Hurry! Hurry! Hurry! Step Right Up!

Have you ever played a carnival game? Better yet, have you ever won a big prize? In this module, you use probability to explore the carnival midway.

Bill Chalgren • Mike Trudnowski • Anne Watkins

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Introduction

A carnival barker beckons to you from the midway: "Step right up! You look like a winner!" The barker's booth is decorated with enticing prizes. The barker tells you that you can choose from one of two games. One requires you to throw a dart at a board while blindfolded, the other involves twirling a spinner on an identical board. To win, either the dart or the arrow must land in the unshaded portion of the board. The two game boards are shown in Figure **1** below.
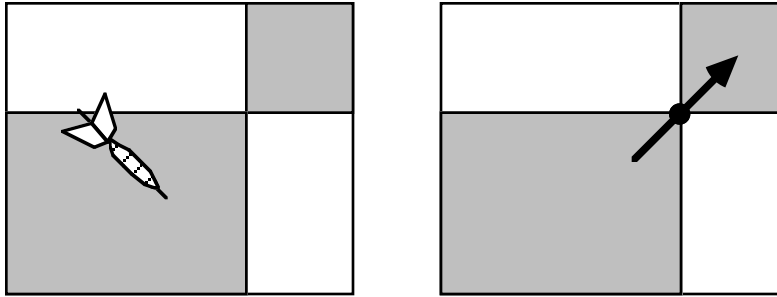

Figure 1: Two carnival game boards

"Put your ticket down," says the barker, "choose a game, and take a chance." When you hesitate, the barker assures you that you continue throwing darts until one hits the board and that the spinner is equally likely to stop at any point on its arc.

Discussion

a. Which has a better chance of landing in the unshaded portion of the board: a dart thrown while blindfolded, or the arrow on the spinner? Explain your response.

b. When determining whether or not the arrow landed in the unshaded portion, why should you consider only the tip of the arrow?

c. **1.** What kinds of carnival games have you played or seen others play?

2. Were those games based on skill or on chance? Explain your reasoning.

d. Which of the games shown in Figure **1** would you rather play? Why?

e. Do you think that carnival games are fair? Explain your response.

Activity 1

As you observed in the introduction, the chances of a dart landing in the unshaded portion of the game board in Figure **1** are not the same as the chances of the spinner landing there. In this activity, you create **geometric models** that will allow you to determine why these probabilities are not the same.

Exploration

In the coin-drop game, each player tries to drop a dime so that it lands entirely inside a 3 cm ∞ 3 cm square on a grid. If the dime lands completely within a square, the player wins. If any part of the dime touches or crosses a line on the grid, the player loses. Figure **2** shows some possible game situations:
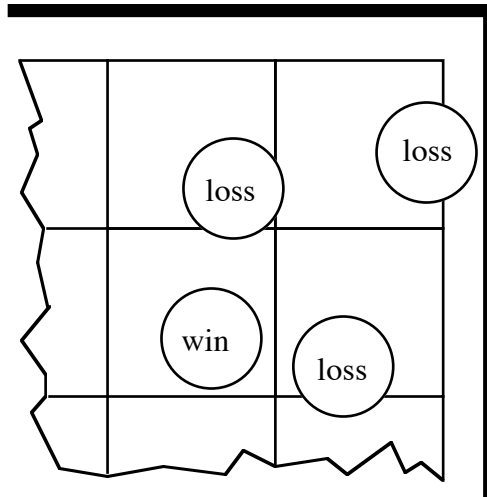

Figure 2: Sample coin-drop outcomes

a.   **1.** Mark a full sheet of graph paper into 3 cm ∞ 3 cm squares.

   2. Use folded cardboard to form walls around the sheet of graph paper, as shown in Figure **3**. Each wall should be placed about 1 cm from the edge of the graph paper. Tape the walls together.

30 cm

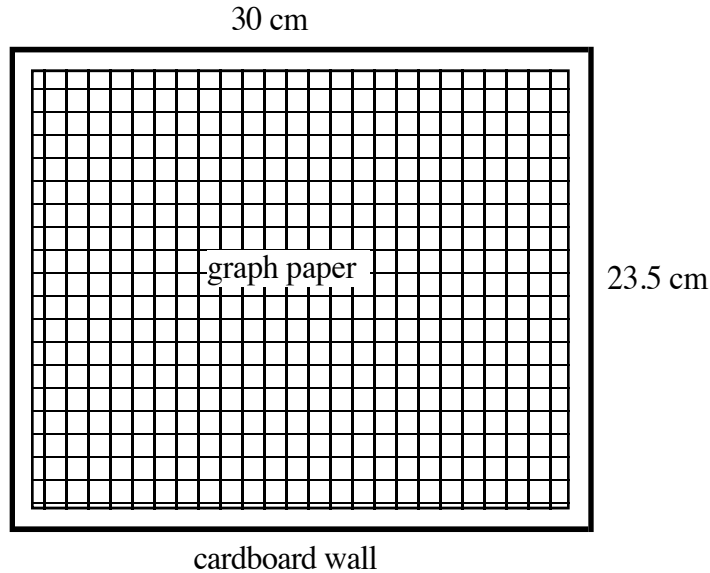graph paper   23.5 cm

cardboard wall

Figure 3: Graph paper with cardboard walls

b.   Drop a dime onto the grid from the height of a chair seat (about 45 cm). Record the result of the drop as a win or loss.

c.   Repeat Part **b** nine more times.

d.   **1.** Compile the total number of wins and losses for the entire class.

2. Use the class data to determine the experimental probability of winning the game.

Mathematics Note

If each outcome in a sample space has the same chance of occurring, then the **theoretical probability** of an event E is determined using the following ratio:

$$P(E) = \frac{\text{number of outcomes in the event}}{\text{total number of outcomes in the sample space}}$$

When the outcomes in an event can be represented as points in a **geometric model**, the theoretical probability of the event may be found using the ratio below:

$$P(E) = \frac{\text{measure of geometric model representing outcomes in the event}}{\text{measure of geometric model representing all outcomes in the sample space}}$$

Some examples of geometric measures are lengths, angle measures, areas, and volumes.

For example, consider a carnival game that involves throwing a dart at balloons. There are 16 balloons with a mean diameter of 10 cm attached to a square board 60 cm on each side. To win a prize, players must break a balloon. As shown in Figure **4**, the balloons can be represented as circles, while the board can be represented as a square.
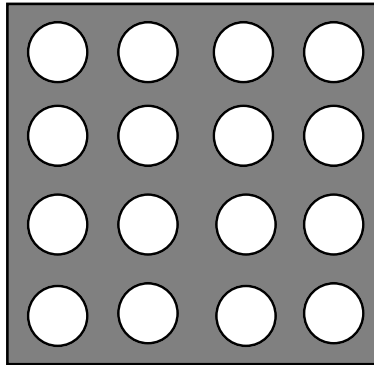

Figure 4: Balloons on a board

Assuming that the dart is equally likely to hit any point on the board, the probability of breaking a balloon (and winning a prize) can be found as follows:

$$P(\text{win}) = \frac{\text{area of 16 circles}}{\text{area of board}} = \frac{16 \bullet \left(\pi \bullet 5^2 \right)}{60 \bullet 60} \approx 0.35 = 35\%$$

e.   Consider one 3 cm ∞ 3 cm square on the grid. To determine the theoretical probability of winning using a geometric model, you must first identify the area that models all winning outcomes.

1. How close can a dime's center come to the side of a 3 cm ∞ 3 cm square and still represent a win?

2. Using the center of the dime as a reference point, determine the area of the region that models all winning outcomes.

3. Use the area determined in Step **2** and the area representing all possible outcomes to find the theoretical probability of winning.

f.   Compare the experimental probability of winning you determined in Part **d** with the theoretical probability from Part **e**.

Discussion

a.   In a spinner game, the tip of the arrow serves as the reference point for determining the outcome of each spin. In the coin-drop game, you used the center of the dime. Why is this an appropriate reference point?

b.   Does the number of 3 cm ∞ 3 cm squares on the grid affect the probability of winning the coin-drop game? Explain your response

c. In Part **f** of the exploration, how did the experimental probability of winning the coin-drop game compare to its theoretical probability?

d. Describe how you could make the coin-drop game impossible to win.

Assignment

1.1 In another version of the coin-drop game, players drop quarters onto the grid instead of dimes.

    a. Describe the area within a 3 cm ∞ 3 cm square that represents a win.

    b. Determine the theoretical probability of winning the coin-drop game using a quarter.

1.2 To attract more players, the carnival decides to allow players to drop dimes onto a grid of 4 cm ∞ 4 cm squares.

    a. Describe the area within a 4 cm ∞ 4 cm square that represents a win.

    b. Determine the theoretical probability of winning this version of the coin-drop game.

1.3 In another kind of carnival game, players throw darts at a circular board. As shown in the diagram below, the board is divided into four concentric regions, each painted a different color.
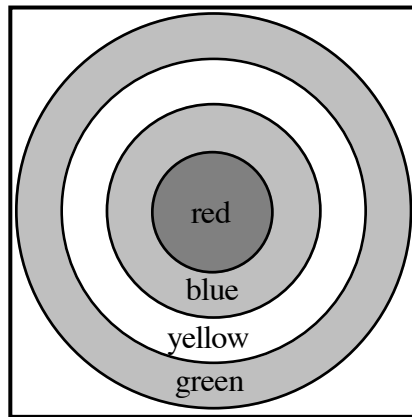
The radius of the red region in the center of the board is 4 cm, while the width of each of the remaining regions is 3 cm.

    a. Assuming that a dart has an equal chance of landing at any point on the board, what is the theoretical probability that it lands in the yellow region? Explain your response.

    b. Is throwing a dart at a target typically a random event? Explain your response?

1.4　In Bowl-o-rama, a bowling pin with a diameter of 8 cm is placed somewhere near the center of a lane 100 cm wide. To keep the ball in play, the lane has rails on each side. To win the game, a blindfolded player must knock down the pin using a bowling ball 20 cm in diameter.

　　a. Assuming that the pin will fall if it is touched by the ball, what is the probability of winning the game? Justify your response.

　　b. Would placing the bowling pin against one of the rails affect the probability of winning the game? Explain your response.

1.5　In the introduction to this module, a carnival barker offered you a chance to play either a dart game or a spinner game. (See Figure **1**.) To win, either the dart or the arrow must land in the unshaded portion of the board. The measurements of the game board are shown below.
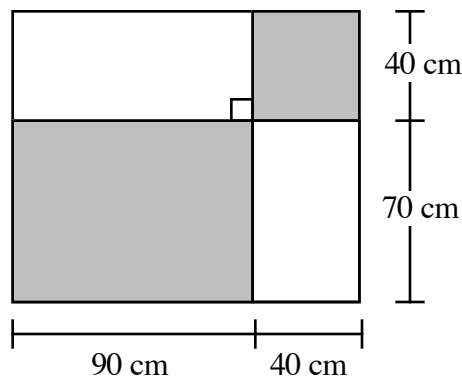
Based on the theoretical probabilities of winning, which game would you rather play? Explain your response.

$$* * * * *$$

1.6　Consider two real numbers, x and y, where $0 \le x \le 5$ and $0 \le y \le 5$.

　　a. Graph all the possible ordered pairs (x,y) on a two-dimensional coordinate system.

　　b. Determine the area of the region that represents all the possible ordered pairs (x,y).

　　c. Graph all the possible ordered pairs (x,y) where $x + y \le 3$, $0 \le x \le 5$, and $0 \le y \le 5$.

　　d. Given a point (x,y) selected at random from Part **a**, determine the probability that $x + y \le 3$. Justify your response.

　　e. Given a point selected at random from Part **a**, determine the probability that $x + y \ge 6$. Justify your response.

1.7   After jumping from an airplane, a parachutist must land safely in a square parking lot 75 m on each side. The parking lot is free of hazards, except for at the corners. In each corner of the lot, there is a utility pole with a street light. If the parachutist lands within 10 m of a utility pole, the parachute's lines will get caught on the light.

    a. Draw a sketch of the parking lot and the utility poles. Shade the areas in which the parachutist should not land.

    b. Determine the area in which it is safe to land.

    c. Assuming that the parachutist is equally likely to land anywhere in the parking lot, determine each of the following probabilities:

        1. the parachutist lands safely

        2. the parachute's lines get caught on a security light.

    d. What is the relationship between the two probabilities in Part **c**?

1.8   Consider two real numbers, x and y, where $1 \le x \le 3$ and $1 \le y \le 3$.

    a. Graph all the possible ordered pairs (x,y) on a two-dimensional coordinate system.

    b. Determine the area of the region that represents all the possible ordered pairs (x,y).

    c. Graph all the possible ordered pairs (x,y) where $y/x \le 1$, $1 \le x \le 3$, and $1 \le y \le 3$.

    d. Given a point (x,y) selected at random from Part **a**, determine the probability that $y/x \le 1$. Justify your response.

    e. Given a point selected at random from Part **a**, determine the probability that $y/x \ge 1$. Justify your response.

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Activity 2

In another carnival game, players roll a six-sided die. The game costs $1.00 to play. The values of the prizes that correspond with the six possible rolls are shown in Table **1** below.

Table 1: Value of prizes in die game

Roll of Die	Value of Prize
1	$1.50
2	$1.00
3	$0.50
4	$0.00
5	$0.00
6	$0.00

Judging from the values of the prizes, do you think this is a fair game? In the following exploration, you use mathematics to help answer this question.

Exploration

    a.   Play the die game 10 times. Record the number of wins for each prize.

    b.   **1.** Compile the data from Part **a** for the entire class.

               2. Use the class data to determine the experimental probability of winning each prize.

    c.   Determine the theoretical probability of winning each prize.

    d.   Compare the experimental probability of winning each prize to the theoretical probability.

Mathematics Note

The mean value of an experiment is the **expected value**. Expected value is determined by adding the products of the value of each event and its corresponding theoretical probability.

For example, suppose a carnival game uses the spinner shown in Figure **5**. The measures of the three central angles are 45°, 135°, and 180°, respectively. Using a geometric model, the probability of winning \$25 is $45/360 = 1/8$, the probability of winning \$15 is $135/360 = 3/8$, and the probability of winning \$0 is $180/360 = 1/2$.
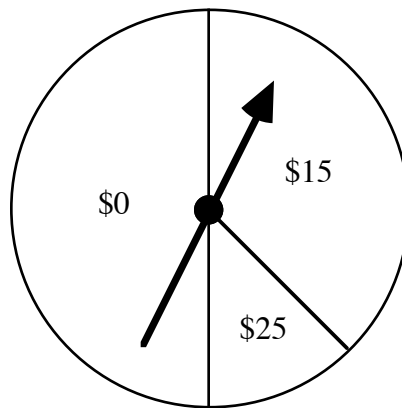

Figure 5: Spinner with three central angles

The expected value for this game is the sum of the products of each prize value and its corresponding probability. This can be calculated as follows:

$$\frac{1}{8} \bullet \$25 + \frac{3}{8} \bullet \$15 + \frac{1}{2} \bullet \$0 = \$8.75$$

This indicates that, for a large number of plays, the mean amount won per play is likely to be close to \$8.75.

e.   Determine the expected value of the die game.

Discussion

a.   When playing the die game, are you equally likely to win each prize? Explain your response.

b.   A **fair game** is one in which the expected value equals the cost to play. Is the die game mathematically fair? If so, justify your response. If not, suggest at least one way to make it fair.

c.   Is it reasonable to expect a carnival game to be mathematically fair? Explain your response.

Assignment

2.1 In another version of the die game, prizes are awarded as shown in the following table.

Roll of Die	Value of Prize
1	$2.00
2	$0.00
3	$0.50
4	$0.00
5	$0.50
6	$0.00

Determine the cost to play that makes this a fair game.

2.2 In one version of the coin-drop game described in Activity **1**, the theoretical probability of winning is $4/25$. The game costs $1.00 to play. Winning players receive a prize worth $5.00. Losing players receive no prize.

  a. Determine the expected value for this game.

  b. Given that the prize value remains $5.00, change the cost of playing to make this a fair game.

  c. Given that the cost of playing remains $1.00, change the value of the prize to make this a fair game.

2.3 In another version of the coin-drop game, the theoretical probability of winning is $1/25$. This game also costs $1.00 to play. However, winning players receive a prize worth $15.00.

  a. Determine the expected value for this game.

  b. Given that the prize value remains $15.00, change the cost of playing to make this a fair game.

  c. Given that the cost of playing remains $1.00, change the value of the prize to make this a fair game.

2.4 Imagine that you are the manager of a carnival. One of the game operators has designed a new game. In this game, players pick 1 card from an ordinary deck of 52 cards. An ace wins $10.00, a face card (king, queen, or jack) wins $1.00, and all other cards win nothing. Determine the cost to play to make this a fair game.

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2.5   Joan and Juan are playing a game that involves randomly selecting two real numbers in the interval [0, 5]. Joan offers to pay Juan $0.50 every time the sum of the two numbers is less than or equal to 3, if he will pay her $0.20 every time the sum is greater than 3.

     a. Before deciding whether to accept these rules, Juan wants to know his chances of winning. Use a geometric model to determine the probability that Juan will win.

     b. Determine the expected value of the game for Juan.

     c. Do you think Juan should accept Joan's rules? Explain your response.

     d. Modify the prize amounts so that the expected values for both players are equal.

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Research Project

Design your own carnival game. Calculate the expected value and determine the cost to play required to make it a fair game. Exchange your game with a classmate. Check each other's designs to verify that the games are fair. Present your game to the class.

Activity 3

In the fishing derby, players hook three rings, one at a time, from a tank of swirling water. The tank contains 10 rings. Four of the rings are marked with a small red dot and six are marked with a small blue dot.

     Players receive 1 point for each blue ring and 2 points for each red ring. After each ring is caught, it is placed back in the tank before the player's next try. Table **2** below shows the total number of points required to win each prize.

Table 2: Prizes in the fishing derby

Prize	Number of Points Needed
rabbit's foot key chain	3
celebrity poster	4
black hat with a pink feather	5
large teddy bear	6

To win a particular prize, a player must have the exact number of points for that prize after hooking three rings. To win the celebrity poster, for example, a player must have exactly 4 points. Players with 5 or 6 points cannot win the poster, even though they may have had exactly 4 points at some time before hooking the last ring. Points earned in one game cannot be added to points earned in another game.

Exploration 1

a. To simulate the fishing derby, mark four balls with a red dot and six balls with a blue dot. Place the balls in a container.

    1. Without looking, draw one ball from the container. Record the color of its dot, then return it to the container.

    2. Repeat Step **1** two more times, mixing the balls before each draw.

    3. Determine the total number of points for the three draws. As in the actual fishing derby, each blue dot is worth 1 point and each red dot is worth 2 points.

b.   **1.** Play the game 10 times. Record the points earned in each game.

    2. Compile the experimental results for the entire class.

    3. Using the compiled results, determine the experimental probability of winning each prize.

c. Create a tree diagram to show the possible outcomes in the fishing derby after a player's first catch. Label each branch with a theoretical probability.

d. Create a tree diagram to show the possible outcomes after all three catches. Label each branch with a theoretical probability.

Mathematics Note

Two events A and B are **independent** if $P(\text{A and B}) = P(\text{A}) \bullet P(\text{B})$. Three events A, B, and C are independent if each pair of events is independent and $P(\text{A and B and C}) = P(\text{A}) \bullet P(\text{B}) \bullet P(\text{C})$. This definition can be extended to any number of independent events.

Given two independent events, the occurrence of one has no effect on the likelihood of the occurrence of the other. Two events that are not independent are said to be **dependent**.

For example, consider tossing two coins—a penny and a nickel—one at a time. Since the outcome of tossing the penny does not affect the outcome of tossing the nickel, the two tosses are independent events.

The probability of obtaining heads on either coin, or $P(\text{H})$, is $1/2$. The probability of obtaining heads on the penny and the nickel, or $P(\text{H and H})$, is:

$$P(\text{H and H}) = P(\text{H}) \bullet P(\text{H}) = \frac{1}{2} \bullet \frac{1}{2} = \frac{1}{4}$$

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e. In the fishing derby, hooking the first ring, hooking the second ring, and hooking the third ring are independent events. Use your tree diagram from Part **d** and the definition given in the mathematics note to determine the theoretical probability of each outcome.

f. **1.** Determine the theoretical probability of winning each prize in the fishing derby.

   2. Compare these probabilities with the experimental probabilities you found in Part **b**.

Discussion 1

a. From your tree diagram in Part **d** of Exploration **1**, how many possible outcomes are there in the fishing derby?

b. Describe the different sets of three rings that result in each of following numbers of points:

   1. 3

   2. 4

   3. 5

   4. 6

   5. 7

c. Why do you think the rings with red dots are worth more points than the ones with blue dots?

d. **1.** Does hooking a red ring on the first try affect the probability of hooking a blue one on the second try? Explain your response.

   2. Does it affect the probability of hooking a red ring on the second try? Why or why not?

e. Which prize do you think is awarded most often? Why?

f. **1.** Describe how you could use technology to model the fishing derby.

   2. Would you expect this simulation to produce the same results as a the simulation in Exploration **1**? Explain your response.

g. Which information is likely to better represent the long-term results of playing the fishing derby: the data from 10 simulations, or the compiled class data? Explain your reasoning.

Exploration 2

In the next town on the carnival circuit, the operator of the fishing derby changes the rules of the game. The change requires players to keep each ring hooked instead of returning it to the tank. This new version of the derby is supposed to give players a better chance of winning a hat or a teddy bear. The rest of the game remains the same.

a. Predict how the rule change will affect the probability of winning each prize.

b. Create a tree diagram to show the possible outcomes in the new fishing derby. Label each branch with a theoretical probability.

Mathematics Note

Conditional probability is the probability of an event occurring given that an initial event has already occurred. The probability that event B occurs, given that event A has already occurred, is denoted by $P(B|A)$.

In an experiment involving conditional probabilities, the probability of both A and B occurring is found by multiplying the probability of A by the conditional probability of B given A:

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

For example, consider a game in which players draw two chips from a box containing three chips: two red and one blue. Each player draws a chip then, without replacing the first chip, draws another. Since the first chip drawn affects the probabilities in the second draw, they are not independent events. Figure **6** shows a tree diagram for this game. The probability of drawing a red on the second draw, given a red on the first draw is $1/2$. The probability of drawing a red on the second draw, given a blue on the first draw, or $P(B|R) = 2/2$.
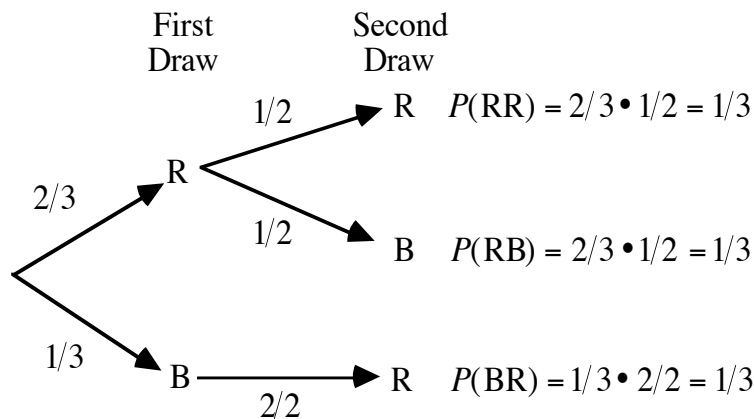

First Draw   Second Draw

$2/3$ R $1/2$ R $P(RR) = 2/3 \cdot 1/2 = 1/3$

$1/2$ B $P(RB) = 2/3 \cdot 1/2 = 1/3$

$1/3$ B $2/2$ R $P(BR) = 1/3 \cdot 2/2 = 1/3$

Figure 6: Conditional probabilities

c. Use your tree diagram to determine the theoretical probability of winning each prize.

d. Create a simulation for this version of the game.

e. 1. Conduct your simulation a total of 10 times. Record the points earned and the prize won each time.

2. Compile the results from the entire class.

3. Using the compiled results, determine the experimental probability of winning each prize.

4. Compare these experimental probabilities with the theoretical probabilities you determined in Part **c**.

Discussion 2

a. Which part of your simulation models the rule that each ring caught must be removed from the tank?

b. Are the chances of getting 6 points in the new fishing derby better or worse than in the original version of the game?

c. Does the rule change appear to have improved the chances of winning a hat or a teddy bear?

d. 1. In the game modeled in Figure **6**, why are the probabilities of obtaining a red chip on the second draw different from the probability of obtaining a red chip on the first draw?

2. What is the probability of drawing two blue chips in this game?

Assignment

3.1 **a.** What is the theoretical probability of winning a prize in the original fishing derby? Explain your response.

b. What is the theoretical probability of earning 7 points in the original fishing derby? Explain your response.

c. What is the expected value, in points, of the original fishing derby?

3.2 **a.** What is the expected value, in points, of the new fishing derby?

b. Which version of the fishing derby would you rather play? Explain your response.

3.3 Both versions of the fishing derby cost $1.00 to play. The values of the prizes are shown in the following table.

Prize	Value
rabbit's foot key chain	$0.25
celebrity poster	$0.60
black hat with pink feather	$0.80
large teddy bear	$2.00

    a. Is the original fishing derby a fair game? If so, justify your response. If not, describe how you could modify the game to make it fair.

    b. Is the new fishing derby a fair game? If so, justify your response. If not, describe how you could modify the game to make it fair.

3.4 Guinn and Ebdul are playing the stair-step game. This game starts on the middle step of a long staircase. Each player's turn consists of flipping two coins and moving up or down according to the following rules: if no heads appear on a player's two coins, the player does not move; if one head appears, the player moves up one stair; if two heads appear, the player moves down one stair.

    The staircase has 25 steps. The game is over when one player wins by reaching the top step, or loses by reaching the bottom step.
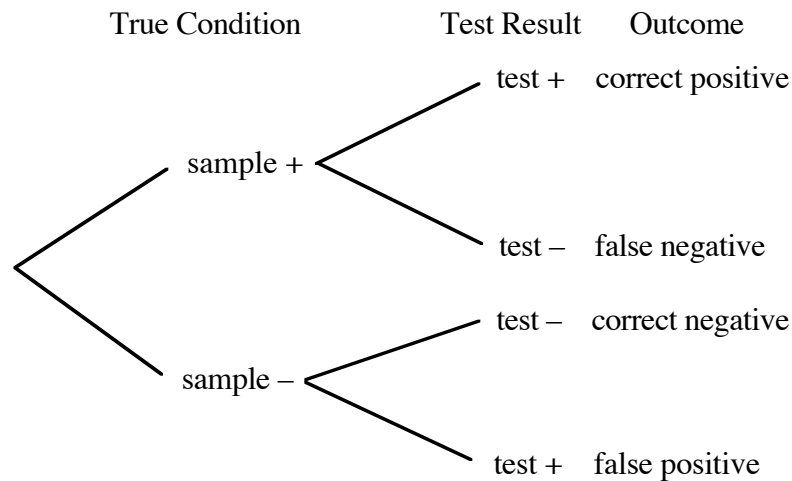
    a. If movement is described in terms of the number of stairs moved upwards, then each player's turn in the game has three possible results: 1 step, 0 steps, and –1 step.

        What is the probability of each of these results?

    b. What is the expected value, in number of stairs moved upwards, for each turn in the stair-step game?

    c. Where on the staircase do you think Guinn will be after 10 turns?

    d. How many turns do you think it will take for a player to reach the top step? Explain your response.

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3.5   In 1992, the Centers for Disease Control (CDC) reported that 1 out of every 250 people in the United States had the HIV virus. For people in either group (HIV positive and HIV negative), the HIV test result is correct 99.8% of the time and false 0.2% of the time.

The tree diagram below shows the possible outcomes of the test for a person selected at random from the U.S. population. "Sample +" indicates that the person who gave the sample has the HIV virus. "Test +" indicates that the sample tested positive for HIV.

A test that incorrectly characterizes the sample is called a false test. A "false negative" means that the sample tested negative for HIV, but actually contains the virus. A "false positive" means that the sample tested positive, but does not contain the HIV virus.

True Condition	Test Result	Outcome
sample +	test +	correct positive
	test −	false negative
sample −	test −	correct negative
	test +	false positive

a. Using a tree diagram like the one above, calculate the probability of each outcome in the sample space.

b. Use your results from Part **a** to answer the following questions. If a random sample of 1,000,000 people were tested for HIV, how many of them do you think would test:

   1. positive and actually have HIV (correct positive)?

   2. negative and actually have HIV (false negative)?

   3. negative and not have HIV (correct negative)?

   4. positive and not have HIV (false positive)?

c. Use your results in Part **b** to estimate the probability that a sample contains the HIV virus, given that the test result is positive.

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1. As part of its annual fund-raiser, a local charity has designed a game called Auto Dart. Each player throws one dart at a computerized board. After the dart lands, the board lights up, displaying three red zones and one blue zone. The dimensions of the board, along with one possible arrangement of the zones, is shown in the diagram below.

200 cm

150 cm

Red Zones
30 cm by 35 cm

Blue Zone
radius of 4 cm
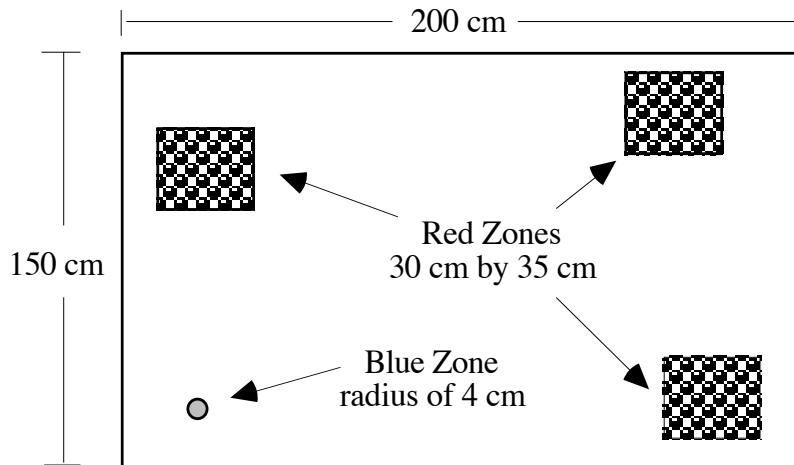
If the dart landed in the blue zone, the player wins a car worth $10,000. If the dart landed in any of the three red zones, the player wins a miniature replica of the car valued at $25. If the dart landed anywhere else on the board, the player wins nothing. After each throw, the computer randomly relocates the winning zones. These zones are always the same size and never overlap.

Imagine that you are the charity's financial advisor. Analyze the Auto Dart game and decide if the charity should use it in the fund-raiser. Write a letter to the charity explaining your recommendation. Your letter should include the following:

a. the probability of winning each prize in Auto Dart

b. the expected value for the game

c. the lowest price, rounded to the nearest dollar, that the charity could charge for the game and still expect to make a profit

d. the number of miniature replicas and full-sized cars the charity can expect to award to the first 1000 players

e. the probability of two players in a row winning full-sized cars.

2. A carnival manager has created a new game called Blind Draw. In this game, players draw three small balls from a container. The container holds 20 balls: 10 green balls worth 1 point each, 8 blue balls worth 2 points each, and 2 red balls worth 5 points each.

Players cannot see the contents of the container and remove each ball after it is drawn. The point total for three balls determines the prize won. A total of 6 through 8 points wins a pennant, 9 through 11 points wins a small stuffed animal, and 12 or more points wins a large stuffed animal. Points from one game cannot be carried over to another game.

The pennants cost the carnival $0.40 each, the small stuffed animals cost $1.75 each, and the large stuffed animals cost $3.00 each. The manager plans to charge $1.00 to play the game.

As the carnival's financial advisor, write a letter to the manager describing your recommendations on this game. Use mathematics to support your position.

- If each outcome in a sample space has the same chance of occurring, then the **theoretical probability** of an event E is determined using the following ratio:

$$P(\text{E}) = \frac{\text{number of outcomes in the event}}{\text{total number of outcomes in the sample space}}$$

- When the outcomes in an event can be represented as points in a **geometric model**, the theoretical probability of the event may be found using the ratio below:

$$P(\text{E}) = \frac{\text{measure of geometric model representing outcomes in the event}}{\text{measure of geometric model representing all outcomes in the sample space}}$$

Some examples of geometric measures are lengths, angle measures, areas, and volumes.

- The mean value of an experiment is the **expected value**. Expected value is determined by adding the products of the value of each event and its corresponding theoretical probability.

- A **fair game** is one in which the expected value equals the cost to play.

- Two events A and B are **independent** if $P(\text{A and B}) = P(\text{A}) \bullet P(\text{B})$. Three events A, B, and C are independent if each pair of events is independent and $P(\text{A and B and C}) = P(\text{A}) \bullet P(\text{B}) \bullet P(\text{C})$. This definition can be extended to any number of independent events.

    Given two independent events, the occurrence of one has no effect on the likelihood of the occurrence of the other. Two events that are not independent are said to be **dependent**.

- **Conditional probability** is the probability of an event occurring given that an initial event has already occurred. The probability that event B occurs, given that event A has already occurred, is denoted by $P(\text{B}|\text{A})$.

- In an experiment involving conditional probabilities, the probability of both A and B occurring is found by multiplying the probability of A by the conditional probability of B given A:

$$P(\text{A and B}) = P(\text{A}) \bullet P(\text{B}|\text{A})$$

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