What's Your Bearing?

In this module, you'll explore how surveyors locate property lines, find the lengths of boundaries, and calculate the area of land parcels.

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Introduction

For anyone hiking in unfamiliar territory, a good compass can be just as essential as a good pair of shoes. Combined with a map, a compass allows you to determine not only your direction of travel, but also your location. A compass is an important tool in many professions—from forestry to ocean navigation.

Science Note

An **azimuth** can be described as the angle formed by rotating a ray clockwise from the ray representing north. In Figure **1**, for example, the ray from point *A* to point *B* has an azimuth of 60°. The ray from point *B* to point *C* has an azimuth of 120°.

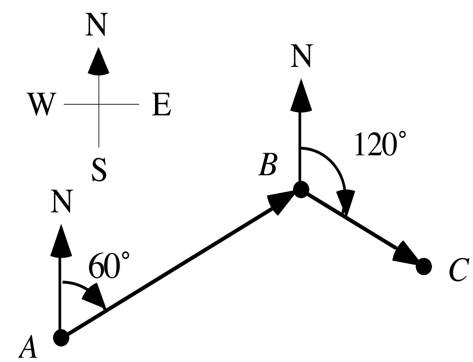


Figure 1: Azimuths of 60° and 120°

Exploration

Figure **2** shows the parts of a typical orienteering compass. The freely suspended magnetic needle indicates the direction of magnetic north. **Note:** To simplify the calculations in this module, *north* always refers to magnetic north.

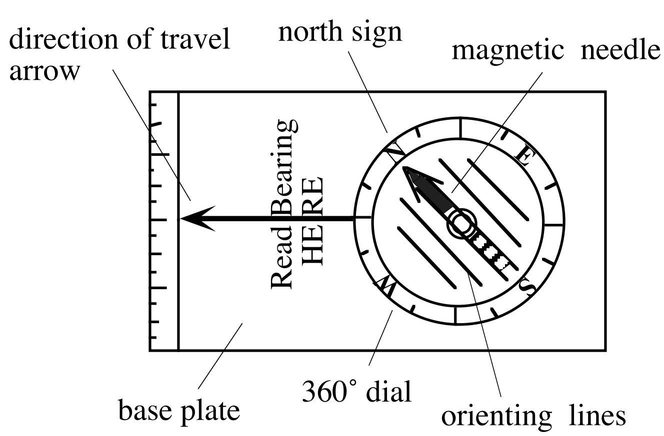


Figure 2: Parts of an orienteering compass

a. In this part of the exploration, you practice following a designated azimuth.

1. Select an azimuth. Turn the 360° dial on your compass until the desired degree measure aligns with the arrow indicating direction of travel.
2. Hold the compass level in your hand to permit the magnetic needle to swing freely. Point the arrow indicating direction of travel directly away from you.
3. While holding the compass steady, turn yourself until the north end of the magnetic needle points to the letter N on the dial. You are now facing your chosen azimuth.
4. Lift the compass close to eye level. Using the arrow indicating direction of travel as a line of sight, choose a distinct landmark in the direction of your azimuth. Lower the compass and walk the number of steps specified by your teacher toward the landmark.

b. Now that you have practiced with your compass, you can use it to follow a prescribed path.

1. Select a starting point and place a marker on the ground between your feet.
2. Select and record an initial azimuth. Walk along this initial azimuth for 30 steps, then stop.
3. Turn 120° clockwise from the direction in which you just walked. Record the new azimuth. Walk 30 steps along this azimuth and stop.
4. Turn another 120° clockwise from the last azimuth. Record the new azimuth. Walk 30 more steps in this direction, then stop.
5. You should now be back at your original starting point—indicated by the marker. If you did not return to the marker, try again.

c. Draw a detailed map of the path you completed, including the azimuth and length of each segment walked.

Discussion

a. In Part **b** of the exploration, what types of errors might have caused you to miss the marker?

b.
1. What type of geometric figure should be created by the path described in Part **b** of the Exploration? Explain your response in terms of the increase in azimuth at each turn.
2. How does the length of your steps affect this figure?

c. At the point where you stopped, how many more degrees should you turn in order to be facing in the same direction in which you started? Explain your response.

d. Would you expect to end up in the same starting place and with the same azimuth every time you followed a path that made three turns of 120˚? Explain your response.

e. How could you use a process like the one described in Part **b** of the exploration to make a rectangular path?

Activity 1

The directions indicated by azimuths also can be used to describe property boundaries. When a conflict arises over a property line, land surveys are often used to settle the dispute. A land survey is a map, similar to the one in Figure **3**, that shows the lengths and **surveying bearings** of the property lines as well as the area of the property.

Science Note

A **surveying bearing** is given as the number of degrees measured either to the east or west from a north or south reference. Since surveying bearings always measure acute angles, they are always greater than 0˚ and less than or equal to 90˚. In this module, *bearing* always refers to a surveying bearing.

Figure **3** shows two examples of bearings. The ray from point *A* to point *B* follows an azimuth of 52˚ and a bearing of N 52˚ E. This bearing means the ray from *A* to *B* is at an angle 52˚ east of north. The ray from point *B* to point *C* follows an azimuth of 111˚. Since bearings are always less than or equal to 90˚, the bearing for this ray is measured from south instead of north. The bearing of this ray is S 69˚ E. This means the ray from *B* to *C* is at an angle 69˚ east of south.

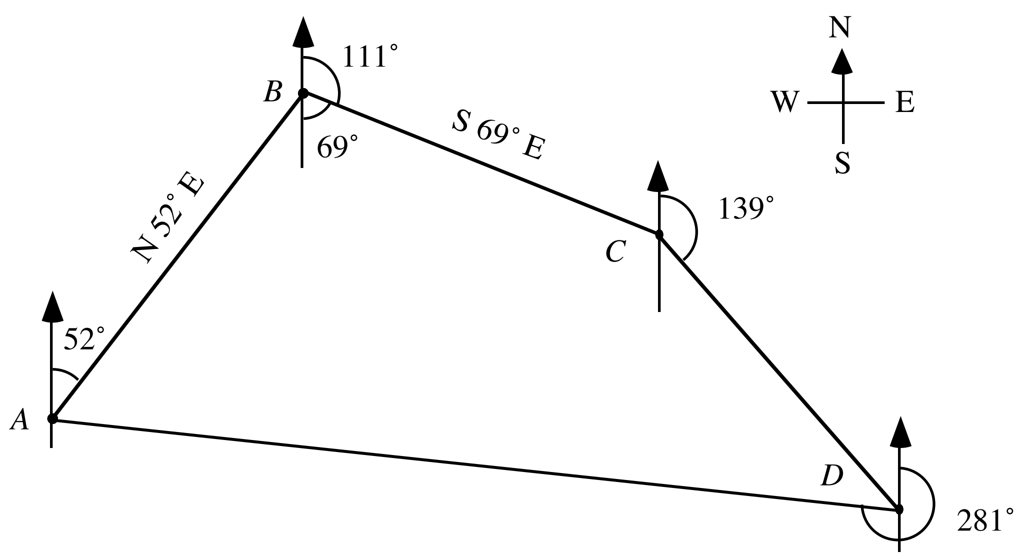


Figure 3: The relationship between azimuth and bearing

Exploration 1

In this exploration, you find the bearings for different azimuths.

a. Using the information in Figure **3**, find the bearings for:

1. the ray from point *C* to point *D*

2. the ray from point *D* to point *A*.

b. Figure **3** shows the bearings for the rays from *A* to *B* and from *B* to *C*. Determine the bearings for each of the following:

1. the ray from point *B* to point *A*

2. the ray from point *C* to point *B*.

Discussion 1

a. Describe how you determined whether a bearing should be measured from north or south.

b. Describe how to determine whether a ray is pointing east or west.

c. Explain why the bearing of the ray from point *A* to point *B* is different than the bearing of the ray from point *B* to point *A*.

d. The ray *MN* has a bearing of N 35° E. What is the bearing of ray *NM*? Describe how you determined your response.

e. Give two bearings for a ray pointing due east. Explain why both bearings are correct.

Exploration 2

In this exploration, you describe the boundaries of a plot of land.

a. One way to estimate distance is by paces. A pace is the distance covered with two normal walking steps. The length of a pace varies from person to person. Determine a method to estimate the length of your pace in meters. Record your estimate.

b. Using visible landmarks such as trees, corners of buildings, or flagpoles, identify a plot of land to be surveyed. Your plot should consist of at least five straight boundary lines.

c. Use a compass to determine the azimuth of each boundary, then estimate the length of each boundary in paces. Record these measurements.

d. Create a scale drawing of your plot. Identify the surveying bearing and length in meters of each boundary line.

e. Exchange maps with another student or group of students. To check the accuracy of the map, use the recorded bearings and lengths to walk the described boundaries. Determine whether or not the measurements match the actual boundaries of the property.

Discussion 2

a. What method did you use for estimating the length of your pace? How could your method be improved?

b. What problems did you encounter when trying to walk the boundaries described by the map in Part **e** of the exploration? What caused these problems?

c. Describe the process you used to calculate a surveying bearing from an azimuth.

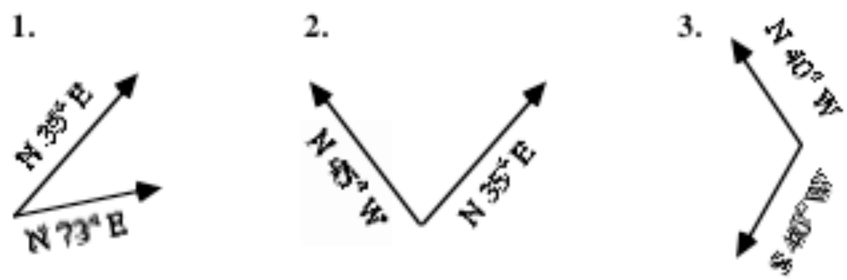
d. The method of surveying used in this exploration could introduce large amounts of error. Where and how might such errors occur?

e. How could you improve the accuracy of your land survey?

f. What tools do professional surveyors use to minimize the amount of error in their land surveys?

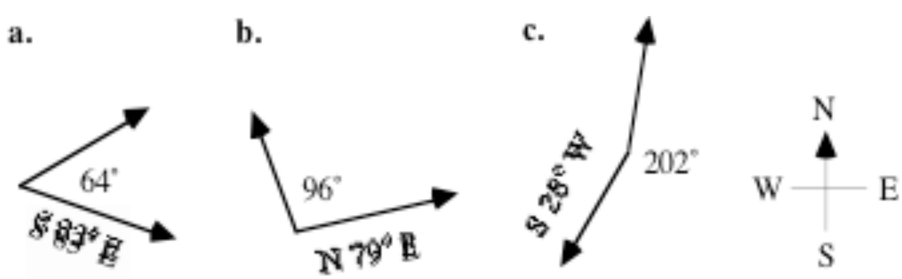
Assignment

1.1 **a.** Without using a protractor, find the measure of the angle between each of the following sets of bearings:

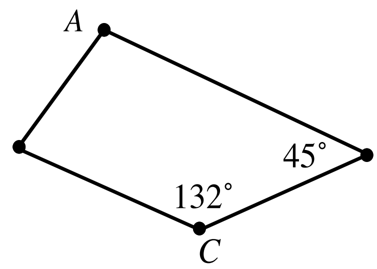


b. Develop a method for calculating the measure of an angle formed by two intersecting lines with known bearings.

1.2 Find the bearing of each unlabeled ray in Parts **a–c** below.



1.3 Use the following figure to find each bearing in Parts **a–c** below.

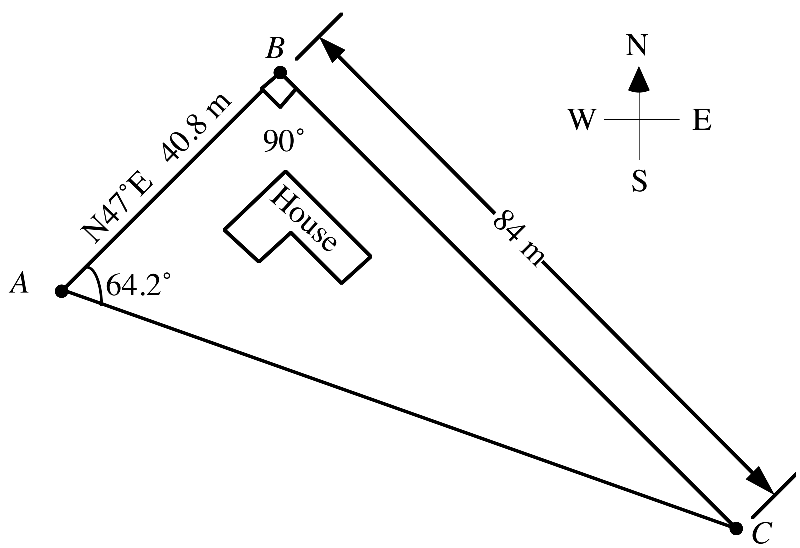


a. ray *BA*

b. ray *CB*

c. ray *CD*

1.4 A map of the Martin family's property shows some of the measurements taken during a land survey.



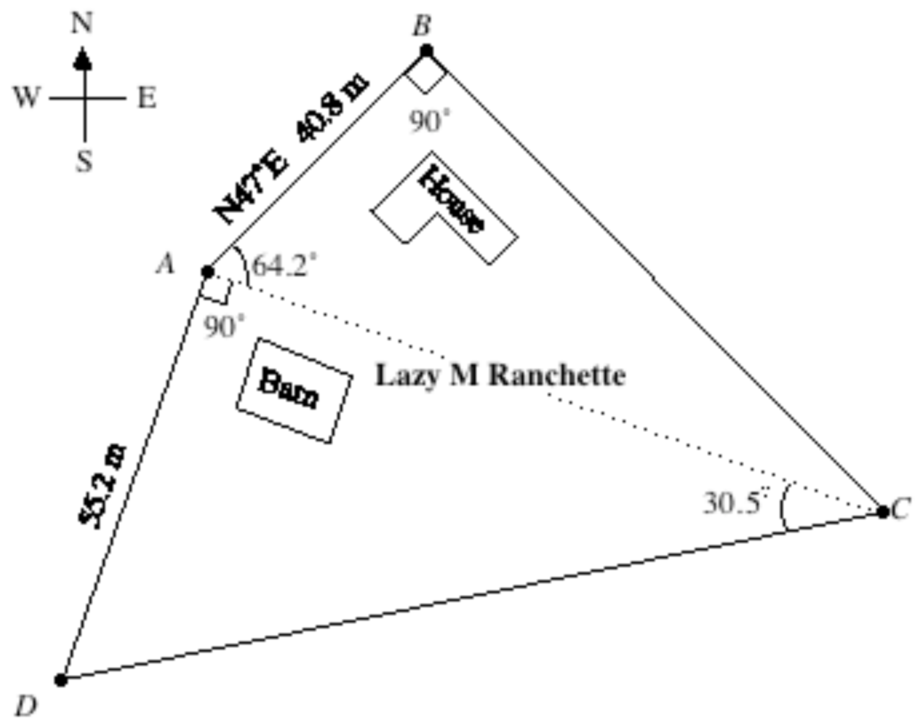
a. **1.** Determine the length of side *CA*.

2. Determine the measure of angle *C*.

b. **1.** Determine the bearing of ray *CA*.

2. Determine the bearing of ray *CB*.

1.5 The Martin family has purchased the lot adjoining their original property. As shown on the map below, they have named their new spread the Lazy M Ranchette. Use the information given on the map and the measurements you determined in Problem **1.4** to complete Parts **a–c**.



a. Calculate the length of side *CD*.

b. **1.** Determine the bearing of ray *CD*.

2. Determine the bearing of ray *DA*.

c. Determine the total area of the Lazy M Ranchette.

Science Note

Dividing a parcel of land into a system of connected triangles is known as **triangulation**. This technique is often used in surveying. The hexagonal plot of land in Figure **4**, for example, can be divided into four triangles so that each vertex of a triangle is also a vertex of the hexagon, and so that each triangle contains at least one side of the hexagon.

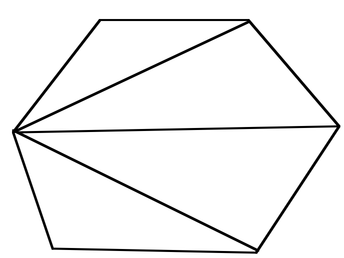


Figure 4: Hexagon divided into triangles

1.6 Describe the difficulties you might have encountered in completing a land survey of the Lazy M Ranchette if it had not been divided into triangles.

1.7 Draw a map of the completed land survey for the Lazy M Ranchette, including the lengths and bearings of all boundary lines, as well as the area of the property.

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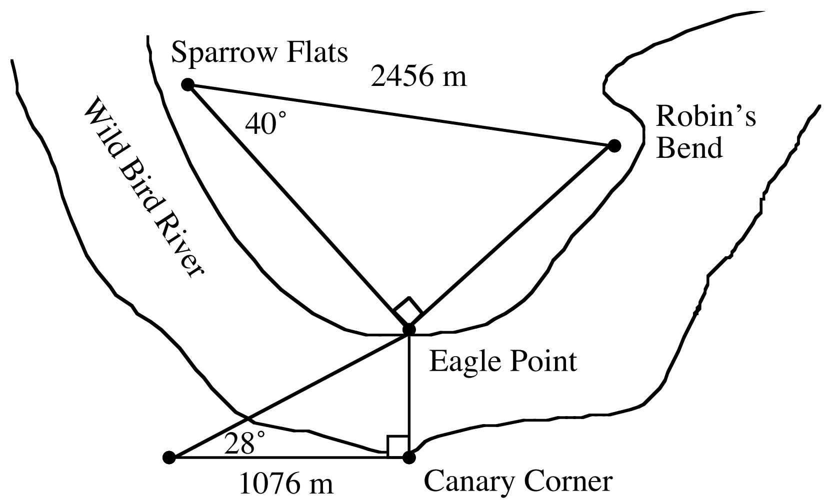
1.8 Imagine that you work for a construction company. Your firm has been hired to help develop the Wild Bird River Estates. A map of the property is shown below.



a. Your company must build a bridge from Canary Corner to Eagle Point. Use your knowledge of right-triangle trigonometry to determine the length of the bridge.

b. After the bridge is completed, the company must install sewer pipe from the pump stations at Sparrow Flats and Robin's Bend to the sewage treatment plant at Eagle Point.

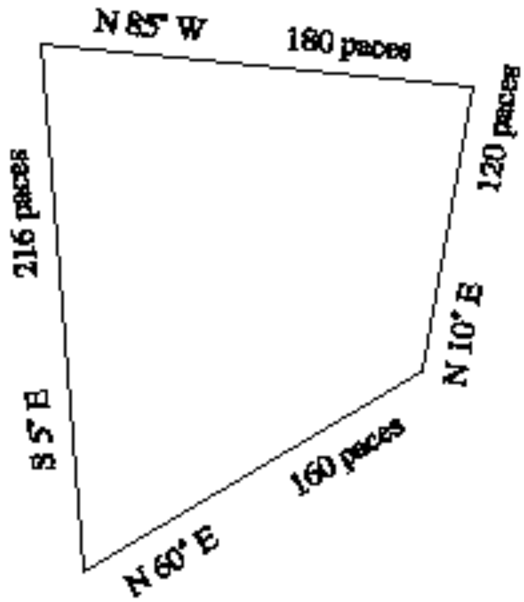
1. Find the distance from Sparrow Flats to Eagle Point and describe the method you used.
2. Find the distance from Robin's Bend to Eagle Point. Describe at least two different ways to determine this distance.

1.9 Measuring all the sides and angles on a piece of property can sometimes be expensive or impractical. In such cases, surveyors often make just a few measurements, then use trigonometry to determine the remaining distances and bearings.

a. Using triangulation, what is the minimum number of triangles into which any polygon can be divided?

b. How does trigonometry allow surveyors to take fewer measurements to complete a survey?

1.10 As part of a class project, Rocky made the following map of a park near his school. By mistake, however, he left the distance measurements in paces.



a. Using this map, Rosa tries to follow the boundaries of the park exactly as Rocky has directed. The size of her pace, however, is three-fourths that of Rocky's. Create a map that shows both the boundaries which Rocky surveyed and the path which Rosa took.

b. **1.** Does Rosa's path end at the same point where she started?

2. How does Rosa's path compare to the park's actual boundaries?

3. How does the area bounded by Rosa's path compare to the area of the park?

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Activity 2

The Westwolff family is considering the purchase of one of two triangular plots of land. To avoid potential boundary disputes, they have hired your company to survey both plots. A surveying team is sent out to take the necessary measurements. When the team returns to the office, the surveyors hand you the map shown in Figure **5** below.

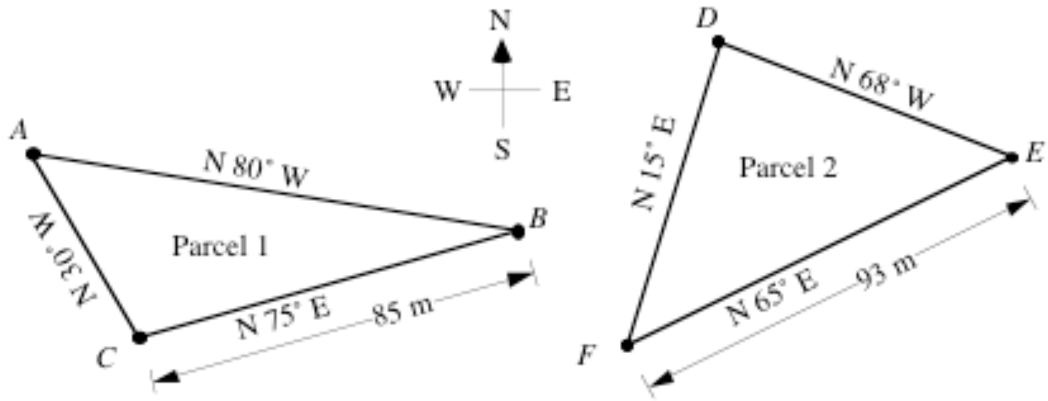


Figure 5: Field data for two land surveys

It is your job to determine the unknown lengths, calculate the areas of the plots, and complete the two surveys. What methods could you use to find this information? In the following explorations, you develop some methods for completing this job.

Exploration 1

As shown in Figure **5**, Parcel 1 appears to be shaped like an obtuse triangle. In this exploration, you use your knowledge of right-triangle trigonometry to interpret sine and cosine values for angle measures greater than 90˚.

a. Figure **6** below shows a triangle *ABC* with one vertex located at the origin of a two-dimensional coordinate system.

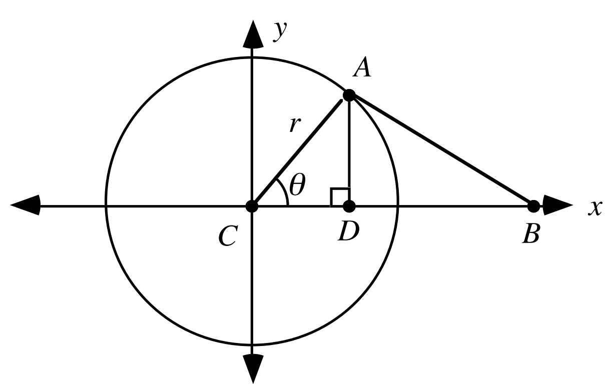


Figure 6: Triangle drawn on a set of coordinate axes

Using a geometry utility, create a drawing similar to the one in Figure **6**. In your construction, point *C* should be the center of the circle and the origin of a two-dimensional coordinate system. (If your geometry utility reports coordinates, make sure that the measure of *r* equals the *x*-coordinate of the point where the circle and the *x*-axis intersect.) Point *A* should be a freely moving point on the circle. Segment *AD* is the altitude of ΔABC through point *A*.

Note: Save this construction for use in Activity **3.**

b. The lengths *CD* and *AD* represent the absolute values of the *x*- and *y*-coordinates, respectively, of a point on the circle.

Use your construction in Part **a**, along with right-triangle trigonometry, to complete Table **1** for several angles with measures between 0 and 90˚.

Table 1: Coordinates of point *A* in terms of *r*

$m\angle ACB$	r	$\cos\angle ACB$	$\sin\angle ACB$	Coordinates of *A*

c. **1.** Describe the relationship between the cosine of θ, the radius *r*, and the *x*-coordinate of point *A*.

2. Describe the relationship between the sine of θ, the radius *r*, and the *y*-coordinate of point *A*.

d. Use the relationships you determined in Part **c** to extend Table **1** to include angle measures between 90˚ and 180˚.

e. Use the data in Table **1** to create a scatterplot of $\sin\angle ACB$ versus $m\angle ACB$.

f. Use the data in Table **1** to create a scatterplot of $\cos\angle ACB$ versus $m\angle ACB$.

Discussion 1

a. **1.** In Figure **6**, why does the length of $\overline{CD}$ represent the absolute value of the *x*-coordinate of point *A*?

2. Why does the length of $\overline{AD}$ represent the absolute value of the *y*-coordinate of *A*?

b. **1.** If $r = 1$ in Figure **6** and $0^\circ \le \theta < 180^\circ$, what does the sine of θ represent?

2. What does the cosine of θ represent?

c. When the measure of $\angle ACB$ in Exploration **1** was between 90° and 180°, point A was in the second quadrant. Figure **7** shows an example of ΔACB with point A in the second quadrant and altitude AD.

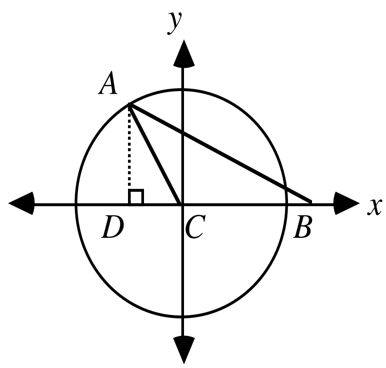


Figure 7: ΔABC with altitude AD

1. What are the signs of the x- and y-coordinates of point A?
2. Write an equation using $m\angle ACB$ and $m\angle ACD$.

d.
1. Considering that $r\sin\angle ACB$ is the y-coordinate of A, how do $\sin\angle ACB$ and $\sin\angle ACD$ compare?
2. Based on your response above, what appears to be the relationship between $\sin\theta$ and $\sin(180 - \theta)$? How does the scatterplot you created in Part **e** of Exploration **1** support this conjecture?

e.
1. Considering that $r\cos\angle ACB$ is the x-coordinate of A, how do $\cos\angle ACB$ and $\cos\angle ACD$ compare?
2. Based on your response above, what appears to be the relationship between $\cos\theta$ and $\cos(180 - \theta)$? How does the scatterplot you created in Part **f** of Exploration **1** support this conjecture?

f. Describe a method for finding the sine and cosine of an angle with a measure of 130°, given the sine and cosine of an angle with a measure of 50°.

g. In general, how could you find the sine and cosine of an angle with a measure of $n°$ given the sine and cosine of an angle with a measure of $(180 - n)°$?

Exploration 2

In this exploration, you apply what you know about right-triangle trigonometry to develop a method for determining the unknown sides and angles of non-right triangles.

a. Using a geometry utility, draw an acute triangle like the one shown in Figure **8**.

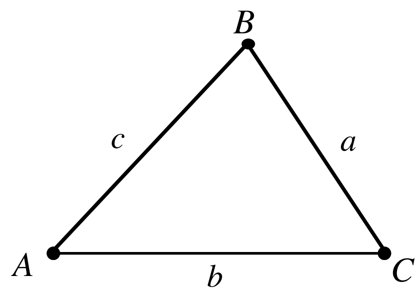


Figure 8: An acute triangle

b. Find the measures of $\angle BAC$, $\angle ABC$, and $\angle ACB$.

c. Find the lengths a, b, and c.

d. Construct an altitude from vertex B to the line containing the opposite side $\overline{AC}$. As shown in Figure **9**, label the altitude h and its intersection with line AC point D. The altitude divides ΔABC into two other triangles. Describe these triangles.

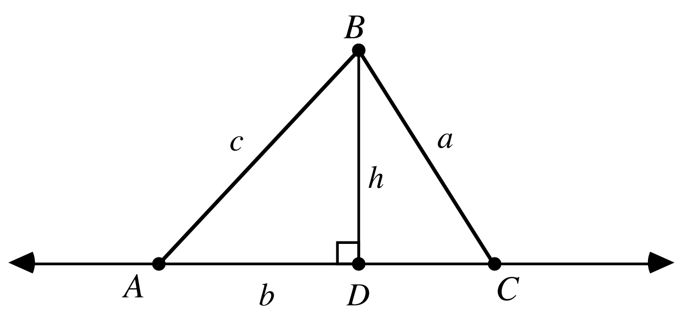


Figure 9: ΔABC with altitude BD

e. Write equations for $\sin\angle BAC$ and $\sin\angle ACB$ in terms of h, c, and a.

f. Solve for h in each equation in Part **e**.

g.
1. Use your equations from Part **f** and the geometry utility to calculate a value for the altitude.
2. Measure the altitude.
3. Observe how the values for h in Step **1** compare to the measured altitude as you drag any vertex of the triangle.

h. Since the two expressions from Part **f** are both equal to h, they are equal to each other. Set these two equations equal to each other and solve for the following ratio:

$$\frac{\sin\angle ACB}{c}$$

Discussion 2

a. What does the relationship you found in Part **h** of Exploration **2** indicate about the ratio of the sine of each angle in a triangle to the length of its opposite side?

Mathematics Note

The **law of sines** states that the lengths of the sides of a triangle are proportional to the sines of the opposite angles.

For example, Figure **10** shows a triangle ABC.

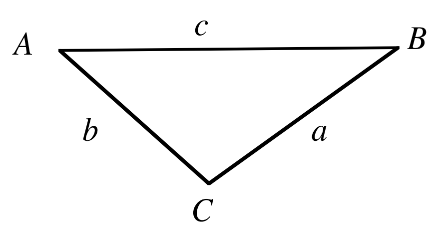


Figure 10: Triangle *ABC*

According to the law of sines,

$$\frac{a}{\sin\angle A}=\frac{b}{\sin\angle B}=\frac{c}{\sin\angle C}$$

b. Describe how the law of sines can be used to find the unknown values in Figure **11**.

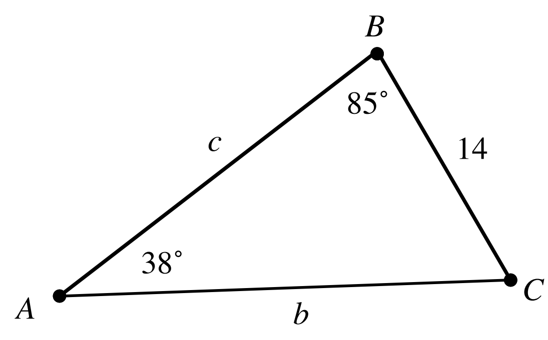


Figure 11: A triangle with some unknown measures

c. In one of the triangles in Figure **12**, the law of sines cannot be used to find the unknown sides and angles. Identify this triangle and explain why the law of sines cannot be used to determine these measurements.

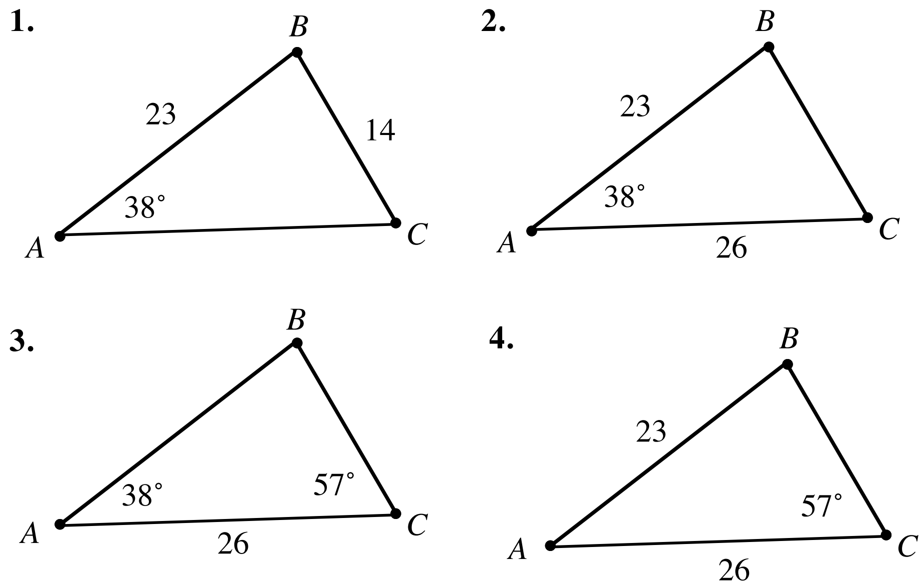


Figure 12: Triangles with unknown measures

d. Describe the minimum information needed to use the law of sines to determine the measures of all angles and sides of a triangle.

e. In Exploration **2**, you discovered a relationship between the sine of an angle and the altitude of a triangle. Describe how you could use this relationship to determine the area of the triangle in Figure **13**.

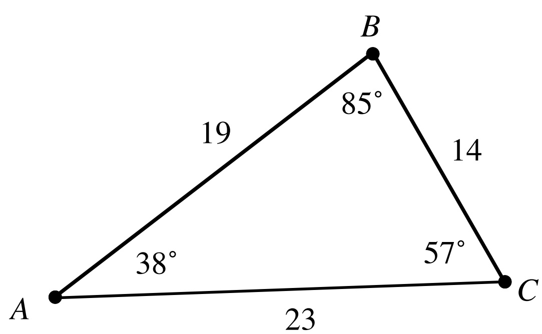
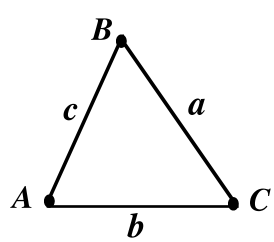


Figure 13: A triangle

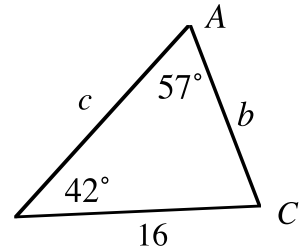
f. Write a formula to find the area of any general triangle ABC.



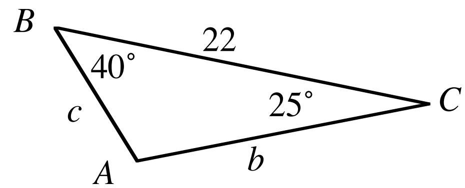
Assignment

2.1. Determine the measures of the unknown sides and angles in each of the following triangles. If any of the missing values cannot be determined using the law of sines, explain why not.

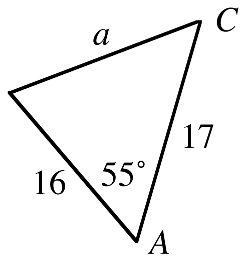
a.



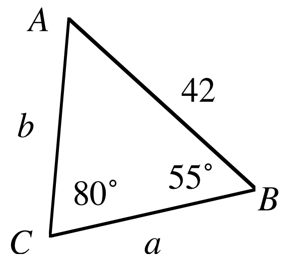
b.



c.

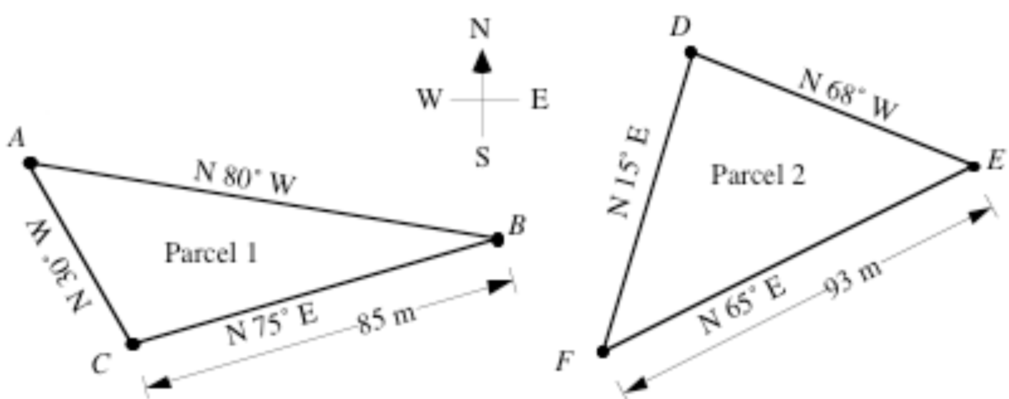


d.



2.2 Find the areas of the four triangles in Problem **2.1**.

2.3 As mentioned in the introduction to this activity, the Westwolff family plans to buy one of two triangular plots of land. The field data from the surveys of the two properties are shown in the figure below.



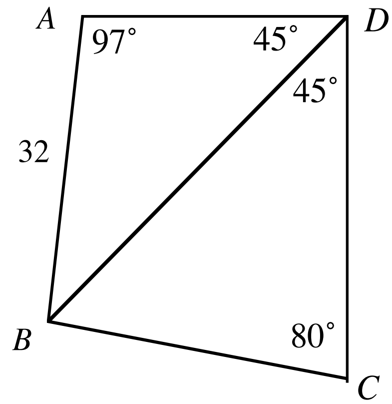
a. Determine the measure of each interior angle and the length of each side of Parcel 1.

b. Determine the measure of each interior angle and the length of each side of Parcel 2.

c. Calculate the areas of Parcels 1 and 2.

d. If Parcel 1 costs \$21,000 and Parcel 2 costs \$26,000, which one do you think is the better buy based on the cost per unit of area? Justify your response.

2.4 The diagram below shows a polygon *ABCD*.

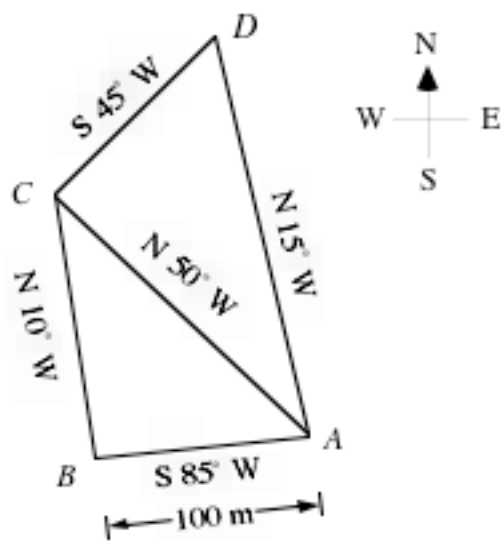


a. Use the information given in the diagram to calculate each of the following lengths:

1. *AD*
2. *BD*
3. *BC*
4. *DC*

b. Determine the area of polygon *ABCD*.

2.5 A surveying crew has collected the data shown on the following map.



a. Determine the measures of all interior angles on the map.

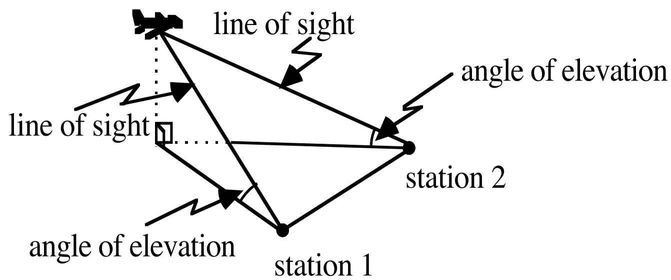
b. Determine the measures of the sides of all triangles on the map.

c. Calculate the area of each triangle.

d. Calculate the area of the plot of land described by polygon *ABCD*.

* * * * *

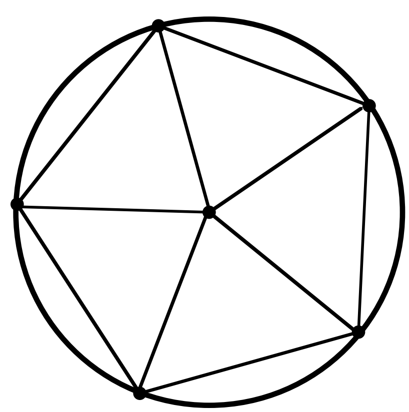
2.6 The process of triangulation also can be used to determine the location and altitude of an airplane from the ground. For example, the diagram below shows two tracking stations monitoring the location of a plane. Observers at each station measure the angle of elevation (the angle measured from horizontal) between themselves and the plane, as well as the angle formed by the line of sight to the plane and the line connecting the stations. Given these angles and the distance between the stations, they can calculate the location of the plane.



Suppose that the distance between the two stations is 2 km, the plane's angle of elevation at station 1 measures 39°, and the plane's angle of elevation at station 2 measures 60°. At station 1, the angle formed by the line of sight to the plane and the line between the stations reasures 43°; at station 2, this angle measures 65°.

What is the plane's altitude at the time these observations were recorded? Explain your response.

2.7 As part of her design for a new park, a city planner has decided to build a circular garden. The garden will feature five walkways radiating from its center like the spokes of a wheel, as shown in the diagram below. Five more walkways will connect the spokes to form a regular pentagon. If the radius of the circle is 40 m, find the sum of the lengths of all the walkways.



2.8 Imagine that you and an associate are on opposite sides of a river, surveying land for a new city bridge. There is a large tree on your side of the river. From the point where you are standing, the angle between your associate and the tree measures 48°. From the point where your partner is standing, the angle between you and the tree measures 62°. The distance from you to the tree is 160 m.

a. Make a sketch of this situation.

b. What is the distance between you and your associate? Explain your response.

* * * * * * * * * *

Activity 3

In Activity **2**, you used the law of sines to determine unknown measurements in triangles. However, you also discovered that the law of sines has some limitations. As a surveyor, you have been asked to finish a survey for the plot of land shown in Figure **14**. The results of the field measurements are indicated on the map. In this case, the lengths of two sides are known, as well as the measure of the angle between those sides. Is this enough information to proceed without taking more measurements?

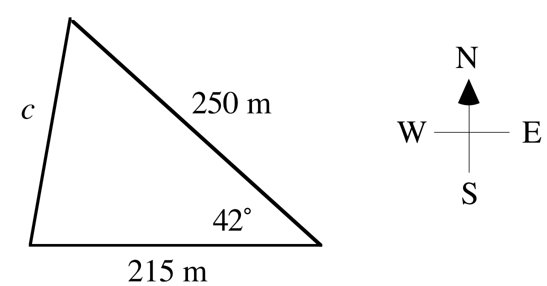


Figure 14: A partially completed land survey

Exploration 1

The Pythagorean theorem states that the sum of the squares of the legs of a right triangle is equal to the square of the hypotenuse. As you have seen in other modules, this relationship can be represented by constructing squares on each side of a right triangle and comparing the areas of the squares. In this exploration, you use a similar model to explore the relationships among the sides of acute and obtuse triangles.

a. Using a geometry utility and your construction from Activity **2**, create a diagram similar to the one shown in Figure **15**. In this construction, point C is the center of the circle, point A is a moveable point on the circle, and ΔABC is a right triangle with $\overline{AB}$ as its hypotenuse.

The quadrilaterals on each side of the triangle are squares. The sides of each square are congruent to the corresponding side of the triangle. These squares should be created so that they remain squares when the sides of the triangle are moved.

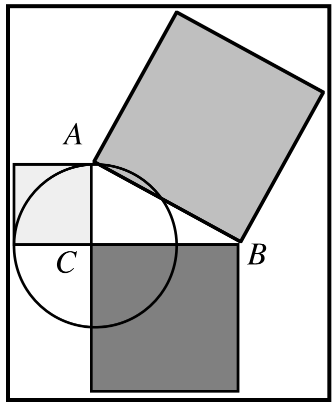


Figure 15: A right triangle with squares on its sides

b. Record the area of each square in Part **a** and the measure of $\angle ACB$.

c. Compare the area of the square constructed on $\overline{AB}$ to the sum of the areas of the other two squares.

d. Move point *A* to several different locations on the circle, repeating Parts **b** and **c** at each location. **Note:** Save this construction for use in Exploration **2**.

Discussion 1

a. **1.** In general, what is the relationship between the area of the square constructed on $\overline{AB}$ to the sum of the areas of the other two squares when the measure of $\angle ACB$ is greater than 90˚?

2. What is this relationship when the measure of $\angle ACB$ is less than 90˚?

b. Describe your generalizations from Part **a** of Discussion **1** in terms of the type of triangle—right, obtuse, or acute— and the lengths of its sides.

c. Recall that the converse of a statement in the form "If A, then B" is the statement "If B, then A." The converses of the generalizations you made in Part **a** also are true. How could you use these statements, along with the converse of the Pythagorean theorem, to classify a triangle as acute, obtuse, or right knowing only the measures of its sides?

Exploration 2

In Exploration **1**, you found that in an obtuse triangle, the square of the length of the side opposite the obtuse angle is greater than the sum of the squares of the lengths of the other two sides. You also found that in an acute triangle, the square of the length of one side is less than the sum of the squares of the lengths of the other two sides. In this exploration, you discover a more precise way to describe these generalizations.

a. Remove the squares from the construction you created in Exploration **1**. Move point *A* so that $\angle CAB$ is obtuse.

b. Construct an altitude from vertex *A* to the opposite side $\overline{BC}$ to form two right triangles. Label the sides of ΔABC, the altitude, and the lengths of the two segments formed on side $\overline{BC}$ as shown in Figure **16**.

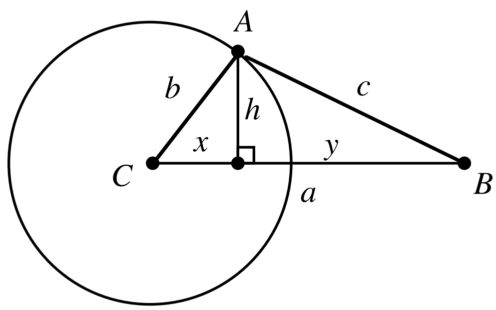


Figure 16: Labeled construction

c. Use trigonometry to express *x* in terms of *b* and $\angle ACB$.

d. Use the expression for *x* found in Part **c** and the Pythagorean theorem to write an expression for h^2 in terms of *b* and $\angle ACB$.

e. Use the Pythagorean theorem to express h^2 in terms of *c* and *y*.

f. Express *y* in terms of *x* and *a*.

g. Substitute the expression for *y* from Part **f** and the expression for *x* from Part **c** into the expression for h^2 from Part **e** to write h^2 in terms of the sides of ΔABC and $\angle ACB$.

h.
1. Since the expressions from Parts **d** and **g** are both equal to h^2, they are equal to each other. Set these two expressions equal to each other and solve for c^2.
2. How does the equation from Step **1** appear to be related to the Pythagorean theorem?

Mathematics Note

The **law of cosines** states that the square of the length of any side of a triangle is equal to the sum of the squares of the lengths of the other two sides, minus twice the product of the lengths of these sides and the cosine of the angle included between them. In Figure **17**, for example, $c^2 = a^2 + b^2 - 2ab\cos\angle C$.

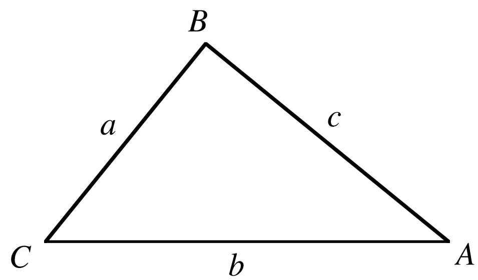


Figure 17: Triangle *ABC*

The law of cosines can often be used to determine an unknown length or angle measure in a triangle. For example, if $a = 10$ cm, $b = 12$ cm, and $m\angle ACB = 35°$, then the law of cosines can be used to find c as follows:

$$c^2 = a^2 + b^2 - 2ab\cos\angle C$$
$$c^2 = 10^2 + 12^2 - 2(10)(12)\cos 35°$$
$$c^2 \approx 47.40$$
$$c \approx 6.89 \text{ cm}$$

Discussion 2

a. Compare the equation you wrote in Part **h1** of Exploration **2** to the law of cosines described in the mathematics note.

b. **1.** When $\angle ACB$ is acute, what is the sign of its cosine?

2. When $\angle ACB$ is obtuse, what is the sign of its cosine?

c. Describe how you could use the law of cosines to find a in Figure **18**.

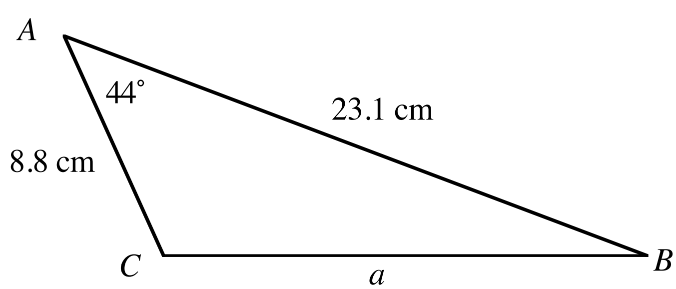


Figure 18: Triangle with an unknown side

d. Given the information in Figure **18** and the value of a, how could you determine the measure of one of the remaining angles using:

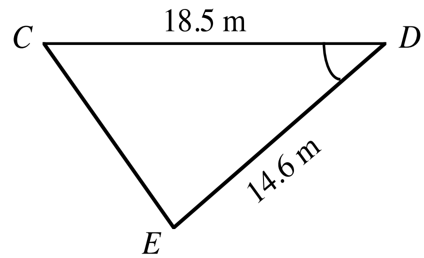
1. the law of cosines?

2. the law of sines?

e. The law of cosines works with acute triangles and obtuse triangles. Does it also work for right triangles? Justify your response.

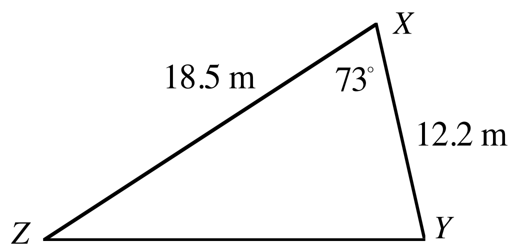
Assignment

3.1 **a.** Find the length of $\overline{CE}$ in the map below.



b. Using your response to Part **a**, describe how to find the measures of the other two angles of the triangle.

3.2 The diagram below shows a map of the Martin family's garden plot.



Use the measurements given to find each of the following:

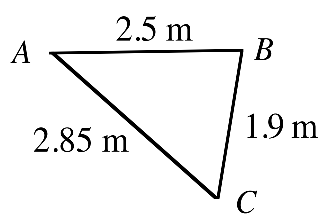
a. YZ

b. $m\angle XZY$

c. $m\angle XYZ$

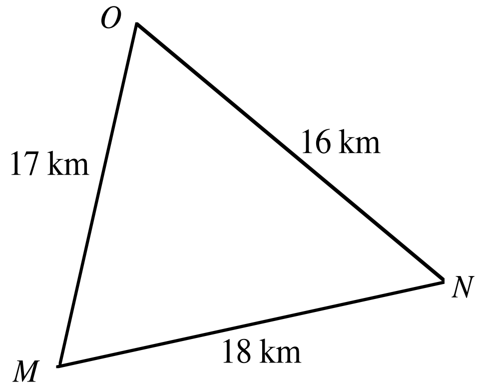
d. the area of the triangle

3.3 **a.** Use the law of cosines to find $m\angle BAC$ in the triangle below.



b. Given $m\angle BAC$, describe two methods for determining $m\angle ABC$ and $m\angle ACB$.

3.4 The following sketch shows some measurements taken during a survey of a triangular plot of land.



Use the information given to determine each of the following:

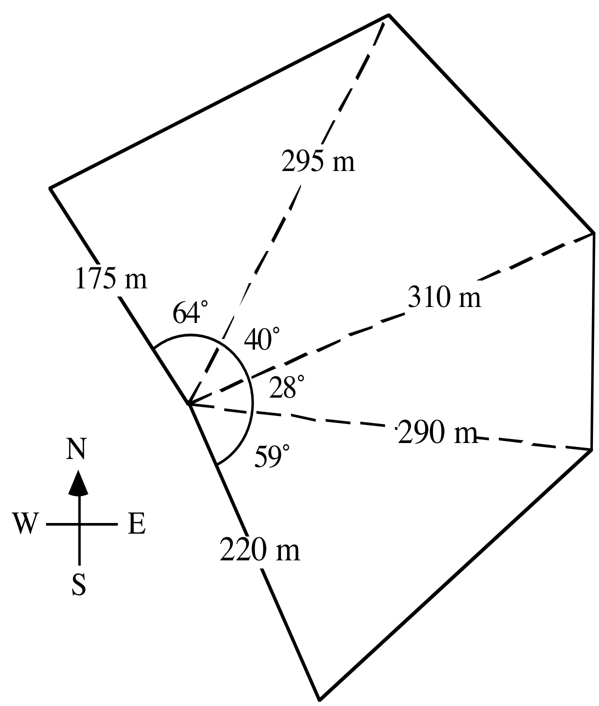
a. $m\angle NMO$

b. $m\angle MNO$

c. $m\angle MON$

d. the area of the triangle

3.5 The following map shows the data collected during a field survey.



Using the information recorded on the map, determine:

a. the length of each segment along the perimeter of the property

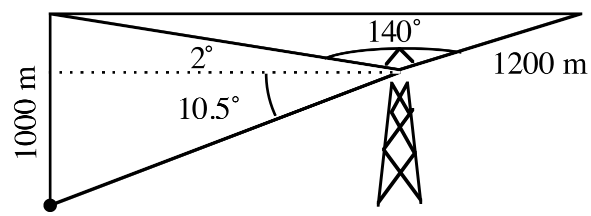
b. the total area of the property.

* * * * *

3.6 Two ships leave San Francisco Bay at the same time. As they pass under the Golden Gate Bridge, one ship is approximately 0.5 km due north of the other, cruising at 21 knots (approximately 39 km/hr) at a survey bearing of S 70° W. The other ship is traveling due west at 24 knots (approximately 44 km/hr). How far apart are the two ships after 3 hr?

3.7 Fire lookouts live and work in small, one-room observation towers high in the mountains. Part of their job involves reporting forest fires and helping to coordinate firefighting efforts.

During one fire, a plane delivering a load of fire retardant passes 1000 m above the flames. The lookout measures the angle of elevation to the plane and the angle of depression to the fire. At that moment, a second plane flying at the same altitude reports its location as 1200 m from the observation tower. Given the measurements shown in the diagram below, how far is the second plane from the first plane?



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Research Project

Obtain a survey map of a property that interests you. Verify that the areas given for each plot of land on the survey are correct and that the sum of all the angles of each plot of land produces a closed polygon. In your report, sketch a map of the property and include a detailed description of how you completed your calculations.

Summary Assessment

1. Use the following steps to complete a survey of a plot of land specified by your teacher.

a. Devise a method of dividing the plot into triangles that will minimize the number of measurements required.

b. Using a compass, measure and record the necessary bearings.

c. Determine the lengths of all boundaries.

d. Find the total area of the plot of land.

2. Write a summary of the procedure you used to survey the plot, including:

a. a field sketch showing your preliminary measurements

b. a scale drawing of the completed survey with all appropriate information

c. descriptions of where and how you used trigonometric ratios, the law of sines, and the law of cosines to complete the survey.

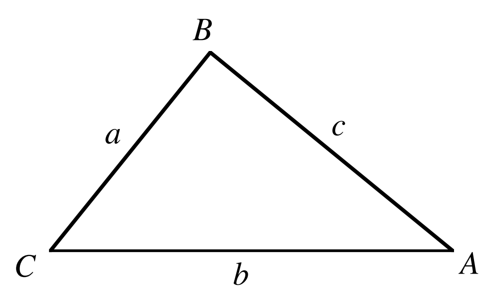
Module Summary

- An **azimuth** can be described as the angle formed by rotating a ray clockwise from the ray representing north.
- A **surveying bearing** is given as the number of degrees measured either to the east or west from a north or south reference. Since surveying bearings always measure acute angles, they are always greater than 0˚ and less than or equal to 90˚.
- The **law of sines** states that the lengths of the sides of a triangle are proportional to the sines of the opposite angles. In a triangle *ABC*,

$$\frac{a}{\sin\angle A} = \frac{b}{\sin\angle B} = \frac{c}{\sin\angle C}$$

- The **law of cosines** states that the square of the length of any side of a triangle is equal to the sum of the squares of the lengths of the other two sides minus twice the product of the lengths of these sides and the cosine of the angle included between them. In a triangle *ABC*,

$$c^2 = a^2 + b^2 - 2ab\cos\angle C$$



- The area of any triangle can be calculated if the measure of two sides and the included angle are known. In the triangle ΔABC,

$$Area = \frac{1}{2}ab\sin C$$

where a and b are the length of the sides and C is the measure of the included angle.

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