From Here to There

If you've ever hiked over unfamiliar terrain, chances are you consulted a topographic map to help find your way. (If you didn't, you may have wished that you had.) In this module, you use topography to investigate three-dimensional coordinate systems and the distances between points.

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From Here to There

Introduction

Topographic maps are two-dimensional representations of regions of the earth's surface. Besides identifying roads, trails, rivers, and other landmarks, topographic maps also contain information about the elevation of the terrain.

By knowing how to read a topographic map, you can determine the height of a mountain—and make a good guess about the difficulty of the climb. For example, Figure **1** shows a picture of the south side of a mountain. The horizontal lines are **level curves**. Each level curve represents points of equal elevation above sea level.

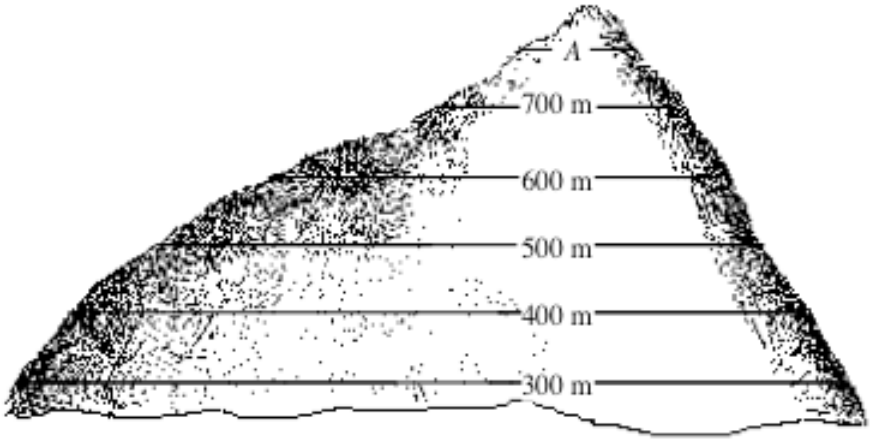


Figure 1: Level curves on the south side of a mountain

Although the view in Figure **1** provides some information about the landscape, a topographic representation of the mountain, as shown in Figure **2**, can be much more helpful. On a topographic map, the curved lines indicating points of equal elevation are **contour lines**.

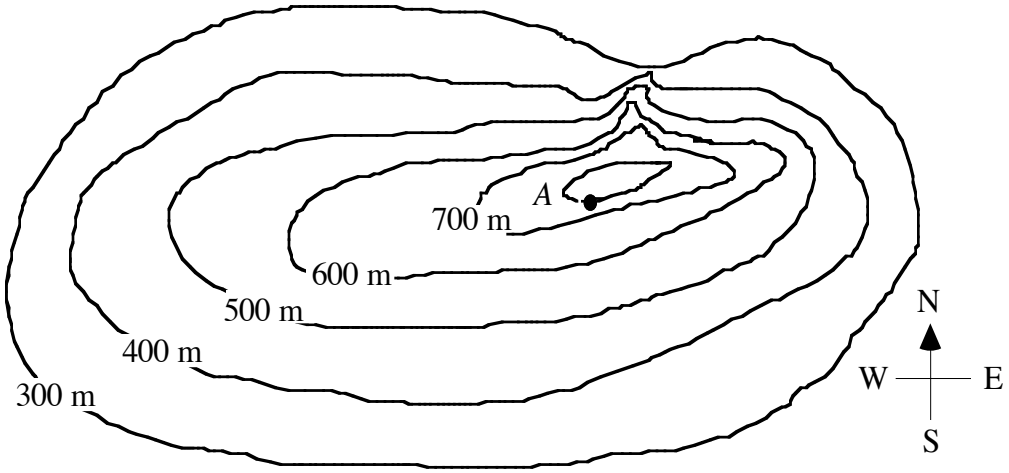


Figure 2: Topographic map of the mountain in Figure 1

Geography Note

When these terms are used to describe features of mountainous terrain, a **summit** is the highest point of the surrounding elevations and a **saddle** is a pass between two higher elevations. Some examples of these features are shown in Figure **3**.

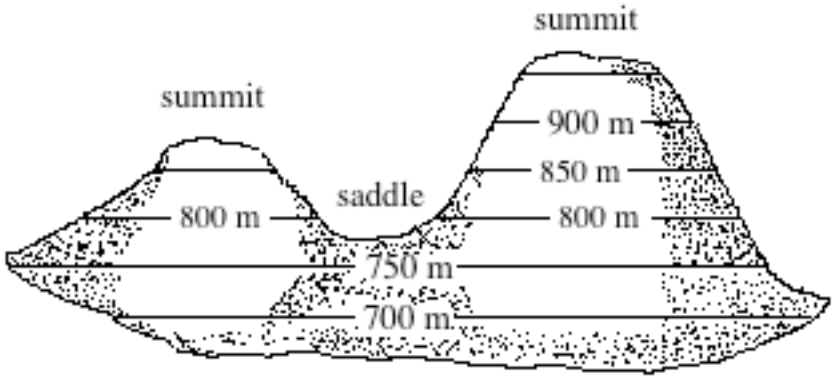


Figure 3: Some geographic features

Discussion

a. From what perspective does a topographic map show the earth's surface?

b. In Figure **1**, what do the quantities expressed in meters represent?

c. In Figure **2**, what is the elevation at contour line *A*?

d. On a topographic map, moving from one contour line to the next consecutive line indicates a constant change in altitude. However, the contour lines themselves may not be evenly spaced.

What is indicated by the space between two consecutive contour lines? Explain your response.

e. **1.** How would you estimate the elevation of a point between two contour lines?

2. What physical characteristics of the terrain could make this estimate inaccurate?

f. How does the south side of the mountain in Figure **1** compare with its north side?

Activity 1

In this activity, you create a topographic map for a three-dimensional surface of your own design. To investigate some of the features of your map, you use a three-dimensional coordinate system.

Exploration

When trying to visualize a mountain from a topographic map, it may help to build a three-dimensional model of the terrain. In this exploration, you build a model that contains a saddle, a summit, a cliff, and a lake. At the same time, you create a topographic map of your model on a coordinate plane. **Note:** Read the instructions in Parts **a–g** before beginning your model.

a. Draw a pair of coordinate axes on a large sheet of paper (at least 60 cm by 28 cm).

b. **1.** On a sheet of cardboard slightly smaller than the paper in Part **a**, sketch the outline of the base of a mountain, such as the example shown in Figure **4**. Cut out this base.

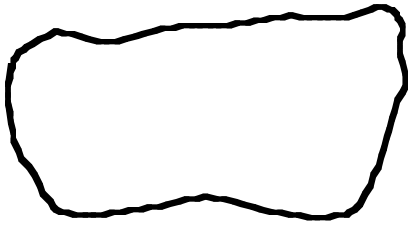


Figure 4: Outline of the base of a mountain

2. Trace the outline of the cardboard base on the coordinate plane, as shown in Figure **5**. This is the first contour line of your topographic map.

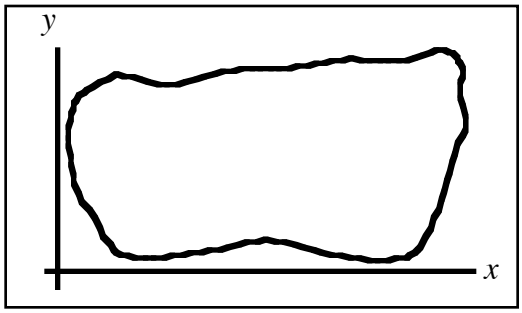


Figure 5: Coordinate plane with first contour

c. **1.** Trace the outline of the base on another sheet of cardboard. Using the tracing as a guide, draw the next contour of the mountain. As shown in Figure **6**, make this contour slightly smaller than the base.

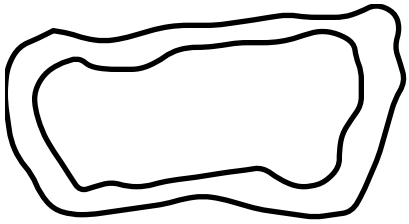


Figure 6: Cardboard with first and second levels

2. Cut out this second level and place it on the coordinate plane inside the outline of the base. Trace the outline of the second contour on the plane, as shown in Figure **7**.

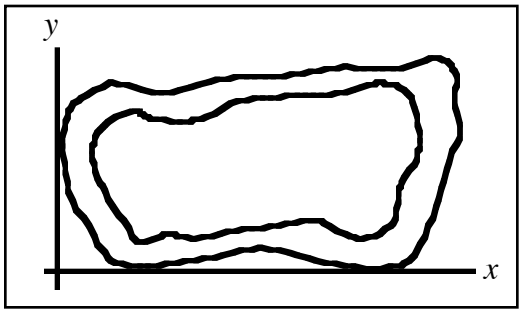


Figure 7: Topographic map of first two levels

d. Trace the outline of the second level on another sheet of cardboard to create a guide for the third contour. Then tape or glue the second level to the base in the same relative position as the contour lines on the coordinate plane.

e. Cut out the third level and place it on the coordinate plane inside the second contour. Trace the outline of the third contour on the plane.

f. Repeat the process described in Parts **d** and **e** until you have created a complete model of a mountain along with its topographic map. Your model should consist of 10 layers of cardboard; the map should have 10 contour lines. Both should include the following geographic features:

1. a lake

2. a summit

3. a saddle

4. a cliff at least half as high as the summit.

g. Measure the thickness of one layer of cardboard. Let this distance represent 100 m in elevation as well as 100 m along the *x*- and *y*-axes on your topographic map.

h. Use your map to identify an ordered pair (x,y) that represents the location of the summit.

i. Identify an ordered triple (x,y,z), where z represents elevation, that designates the location of the summit.

Discussion

a. Describe how the cliff, lake, saddle, and summit are represented on your topographic map.

b. Why does an ordered triple provide more information about the summit than an ordered pair?

c. Compare the method you used to find the thickness of one layer of cardboard with the methods used by others in your class.

d. Describe the z-coordinate of all the points that represent the surface of your lake.

e. Describe how to draw a path that provides a gradual ascent from the lake to the summit on your topographic map.

Assignment

1.1 Using your cardboard model from the exploration, locate a point on the top of the cliff. Locate another point at the bottom of the cliff, directly below the first one. Label the two points C_1 and C_2.

a. Identify an ordered triple for each point.

b. How do the coordinates of the ordered triples $C_1(x_1,y_1,z_1)$ and $C_2(x_2,y_2,z_2)$ compare?

c. What geometric figure is formed by the set of all points with coordinates (x,y,z) where the x- and y-coordinates are held constant? Explain your response.

d. Where are the points that correspond to ordered triples such as C_1 and C_2 located on a topographic map?

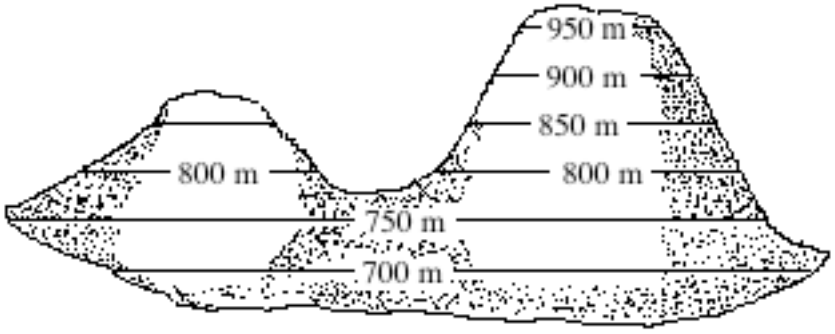
1.2 Using your cardboard model from the exploration, label two points L_1 and L_2 on opposite shores of the lake.

a. Identify an ordered triple for each point.

b. How do the coordinates of the ordered triples $L_1(x_1,y_1,z_1)$ and $L_2(x_2,y_2,z_2)$ compare?

c. What geometric figure is formed by the set of all points with coordinates (x,y,z) where the z-coordinate is held constant? Explain your response.

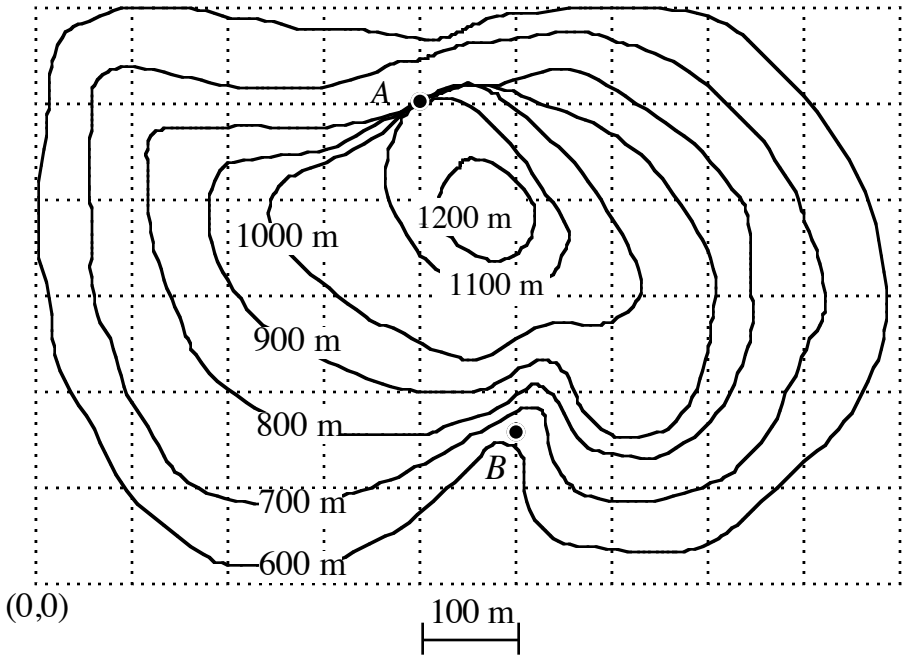
1.3 The diagram below shows the north side of a mountain. Draw a topographic map that might represent this terrain.



1.4 Use the topographic map below to complete Parts **a** and **b**.

a. Describe the features of the terrain at points *A* and *B*.

b. Identify the locations of points *A* and *B* using ordered triples.



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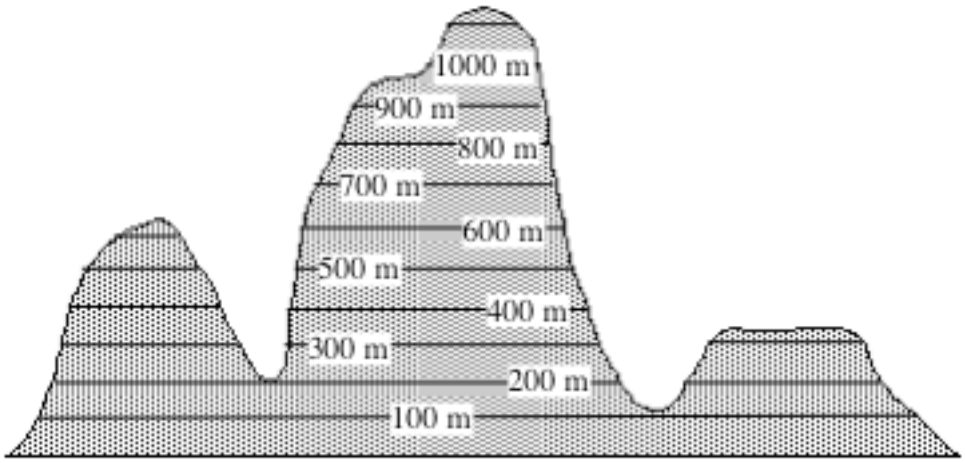
1.5 Create a topographic map for a right circular cone using at least five contour lines and your own grid system.

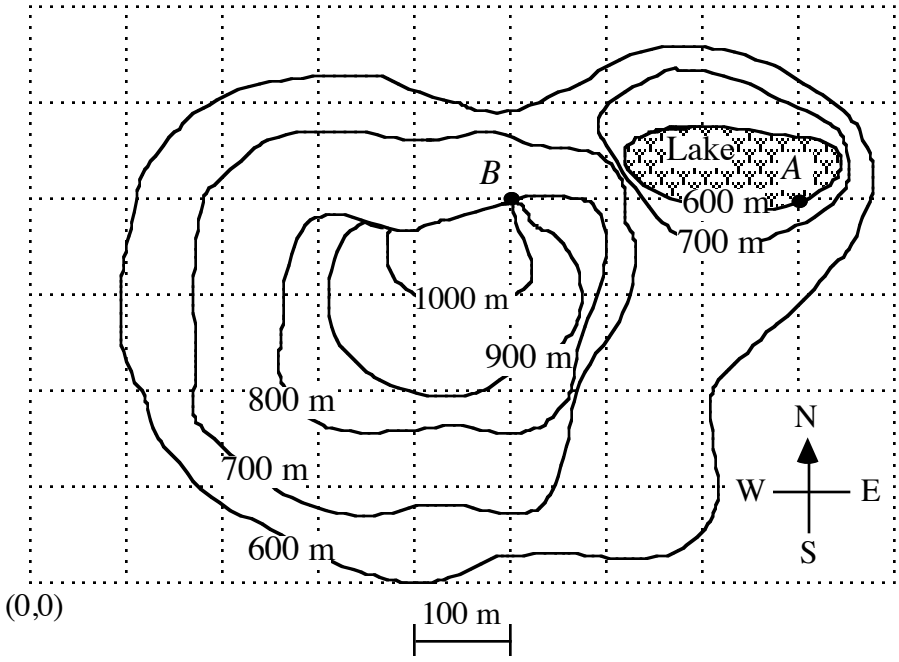
1.6 The diagram below shows the south side of a mountain range. Draw a topographic map that might represent this terrain.



1.7 Use the topographic map below to complete Parts **a** and **b**.

a. Describe the features of the terrain around points *A* and *B*.

b. Identify the locations of points *A* and *B* using ordered triples.



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Activity 2

In previous modules, you calculated distances in one or two dimensions. Finding the distance from the base of a mountain to its summit, however, could require the use of three-dimensional coordinates.

Exploration

In this exploration, you develop a method for finding the distance between any two points in a three-dimensional coordinate system.

Figure **8** shows a three-dimensional coordinate system and three points, P_1, P_2 and P_3. Each unit on the coordinate system represents 1 m. Each edge of the rectangular prism in Figure **8** either coincides with or is parallel to one of the axes.

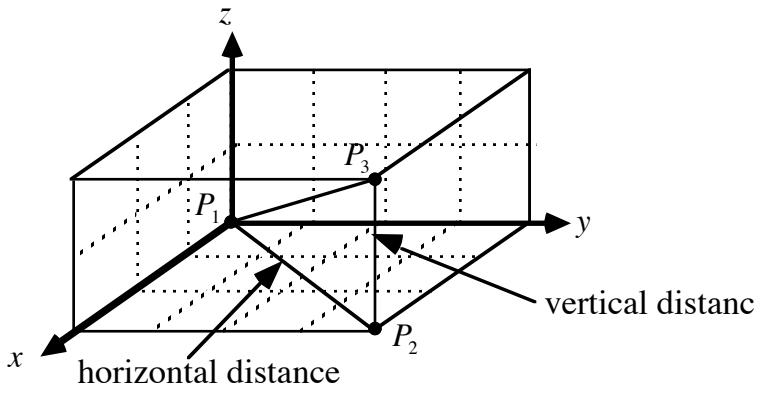


Figure 8: A three-dimensional coordinate system

a. One way to find the length of the segment with endpoints P_1 and P_3 is through the following steps.

1. Identify the coordinates of P_1, P_2, and P_3.

2. Find the distance between P_1 and P_2. This is the horizontal distance between P_1 and P_3.

3. Find the distance between P_2 and P_3. This is the vertical distance between P_1 and P_3.

4. Use the fact that triangle $P_1P_2P_3$ is a right triangle to find the distance between P_1 and P_3.

b. The three-dimensional coordinate system in Figure **9** shows two right triangles $P_1P_2P_3$ and $P_1P_3P_4$, with right angles at P_2 and P_4. The coordinates of P_3 are $(6,3,4)$, $\overline{P_2P_3}$ is parallel to the z-axis, and $\overline{P_3P_4}$ is parallel to the y-axis.

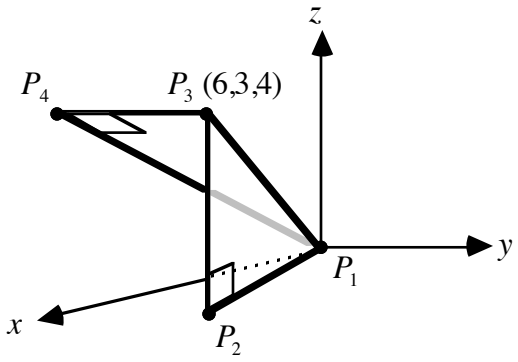


Figure 9: Two right triangles

1. Find the coordinates of P_2, a point in the xy-plane. Use these coordinates to find the distance from P_1 to P_3.

2. Find the coordinates of P_4, a point in the xz-plane. Use these coordinates to find the distance from P_1 to P_3.

3. Compare the distance found in Step **2** with the distance in Step **1**.

c. Draw and label a three-dimensional coordinate system.

1. Plot the two points with coordinates $(4,3,5)$ and $(0,2,8)$.

2. Find the distance between these two points. Hint: Draw right triangles.

d. The three-dimensional coordinate system in Figure **10** shows the locations of two points with coordinates (x_1,y_1,z_1) and (x_2,y_2,z_2). Find the distance between the two points.

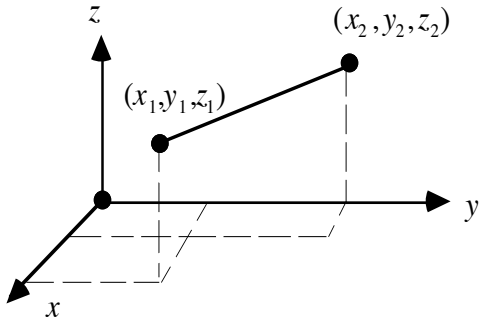


Figure 10: Two points

Discussion

a. How many times did you use the Pythagorean theorem to find the distance between P_1 and P_3 in Figure **8**?

b. How would you find the distance between any two points (x_1, y_1, z_1) and (x_2, y_2, z_2)? Express your method using a mathematical formula.

c. Describe how you could find the distance between points *A* and *B* on the topographic map in Figure **11**.

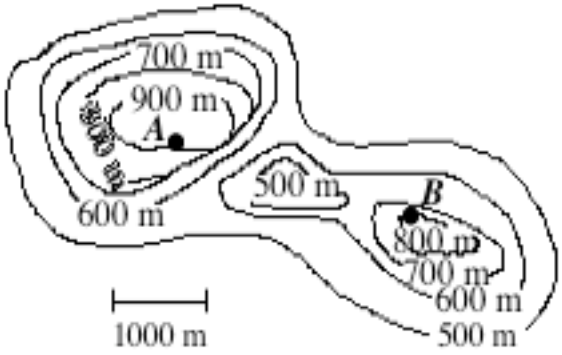


Figure 11: A topographic map

Assignment

2.1 Draw and label a three-dimensional coordinate system.

a. Plot the two points with coordinates $(5,0,2)$ and $(2,2,4)$.

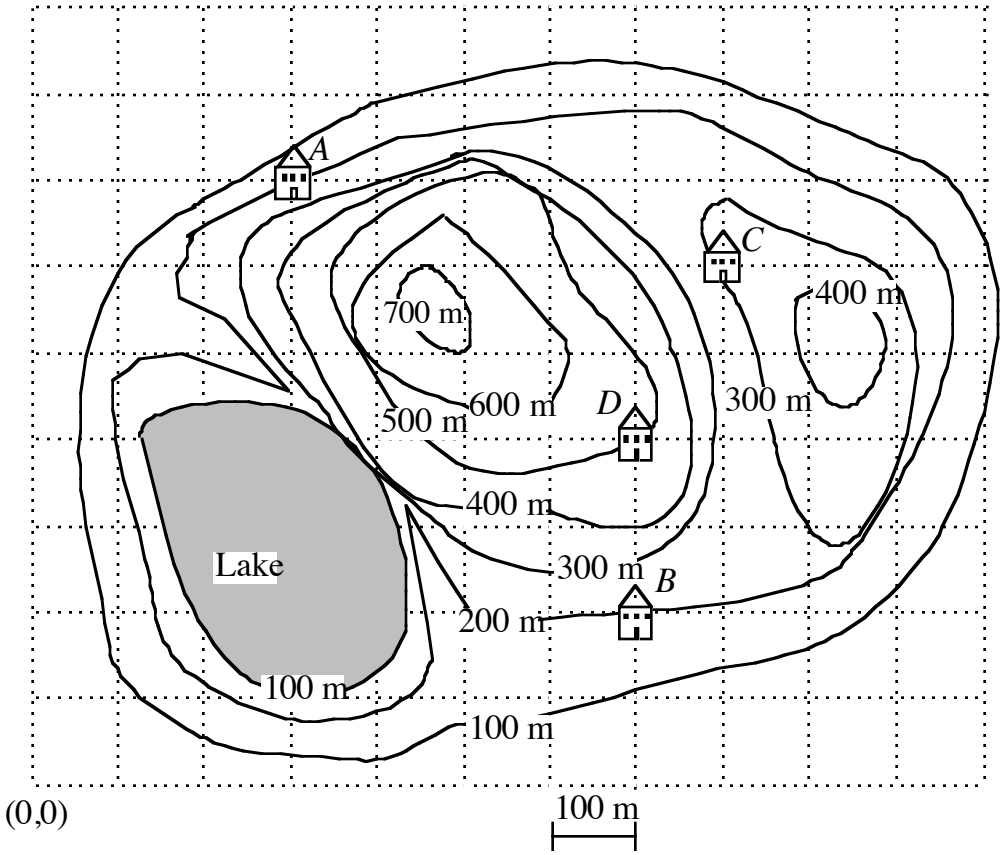
b. Find the distance between these points.

2.2 Draw and label a three-dimensional coordinate system.

a. Plot the two points with coordinates $(3,4,2)$ and $(5,4,0)$.

b. Find the distance between these points.

2.3 The figure below shows a topographic map of the terrain surrounding a lake, including four houses designated *A*, *B*, *C*, and *D*.



Use the map to find the distance between each of the following pairs of houses:

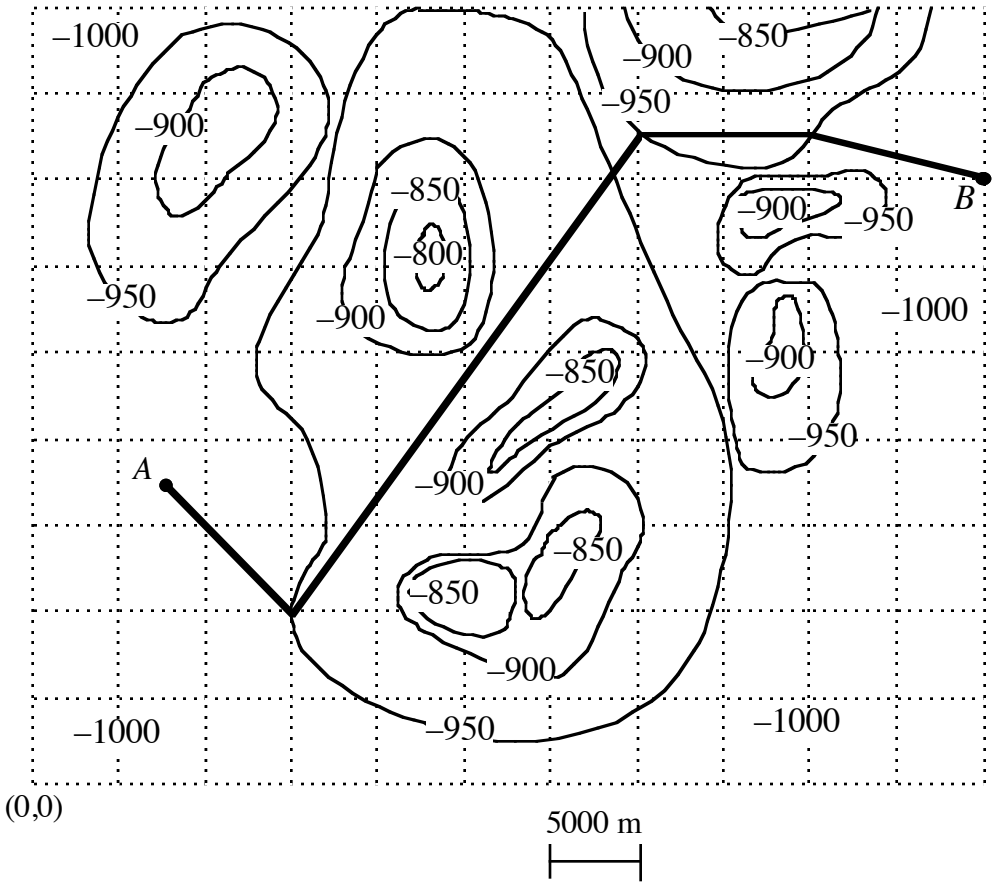
a. *A* and *B*

b. *B* and *C*

c. *C* and *D*

d. *A* and *D*.

2.4 The following diagram shows a topographic map of a region of the ocean floor. In this case, the contour lines represent distances below the surface of the ocean. For example, –850 indicates 850 m below sea level. The ocean floor for this region is 1000 m deep.



a. The captain of a submarine must navigate from point *A* to point *B* along the path shown. Assume the submarine is 10 m above the floor at each point along the path. What are the coordinates of point *A*, point *B*, and each point where the path turns?

b. The submarine must travel no closer than 10 m but no more than 25 m above the ocean terrain. What is the total distance the sub will travel from point *A* to point *B* along the given path?

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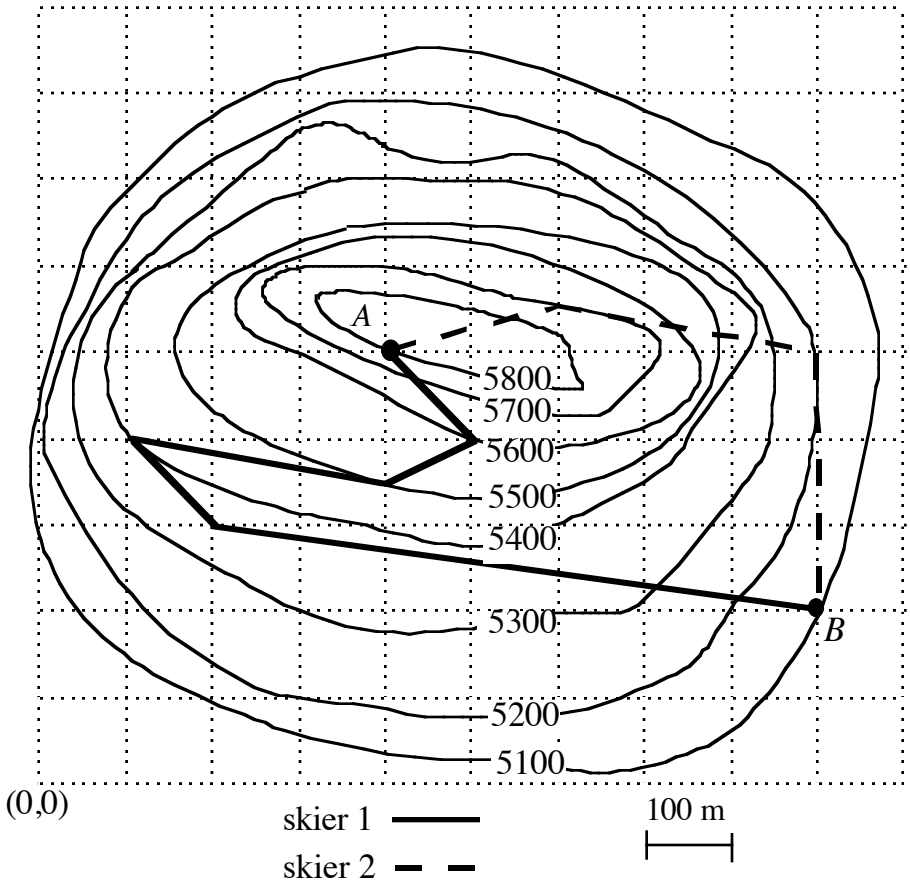
2.5 The topographic map below shows the routes taken by two skiers traveling down a mountain from point *A* to point *B*. Create a story for each skier's descent. In each story, include a description of the terrain and the approximate distance traveled. Assume that the terrain between contour lines is smooth, with no drops or rises other than those indicated on the map.



2.6 Use the map in Problem **2.5** to complete Parts **a–c** below.

a. **1.** Find the vertical distance between points *A* and *B*.

2. Find the horizontal distance between points *A* and *B*

3. Find the distance between *A* and *B* using the Pythagorean theorem.

b. **1.** Determine the coordinates of points *A* and *B*.

2. Find the distance between *A* and *B* using the coordinates of the points.

c. Compare your results in Parts **a** and **b**.

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Activity 3

Microwave signals are used in a variety of applications, including telephone transmissions. Since microwaves travel in a straight line and weaken considerably over distances greater than 5 km, engineers must place microwave repeaters along the transmission path to receive and relay signals.

In this activity, you use a topographic map to create a **profile** of mountainous terrain. Profiles can help provide a visual image of the landscape between two points on a topographic map.

Geography Note

A **profile** shows a vertical cross section or "side view" of the terrain. For example, Figure **12** shows a profile of a mountain valley.

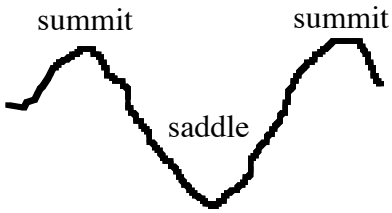


Figure 12: A profile of a mountain valley

Exploration

a. **1.** On a topographic map provided by your teacher, label two points at least 10 km apart and located on different contour lines.

2. As shown in Figure **13**, connect the points with a **profile segment**.

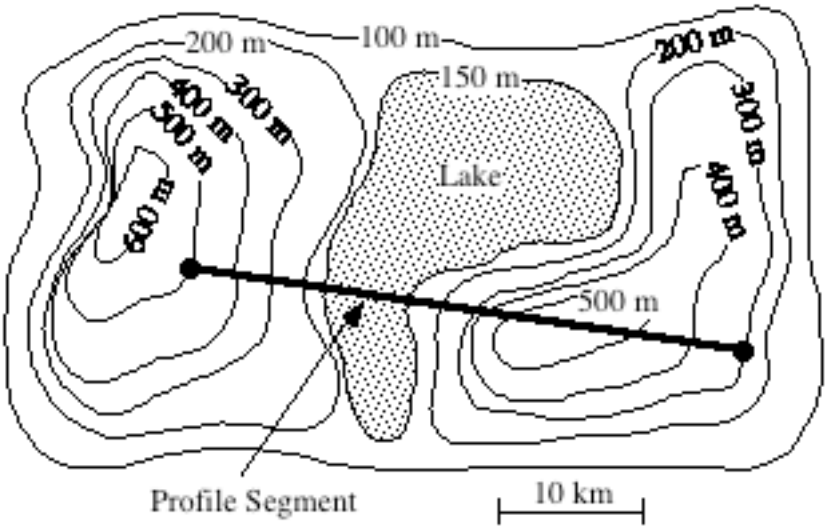


Figure 13: Profile segment drawn on a topographic map

b.

1. Label the top line of a sheet of notebook paper with the highest elevation crossed by the profile segment.
2. Determine the change in elevation between consecutive contour lines on the map. This change in elevation is the **contour interval** (*I*). In Figure **13**, for example, the contour interval is 100 m.
3. Label each successive line on the notebook paper with an elevation *I* units less than the line above it. Continue this process until a line has been labeled with the lowest elevation crossed by the profile segment.

c.

1. Place the top edge of your notebook paper along the profile segment, as shown in Figure **14**.
2. At each point where a contour line crosses the profile segment, draw a segment perpendicular to the profile segment. Extend each perpendicular segment to the line on the notebook paper that represents the same elevation as the corresponding contour line.
3. Mark the intersections of the perpendicular segments and their corresponding lines on the notebook paper.

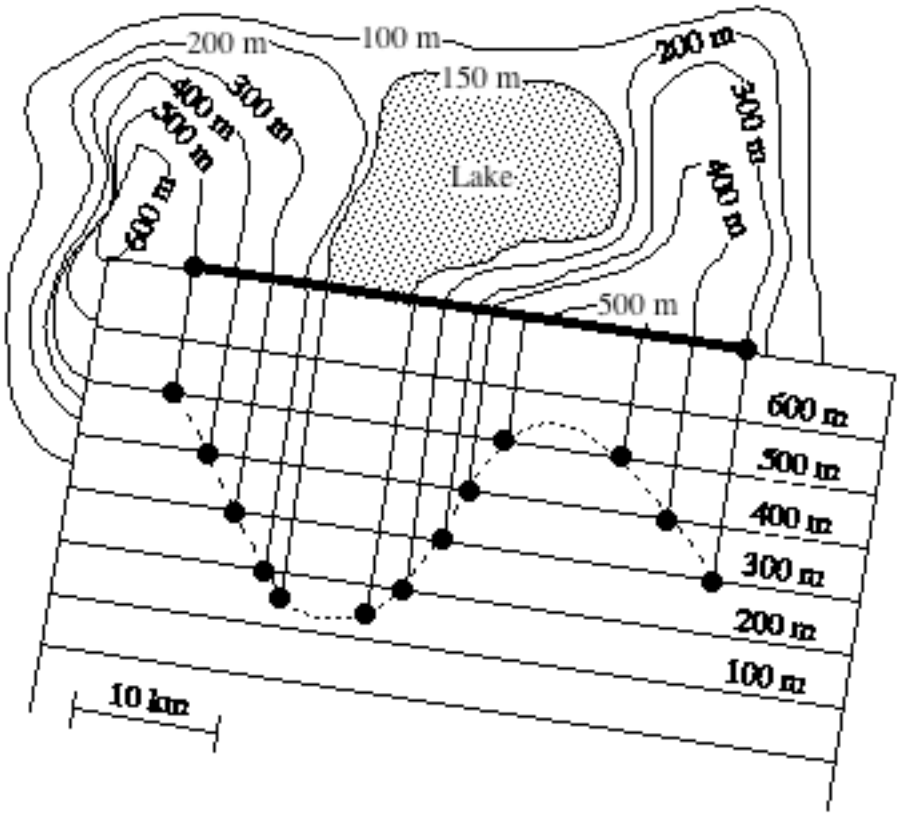


Figure 14: Creating a profile

d. Connect the points of intersection with a smooth curve. This curve represents a profile of the terrain between the two points you located on the map.

e. Compare the vertical scale on the profile with the horizontal scale. If they are different, make a sketch of the profile using the same vertical and horizontal scales.

f. Describe any similarities or differences you observe in the two profiles in Parts **d** and **e**.

Discussion

a. What information does a profile provide about a landscape?

b. Given a profile between two points, could you recreate the topographic map from which it was derived? Why or why not?

c. Microwaves travel in a straight line and the distance between repeaters must be less than 5 km along an unobstructed path.

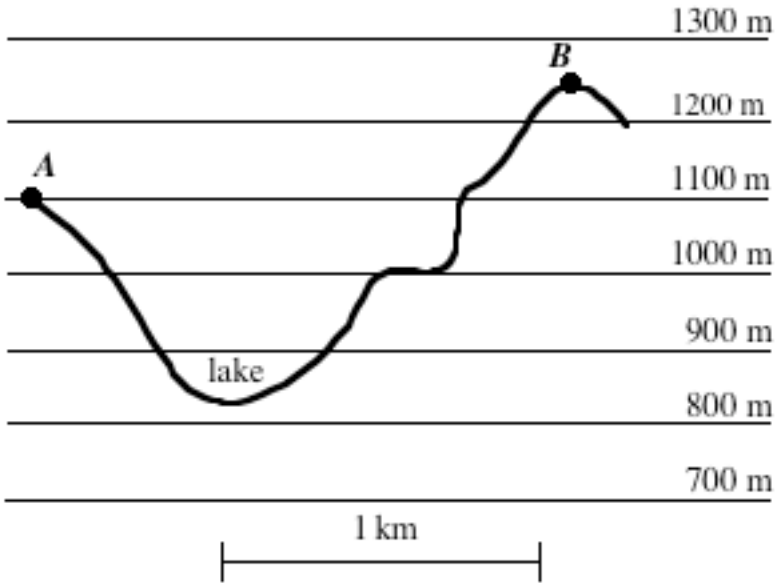
Imagine that the two points you located in the exploration are the sites of a microwave transmitter and receiver, respectively. Describe how you could use the profile to help locate appropriate sites for microwave repeaters.

d. Describe how the distance between two points on a topographic map can be approximated using a profile of the terrain.

e. Could you use a profile to determine the angle of elevation between two points (the angle formed by the segment joining the two points and a horizontal line)? Explain your response.

Assignment

3.1 The figure below shows a profile of a mountain landscape.



a. What is the elevation of the summit?

b. Approximately how tall is the cliff?

c. At what elevation is the lake?

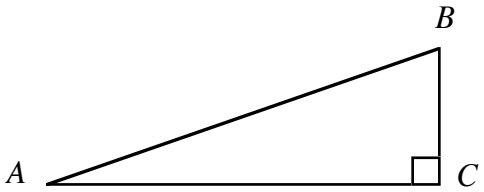
d. What is the distance from point *A* to point *B*?

e. **1.** Sketch a profile of this mountain on a coordinate grid, using the same scale for both the horizontal and vertical axes.

2. Use the ordered pairs that describe the locations of points *A* and *B* to verify the distance found in Part **d**.

3.2 Imagine that a microwave transmitter is located at point *A* on the profile in Problem **3.1**. A receiver is located at point *B*.

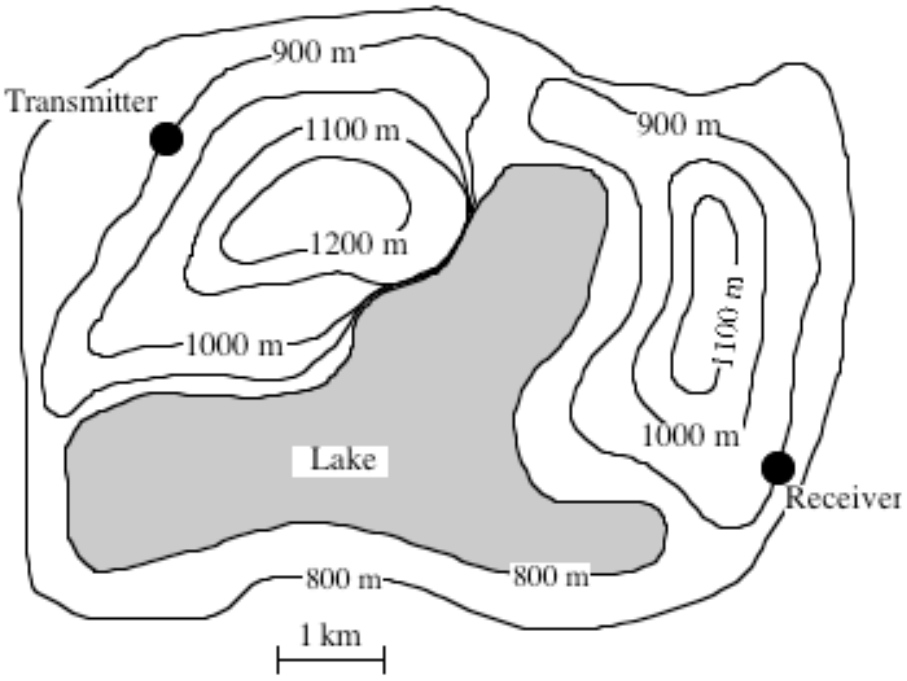
a. Using the distances found in Problem **3.1**, sketch a right triangle like the one in the diagram below to model this situation.



b. In the diagram, what trigonometric ratio is defined by BC/AB? Explain your response.

c. Using the trigonometric ratio you identified in Part **b**, determine the angle necessary to transmit a microwave signal directly from *A* to *B*.

3.3 The topographic map below shows the locations of a microwave transmitter and receiver. Both are built on towers 20 m above the ground.



a. In order to relay signals from the transmitter to the receiver, two microwave repeaters must be built between them. Identify possible locations for the repeaters. (Recall that microwaves travel in straight lines and require unobstructed paths no greater than 5 km long.)

b. Demonstrate that your proposed locations are adequate by:

1. drawing profiles of the terrain

2. finding the appropriate distances using ordered triples.

3.4 Microwave signals travel at the speed of light, about $3 \cdot 10^8$ m/sec. Determine the time required for a signal to travel from the transmitter to the receiver in Problem **3.3**.

3.5 The terrain represented by the topographic map in Problem **3.3** features two summits. Write a detailed paragraph describing a journey from one summit to the other. Include an estimate of the distance traveled and describe how you determined this distance.

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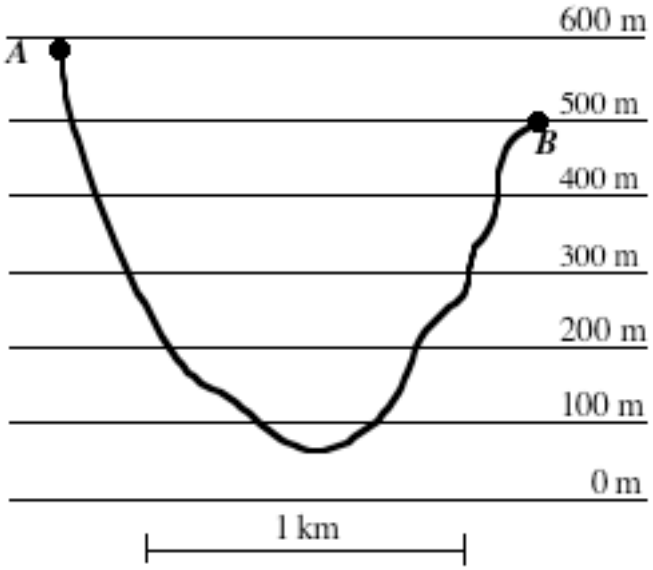
3.6 Create a topographic map that corresponds with the profile given in Problem **3.1**.

3.7 The figure below shows a profile of a valley.



a. What is the elevation of the lowest part of the valley shown in the profile?

b. What is the distance from point *A* to point *B*?

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Research Project

The U.S. Geological Survey (USGS) has published topographic maps of nearly the entire United States. Obtain a topographic map of a region near your school.

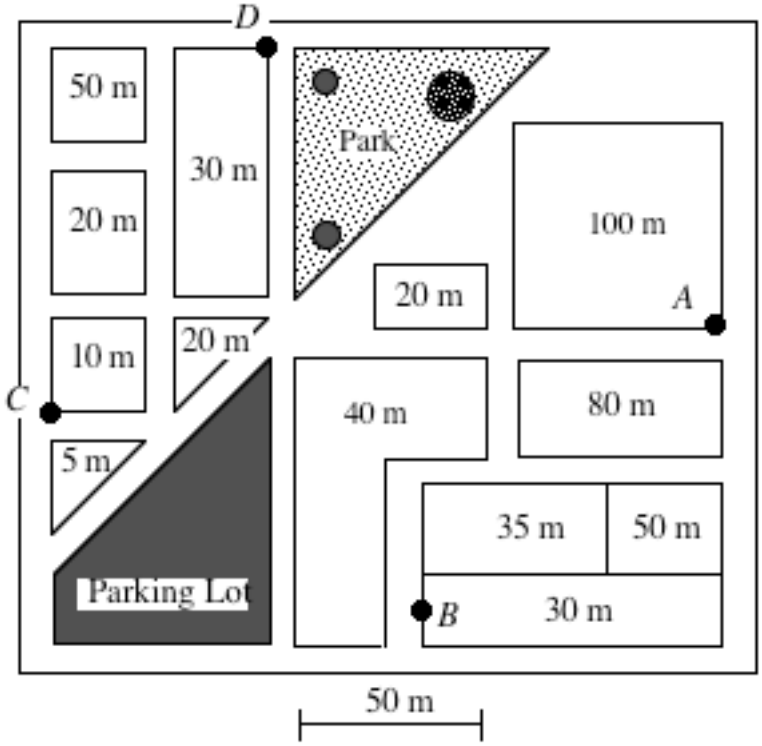
a. Identify several key features of the terrain. In a paragraph, describe how contour lines help characterize these features on the map.

b. Create a profile of the terrain.

c. Determine the straight-line distance and angle of elevation between the two highest points on the map.

Summary Assessment

The diagram below shows a map of a business district in a large city. Each polygon not otherwise labeled represents a building in the district. Each measure indicates the height above street level of the corresponding building. Use this map to complete Problems **1** and **2** below.



1. The local telephone company plans to build a microwave transmitter on the top of the building at point *A*. The transmitter will send signals to a receiver located on the top of a building at point *C*.

a. Determine whether or not there is an unobstructed line of sight between points *A* and *C* by creating a profile of the terrain.

b. Determine the distance between points *A* and *C* using the coordinates of ordered triples.

c. Determine the angle of elevation (measured from the horizontal) necessary to transmit the signal directly from point *A* to point *B*.

2. A cable television company is trying to determine the most cost-efficient way to relay signals from its headquarters at point *B* to a substation at point *D*. The company has two choices: burying a cable underground, or installing a system on the roofs of buildings in order to transmit signals through the air.

a. Burying a cable underground will cost $450 per meter. To minimize the disruption of traffic, the cable may not pass under more than three streets and must run alongside buildings.

1. Determine a possible route between points *B* and *D*.

2. Calculate the cost of this plan.

b. The transmitters required to send the signal through the air cost $35,000 each. The repeaters cost $15,000 each. The path between a transmitter and a repeaters must be unobstructed by other buildings. If a transmitter or repeaters must be located on the roof of a building not already owned by the company, the space must be purchased for $3000.

1. Use a profile to determine the number and location of the transmitters and repeaters necessary to relay signals between points *B* and *D*.

2. Calculate the cost of this plan.

c. Write a letter to the president of the company describing both options. Include appropriate maps and profiles, a summary of the costs of each plan, and a report of your recommendations.

Module Summary

- The lines that indicate elevation on a topographic map are **contour lines**.
- A **summit** is the highest point of the surrounding elevations.
- A **saddle** is a pass between two higher elevations.
- The coordinates of a point in a three-dimensional rectangular coordinate system are written as an **ordered triple**.
- The **origin** of a three-dimensional rectangular coordinate system is the point where the three axes intersect. The origin has coordinates $(0,0,0)$.
- The distance *d* between two points with coordinates (x_1, y_1, z_1) and (x_2, y_2, z_2) can be found using the following formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

- A **profile** shows a vertical cross section or "side view" of terrain.

Selected References

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