Strive for Quality

The quality of the products we buy affects our lives in many different ways. What is quality and how can we control it? In this module, you examine some statistical methods for evaluating quality.

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Introduction

As a consumer, you expect every product you buy to meet certain standards of quality. You want your food to be free from contamination, your clothes to be stylish and durable, and your stereo system to sound clear and lifelike.

To monitor product quality, manufacturers typically use a process called **quality control**. This process involves four steps:

- defining the characteristics of a quality product
- monitoring the product for these characteristics
- using statistics to analyze the results obtained from monitoring
- making needed adjustments in production to improve quality.

In this module, you investigate the mathematics involved in the third step: using statistical techniques to analyze samples taken during the manufacturing process.

Mathematics Note

All the members of a group can be referred to as a **population**. A **sample** is a subset of a population. Typically, a sample includes only some members of the population, not all of them.

A **parameter** is a numerical characteristic of a population. A **statistic** is a numerical characteristic of a sample. Statistics are used to estimate the corresponding parameters of the population.

For example, the junior class in your school can be considered a population. The mean number of courses taken by all juniors is a parameter of that population. You can estimate the population mean by selecting a sample of the junior class and determining the mean number of courses taken by the students in the sample.

Sampling is the process of choosing a subset of a population. In a **simple random sample**, each member of the population must have the same chance of being included in the sample.

For example, suppose that the name of each student in the junior class is written on a slip of paper. If the slips are placed in a container and mixed thoroughly, then each name would have the same chance of being drawn from the container. By drawing 30 names from the container, you could obtain a simple random sample of 30 students from the population. **Note:** For the remainder of this module, each mention of a "random sample" refers to a simple random sample.

Exploration

Manufacturers often monitor product quality by taking samples, then testing those samples to determine if an appropriate number of products pass inspection. Imagine that you work for a company that makes paper coffee filters. Your most popular product is a circular filter 20 cm in diameter.

a. Using only a sheet of paper, a ruler, a pencil, and scissors, simulate the production of a coffee filter by completing Steps **1–3** below.

1. Draw two approximately perpendicular diameters of a filter on the sheet of paper.
2. Using the diameters from Step **1** as a guide, draw an outline of the filter as shown in Figure **1**.

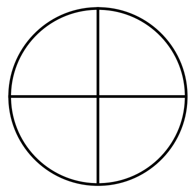

Figure 1: Outline of coffee filter

3. Cut out the filter with scissors.

b. Repeat Part **a** four more times to produce a total of five coffee filters.

c.
1. On each filter, draw several diameters at intervals of approximately 30°.
2. In order for a filter to pass inspection, every diameter must be within 5 mm of the desired length of 20 cm. Inspect each filter you manufactured and determine the number of filters that fail inspection.

d. The president of your company has decided to improve the quality of the firm's products. The diameters of all new filters must now fall within 2 mm of the desired 20 cm. To meet the new requirement, you may change the manufacturing process by adding new equipment and additional steps.

1. Describe the steps you would use to enhance the manufacturing process.
2. Manufacture five filters using the new process described in Step **1**.
3. Exchange filters with another manufacturer. Inspect the filters by drawing and measuring diameters as described in Part **c**. Each diameter must now fall within 2 mm of the desired 20 cm. Identify filters that fail inspection by writing the word "defective" on them.
4. Return the filters to the manufacturer, along with a record of the number that failed inspection.

e. Collect the class data from Part **d**, compile the results, and determine the percentage of failures for the entire class.

f. How many filters do you think would fail inspection in a sample of 10,000 produced by your class?

Discussion

a.
  1. What characteristic did you use to measure quality in the exploration?

  2. What other criteria might a coffee-filter manufacturer use to determine quality?

b.
  1. Was it difficult to produce filters that would pass inspection using the process described in Part **a** of the exploration? Why or why not?

  2. Do you think that it is reasonable to allow a variation of 5 mm if the desired diameter is 20 cm? Why or why not?

c. What factors might account for any differences in the numbers of failures observed in Part **d** of the exploration?

d. How could you improve the manufacturing process used in Part **d** of the exploration?

e.
  1. Do you think the failure rate for the class data would be acceptable to a real manufacturer?

  2. What factors should a company consider when making this decision?

f. In Part **f** of the exploration, how did you determine the number of filters you expected to fail?

g. Why is it often impractical to test every item produced for quality?

h. If your company manufactured 10,000 filters a day, describe a process you might use to monitor the quality of the filters.

Activity 1

Analyzing random samples of their products can provide manufacturers with important information about product quality. For example, the mean number of defective items that occur over many samples can help companies determine how many defective items to expect in any one sample.

If more than the expected number fail inspection, this may mean that the production process should be reviewed or modified. In this activity, you examine one model used to analyze such situations.

Exploration

A manufacturing company has found that the percentage of defective items produced by its factory is about 30%. As a quality control specialist, you decide to verify this figure by inspecting some samples.

The company produces 1000 items a day. To monitor product quality, you select one item from this population, inspect it, determine whether or not it is defective, then remove it from the population. To obtain a sample of 4 items, you repeat this process 4 times. This process involves **conditional probability**.

Mathematics Note

Conditional probability is the probability of an event occurring given that an initial event has already occurred. The probability that event B occurs, given that event A has already occurred, is denoted $P(\text{B}|\text{A})$.

For example, consider a paper bag containing three chips: two red and one blue. Suppose that you draw one chip from the bag. The probability that this chip is blue is $1/3$. However, suppose that you obtain a red chip on your first draw. Without replacing the first chip, you then draw a second. Since only two chips remain in the bag, the conditional probability of drawing a blue chip, given that a red one has already been drawn, is $1/2$.

In an experiment involving conditional probabilities, the probability of both A and B occurring is found by multiplying the probability of A by the conditional probability of B given A has already occurred:

$$P(\text{A and B}) = P(\text{A}) \bullet P(\text{B}|\text{A})$$

For example, again consider the paper bag containing three chips: two red and one blue. Suppose that you draw one chip from the bag. Without replacing the first chip, you then draw a second. The tree diagram in Figure **2** shows all the possible outcomes in this situation, along with their probabilities. In this case, the probability of drawing a red chip followed by a blue chip, or $P(\text{RB})$, is $1/3$.

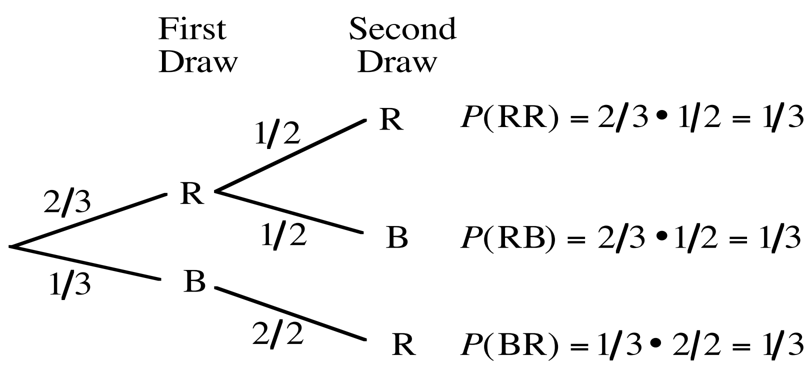

Figure 2: Possible Outcomes and Probabilities

a. Assuming that the percentage of defective items is 30%, how many defective items would there be in a population of 1000?

b. Create a tree diagram that shows all the possible outcomes for a sample of 4 items taken from this population of 1000 items. Determine the probability of each outcome.

Mathematics Note

Two events are **mutually exclusive** if they cannot occur at the same time in a single trial. If A and B are mutually exclusive events, then $P(\text{A and B}) = 0$.

When two events are mutually exclusive, the probability that one or the other occurs is the sum of the probabilities of the individual events. This can be written symbolically as follows: $P(\text{A or B}) = P(\text{A}) + P(\text{B})$.

For example, each outcome shown in the tree diagram in Figure **2** is mutually exclusive of the others. The probability of obtaining BR *and* RB in a single sample of 2 chips is 0, since one excludes the other. However, the probability of obtaining either BR *or* RB—in other words, one red chip and one blue chip, regardless of order—is $1/3 + 1/3 = 2/3$.

c. Determine the probabilities of obtaining each of the following numbers of defective items in a sample of 4 items from the company's daily production.

1. 0
2. 1
3. 2
4. 3
5. 4

d. Recall that two events A and B are **independent** if

$$P(\text{A and B}) = P(\text{A}) \bullet P(\text{B})$$

Three events A, B, and C are independent if each pair of events is independent and $P(\text{A and B and C}) = P(\text{A}) \bullet P(\text{B}) \bullet P(\text{C})$. This definition can be extended to any number of independent events.

When each item drawn in a sample is replaced in the population before drawing another, the draws are independent events. Repeat Parts **b** and **c** assuming that each item is returned to the population after inspection.

e. Compare your results from Parts **c** and **d** and record your observations.

Discussion

a. What differences did you observe in the probabilities calculated in Parts **c** and **d** of the exploration?

b. In which situation were the expressions representing the probabilities easier to write? Explain your response.

Mathematics Note

A **binomial experiment** has the following characteristics:

- It consists of a fixed number of repetitions of the same action. Each repetition is a **trial**.
- The trials are independent of each other. In other words, the result of one trial does not influence the result of any other trial in the experiment.
- Each trial has only two possible outcomes: a success or a failure.
- The probability of a success remains the same from trial to trial.
- The total number of successes is observed.

For example, consider an experiment that consists of tossing a six-sided die 10 times and observing the number of times that a 6 appears. In this case, there is a fixed number of trials, 10. For each trial, there are only two possible outcomes: either a 6 or not a 6. The probability that a 6 appears remains constant for each toss, and the result of one toss does not influence the result of any other. Therefore, this represents a binomial experiment.

c. Which of the samplings in the exploration can be considered a binomial experiment? Explain your response.

d. Why do you think that quality control specialists typically take samples without replacing the items?

e. Refer to the tree diagram you created in Part **d** of the exploration.

1. Describe the different outcomes in which exactly 2 defective items occur in a sample of 4.
2. What is the theoretical probability of each of these outcomes?
3. How did you determine the total probability of obtaining 2 defective items in a sample of 4?
4. How did you determine the total probability of obtaining 3 defective items in a sample of 4?

f. Even though sampling without replacement does not fit the definition of a binomial experiment, quality control specialists typically use binomial experiments to model their inspections.

1. Based on your observations in the exploration, why do you think this is acceptable?
2. When do you think this would not be acceptable?

g. Given that A and B are independent events, how could you demonstrate that $P(B|A) = P(B)$?

Assignment

1.1 As part of its quality control procedure, a compact disc (CD) maker uses the following process: A CD is randomly selected from the population, tested to determine whether or not it meets a certain standard, then returned to the population. The manufacturer repeats this process 10 times.

a. What is a trial in this experiment?

b. Are the trials independent events? Explain your response.

c. What could be considered a success in this experiment?

d. Is this process a binomial experiment? Explain your response.

1.2 A computer firm produces 5000 computer chips in one day. Of that total, 1050 are defective.

a. What is the probability that a chip selected at random from the day's production will be defective?

b. If the chip selected in Part **a** is removed from the population, what is the probability that a second chip selected at random will be defective?

c. Suppose that you select a sample of 10 chips from the day's production and determine that 4 are defective. If this sample is not replaced, what is the probability of selecting a defective chip from the remaining population?

d. Considering your responses to Parts **a–c**, write a paragraph evaluating the following statement: "If a small random sample is taken without replacement from a large population, then the probability of selecting a defective item is essentially unchanged from trial to trial and the experiment can be modeled by a binomial experiment."

1.3 Determine whether or not each of the following procedures is a binomial experiment. If the procedure is not a binomial experiment, can it be reasonably modeled by one? Explain your responses.

a. You select a random sample of 6 computer chips from a batch of 20. You replace each chip before selecting the next one.

b. You select a random sample of 6 computer chips from a batch of 20. You do not replace the sample.

c. You select a random sample of 500 American teenagers and determine their favorite brand of tennis shoes from a list of 10 brands.

d. You select a random sample of 5 electric motor shafts from an assembly line that produces 2000 shafts a day and determine if the shaft diameter is between 1.71 cm and 1.73 cm.

e. You roll a pair of dice 5 times and determine the number of times their sum is greater than 8.

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1.4 Rindy rolls a fair die 10 times and records the number of times the result is greater than 4.

a. What is a trial in this experiment?

b. Are the trials independent? Explain your response.

c. What could be considered a success in this experiment?

d. What is the probability of a success?

e. Is this a binomial experiment? Explain your response.

1.5 Design a binomial experiment in which the probability of success does not equal the probability of failure. Describe how your design meets the criteria required for a binomial experiment.

1.6 Consider a box of 30 light bulbs in which 5 bulbs are defective.

a. If sampling from the box is done without replacement, what is the probability that in a sample of 30 bulbs,

1. exactly 4 bulbs will be defective?

2. exactly 5 bulbs will be defective?

b. If sampling from the box is done with replacement, the probability that 4 bulbs will be defective in a sample of 30 bulbs is approximately 0.18. The probability that 5 bulbs will be defective in a sample of 30 is approximately 0.19.

How do these values compare with your responses in Part **a**?

c. What must be true of the relative sizes of a sample and a population if the sampling process is to be reasonably modeled by a binomial experiment?

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Activity 2

As a quality-control consultant, you have been hired to help a computer disk manufacturer evaluate its production process. Due to vigorous competition, the company wants to reduce manufacturing costs and market a less expensive product. However, the company's executives worry that cutting costs may also reduce the quality of their disks.

They want you to determine the percentage of defective disks produced by the current manufacturing process and estimate the percentage of defects that will occur if they modify that process. Using your findings, they will decide whether or not to make any changes.

Discussion

a. Describe the difficulties you might encounter in using a tree diagram to show all the possible outcomes in a sample of 10 disks.

b. In the exploration in Activity **1**, you created a tree diagram of all the possible outcomes for a sample of 4 items from a population of 1000. For example, one possible outcome is DNDD, where D represents a defective disk and N represents a disk that is not defective.

1. If the rate of defective disks is 30%, how could you express the theoretical probability for this outcome using exponents?
2. What do the exponents represent?
3. How many other possible outcomes have exactly 3 defective disks?
4. Do each of these have the same probability as the outcome DNDD? Explain your response.
5. Describe how you could use combinations to count the number of possible outcomes that contain exactly 3 defective disks.
6. Using your responses to Steps **1** and **5** above, describe how to determine the theoretical probability of obtaining exactly 3 defective disks in a random sample of 4 disks.

Mathematics Note

The **binomial formula** can be used to determine the probability of obtaining r successes in n trials in a binomial experiment. Symbolically, the binomial formula can be written as follows, where p is the probability of success in any one trial:

$$P(r \text{ successes in } n \text{ trials}) = C(n,r) \bullet p^r \bullet (1-p)^{n-r}$$

For example, if 25% of a population of computer disks are defective, then $(1-25\%) = 75\%$ are not. The theoretical probability that exactly 4 defective disks will occur in a sample of 10 is:

$$\begin{aligned} P(4 \text{ successes in } 10 \text{ trials}) &= C(10,4) \bullet (0.25)^4 \bullet (0.75)^{10-4} \\ &= 210 \bullet (0.25)^4 \bullet (0.75)^6 \\ &\approx 0.15 \end{aligned}$$

c.

1. In the binomial formula, what does the quantity $(1-p)$ represent? Explain your response.
2. What does the expression $p^r \bullet (1-p)^{n-r}$ represent?

d. If the average rate of defective disks is 40%, describe how to determine the theoretical probability of obtaining each of the following in a random sample of 10 disks:

1. exactly 7 defective disks
2. at least 7 defective disks.

Assignment

2.1 Suppose that 30% of the computer disks manufactured by a company are defective. A single sample of 8 disks, recorded as NNDNDDNN, indicates that 3 disks are defective and 5 disks are not defective.

- **a.** Give an example of a different sample in which 3 of 8 disks are defective.
- **b.** Describe how to determine the probability of the outcome you listed in Part **a**.
- **c.** Use combination notation, $C(n,r)$, to express the number of ways in which exactly 3 defective disks can occur in a sample of 8 disks.
- **d.** Write an expression that represents the probability of obtaining exactly 3 defective disks in a sample of 8 disks.
- **e.** Determine the probability of obtaining 3 defective disks in a sample of 8 disks.

2.2 By modifying the production process (and increasing its production costs), a manufacturer can reduce the rate of defective disks to 5%.

- **a.** What is the probability that a disk selected at random from this population will pass inspection?
- **b.** What is the probability that a disk selected at random from this population will not pass inspection?
- **c.** Consider a sample of 100 disks drawn from this population. In how many ways can 20 defective disks occur in a sample of 100 disks?
- **d.** In how many different ways can a sample of 100 disks contain exactly 80 disks that pass inspection?
- **e.** Determine the probability that a sample of 100 disks consists of 20 defective disks and 80 nondefective disks.

2.3 A consulting firm has determined that an average of 20% of a manufacturer's computer disks are defective.

a. If you select a random sample of 4 disks from this population, what is the theoretical probability that:

1. none of the disks is defective?

2. exactly 1 of the disks is defective?

3. exactly 2 of the disks are defective?

4. exactly 3 of the disks are defective?

5. exactly 4 of the disks are defective?

b. Determine the sum of the probabilities in Part **a**. Explain why this sum occurs.

c. What is the theoretical probability that at least 3 of the disks are defective?

2.4 In a sample of 20 chips from a manufacturing process with a defect rate of 10%, is it possible for all the chips to be defective? If so, determine the probability that this occurs. If not, explain why not.

2.5 When a CD production line is working properly, 90% of the compact discs pass inspection. Determine the theoretical probability of obtaining at least 18 discs that pass inspection in a random sample of 20.

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2.6 A census of the student population at a large school revealed that 75% support the opening of a concession booth at football games. If you select a random sample of 5 students from this population, what is the probability that:

a. the majority of those sampled are in favor of opening the booth?

b. none of those sampled are in favor of opening the booth?

c. all of those sampled are in favor of opening the booth?

d. less than half of those sampled are in favor of opening the booth?

2.7 During basketball season, Clyde makes 80% of his free throws. Assuming this rate continues and that free throws are independent events, what is the probability that Clyde will make:

a. at least 9 of his next 10 free throws?

b. at least 8 of his next 10 free throws?

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Activity 3

As a quality control specialist, it is your job is to determine if a company's quality control procedures are working well. After you have inspected some samples and compiled the results, how do you decide what action to take? In this activity, you investigate how to interpret the results of your inspections.

Mathematics Note

A **probability distribution** is the assignment of probabilities to each possible outcome of an experiment. If the set of outcomes is either finite or can have a one-to-one correspondence with the natural numbers, the distribution is a **discrete distribution**.

For example, Table **1** below shows the discrete probability distribution for the numbers of heads that can occur when flipping two fair coins.

Table 1: A probability distribution

No. of Heads	Theoretical Probability
0	0.25
1	0.5
2	0.25

Exploration

Imagine that you have been hired by Compuquartz Corporation, a computer chip manufacturer, to test their chips for defects. The company believes that an average of 35% of their chips are defective.

a. Table **2** shows the probabilities of finding 0, 1, or 2 defective chips in a sample of 20 chips. Complete this discrete probability distribution table for up to 20 defective chips.

Table 2: Theoretical probability of defective chips

No. of Defective Chips	Theoretical Probability
0	0.0002
1	0.0020
2	0.0100

b. **1.** Use your completed table to create a histogram that shows probabilities versus the numbers of defective chips.

2. Using your histogram, determine the number of defective chips most likely to occur in a sample of 20 chips. Record the probability of this occurrence.

c. In previous modules, you calculated the expected value of an experiment by multiplying each possible outcome by its corresponding probability, then adding all of the resulting products. Use this method to calculate the expected number of defective chips in a sample of 20.

Mathematics Note

The **expected value** of a binomial experiment is the theoretical mean number of successes in n trials.

If a binomial experiment consists of n trials and p is the theoretical probability of success on any trial, then the expected value (or expected number of successes) is $n \bullet p$.

For example, consider an experiment that consists of flipping a fair coin 40 times. If a head is considered a success, the expected number of successes for this experiment is $40 \bullet 0.5 = 20$.

d. Use the formula for the expected value of a binomial experiment described in the mathematics note to calculate the expected number of defective chips in a sample of 20. Compare this value to the one determined in Part **c**.

e. For large sample sizes, the probability that a specific number of defective items will occur tends to be small. For this reason, quality control specialists often focus on the probability that the number of defective items in a sample will fall in a certain interval of values.

Identify the interval that describes each of the following:

1. the numbers of defective chips within 1 of the expected number

2. the numbers of defective chips within 2 of the expected number

3. the numbers of defective chips within 3 of the expected number

4. the numbers of defective chips within 4 of the expected number

f. Determine the probability that the number of defective chips in a sample will fall in each interval you identified in Part **e**.

g. To help interpret the statistics generated by sampling, quality control specialists often use graphs and charts. For example, the chart shown in Figure **3** below shows an **expected region** for the number of defective items in a sample.

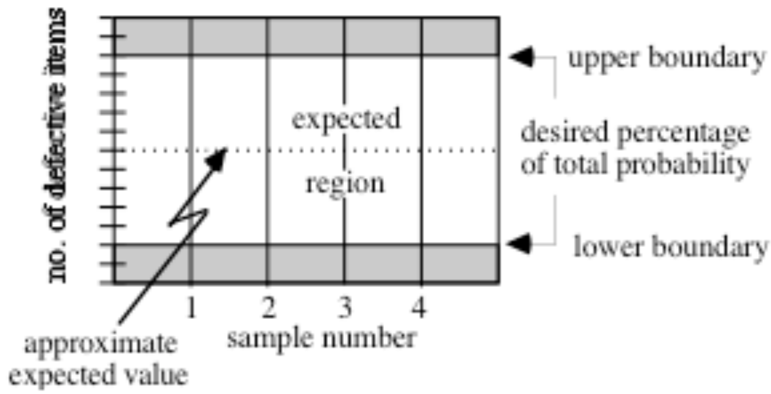

Figure 3: A chart showing an expected region

The boundaries of an expected region enclose a desired percentage of the total probability in an experiment. The dotted line drawn through the middle of the region represents the whole number of defective items closest to the expected value.

1. Which interval from Part **e** encloses approximately 90% of the probability in this experiment?
2. Use this interval, along with the expected number of defective chips, to draw a quality-control chart that shows a 90% expected region. Allow enough room on your chart to plot the results of four samples.

h. In quality control, the number of defective items in a sample is reasonable if the number falls within the expected region. If a sample statistic falls outside the expected region, this may indicate a need to examine the assumed defect rate.

1. Suppose that you take four samples of 20 chips from the population and obtain the following numbers of defective chips: 2, 9, 6, and 12, respectively. Plot this data on your quality-control chart.
2. Based on these results, what recommendations would you make to the company?

i. Compuquartz finds a 35% defect rate in their chips unacceptable. After adjusting their manufacturing process, they obtain a new defect rate of 13%. Repeat Parts **a–h** using the company's new defect rate.

Discussion

a. Describe how you would determine the expected number of defective chips in a sample of size *x* given a defect rate of *y*.

b. Describe any similarities or differences you observe in the probability histograms for the 35% and 13% defect rates. What defect rate do you think would correspond with a perfectly symmetrical histogram?

c. Can you identify the expected value for an experiment by examining its probability histogram? Defend your response.

d. Consider an experiment that involves rolling a single die one time. The probability of each face occurring is $1/6$. There are 6 faces. Explain why the expected value is not $6 \bullet 1/6 = 1$.

e. If the number of chips sampled had been 1000, what would have happened to the probability of obtaining a specific number of defective chips in the probability distribution table?

f. What percentage of the total probability is represented by the shaded regions in Figure **4** below?

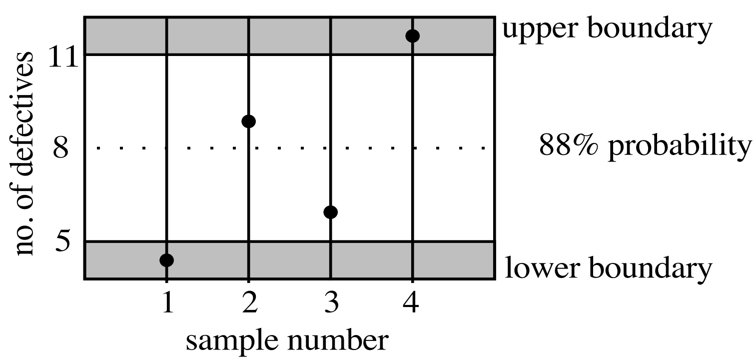

Figure 4: A completed quality-control chart

g. What is indicated by a sample in which the number of defective items falls above the upper boundary of the expected region?

h. What recommendations did you make to Compuquartz about their manufacturing process in Part **i** of the exploration?

Assignment

3.1 As part of its quality-control procedure, a company samples 100 ball bearings to see if their masses fall within an acceptable range.

a. Assuming the population has a 10% defect rate, determine the expected number of defective ball bearings in the sample.

b. Let d be the expected number of defective ball bearings found in Part **a**. Determine the probability that the number of defective ball bearings in the sample will fall in the interval $[d - 1, d + 1]$.

c. Determine the probability that the number of defective ball bearings will fall in the interval $[d - 2, d + 2]$.

d. If your sample contained more than 24 defective ball bearings, would you question the assumed defect rate of 10%? Why or why not?

3.2 Imagine that a manufacturer of coffee filters has hired you to establish a quality control procedure. The manufacturer thinks the defect rate in the manufacturing process is about 20%. You plan to inspect a random sample of 25 filters.

a. Assuming a defect rate of 20%, create a table and histogram showing the probability distribution for defective filters in the sample.

b. Determine the expected number of defective filters in the sample. Mark this value on the histogram.

c. According to your probability distribution, what are the chances that your random sample will have more than 8 defective filters?

d. If your sample contained more than 8 defective filters, would you question the assumed defect rate of 20%? Why or why not? Create a quality control chart to support your claim.

3.3 In a study on brand recognition, a marketing firm determined that one brand of tennis shoes, Rocket Shoes, was recognized by 60% of consumers.

a. Given a sample of 30 people, what is the probability that from 13 to 23 of them will recognize Rocket Shoes?

b. What interval represents an 80% expected region in this situation?

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3.4 While writing an article on women in the workplace, a reporter for the local newspaper discovers the following statistic: According to the *Statistical Abstract of the United States*, 33.2% of the mathematical and computer scientists in 1992 were female. To see how the local situation compares to the national one, the reporter decides to collect data from a sample of 50 local scientists.

a. In a random sample of 50 mathematical and computer scientists, how many would you expect to be female?

b. Find an interval that corresponds to an expected region that contains approximately 90% of the probability in this situation. Create a chart of this region.

c. Use a simulation to generate data for 3 samples of 50 scientists each, assuming that 33.2% of the population of scientists is female. Plot the results of each sample on the chart from Part **b**. Report what these samples indicate.

3.5 As the old saying goes, "one bad apple spoils the whole bushel." Assume that there are 50 apples in a bushel. The probability of any individual apple going bad before the bushel is sold is 0.001. What is the probability that a bushel of apples will spoil before it is sold?

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Summary Assessment

In 1993, a nationally known clothing manufacturer found that only 248,000 of the 303.6 million items it sold were defective.

1. In a shipment of 10,000 items sold by this company in 1993, how many would you expect to be defective?

2. The following table shows an incomplete probability distribution for the number of defective items in a shipment of 10,000.

No. of Defective Items	Probability
0	0.00033439
1	0.00267726
2	0.01071656
⋮	⋮
20	0.00015799

a. Complete the table above by determining the probabilities of obtaining 3, 4, 5, …, 19 defective items.

b. What is the sum of the probabilities for obtaining from 0 to 20 defective items? Explain what this sum indicates.

3. Given a shipment of 10,000 items, what is the probability that the number of defective items falls in each of the following intervals:

a. [0, 4]

b. [20, 10,000]

4. **a.** Create a histogram that shows the probabilities of finding from 1 to 20 defective items in a shipment of 10,000.

b. What numbers of defective items appear to be most probable?

c. Is there a chance that 1% of the shipment of 10,000 items could be defective? Explain your response.

5. Considering a sample of 10,000 items, how many defective items would indicate to you that the company should examine its manufacturing process? Defend your response.

Module Summary

- All the members of a group can be referred to as a **population**. A **sample** is a subset of a population. Typically, a sample includes only some members of a population, not all of them.
- A **parameter** is a numerical characteristic of a population. A **statistic** is a numerical characteristic of a sample. Statistics are used to estimate the corresponding parameters of the population.
- **Sampling** is the process of choosing a subset of a population.
- In a **simple random sample**, each member of the population must have the same chance of being included in the sample.
- **Conditional probability** is the probability of an event occurring given that an initial event has already occurred. The probability that event B occurs, given that event A has already occurred, is denoted $P(\text{B}|\text{A})$.

  In an experiment involving conditional probabilities, the probability of both A and B occurring is found by multiplying the probability of A by the conditional probability of B given A has already occurred:

  $$P(\text{A and B}) = P(\text{A}) \bullet P(\text{B}|\text{A})$$
- Two events are **mutually exclusive** if they cannot occur at the same time in a single trial. If A and B are mutually exclusive events, then $P(\text{A and B}) = 0$.
- If two events are mutually exclusive, then the probability that one or the other occurs is the sum of the probabilities of the individual events. This can be written symbolically as follows: $P(\text{A or B}) = P(\text{A}) + P(\text{B})$.
- Two events A and B are **independent** if $P(\text{A and B}) = P(\text{A}) \bullet P(\text{B})$. Three events A, B, and C are independent if each pair of events is independent and $P(\text{A and B and C}) = P(\text{A}) \bullet P(\text{B}) \bullet P(\text{C})$. This definition can be extended to any number of independent events.
- A **binomial experiment** has the following characteristics:
  1. It consists of a fixed number of repetitions of the same action. Each repetition is a **trial**.
  2. The trials are independent of each other. In other words, the result of one trial does not influence the result of any other trial in the experiment.
  3. Each trial has only two possible outcomes: a success or a failure.
  4. The probability of a success remains the same from trial to trial.
  5. The total number of successes is observed.

- The **binomial formula** can be used to determine the probability of obtaining r successes in n trials in a binomial experiment. Symbolically, the binomial formula can be written as follows, where p is the probability of success in any one trial:

$$P(r \text{ successes in } n \text{ trials}) = C(n, r) \bullet p^r \bullet (1 - p)^{n-r}$$

- The **expected value** of a binomial experiment is the theoretical mean number of successes in n trials. If a binomial experiment consists of n trials and p is the theoretical probability of success on any trial, then the expected value (or expected number of successes) is $n \bullet p$.
- A **probability distribution** is the assignment of probabilities to each possible outcome of an experiment. If the set of outcomes is either finite or can have a one-to-one correspondence with the natural numbers, the distribution is a **discrete distribution**.

Selected References

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