Banking on Life

The price you pay for borrowing money—as well as the amount you earn by saving it—should be of interest to you. In this module, you learn how financial institutions calculate the interest they pay on savings accounts and the interest they charge on consumer loans.

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Introduction

In order to pay for life's major expenses, you have two options: save money or borrow it. If you decide to save money now to make purchases later, your investments may earn interest. If you decide to borrow money now and make payments later, you will owe interest on your loans. In this module, you explore the mathematics of investing and lending.

Activity 1

Principal is the amount of money invested or borrowed. **Interest** is the amount earned on invested money or the amount paid for borrowed money. The amount of interest earned or paid depends on three quantities: principal, interest rate, and time. Interest also varies according to the method used to calculate it. In this activity, you begin to investigate how savings accounts earn money.

Mathematics Note

When you invest money, one method for calculating interest involves **simple interest**. In this case, interest is paid only on the original principal. The formula for calculating simple interest I, where P represents the principal, r represents the interest rate, and t represents time, is:

$$I = Prt$$

To use this formula, t must be expressed in the same units as time in the interest rate r. For example, if the interest rate is 3% per year, then t also must be expressed in years. If \$1000 is invested at an interest rate of 3% per year for 10 yr, the interest earned can be calculated as follows:

$$I = \$1000(0.03)(10) = \$300$$

Exploration 1

Suppose that two new parents begin saving for their child's education on the day of birth. They invest \$1000 earning simple interest at an annual rate of 4%. The interest on this investment is paid once a year, on the child's birthday. In the following exploration, you investigate the income earned from this investment each year and the cumulative value of the original investment plus interest.

a. Calculate the interest earned on \$1000 in 1 yr at an annual interest rate of 4%.

b. Determine the amount of interest earned each year, the total interest earned, and the principal plus total interest for the investment in each of the next 18 years. Record this information in a copy of Table **1** below.

Table 1: Interest by year

Year	Principal	Annual Interest	Total Interest	Principal plus Total Interest
1	\$1000.00			
2	\$1000.00			
3	\$1000.00			
⋮	⋮			
18	\$1000.00			

c. On the same set of axes, create a scatterplot of each of the following:

1. total interest versus time

2. principal plus total interest versus time.

d. Determine an equation to model each scatterplot in Part **c**.

Discussion 1

a. Compare the two scatterplots you created in Part **c** of Exploration **1**.

b. What type of function seems to best model the scatterplots? Explain your response.

c. How do the equations found in Part **d** of Exploration **1** describe the differences in the graphs?

d. What do the slope and *y*-intercept of each graph represent in terms of the investment?

e. The term *account balance* sometimes refers to the total of the principal and the interest earned. When interest is paid only on the original principal using simple interest, the balance B_t of the investment after *t* years can be calculated using the explicit formula below:

$$\begin{aligned} B_t &= P + I \\ &= P + Prt \\ &= P(1 + rt) \end{aligned}$$

where *P* represents the principal, *r* represents the annual interest rate, and *t* represents time in years.

1. Explain why the three expressions given for B_t are equivalent.

2. Use the explicit formula for B_t to determine the balance of the investment after 18 yr. How does the balance compare to the sum of the principal and the total interest after 18 yr?

Exploration 2

If the simple interest earned in Exploration **1** had been added to the original principal each year and no money removed from the investment, most banks would have paid interest on each year's balance. This is **compound interest**. When interest is compounded, the interest earned for each period is calculated using the balance from the previous period. In other words, the balance from one period becomes the principal for the next period.

a. Suppose that the new parents described in Exploration **1** deposited \$1000 in a savings account with an interest rate of 4% per year, compounded annually.

Complete Table **2** for years 3–18. Notice that in this account the balance at the end of each year becomes the principal for the following year.

Table 2: Value of account using compound interest

Year	Principal (*P*)	Interest (*Prt*)	Total Interest	Balance (*P* + *Prt*)
1	\$1000.00	\$40.00	\$40.00	\$1040.00
2	\$1040.00	\$41.60	\$81.60	\$1081.60
3	\$1081.60			
⋮				
18				

b. Write a recursive formula that describes the balance B_t at the end of any year.

c. Create a scatterplot of the account balance versus the year.

d. One way to find an equation that models the scatterplot in Part **c** is to determine an explicit formula for the sequence of values for the account balance.

1. Write an expression for the balance (B_1) at the end of year 1 in terms of the original principal (P_0) and the annual interest rate (r).
2. Write an expression for the balance (B_2) at the end of year 2 in terms of B_1 and r.
3. Substitute the expression for B_1 from Step **1** into the expression you wrote in Step **2**. Rewrite the expression using an exponent.
4. Repeat the process described in Steps **2** and **3** for the balance in years 3, 4, and 5.

e. Generalize the formula found in Part **d** for any balance B_t after t years in terms of P_0, r, and t. Test your formula using the information in Table **2**.

Discussion 2

a. Describe how the difference between simple interest and compound interest affects the value of an $1000 investment after 18 yr.

b. How is the formula for simple interest used in calculating compound interest?

c. How could you determine the interest earned on an investment given the original principal and the final balance?

d. Imagine that you must save $1000 in 5 yr. Your savings account pays 5% interest per year, compounded annually. Describe how you could use a symbolic manipulator to determine the initial deposit you should make.

Assignment

1.1 In Exploration **1**, the child's parents invested $1000 earning simple interest at a rate of 4% per year.

- **a.** Determine the total amount of interest earned after 30 yr.
- **b**. How many years would it take for the sum of the principal and the total interest earned to reach $5000?
- **c.** How much would the parents have had to invest to produce a balance of $15,000 (principal and interest) after 18 yr?

1.2 In Exploration **2**, the child's parents deposited $1000 in a savings account with an interest rate of 4% per year, compounded annually.

- **a.** Assuming that they make no withdrawals, determine the total amount of interest earned in this account after 30 yr.
- **b**. How many years would it take for the account balance to reach $5000 if no withdrawals are made?
- **c.** **1.** How much would the parents have had to deposit to produce a balance of $15,000 after 18 yr?
  - **2.** How does this amount compare with your response to Problem **1.1c**?

1.3 Consider the investment account described in Exploration **1**, earning simple interest at a rate of 4% per year. On the child's 10th birthday, an aunt adds $500 to the principal in the account.

- **a.** Considering the original investment of $1000 and all previous interest earned, what is the sum of the principal and interest after 18 yr?
- **b.** Create a scatterplot that shows the effect of this deposit on the sum of the principal and the interest over time. Describe how the graph shows this effect.

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1.4 A local department store is going out of business. In order to attract customers, the owners decide to discount everything left in the store 15% each week until all merchandise is sold.

a. If an item regularly sells for \$150, what will its price be during the third week of the sale?

b. During what week will the sale price of this item fall below \$10?

c. Some of the store's customers think that, if they wait long enough, the items they want will be free. Is this true? Justify your response.

1.5 A small museum has 17,000 items in its collection. Of these items, 2000 are on permanent display. The remainder of the collection is placed on display on a rotating basis, 12% every six months. Using this system, how many years will pass before museum visitors have had the opportunity to see the entire collection? Explain your response.

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Activity 2

In most savings accounts, interest is compounded more than once per year. To calculate the balance in such cases, the interest rate must be adjusted to correspond with the compounding period. This adjusted rate is referred to as the **interest rate per compounding period** and equals r/n, where r is the annual interest rate and n is the number of times interest is compounded per year.

For example, if the annual interest rate is 8% and the interest is compounded quarterly (4 times per year), then the interest rate per quarter is $8\%/4 = 2\%$. In this activity, you explore how increasing the number of compoundings affects the account balance over time.

Exploration

a. Imagine that the parents described in Activity **1** had deposited \$1000 in a savings account at an annual interest rate of 4%, compounded monthly. What is the interest rate per compounding period for this account?

b. Assuming that they make no withdrawals, determine the account balance at the end of each month for the next 18 yr.

c. Create a scatterplot of account balance versus time in months.

d. Determine an explicit formula that calculates the balance B_t after t years in terms of the original principal (P_0), the interest rate per compounding period (r/n), and the number (n) of times interest is compounded per year. Use the data from Part **b** to test the formula.

e. On the same graph, plot the equation found in Part **d** using an original principal of \$1000, an annual interest rate of 4%, and each of the following numbers of compoundings per year: 5, 10, 100, and 1000.

f. As the number of compoundings increases without bound, the balance after a given time period approaches a limiting value. To investigate what happens when the number of compounding periods becomes very large, consider an investment of \$1.00 at an annual interest rate of 100%, compounded *n* times per year for 1 yr.

Create a spreadsheet with columns similar to those in Table **3**. Use the formula found in Part **d** to complete the spreadsheet.

Table 3: Account balances for different compoundings

No. of Compoundings per Year (*n*)	Balance at End of Year (in dollars)
1	
10	
100	
1000	
10,000	
100,000	
1,000,000	
10,000,000	
100,000,000	

g. The values in the right-hand column of Table **3** can be thought of as the terms of a sequence. As the number of compoundings per year increases, what limit does this sequence appear to approach?

Mathematics Note

The limit of the following expression, as *n* approaches infinity, is an irrational number approximately equal to 2.71828:

$$\left(1+\frac{1}{n}\right)^n$$

This irrational number is represented as ***e***, in honor of Swiss mathematician Leonhard Euler.

h. For an initial principal of \$1.00 and a period of 1 yr, the formula for account balance when interest is compounded *n* times a year becomes:

$$B_1=\left(1+\frac{r}{n}\right)^n$$

To investigate how a change in interest rate affects the balance at the end of the year, create and complete a spreadsheet with columns like those in Table **4** below.

Table 4: Balance in dollars for different interest rates

No. of Compoundings per Year (n)	$B_1 = \left(1 + \frac{1}{n}\right)^n$	$B_1 = \left(1 + \frac{2}{n}\right)^n$	$B_1 = \left(1 + \frac{3}{n}\right)^n$
1			
10			
100			
1000			
10,000			
100,000			
1,000,000			
10,000,000			
100,000,000			

i. **1.** Calculate e^2. Compare it to the values in the spreadsheet in Part **h**.

2. Calculate e^3. Compare it to the values in the spreadsheet in Part **h**.

Discussion

a. When interest is compounded annually, an investment of $1000 at an annual interest rate of 4% for 18 years results in a balance of $2025.82. How does this compare with the account balance when interest is compounded monthly?

b. **1.** Describe how the graphs of the four equations in Part **e** of the exploration are related.

2. What do you think a graph of the corresponding equation would look like if interest were compounded 1,000,000 time per year or if interest were compounded continuously?

c. Considering your responses to Part **i** of the exploration, predict the relationship between e^r and the following equation as n becomes extremely large.

$$B_1 = \left(1 + \frac{r}{n}\right)^n$$

d. Using a calculator, a student evaluated the equation in Part **c** above for $n = 10^{14}$. The calculator reported that $B_1 = 1$. Why did this occur?

e. How could you use your response to Part **c** of the discussion, along with the formula from Part **d** of the exploration, to write a formula for account balance when interest is compounded continuously?

Assignment

2.1 **a.** Imagine that you have invested \$1000 at an annual interest rate of 5% and made no withdrawals. Determine the balance after 10 yr when interest is compounded:

1. every year
2. every month
3. every day
4. every hour
5. every second

b. Repeat Part **a** for the balance after 20 yr.

c. How does increasing the number of compoundings per year affect the account balance?

2.2 What has more impact on the balance of a savings account from which the owner makes no withdrawals: the annual interest rate or the number of compoundings per year? Justify your response.

2.3 To help consumers compare earnings among accounts with different compounding periods, the federal government requires financial institutions to report the **annual percentage yield** or **APY** for all accounts that earn interest. Annual percentage yield is the interest rate that the account would earn if interest were compounded annually.

a. Consider an investment of \$1000 at an annual interest rate of 3%, compounded quarterly. Calculate the account balance after 1 yr if no withdrawals are made.

b. To find the annual percentage yield for this account, you can solve the formula for compound interest for r

$$B_t = P\left(1 + \frac{r}{n}\right)^{nt}$$

where B_t equals the value determined in Part **a**, $P = 1000$, $n = 1$, and $t = 1$. Determine the APY for the 3% annual interest rate when it is compounded quarterly.

c. Determine the APY for a savings account with an annual interest rate of 5%, compounded monthly.

2.4 **a.** After 1 year, will the balance of an account be significantly more if interest is compounded every hour rather than every day? Use an example to support your response.

b. In general, what effect does increasing the number of compoundings per year have on the account balance?

2.5 For small values of n, how do the values of e^r and the following expression compare?

$$\left(1+\frac{r}{n}\right)^n$$

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2.6 One general equation used to model the growth or decay in a quantity is $N_t = N_0 e^{nt}$, where N_t represents the final amount, N_0 represents the initial amount, n represents some constant, and t represents time. When $n > 0$, the equation can be used to model growth; when $n < 0$, the equation can be used to model decay.

A population of bacteria has a constant n of 0.538 when t is measured in days. How many days will it take an initial population of 8 bacteria to increase to 320?

2.7 A population of bacteria grows at a rate of 5% per hour. If the initial population is 100, what is the population after 10 hr?

2.8 A population of song birds is decreasing at a rate of 8% per year. If this trend continues, how many years will it take for the current population of 1000 birds to reach 100 birds?

2.9 The number of employees at Sky High, a local parachute company, has increased exponentially in the past three years, from 25 to 115. What was the annual growth rate for the company during this time?

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Activity 3

To make large purchases such as a car or a home, many people use a portion of their savings as a down payment, then borrow the rest. In this activity, you investigate the methods banks use to calculate the monthly payments on loans.

Business Note

Loans are generally repaid over a period of time by making regular monthly payments, or **installments**. **Amortization** is the process of repaying a loan by installments. Each month, the borrower pays the interest on the unpaid balance of the loan as well as a portion of the principal.

The **annual percentage rate** or **APR** is used to calculate the interest paid on an installment loan. (Note that APR refers to the interest rate paid on loans while APY refers to the interest rate earned on savings accounts.)

Exploration

A high school senior has approximately $2000 in a savings account. The student decides to use $1000 for a down payment on a used car and $1000 for the first semester of college. Since the car costs $5000, the student must borrow an additional $4000.

a. A local credit union offers car loans at an APR of 9% with 24 monthly installments. Estimate the size of the monthly payment.

b.

1. Part of each monthly installment pays the interest on the balance of the loan. Determine the interest for 1 month on a principal of $4000 at an APR of 9%. This is the interest due for the first month of the loan.
2. The difference between the monthly installment and the interest due is applied to the principal. Use the estimated payment from Part **a** to calculate the amount the principal will be reduced after the first month.
3. The balance of the loan remaining is the difference of the previous balance and the amount applied to the principal. Determine the balance of the loan remaining after one installment. This balance is then used to continue the process.

c. Create a spreadsheet with headings like those in Table **5** below. Use the spreadsheet to determine the size of the monthly installment necessary to pay off the loan in 24 months.

Note: Write the formulas for individual cells so that when you change the estimated payment, the APR, or the amount borrowed, the spreadsheet will automatically update the remaining cells.

Table 5: Monthly payment for car loan

Amount Borrowed	**APR**	**Estimated Payment**	
4000	0.09		
Payment No.	**Loan Balance after Payment**	**Interest Payment**	**Principal Payment**
	4000		
1			
2			
3			
⋮			
24			

d. The amount of interest paid over the life of the loan is referred to as the **finance charge**.

1. Determine how much the student will pay in finance charges on a loan of \$4000 at an APR of 9% for 24 months.
2. Including the down payment, find the total costs for this car.

e. With the salary from a part-time job, the student can afford a monthly car payment of no more than \$100. Use the spreadsheet from Part **c** to determine the maximum amount of money the student can borrow for 24 months at 9%.

f. After shopping for a less expensive model, the student decided to buy the \$5000 car anyway. Using a monthly payment of \$100, how many months will it take to repay a \$4000 loan at an APR of 9%?

Discussion

a. How did you use the formula for simple interest in Part **b** of the exploration?

b. How much interest would the student pay over 24 months on a loan of \$4000 at an APR of 9%?

c. What can the student do to reduce the size of the monthly payments?

d. How does increasing the time to repay a loan affect the finance charges?

Mathematics Note

The **monthly payment** M for an installment loan can be calculated using the explicit formula below, where r is the monthly interest rate $(\text{APR}/12)$, P is the principal of the loan, and t is the time in years:

$$M = P\left[\frac{r}{1-\left(\frac{1}{1+r}\right)^{12t}}\right]$$

For example, the monthly payment for a \$10,000 loan at an APR of 8% over 4 yr can be calculated as follows:

$$M = \$10{,}000\left[\frac{\frac{0.08}{12}}{1-\left(\frac{1}{1+\frac{0.08}{12}}\right)^{12 \cdot 4}}\right] = \$244.13$$

The balance B_n remaining after n monthly payments, where r is the monthly interest rate, can be found using the following recursive formula:

$$B_n = B_{n-1} + rB_{n-1} - M, n > 1$$

or by using the explicit formula below:

$$B_n = P(1+r)^n - M\left[\frac{(1+r)^n - 1}{r}\right]$$

Using the example given above, the balance remaining after the first month's payment can be calculated recursively as shown below:

$$B_1 = \$10,000 + \frac{0.08}{12} \bullet \$10,000 - \$244.13 = \$9822.54$$

Using the explicit formula, the balance remaining after 18 monthly payments can be calculated as follows:

$$B_{18} = \$10,000\left(1 + \frac{0.08}{12}\right)^{18} - \$244.13\left[\frac{\left(1 + \frac{0.08}{12}\right)^{18} - 1}{\frac{0.08}{12}}\right] = \$6618.05$$

e. How does the recursive formula for the balance remaining on a loan compare to the recursive formula for compound interest below?

$$B_t = B_{t-1} + rB_{t-1}, t > 1$$

Assignment

3.1 Since the costs of buying a home are so high, many people finance their purchases over a period of 30 yr.

a. What is the monthly payment on a \$50,000 loan for 30 yr at an APR of 8%?

b. Determine the total amount of interest paid on the loan in Part **a**.

c. Calculate the balance remaining after 5 yr.

3.2 A typical home loan can be repaid over 15, 20, or 30 years. Imagine that you could afford a monthly house payment of \$450.

a. How much money could you borrow for 30 yr at an APR of 8%?

b. How much could you borrow for 15 yr at an APR of 8%?

c. How much could you borrow for 30 yr at an APR of 16%?

3.3 **a.** Lisa plans to borrow $70,000 to purchase a home. If she can afford a $500 monthly payment, how long will it take her to repay an installment loan at an APR of 8%?

b. Banks usually limit the repayment period to 30 yr. How much would Lisa's monthly payment have to be in order to repay the loan in 30 yr?

c. Lisa can reduce the size of the loan by increasing the size of her down payment. How much money should Lisa use as a down payment in order to reduce her monthly payment to $500?

3.4 The Jakes family wants to borrow $45,000 for 20 yr. After considering their other expenses, they decide that they can make monthly payments of no more than $350. What annual percentage rate (to the nearest 0.1%) do the Jakes need to be able to afford the loan?

3.5 Imagine that you plan to borrow $65,000 at an APR of 7.5%. The bank will allow you to borrow the money either for 15 yr or for 30 yr.

a. Calculate the monthly payment for each loan.

b. Compare the interest paid on each loan.

c. Determine the amount of money you will still owe after 15 yr if you take the 30-yr loan.

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3.6 **a.** Wynette plans to buy a new car that costs $22,000. Her bank will loan her up to 80% of the purchase price. What is the maximum amount of money she can borrow for the car?

b. The APR for a 36-month loan is 8%. If Wynette borrows the maximum amount, determine:

1. the monthly payment

2. the total interest cost

3. the total cost of the car.

c. The APR for a 60-month loan is 9.5%. Repeat Part **b** for this loan option.

3.7 One loan option available at some financial institutions involves a **balloon payment**. In this type of loan, the borrower pays relatively small installments for a certain time, then pays the balance remaining at the end of the loan in one large payment (the balloon).

For example, the monthly payments on a loan of $5000 for 2 yr at an APR of 8% are $226.14. In a loan involving a balloon payment, the consumer might pay only $150 a month for 2 yr, then pay the remaining balance in one payment at the end of the 2 yr.

a. Imagine that you have borrowed $5000 for 2 yr at an APR of 8%. If you make monthly payments of $150 on this loan for 2 yr, what will be the value of your balloon payment?

b. Compare the finance charges on the balloon-payment loan in Part **a** to the finance charges on a traditional installment loan for the same amount. Explain any differences you observe.

c. One drawback to a loan with a balloon payment is the large unpaid balance at the end of the repayment period. Describe some situations in which you think this type of loan might be useful to a borrower, despite this drawback.

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Research Project

Compare the interest rates and compounding periods for savings accounts at several financial institutions in your community or region. Also compare the interest rates and repayment times offered for loans on both new and used cars. Use examples to illustrate the long-term effects of any differences you observe.

Summary Assessment

1. Whenever consumers use credit cards to make purchases, they are borrowing money. If the account is not paid in full each month, the credit-card company charges interest on any balance remaining.

Since the balance in a credit-card account changes whenever a payment is made or a purchase occurs, credit-card companies typically use the **average daily balance** to calculate interest charges. The average daily balance is determined by recording the account balance at the beginning of each day in a month, totaling these daily balances, and then dividing the total by the number of days in the month.

Calculate the average daily balance for the account described in the table below. Assume that each purchase or payment does not affect the average daily balance until the day after it occurs.

Balance at Beginning of Month: $250			
Month	**Day**	**Purchases ($)**	**Payments ($)**
Sept.	4	15.00	
Sept.	5	150.00	
Sept.	7	42.00	
Sept.	8		50.00
Sept.	11	50.00	
Sept.	21	58.00	
Sept.	22	25.00	
Sept.	23	37.00	
Sept.	24	98.00	
Sept.	29	125.00	

2. The APR for the account in Problem **1** is 18%. Use the average daily balance to calculate the finance charge on the account for the month of September.

3. Many banks do not use the amortization formula to calculate the monthly payment for credit-card balances. Instead, the minimum payment is 2% of the balance due or $50, whichever is greater. In this case, the balance due is the average daily balance plus the finance charges for the month.

a. Calculate the minimum payment due on the account in Problem **1**.

b. Calculate the balance due necessary for the minimum payment to be greater than $50.

4. When the balance due on a credit-card account is higher than the amount calculated in Problem **3b**, the minimum monthly payment is calculated using the 2% rule. After the balance due decreases to the amount calculated in Problem **3b**, the minimum monthly payment is \$50. Use this method of repaying the loan to complete the following.

a. If no other purchases are made, how many years would it take to pay off a credit-card balance of \$7500 with an APR of 18%?

b. What is the total interest paid while repaying \$7500 in this manner?

5. If a credit-card company used the amortization formula to determine monthly payments, the payments would be the same each month. Recall that the monthly payment M for an installment loan can be calculated using the explicit formula below, where r is the monthly interest rate $(\text{APR}/12)$, P is the principal, and t is the time in years:

$$M = P\left[\frac{r}{1-\left(\frac{1}{1+r}\right)^{12t}}\right]$$

a. Using 2% of \$7500 as the monthly payment, how many years would it take to pay off a credit-card balance of \$7500 with an APR of 18%?

b. What is the total interest paid while repaying \$7500 in this manner?

c. Compare the finance charges in Problems **4** and **5**. Which method would you rather use to pay off a loan?

Module Summary

- **Principal** is the amount of money invested or borrowed.
- **Interest** is the amount earned on invested money or the amount paid for borrowing money.
- **Simple interest** is paid only on the original principal. The formula for calculating simple interest *I*, where *P* represents the principal, *r* represents the interest rate, and *t* represents time, is:

  $$I = Prt$$

  When interest is **compounded**, the interest earned for each period is calculated using the balance from the previous period. In other words, the balance from one period becomes the principal for the next period.
- The **interest rate per compounding period** equals r/n, where *r* is the annual interest rate and *n* is the number of times interest is compounded per year.
- When interest is compounded, the balance B_t after *t* years can be calculated using the following explicit formula, where *P* is the original principal, *r* is the annual interest rate, and *n* is the number of compoundings per year:

  $$B_t = P\left(1 + \frac{r}{n}\right)^{nt}$$
- The limit of the following expression, as *n* approaches infinity, is an irrational number approximately equal to 2.71828:

  $$\left(1 + \frac{1}{n}\right)^{n}$$

  This irrational number is represented as ***e***, in honor of Swiss mathematician Leonhard Euler.
- The **annual percentage yield** or **APY** is the interest rate that an account would earn if interest were compounded annually.
- Loans are generally repaid over a period of time by making regular monthly payments or **installments**. **Amortization** is the process of repaying a loan by installments. Each month, the borrower pays the interest on the unpaid balance of the loan as well as a portion of the principal.
- The **annual percentage rate** or **APR** is used to calculate the interest paid on an installment loan.
- The amount of interest paid over the life of the loan is referred to as the **finance charge**.

- The **monthly payment** M for an installment loan can be calculated using the explicit formula below, where P is the principal of the loan, r is the monthly interest rate $(\text{APR}/12)$, and t is the time in years:

$$M = P\left[\frac{r}{1-\left(\frac{1}{1+r}\right)^{12t}}\right]$$

- The balance B_n remaining after n monthly payments on an installment loan can be found using the following explicit formula, where P is the principal, r is the monthly interest rate $(\text{APR}/12)$, and M is the monthly payment:

$$B_n = P(1+r)^n - M\left[\frac{(1+r)^n - 1}{r}\right]$$

Selected References

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