagram below shows the regular hexagonal floor of a modern domed camping tent. Given that each side is 1 m long, determine the area of the floor.



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Research Project

Choose one American Indian tribe—or several tribes from a single region—and study their traditional buildings. Describe each structure's practical, seasonal, and cultural uses. Then describe each structure from a mathematical point of view, including its shape, geometric properties, and typical dimensions.

Activity 2

When an architect designs a house, the desired amount of floor space is only one consideration. Available building materials also play a role in determining an appropriate design. In some situations, appearance is a primary consideration; in others, durability and cost are the main concerns. In any case, estimating the amount of materials required is an important part of the planning process. When determining the amount of material needed to build a house, it is helpful to calculate the area of the walls, floor, and roof.

American Indian families in the Yukon region of Alaska and Canada often built double lean-tos such as the one shown in Figure **4**.

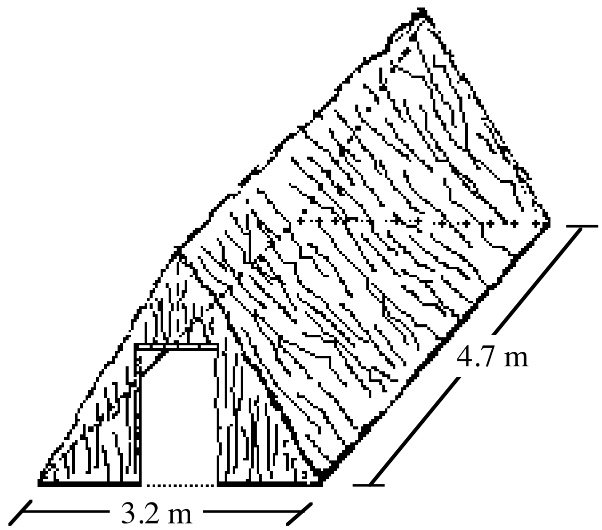


Figure 4: A double lean-to

A double lean-to can be described as a right triangular prism with bases that are equilateral triangles. The total surface area of a prism is the sum of the lateral surface area and the areas of the two bases. For example, Figure **5** shows a net for a right triangular prism. In this case, the three rectangles in the net are the lateral faces of the prism. The sum of their areas is the prism's lateral surface area.

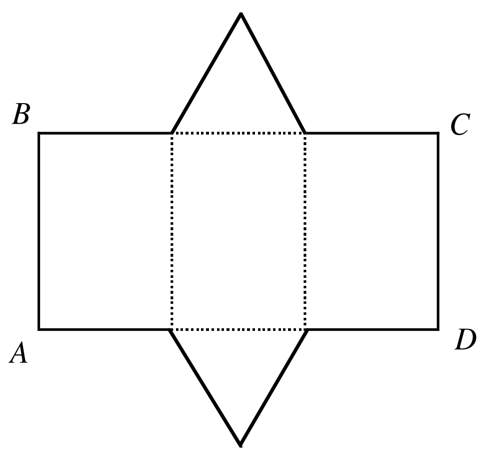


Figure 5: Net for a right triangular prism

Discussion

a. **1.** Why is it incorrect to consider a rectangle as a base of the prism in Figure **5**?

2. Why is this prism called a "right" prism?

b. How are the dimensions of rectangle *ABCD* in Figure **5** related to the perimeter of the prism's base?

c. Describe how to find the surface area of a right prism with a square base.

d. Figure **6** shows a net for a right circular cylinder.

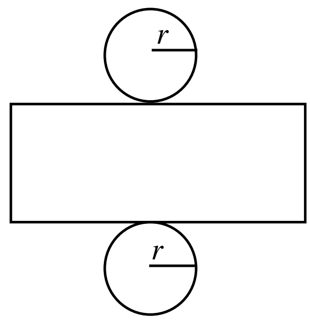
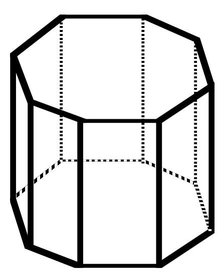


Figure 6: Net for a right circular cylinder

1. What does the length of the rectangle's longer side represent?
2. What part of the net represents the height of the cylinder?
3. What do the circles represent?
4. Express the length of the rectangle's longer side in terms of the radius of the circle.
5. Describe how to find the total surface area of a cylinder.

Assignment

2.1 The Powder House described in Problem **1.2** can be modeled by a right regular octagonal prism. A similar building, shown in the diagram below, is 11 m high. Each side of a base is 6 m long.



a. Sketch a net for the building.

b. Label the dimensions of one of the net's lateral faces.

c. Determine the building's lateral surface area.

d. Calculate the area of the bases.

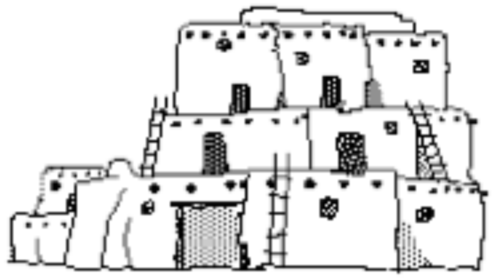
e. Determine the total surface area of the building.

2.2 **a.** Sketch a net for a prism whose bases are regular heptagons.

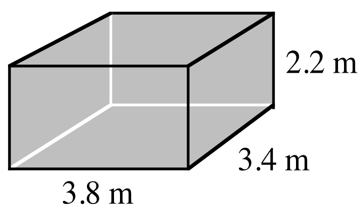
b. As the number of sides in the bases of a prism increase, what geometric shape does the prism approach?

2.3 To help estimate the amount of material needed to build the double lean-to shown in Figure **4**, determine its total surface area.

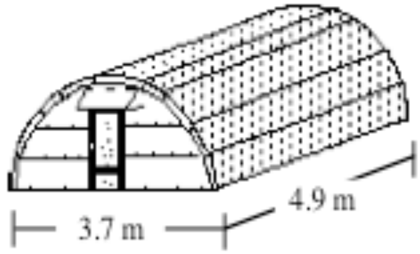
2.4 The Pueblo Indians of the Rio Grande region lived in rectangular flat-roofed homes. As shown in the diagram below, these dwellings resemble an early version of the modern apartment building.



The right rectangular prism below shows some possible dimensions for a single-family unit in a Pueblo village. Use these measurements to determine the total surface area of one family's home.



2.5 A traditional wigwam of the Chippewa tribe and a modern Quonset hut share the same basic shape: a half cylinder. The Chippewas used wigwams as sweat lodges. Quonset huts with translucent walls are commonly used as greenhouses. Complete Parts **a–d** for the Quonset hut shown below.



a. What is the area of the floor?

b. What is the area of the two walls formed by the semicircles?

c. What is the lateral surface area?

d. What is the total surface area, including the floor?

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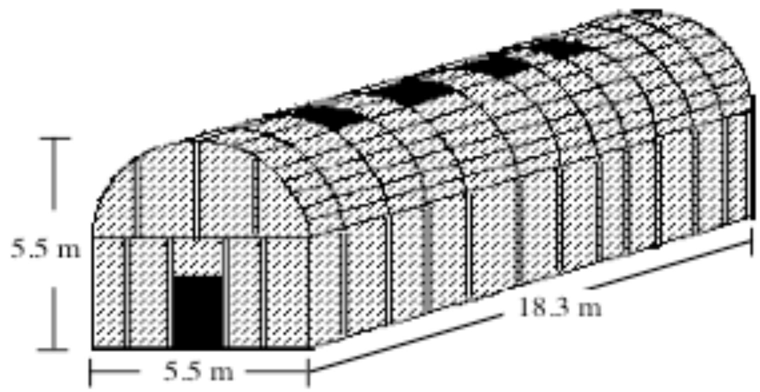
2.6 The Iroquois Indians of New York lived in rectangular barrel-roofed houses. The roof of each structure was made by bending poles into a semicircular shape (a half barrel). The entire building, except for the floor, was covered with bark. How much tree bark would be needed to cover the house shown in the diagram below?



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Activity 3

Like today's families, early American Indian families lived in dwellings of various shapes and sizes. The ease of heating and cooling these structures was an important part of their design. Efficient heating and cooling are also major considerations for nearly all modern homes.

Since you heat the space inside a home (volume) and lose heat through the outside walls, roof, and floor (surface area), you must consider both surface area and volume when estimating efficiency. One way to do this involves calculating the ratio of a building's surface area to its volume. Although this rule of thumb is not always applicable, it does give one indication of a home's potential efficiency.

Exploration

a. **1.** Arrange eight cubes so that each cube shares at least one entire face with another cube.

2. Describe the shape you created in Step **1** and make a sketch of it. For example, Figure **7** shows a sketch of one possible arrangement.

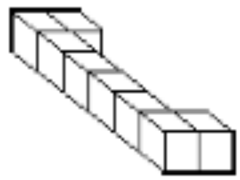


Figure 7: An arrangement of eight cubes

b. Repeat Part **a** for several different arrangements of the cubes.

> **Mathematics Note**
>
> The **volume** (V) of a right prism can be found by multiplying the area of one of its bases (B) by its height (h). The height of a prism is the distance between the two bases. In general, the formula for the volume of a right prism is:
>
> $$V = B \bullet h$$
>
> For example, consider a right hexagonal prism with a height of 15 m. If the area of each base is 256 m^2, the volume of the prism can be found as follows:
>
> $$V = B \bullet h = 256 \text{ m}^2 \bullet 15 \text{ m} = 3840 \text{ m}^3$$

c. For each shape you created In Parts **a** and **b**, determine the volume, the total surface area, and the ratio of surface area to volume. Record these values in a table. For example, the volume (V) of the shape in Figure **7** is 8 units^3, its surface area (S) is 34 units^2, and the ratio S/V is 4.25.

Discussion

a. **1.** Which of the shapes you created in the exploration has the greatest surface area?

2. Which has the least surface area?

3. Which has the greatest volume?

b. Why might the ratio of surface area to volume provide a reasonable way to evaluate the heating and cooling efficiency of a house? (Disregard the impacts of different building materials or insulation when considering your response.)

c. **1.** Which of the shapes you created in the exploration has the greatest ratio of surface area to volume?

2. Which has the smallest ratio of surface area to volume?

d. If you were building a house in a cold climate and needed to conserve heating fuel, which shape would you choose? Explain your response.

e. If you were building a house to collect and store solar energy, which shape would you choose? Explain your response.

Assignment

3.1 The Yuma people of the Colorado River region lived in "Mohave-type" houses. Each one-story dwelling was a right rectangular prism with a square floor. These houses were about 2.2 m tall and had flat roofs. The square floors ranged from 6 m to 7.5 m on each side.

a. What is the range of the volumes for these dwellings?

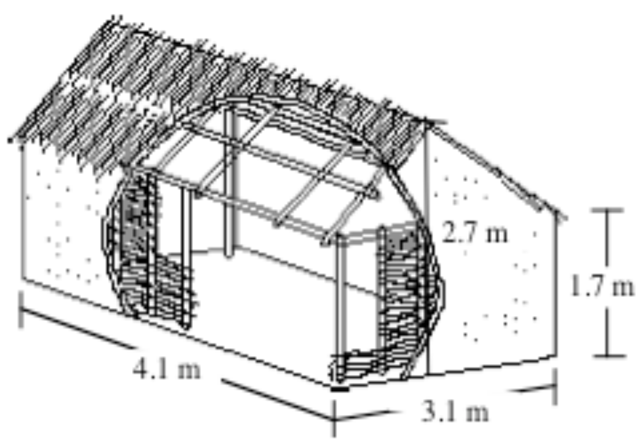
b. What is the ratio of surface area to volume for a Yuma dwelling with a square floor 6 m on each side?

3.2 As mentioned in Problem **2.4**, the traditional homes of the Pueblo Indians are rectangular flat-roofed structures that resemble apartment buildings. A typical family room was about 2.2 m high. The dimensions of the floor were approximately 4 m by 3.25 m.

a. What is the ratio of surface area to volume for this room?

b. How does the amount of space in a Pueblo dwelling compare with that of the smallest Yuma dwelling?

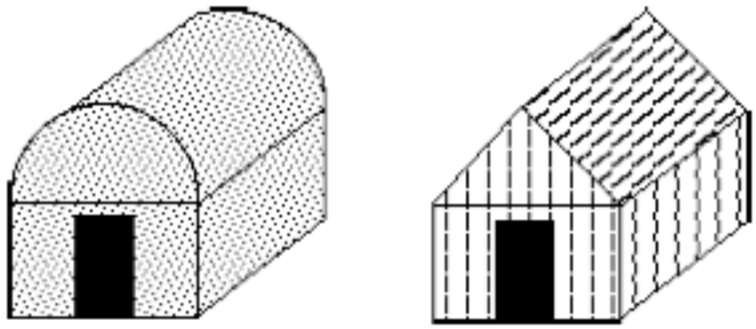
3.3 The summer houses of one Southeastern tribe were rectangular, gabled dwellings with thatched roofs and mud walls. The gabled roof was open, with smoke holes located along the ridge. One example of this type of house is shown in the diagram below.



a. What is the volume of this house?

b. What is its ratio of surface area to volume?

3.4 The basic shapes of the two structures shown below were used by many different American Indian tribes.



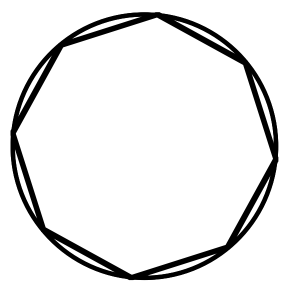
Consider a structure of each type with identical, rectangular bases measuring 3.0 m by 3.8 m. Each structure has walls of the same height, as well as the same overall height, 3.4 m.

a. Which structure has the greater surface area? Explain your response.

b. Which has the greater volume? Explain your response.

c. If the two structures were built of the same materials, which one would you expect to retain heat more efficiently? Explain your response.

3.5 The figure below shows the top view of a right prism inscribed in a cylinder. The bases of the prism are many-sided regular polygons.



a. As the number of sides in a base of the prism increases, how does the volume of the prism compare to the volume of the cylinder?

b. The formula for the volume of a right prism is $V = B \bullet h$, where B represents the area of a base and H represents the height.

1. Can this formula be used to find the volume of a right cylinder? Explain your response.

2. What formula would you use to determine the area of a base of the cylinder?

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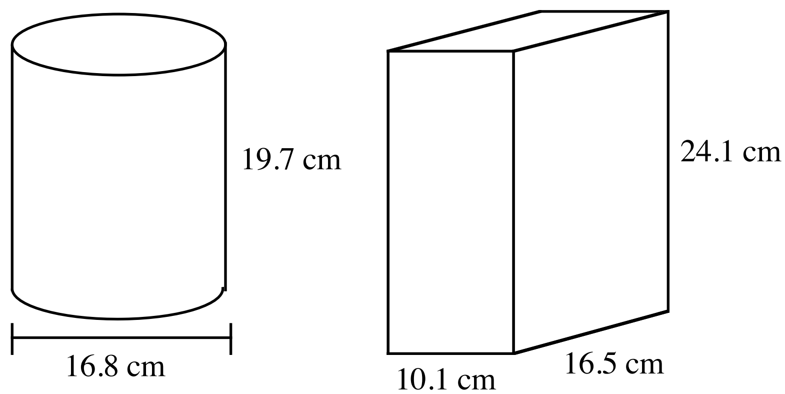
3.6 A manufacturer of painting supplies uses a cylindrical container for paint and a container shaped like a rectangular prism for paint thinner. The dimensions of these two containers are shown in the diagram below.



a. Determine the lateral surface area, total surface area, and volume of the cylindrical container.

b. Determine the lateral surface area, total surface area, and volume of the container shaped like a rectangular prism.

c. If both containers are made from the same material, which one do you think would be cheaper to make? Explain your response.

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Activity 4

For the American Indian tribes who followed the bison herds, a tipi was the ideal home. It was warm in winter, cool in summer, portable, and easily made from available materials. Figure **8** shows one example of a tipi.

Figure 8: A tipi

Some tipis approximated the shape of a regular pyramid. The base was a regular polygon, and the number of sides ranged from 6 in smaller tipis to 18 in larger ones. For example, the pyramid in Figure **9** represents a tipi with a hexagonal base 6 m on each side. The **height** of a pyramid is the distance from the vertex to the base. The **slant height** of a pyramid is the height of its lateral faces. Each lateral face is a triangle.

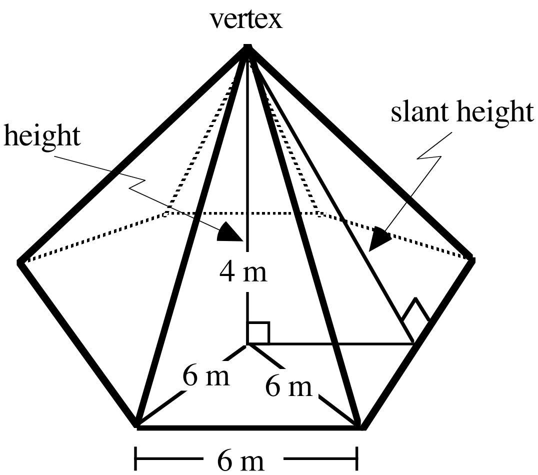


Figure 9: A pyramid with a regular hexagonal base

Discussion 1

a. What type of triangle is each of the lateral faces of a regular pyramid?

b. As the number of lateral faces in a pyramid increases, describe what happens to each of the following:

1. the shape of the base
2. the shape of the lateral faces
3. the shape of the pyramid.

c. Describe how to find the surface area of a pyramid.

Exploration

In this exploration, you construct a net for a model of a tipi.

a.
1. Construct a net, including the base, for a regular hexagonal pyramid**.**
2. Verify that the net forms a pyramid.

b.
1. Cut out the lateral faces of the net. Arrange them so that the vertex angles of the triangles all meet at a single point and there are no gaps between the sides.
2. Tape the triangles together to form a net of the **lateral surface** of the pyramid.

Discussion 2

a. **1.** Describe the net of the lateral surface of the pyramid.

2. As the number of lateral faces in a pyramid increases, what happens to the shape of this net?

Mathematics Note

An **arc** of a circle is a part of the circle whose endpoints are the intersections, with the circle, of the sides of a central angle.

The **measure of an arc** is the measure of its central angle. A **minor arc** has a measure less than 180°; a **major arc** has a measure greater than 180°; a **semicircle** has an arc measure of exactly 180°.

The **length of an arc** is the distance on the circle between the arc's endpoints. This distance can be found by multiplying the circle's circumference by the fractional part of the circle that the arc represents.

A **sector** of a circle is a region bounded by the sides of a central angle and an arc of the circle. The **area of a sector** can be found by multiplying the circle's area by the fractional part of the circle that the sector represents.

For example, Figure **10** shows a circle with center at *O* and a radius of 10 cm. The shaded sector is bounded by a central angle of 40° and the minor arc *AB*. The unshaded sector has a central angle of 320° and is bounded by the major arc *ACB*.

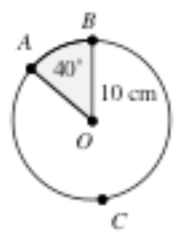


Figure 10: A sector of a circle

Since the measure of arc *AB* is 40° and the central angle of the entire circle is 360°, the length of arc *AB* can be found as follows:

$$\frac{40}{360} \bullet 2\pi(10) \approx 7 \text{ cm}$$

Similarly, the area of the shaded sector is:

$$\frac{40}{360} \bullet \pi(10)^2 \approx 35 \text{ cm}^2$$

b. Consider the expression used to find the length of arc *AB* in the previous mathematics note. In this expression, what does each of the following quantities represent?

1. $2\pi(10)$

2. $40/360$

c. Consider the expression used to find the area of the shaded sector in the previous mathematics note. In this expression, what does the quantity $\pi(10)^2$ represent?

d. As the number of lateral faces in a pyramid increases, how does the net of the lateral surface compare to the sector of a circle?

e. The lateral surface of a cone can be represented by a sector of a circle. For example, Figure **11** shows a sector of a circle of radius *r*, along with the corresponding cone of slant height *s*.

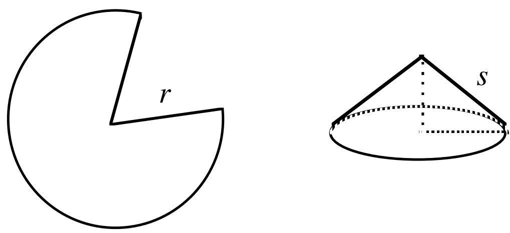


Figure 11: A sector formed into a cone

1. What part of the sector determines the slant height of the cone?

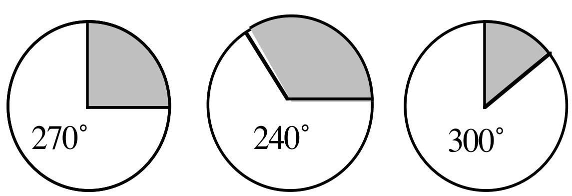
2. What part of the sector determines the circumference of the base of the cone?

3. Describe how to find the radius of the cone's base given the radius of the sector and the measure of its central angle.

Assignment

4.1 Determine the total surface area of the regular hexagonal pyramid in Figure **9**.

4.2 The diagram below shows three circles, each with a radius of 5 cm, along with the measures of the central angles of the unshaded sectors.



a. 1. What is the area of each circle?

2. What is the circumference of each circle?

b. **1.** What fraction of the circle is represented by each unshaded sector?

2. What is the area of each unshaded sector?

3. What is the length of the arc that bounds each unshaded sector?

c. Consider a cone constructed from the sector with a central angle that measures 270°.

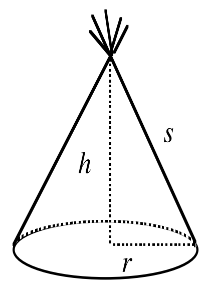
1. What part of the cone is represented by the length of the arc that bounds the sector?

2. What is the height of the cone?

3. What is the area of the cone's base?

4. What is the total surface area of the cone?

4.3 Although tipis are actually pyramids, their shapes can be modeled by cones. As shown in the diagram below, when the covering material for a tipi is cut from a circle and draped over the supporting poles, it resembles a cone without a base. In this case, the covering material represents the lateral surface of the cone.



a. Sketch a net of a cone, including the base.

1. Label the radius of the base r.

2. Label the part of the net that represents the slant height of the cone s.

b. **1.** Determine the length of the "curved" part of the lateral surface in terms of the radius of the cone's base.

2. Find the circumference of a circle with radius s.

3. What fraction of a circle with radius s is represented by the lateral surface of the cone? Express your response in terms of r (the radius of the cone's base) and s (the cone's slant height).

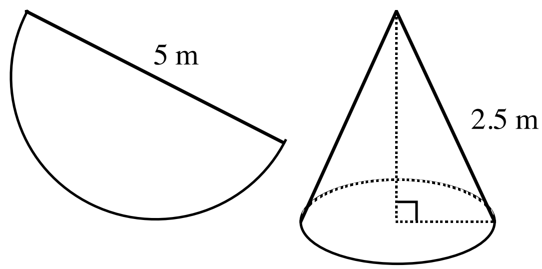
c. **1.** Find the area of a circle with radius s.

2. Determine the area of the lateral surface of the cone.

d. Write a formula for the total surface area of a cone.

e. Describe how to find the height *h* of a cone using *r* and *s*.

4.4 In some modern tipi designs, the covering material is cut in the shape of a semicircle, then draped over the poles. In the model shown below, the cover was cut from a circle of radius 2.5 m.



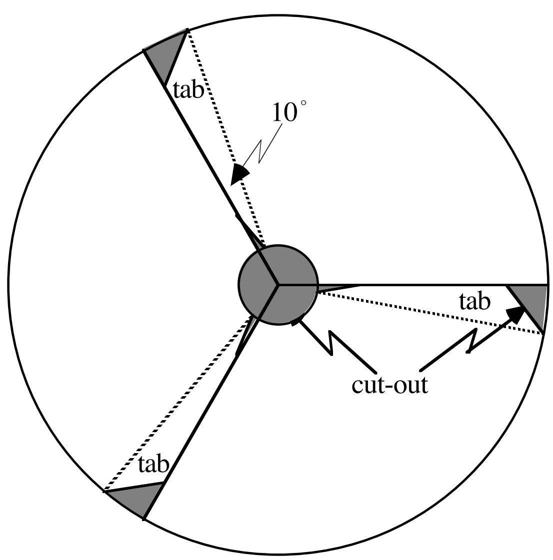
a. What is the diameter of the tipi's floor?

b. What is the height of the tipi?

c. Do these seem like reasonable measurements for a tipi? Explain your response.

* * * * *

4.5 To help customers add engine oil, many service stations provide paper funnels. One manufacturer of such funnels uses the circular template shown in the diagram below. By cutting the template along the solid lines, three funnels (with tabs) can be produced. The dotted lines indicate the folds for the tabs. The circumference of the larger opening of each funnel is 32 cm, while the circumference of the smaller opening is 5 cm.



a. Determine the radius of the circular template.

b. Find the radius of the inner circle that is cut from the template.

c. Determine the height of the paper funnel.

* * * * * * * * * *

Activity 5

A structure's volume—and its ratio of surface area to volume—are important considerations for heating or cooling the space inside. In the previous activity, you modeled tipi coverings using sectors of a circle. In this activity, you experiment with the size of the sectors and observe the resulting effects on a tipi's volume.

Exploration 1

In this exploration, you investigate the relationship between the volumes of two figures with the same height and radius of the base: a right circular cylinder and a right circular cone.

a. Construct a right circular cone by completing the following steps.

1. On a sheet of paper, draw a circle with a radius of at least 4 cm. As shown in Figure **12**, label center *A* and a point on the circle *B*.

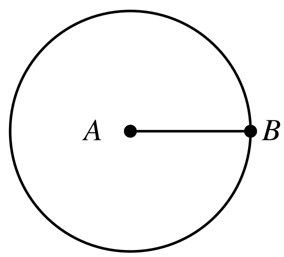


Figure 12: Circle and radius *AB*

2. Cut out the circle, then make a cut along $\overline{AB}$.

3. Investigate the right circular cones you can form by moving point *B* to various points on the circumference of the circle.

4. Select one cone. Tape the overlapping edge to the cone's surface.

b. Measure the height of the cone and the radius of its base in centimeters.

c. **1.** Create a net for the lateral surface of a right circular cylinder with the same height and radius as your cone. Include a tab on one edge, as illustrated in Figure **13**.

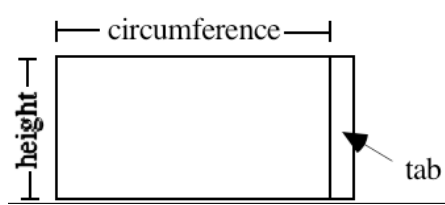


Figure 13: Lateral surface of right circular cylinder

2. Tape the edges of the net together to form the lateral surface of the cylinder.

d.
1. Estimate the ratio of the volume of the cylinder to the volume of the cone.
2. Place the cylinder upright on a flat surface.
3. Fill the cone with rice, then pour the rice into the cylinder. Repeat until the cylinder is full.
4. Record the number of full cones required to fill the cylinder. **Note:** Save the rice for use in Exploration **2**.

Discussion 1

a. Compare the volumes of cones and right circular cylinders with equal heights and radii.

b. Use the formula for the volume of a right circular cylinder and your results in Exploration **1** to determine a formula for the volume of a right circular cone.

Exploration 2

In this exploration, you investigate the relationship between the volumes of two figures with the same height and the same square base: a right prism and a regular pyramid.

a. Use a geometry utility to create a net for a regular pyramid with a square base. Print a copy of the net.

b. Cut the triangular faces from the net and tape them together to form the lateral surface of the pyramid.

c.
1. Measure the height of the pyramid in centimeters.
2. Measure the length of a side of the base in centimeters.

d.
1. Create a net for a right prism with the same height and base as the pyramid from Parts **a–c**.
2. Cut out the net, then cut off one of the bases. Tape the edges together to create an open container.

e.
1. Estimate the ratio of the volume of the prism to the volume of the pyramid.
2. Fill the pyramid with rice, then pour the rice into the prism. Repeat until the prism is full.
3. Record the number of full pyramids required to fill the prism.

Discussion 2

a. Compare the volumes of regular pyramids and right prisms with equal heights and congruent bases.

Mathematics Note

The volume of a pyramid or a cone can be found using the following formula:

$$V = \frac{1}{3}B \bullet h$$

where B is the area of the base and h is the height of the pyramid or cone.

b.
1. Are your results in Exploration **1** consistent with the formula given in the mathematics note? Explain your response.
2. Are your results in Exploration **2** consistent with the formula?

c. Describe how to determine the volume of a regular pyramid with a height of h units and a square base in which each side is s units long.

d. Figure **14** below shows a right circular cylinder, a hemisphere, and a right circular cone, with the same radii and "heights." (For the purposes of this comparison, the radius of a hemisphere also can be thought of as its "height.")

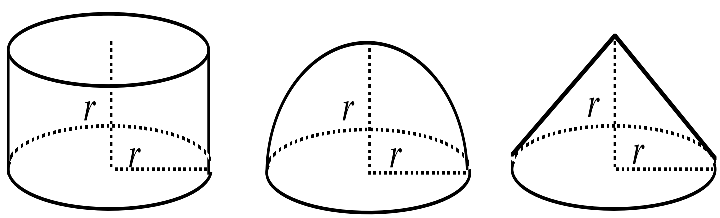


Figure 14: A cylinder, a hemisphere, and a cone

1. Rank the volumes of each figure from largest to smallest.
2. Express the volumes of the cylinder and the cone in terms of r.
3. Make a conjecture about the formula for the volume of a hemisphere with radius r.
4. Based on your conjecture above, what would be the formula for the volume of a sphere of radius r?

Mathematics Note

A **sphere** is the set of all points in space that are the same distance from a given point, the **center** of the sphere. The common distance is the **radius** of the sphere. The **diameter** of a sphere is twice the radius.

The volume of a sphere with radius r can be found using the following formula:

$$V = \frac{4}{3}\pi r^3$$

Assignment

5.1 **a.** Find the volume of a right circular cone with a radius of 10 cm and a height of 15 cm.

b. Determine the height of a right circular cylinder with the same radius and volume as the cone in Part **a**.

c. Determine the height of a right circular cone with a radius of 8 cm and the same volume as the cone in Part **a**.

5.2 **a.** Consider a tipi shaped like a pyramid with a regular dodecagon as its base. (A dodecagon has 12 sides). The tipi's height is 5 m and each side of the base is 2 m long. What is the volume of this tipi?

b. Consider a tipi that can be modeled by a right circular cone. The radius of its base is 3.75 m and its height is 5 m. What is the volume of this tipi?

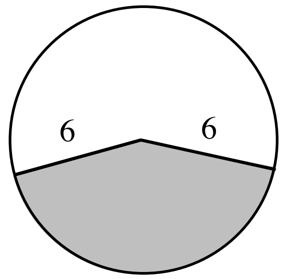
c. Explain why your responses to Parts **a** and **b** are close in value.

5.3 A cone-shaped structure with a small ratio of surface area to volume might be more efficient to heat than one with a larger ratio. Consider a building shaped like a right circular cone with a slant height of 6 m, a height of h meters and a radius of r meters.

a. Write an equation that expresses the height h in terms of the radius r.

b. Use your equation from Part **a** to write a formula for the volume of the cone in terms of r.

c. The shaded sector in the following diagram represents a net for the lateral surface of a cone with a slant height of 6 m. The arc that bounds the sector has a length equal to the circumference of the base of the cone: $2\pi r$.



Write an expression for the lateral surface area of the cone in terms of r.

d. Create a spreadsheet with the following headings. Use the spreadsheet to investigate the ratio of lateral surface area to volume for cones with a slant height of 6 m and radii from 0.1 m to 5.9 m, in increments of 0.1 m.

Radius (m)	**Volume (m^3)**	**Lateral Surface Area (m^2)**	$\dfrac{\textbf{Surface Area}}{\textbf{Volume}}$
0.1			
0.2			
0.3			
⋮			
5.8			
5.9			

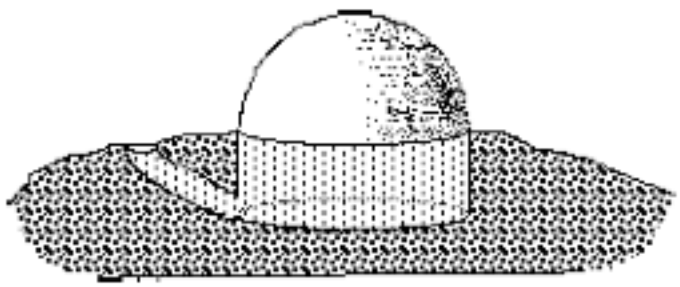
e. Identify the height and radius of the cone in Part **d** with the smallest ratio of surface area to volume.

Mathematics Note

The surface area of a sphere with radius r can be found using the formula below:

$$S = 4\pi r^2$$

5.4 The traditional house of the Maidu and Miwok tribes of central California was a cylindrical pit covered by a hemispherical dome. These pits ranged from 1.5 m to 3.5 m deep, and 3.5 m to 18 m in diameter. The diagram below shows an example of this type of dwelling.



a. The pit in the diagram above is 1.5 m deep and has a diameter of 3.5 m. What is the total volume of this house?

b. What is the total surface area of this house?

c. As shown in the diagram, part of the house lies below ground level. How do you think this will affect the relative efficiency of heating or cooling?

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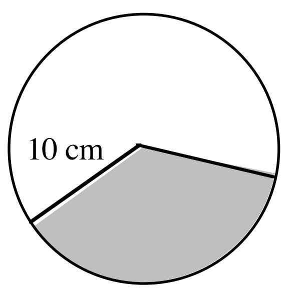
5.5 The diagram below shows a circle with a radius of 10 cm. The shaded sector has a central angle that measures 120˚.



Imagine that the two sectors are cut apart, then folded into cones. In each cone, the cut edges touch, but do not overlap.

a. Find the total surface area of each cone, including the base.

b. Find the volume of each cone.

c. Find the ratio of surface area to volume for each cone.

5.6 The diagram below shows one example of an igloo. The inside diameter of the igloo is 6.0 m, while its walls are made of packed snow 0.1 m thick.

a. The living space inside this igloo is a hemisphere. What is the volume of the living space?

b. What is the volume of the packed snow in the igloo's walls (ignoring the entryway)?

c. What is the ratio of lateral surface area to volume for the inside of the igloo? What might this ratio tell you about the relative heating efficiency of an igloo?

d. Consider a house built in the shape of a half cylinder (such as a Quonset hut) with the same volume as the igloo in Part **a** and a height of 3 m. Disregarding the floor, what is this home's ratio of surface area to volume?

e. Which shape—a hemisphere or a half cylinder—do you think would be more practical for housing in Arctic regions? Explain your response.

5.7 The bathysphere is a spherical steel vessel designed for undersea research. Designed by American naturalist and explorer William Beebe, it made its first dive in 1930.

a. The outside radius of the bathysphere is approximately 75 cm. Determine the volume of water it displaces.

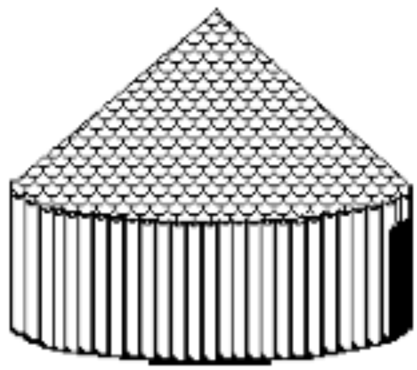
b. Find the volume of the interior of the bathysphere given that its steel walls are 3.8 cm thick.

c. When the first bathysphere was designed, weight was an important concern. Assuming that weight is proportional to surface area, compare the weight of the bathysphere with that of a cube of equal volume.

* * * * * * * * * *

Summary Assessment

1. The Miwok people of central California were one of several tribes who built structures with cylindrical walls and conical roofs known as roundhouses. Roundhouses were used to host ceremonial dances and rituals. The diagram below shows one example of a roundhouse.



The floor of the roundhouse in the diagram has a diameter of 10.6 m. The height of the walls is 2.1 m; the total height of the building is 4.6 m.

a. What is the total surface area of this roundhouse?

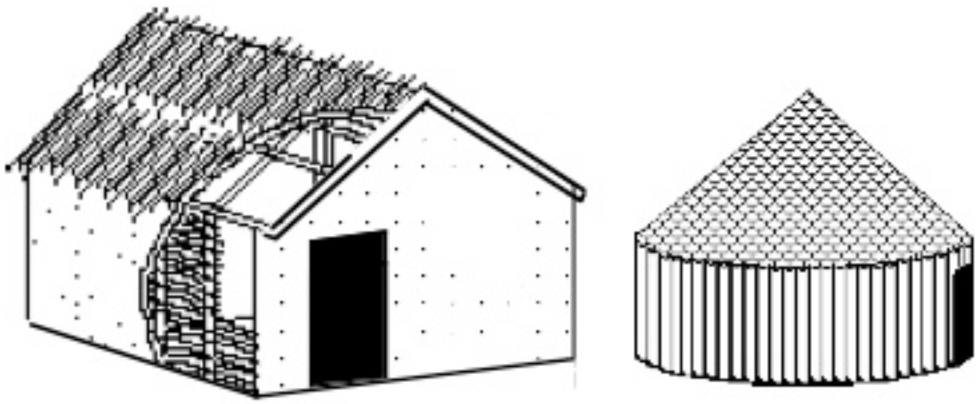
b. What is its volume?

2. As shown in the following diagram, the living space in an igloo is shaped like a hemisphere, while its entryway is shaped like a half-cylinder.

The inside radius of the igloo in the diagram is 2.5 m. The entryway is 1 m long and has an inside radius of 0.5 m.

Determine the inside surface area and volume of the igloo, including the entryway. Describe any assumptions you make in solving this problem.

3. Oklahoma's Delaware tribe did not build roundhouses. Their ceremonial lodges were shaped like square prisms with gabled roofs. In the diagram below, a Delaware lodge is shown on the left, while a Miwok roundhouse appears on the right. For both of these structures, the area of the floor is $49\ \text{m}^2$, the height of the walls is 3 m, and the total height of the building is 4.7 m.



a. Compare the ratios of total surface area to volume (including the floor) for the two buildings.

b. If you lived in a region where building materials were scarce, which type of structure would you choose: the roundhouse or the gabled lodge? Explain your response.

c. Which design might be easier to heat or cool? Explain your response.

Module Summary

- An **inscribed polygon** is one in which each vertex of the polygon lies on a circle. The **radius** of a regular polygon is the radius of the circle in which the polygon can be inscribed.
- A **tangent** to a circle is a line, segment, or ray that intersects a circle in one point and is perpendicular to a radius at that point.
- The **volume** (*V*) of a right prism can be found by multiplying the area of one of its bases (*B*) by its height (*h*). The height of a prism is the distance between the two bases. In general, the formula for the volume of a right prism is:

$$V = B \bullet h$$

- The volume of a right cylinder with radius r and height h can be found using the following formula:

$$V = (\pi r^2)h$$

- An **arc** of a circle is a part of the circle whose endpoints are the intersections, with the circle, of the sides of a central angle.
- The **measure of an arc** is the measure of its central angle. A **minor arc** has a measure less than 180°; a **major arc** has a measure greater than 180°; a **semicircle** has an arc measure of exactly 180°.
- The **length of an arc** is the distance on the circle between the arc's endpoints. This distance can be found by multiplying the circle's circumference by the fractional part of the circle that the arc represents.
- A **sector** of a circle is a region bounded by the sides of a central angle and an arc of the circle. The **area of a sector** can be found by multiplying the circle's area by the fractional part of the circle that the sector represents.
- The lateral surface of a cone is a sector of a circle.
- The volume of a pyramid or a cone can be found using the following formula:

$$V = \frac{1}{3}B \bullet h$$

  where B is the area of the base and h is the height of the pyramid or cone.
- A **sphere** is the set of all points in space that are the same distance from a given point, the **center** of the sphere. The common distance is the **radius** of the sphere. The **diameter** of a sphere is twice the radius.
- The surface area of a sphere with radius r can be found using the formula:

$$S = 4\pi r^2$$

- The volume of a sphere with radius r can be found using the following formula:

$$V = \frac{4}{3}\pi r^3$$

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