Graphing the Distance

Scientists and mathematicians often use graphs to help predict the outcomes of real-life situations from experimental results. In this module, you'll examine the use of graphs in modeling motion.

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Introduction

Have you ever wondered how scientists at the National Aeronautics and Space Administration (NASA) plan the launch of a space shuttle? Or how traffic officers reconstruct the scene of an accident? In this module, you investigate objects in motion and the relationship between distance traveled and time.

Activity 1

One of the tools scientists use to analyze motion is a **distance-time graph**. A distance-time graph displays the distance between two objects (or an object and a fixed point) as a function of time. Typically, time is represented on the *x*-axis and distance on the *y*-axis. By comparing the distance-time graphs of different kinds of motion, you can observe many interesting and useful patterns.

Exploration

A **sonar range finder** uses sound waves to measure the distance from itself to another object. In this exploration, you use a sonar range finder to collect data, then use that data to create distance-time graphs.

a. Connect a sonar range finder to a science interface device as demonstrated by your teacher. As shown in Figure **1**, hold the range finder parallel to the plane of a wall or other flat surface. As a person holding the range finder moves, the science interface device collects data. This data can then be transferred to a graphing utility.

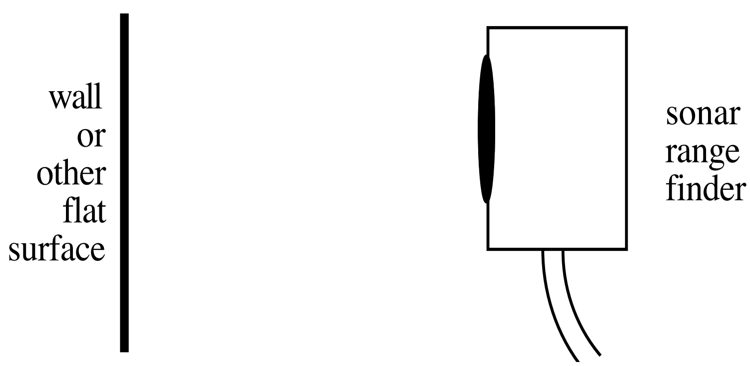
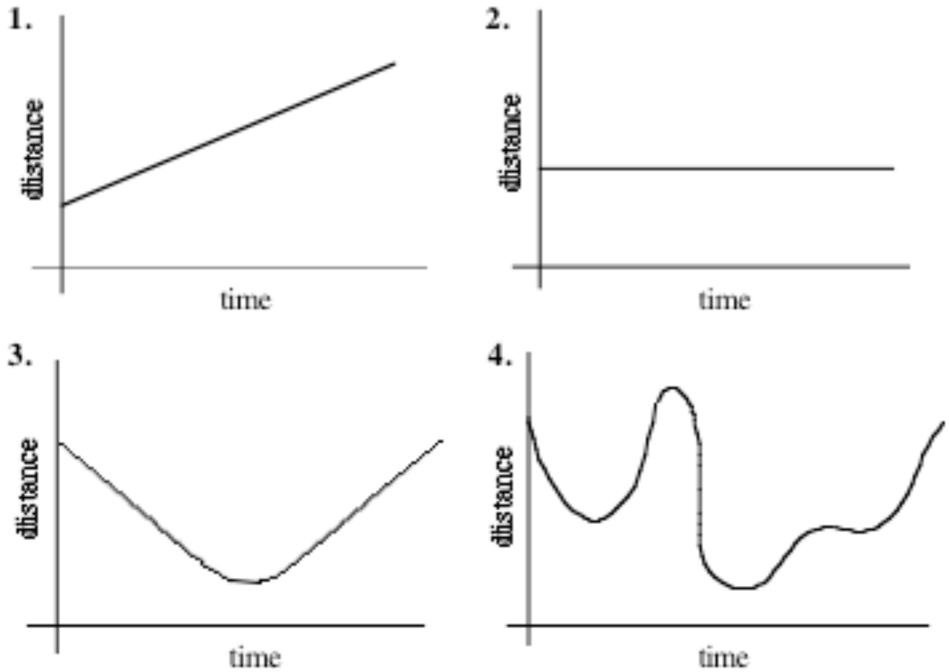


Figure 1: Positioning the sonar range finder

b. Practice using a range finder, science interface, and graphing utility to generate distance-time graphs. Move the range finder toward the wall, then away from it, and observe the resulting graphs. Draw one of the graphs on a sheet of graph paper.

c. Use the range finder to create distance-time graphs that match the ones shown below. (This may take a few trials.) Record the method you used to create each graph.



d. Figure **2** below shows a distance-time graph of data collected during the launch of a model rocket, where the distance is the rocket's height above the ground. After the rocket's engine ignited, it flew straight up. A few seconds after the engine burned out, it began to fall straight back toward the ground. Later, its parachute opened and slowed its descent.

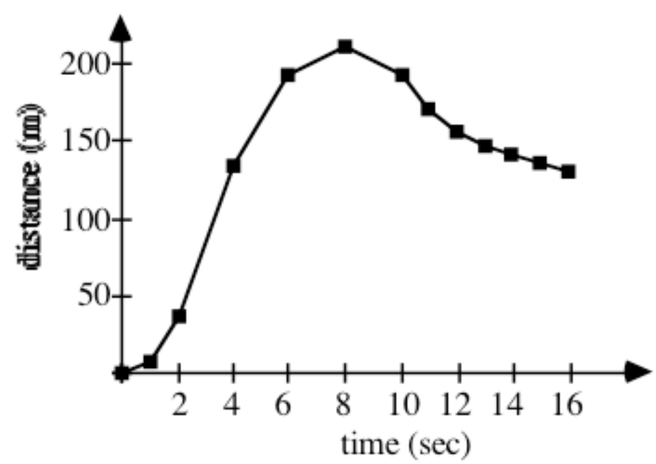


Figure 2: Distance-time graph for a model rocket

Point the range finder at the floor. By raising and lowering the range finder along a vertical path, create a distance-time graph whose shape resembles the graph in Figure **2**.

Discussion

a. In Part **b** of the exploration, you sketched a scatterplot on a sheet of paper. What do the points on the graph represent?

b. On a distance-time graph, what do the x- and y-intercepts represent?

c. How does the motion of the range finder in Part **b** of the exploration affect the resulting distance-time graph?

d. Describe how you moved the range finder to obtain each of the graphs in Part **c** of the exploration.

e. Figure **3** shows a distance-time graph generated by moving a range finder toward and away from a wall. Describe what is happening to the range finder at points P, Q, and R.

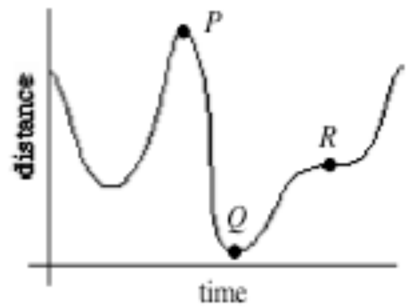


Figure 3: A distance-time graph

f. How does the kind of motion represented by a "curved" section of a distance-time graph differ from the motion represented by a "straight" section?

g. The average speed of an object during a time interval can be calculated by dividing distance traveled by time.

Table 1: Distance-time data for a model rocket

Time (sec)	Distance (m)	Time (sec)	Distance (m)
0	0	11	170.8
1	7.4	12	155.4
2	36.9	13	147.7
4	134.2	14	141.9
6	192.9	15	136.3
8	212.3	16	130.8
10	192.9		

1. Using the data from Table **1**, determine the total distance traveled by the model rocket during the interval [6, 12].
2. What is the rocket's average speed during the interval [6, 12]?

h. Given the current location of an object moving at constant speed, what information would you need to predict the object's location in the future?

Science Note

Displacement is a change in the position of an object. It has both magnitude and direction.

For example, consider the distance-time data for the model rocket in Table **1**. At $t = 2$ sec, the rocket is 36.9 m above the ground. At $t = 6$ sec, it is 192.9 m above the ground. Its displacement during this time is $192.9 - 36.9 = 156.0$ m.

From $t = 6$ sec to $t = 12$ sec, however, the rocket's position changes from 192.9 m above the ground to 155.4 m above the ground. Its displacement over this period is $155.4 - 192.9 = -37.5$ m. In this case, positive values for displacement indicate movement away from the ground, while negative values indicate movement toward the ground.

i. **1.** Using the information in Table **1**, determine the displacement of the model rocket during the time interval [6, 12].

2. Compare this displacement to the distance you determined in Part **g** of the discussion.

Science Note

Velocity is the rate of change in position with respect to time. In other words, an object's velocity is its speed in a specific direction.

The **average velocity** of an object can be calculated by dividing its displacement by the change in time. For example, the model rocket's average velocity between $t = 2$ sec and $t = 6$ sec can be found as follows:

$$\frac{192.3 \text{ m} - 36.9 \text{ m}}{6 \text{ sec} - 2 \text{ sec}} = \frac{156.0 \text{ m}}{4 \text{ sec}} = 39 \text{ m/sec}$$

j. **1.** Use the data in Table **1** to determine the rocket's average velocity during the time interval [6, 12].

2. Compare this average velocity to the average speed you determined in Part **g** of the discussion.

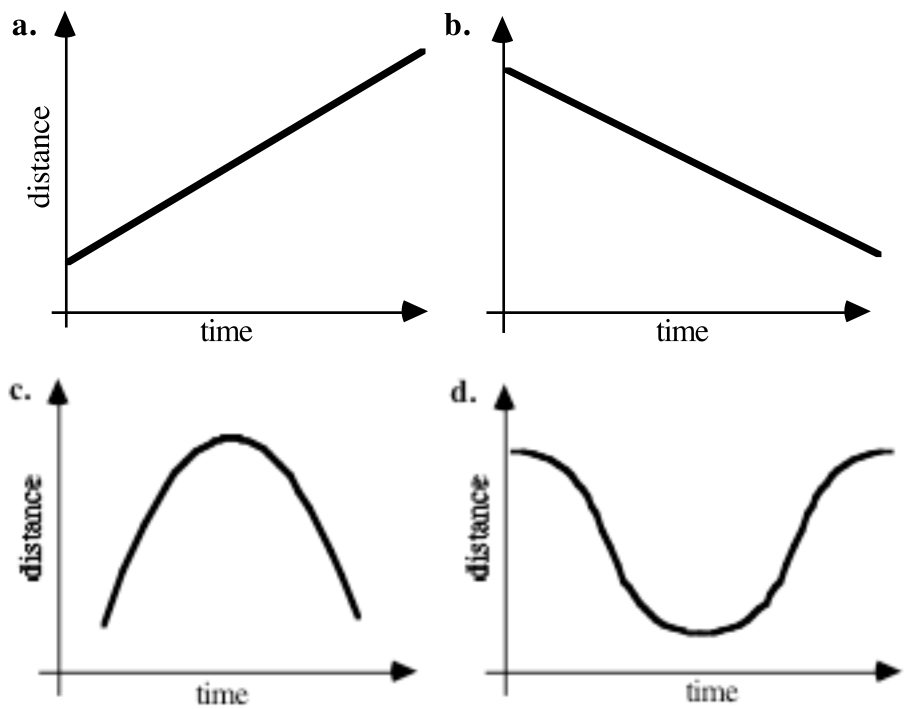
k. Using only your graphs from Parts **c** and **d** of the exploration, how can you tell when the range finder was moving at the greatest velocity?

l. What does a negative value for average velocity indicate about the motion of the rocket?

m. The **instantaneous velocity** of an object is its velocity at a particular instant in time. Describe how you could approximate the instantaneous velocity of the rocket at $t = 11$ sec.

Assignment

1.1 Describe a real-world motion that could be represented by each of the distance-time graphs below.



1.2 Sketch a copy of the distance-time graph in Figure **2.** On your copy, indicate the points at which you think the following events occurred:

a. the rocket takes off

b. the rocket's engine burns out

c. the rocket reaches its highest altitude

d. the parachute opens.

1.3 The table below contains data collected during the flight of a model rocket. Use the table to complete Parts **a–c.**

Time (sec)	Distance (m)	Time (sec)	Distance (m)
0	0	11	170.8
1	7.4	12	155.4
2	36.9	13	147.7
4	134.2	14	141.9
6	192.9	15	136.3
8	212.3	16	130.8
10	192.9		

a. Determine the rocket's average speed during the time interval [6, 10].

b. Determine the average velocity of the rocket during the same interval.

c. Explain why your responses to Parts **a** and **b** are different.

1.4 **a.** Determine the average velocity of the rocket in Problem **1.3** during the interval from $t = 6$ sec to $t = 8$ sec.

b. Estimate the instantaneous velocity of the rocket at $t = 3$ sec. Describe how you determined your estimate.

1.5 The table below shows a space shuttle's distance from earth at various times during its initial vertical ascent.

Time (sec)	Distance (m)	Time (sec)	Distance (m)
24	1791	136	53,355
48	7274	160	66,809
72	15,539	184	78,374
96	27,920	208	88,117
120	43,326		

Source: Johnson Space Center, Houston, Texas.

a. Create a distance-time graph of this data.

b. Based on this data, over what time interval does the space shuttle appear to start slowing down? Justify your response.

c. What is the average velocity of the space shuttle from $t = 24$ sec to $t = 120$ sec?

d. **1.** Estimate the shuttle's instantaneous velocity, in meters per second, 195 sec after liftoff and describe how you determined your estimate.

2. Express your response to Step **1** in kilometers per hour.

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1.6 Sketch a distance-time graph that illustrates the motion of Little Red Riding Hood in the following paragraph:

> Little Red Riding Hood left home and walked briskly down the road towards Grandmother's house. Along the way, she stopped to pick some violets growing beside the road. The Wolf saw her picking flowers and offered to show her a shortcut. He led Little Red Riding Hood on a winding path through the woods. After the path crossed the road for the third time at the place where she had picked the flowers, Little Red Riding Hood realized that she'd been tricked. She got back on the road and ran the rest of the way to Grandmother's house.

1.7 The table below shows the distances between an object and a fixed point over time.

Time (sec)	Distance (m)	Time (sec)	Distance (m)
1	2	14	303
2	5	15	303
3	33	16	313
4	55	17	323
5	94	18	333
6	160	19	104
7	273	20	41
8	283	21	20
9	293	22	11
10	303	23	7
11	303	24	5
12	303	25	3
13	303		

a. Create a distance-time graph of this data.

b. Which ordered pair (t, d), where d represents distance and t represents time, corresponds to the moment when the object first stopped moving away from the fixed point? Explain your response.

c. **1.** When did the object start moving back toward the fixed point?

2. How did its velocity change at this time?

d. Calculate the average velocity of the object during the time interval [4, 10]. What does this value tell you about the object's motion?

e. Calculate the average velocity of the object during the time interval [18, 23]. What does this value tell you about the object's motion?

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Activity 2

In the months before each launch, NASA engineers determine a space shuttle's longitude, latitude, altitude, and weight for every 0.04 sec of the flight. How are they able to predict these values with such accuracy and confidence?

At least some of the credit for scientists' ability to make such predictions must go to the English mathematician and physicist, Sir Isaac Newton (1642–1727). Using three concise laws of motion, Newton described the rules that govern the movement of objects both on earth and in space.

Science Note

A **force** is a physical quantity that can affect the motion of an object. Two familiar forces are gravity and friction.

According to Newton's **first law of motion**, an object in a state of rest or moving in a straight line at a constant speed will continue in that state unless acted on by a force.

Exploration

When the distance between an object and a fixed point changes at a constant rate, the distance-time graph can be modeled by a linear equation. In this exploration, you use a range finder to explore the movement of a ball at a constant velocity.

a. Obtain a track, a ball, and the range-finder apparatus from your teacher. As shown in Figure **4**, place the track on a level surface and position the range finder at one end. Place the ball on the track approximately 0.5 m from the range finder.

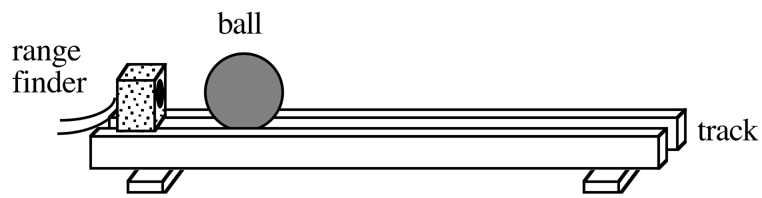

Figure 4: Ball on track with range finder

b. Push the ball away from the range finder just hard enough so that it rolls to the end of the track. Collect distance-time data as the ball rolls.

c. Repeat Part **b** several times, increasing the force of the initial push each time. Observe how changing the ball's speed affects the resulting distance-time graphs.

d. Select a data set and graph from Part **c** that appears to accurately describe the motion of the ball. Determine the average velocity of the ball over the time interval represented by the graph.

e. In the Level 2 module "If the Shoe Fits . . .," you used technology to find a linear regression equation to model data. Determine a linear regression equation that models the graph you chose in Part **d**. **Note:** Save your data, graph, and equation for use in the assignment.

Discussion

a. How did the ball's speed affect the graphs in Part **c** of the exploration?

b. In Parts **d** and **e** of the exploration, you found the ball's average velocity and determined a linear function to model its distance-time data.

1. What does the slope of the line indicate about the ball's movement?
2. What does the y-intercept of the line represent?
3. Should the line pass through the origin? Why or why not?

c. If you placed the ball at the far end of the track and pushed it toward the range finder, what would the resulting distance-time graph look like?

d.
1. How can you tell if an equation provides a good model of a distance-time graph?
2. If the function that models the ball's distance-time data is written in the form $f(x) = mx + b$, which part of the equation represents the ball's average velocity? Explain your response.
3. How could you determine the ball's instantaneous velocity in this situation?

e. Suppose that after collecting distance-time data for a ball on a ramp, the resulting graph can be modeled by a function of the form $d(t) = c$, where c is a constant. Describe the motion of the ball.

f. Describe some real-world situations that could be modeled with linear graphs of distance versus time.

Assignment

2.1 Describe how distance, velocity, and time are related when a ball is moving along a straight track at a constant rate.

2.2 Use the linear equation you found in Part **e** of the exploration to answer the following questions.

a. How far was the ball from the range finder after 1.7 sec?

b. When was the ball 0.75 m away from the range finder?

c.
1. If the track were long enough, how far from the range finder would the ball be after 10 min?
2. Do you think your prediction is accurate? Why or why not?

2.3 The following equation describes the motion of a ball on a level track, where $d(t)$ represents distance in meters from a range finder and t represents time in seconds:

$$d(t) = 0.75t + 0.5$$

a. Make a table showing the ball's distance from the range finder after 0 sec, 1 sec, 2 sec, and 3 sec.

b. Add a column to your table that shows the average velocity of the ball during the following time intervals: [0, 1], [1, 2], and [2, 3].

c. What is the instantaneous velocity of the ball at $t = 2$ sec?

d. How do your responses to Parts **b** and **c** relate to the equation that describes the ball's motion?

2.4 Describe a function that could be used to model the distance-time graph of each of the following:

a. a ball that is not moving

b. a ball moving at a constant velocity of 1 m/sec.

2.5 The table below shows a space shuttle's distance from earth at some specific times after liftoff.

Time (sec)	Distance (m)	Time (sec)	Distance (m)
72	15,539	136	53,355
96	27,920	160	66,809
120	43,326	184	78,374

Source: Johnson Space Center, Houston, Texas.

a. Create a distance-time graph of this data.

b. **1.** Determine a linear equation that closely models the data.

2. What is the average velocity of the shuttle during the interval [72, 184]?

c. Use the equation you found in Part **b** to predict the shuttle's altitude after 520 sec.

d. Would it be reasonable to use this linear model to predict the shuttle's altitude at any time during its flight? Explain your response.

2.6 The table below shows some distance-time data collected as a model rocket returned to the ground under its parachute.

Time (sec)	Distance (m)	Time (sec)	Distance (m)
12	155.4	15	136.3
13	147.7	16	130.8
14	141.9	17	125.2

a. Determine an equation that closely models the data.

b. Use your equation to predict when the rocket will reach the ground.

c. Do you think your prediction is reasonable? Why or why not?

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2.7 Each of the linear equations below models a distance-time data set collected using a range finder:

1. $d(t) = 3.5t + 1.3$
2. $d(t) = -1.2t + 1.3$
3. $d(t) = 3.0t + 1.3$
4. $d(t) = 3.5t + 2.0$
5. $d(t) = 1.2t + 2.0$

a. Which equations represent objects moving at the same average velocity?

b. Which equations represent objects moving at the same average speed?

c. Which equation(s) represents the object which is moving the fastest?

d. Which equation(s) represents the object which is moving the slowest?

e. Which equation(s) corresponds to the object that started nearest to the range finder? farthest from the range finder?

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Activity 3

In the previous activity, you examined distance-time graphs of a ball moving at a constant speed. These graphs could be modeled by linear functions. But how would you model a distance-time graph for an object whose speed was increasing or decreasing? In this activity, you investigate **polynomial functions**, a group of functions that can provide models for other types of motion.

Mathematics Note

A **polynomial** in one variable, x, is an algebraic expression of the general form:

$$a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x^1 + a_0$$

where n is a whole number and the **coefficients** a_i are real numbers for $i = 0,\ 1,\ 2,\ \ldots,\ n$.

The **degree** of a polynomial is equal to the greatest exponent of the variable in the expression with a non-zero coefficient. A polynomial in the general form shown above has a degree of n, provided that $a_n \neq 0$.

For example, the expression $-5x^2 + 3x - 7$ is a polynomial of degree 2. The expression $7x^3 - \sqrt{3}x^2 + 6x - 0.25$ is a third-degree polynomial.

A function f is a **polynomial function** if $f(x)$ is defined as a polynomial in x. The degree of a polynomial function is the same as the polynomial it contains.

For example, the function $f(x) = -5x^2 + 3x - 7$ is a second-degree polynomial function.

Discussion 1

a. Are linear functions of the forms $f(x) = mx + b$ and $f(x) = c$ also polynomial functions? If so, identify the degree of each. If not, explain why not.

b. If you coasted on a bicycle from the top of a hill to the bottom, would you expect a distance-time graph of your movement to be linear? Explain your response.

c. Distance-time graphs of the motion described in Part **b** can be modeled by second-degree polynomial functions, also known as **quadratic functions**. The graph of a quadratic function is a **parabola**. For example, Figure **5** below shows the graphs of two quadratic functions.

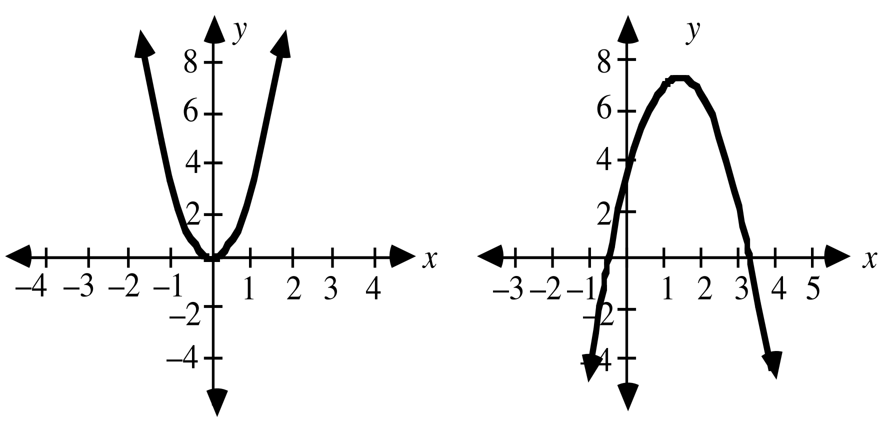


Figure 5: Graphs of two quadratic functions

1. Considering the general form of a polynomial described in the previous mathematics note, what do you think would be the general form of a quadratic function?

2. A parabola is symmetric about a line, known as its **axis of symmetry**. Describe the location of this axis of symmetry.

3. When a parabola opens upward, its **vertex** occurs at the lowest point in the graph; when a parabola opens downward, its vertex occurs at the highest point. Describe how you could locate the vertex of a parabola using the axis of symmetry.

d. A third-degree polynomial function is also known as a **cubic function**. Figure **6** below shows the graphs of two cubic functions.

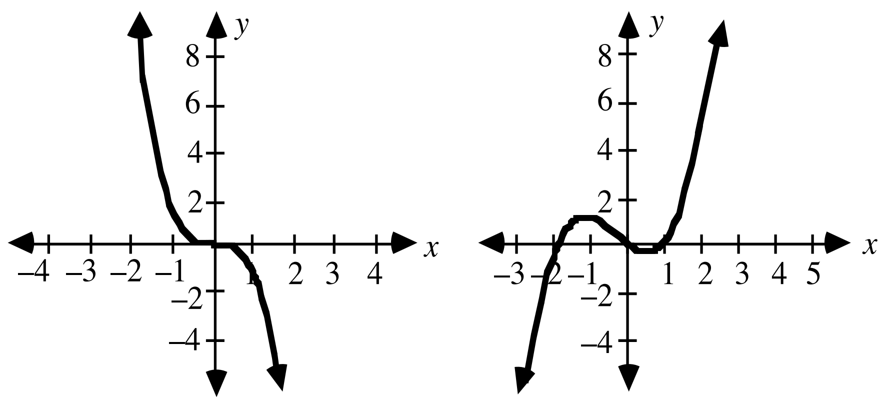


Figure 6: Graphs of two cubic equations

1. Describe the general form of a cubic function.

2. Does the graph of a cubic function appear to be symmetric about a line? Explain your response.

Exploration

Quadratic functions also can be written in the form $f(x) = a(x - c)^2 + d$, where a, c, and d are constants. In the following exploration, you investigate how the values of these constants affect the graph of the functions.

a. Use a graphing utility to create a graph of the quadratic function $f(x) = x^2$.

b. Sketch a copy of the graph on a sheet of graph paper.

c. On the same coordinate system as in Part **b**, sketch the image that results when the graph of $f(x) = x^2$ is translated 1 unit to the right.

d. Use a graphing utility to determine which of the following functions represents the graph you sketched in Part **c**.

1. $f(x) = x^2 + 1$

2. $f(x) = x^2 - 1$

3. $f(x) = (x - 1)^2$

4. $f(x) = (x + 1)^2$

e. Predict the equation of the function that results when $f(x) = x^2$ is translated 2 units to the left.

Verify your prediction using a graphing utility.

f. Compare the graphs of each of the following pairs of functions:

1. $f(x) = x^2$ and $f(x) = -x^2$

2. $f(x) = x^2$ and $f(x) = 3x^2$

3. $f(x) = x^2$ and $f(x) = \frac{1}{3}x^2$

g. Use a graphing utility to compare the graph of the function $f(x) = x^2$ to the graph of each of the following. In each case, experiment with both negative and positive values for the constant. Record your observations.

1. $f(x) = x^2 + d$

2. $f(x) = (x - c)^2$

3. $f(x) = ax^2$

h. Repeat the exploration for a cubic function. To do this, replace the exponent 2 with the exponent 3 in each of the functions in Parts **a–g**.

Discussion 2

a. How does the value of a appear to affect the graphs of functions of the form $f(x) = ax^2$ and $g(x) = ax^3$?

b. How does the value of c appear to affect the graphs of functions of the form $f(x) = (x - c)^2$ and $g(x) = (x - c)^3$?

c. How does the value of d appear to affect the graphs of functions of the form $f(x) = x^2 + d$ and $g(x) = x^3 + d$?

d. Describe how you could use the symmetry of a parabola to determine the translation of the graph of $f(x) = x^2$ that results in the graph of a given quadratic function.

e. Compare the graph of the function $f(x) = -0.5(x - 3)^2 + 4$ to the graph of $f(x) = x^2$.

f. Figure **7** below shows a scatterplot of the data in the table on the left.

What quadratic function would you use to model this data? Justify your response.

x	y
–4	1
–3	2
–2	1
–1	–2
0	–7
1	–14
2	–23
3	–34
4	–47

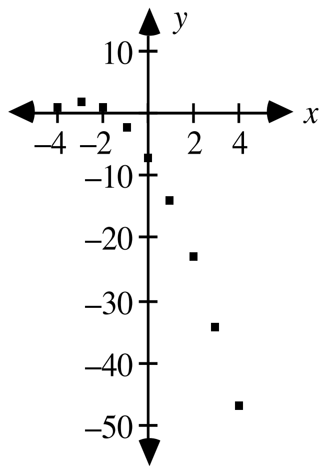


Figure 7: Data and scatterplot

g. Describe how you could rewrite a quadratic function of the form $f(x) = a(x - c)^2 + d$ in the general form $f(x) = a_2x^2 + a_1x + a_0$.

h. Is the expression shown below a polynomial? Justify your response.

$$\frac{x^2 - 1}{x - 1}$$

Assignment

3.1 **a.** What is the degree of the following polynomial: $3x^7 + 1$?

b. In the polynomial in Part **a**, what are the coefficients of each of the following terms: x^7, x^6, x^5, x^4, x^3, x^2, x^1, and x^0?

c. What is the degree of the polynomial below?

$$\frac{x^2}{1000} + 10^6$$

d. Is 6 a polynomial? If so, identify its degree. If not, explain why not.

3.2 Which of the following expressions are polynomials? Justify your response.

a. $\sqrt{x}$

b. $x^{-3} + 6$

c. $x(x + 1)(x + 2)$

3.3 Determine a function of the form $f(x) = a(x - c)^2 + d$ that represents each of the following transformations of the function $f(x) = x^2$. Use graphs to support your responses.

a. a translation 3.5 units to the right

b. a reflection in the x-axis

c. a vertical "stretch"

3.4 Determine a function of the form $g(x) = a(x - c)^3 + d$ that represents each of the following transformations of the function $g(x) = x^3$. Use graphs to support your responses.

a. a translation $2\frac{2}{3}$ units to the left

b. a translation 4.2 units upward

c. a vertical "shrink"

3.5 **a.** Write the function whose graph results in the following transformations of the graph of $f(x) = x^2$: a translation 3 units to the right, a translation 2 units down, and a reflection in the x-axis.

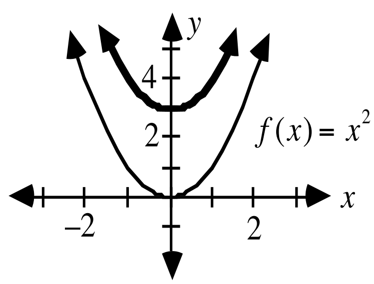
b. To verify your response, graph $f(x) = x^2$ and the function from Part **a** on the same coordinate system.

c. Rewrite the function in Part **a** in the general form of a polynomial.

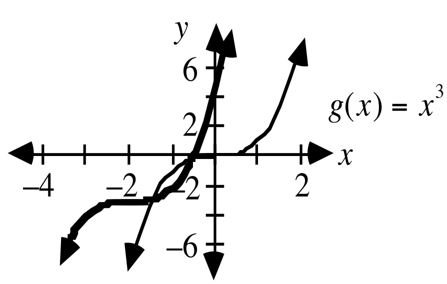
d. Demonstrate that the functions found in Parts **a** and **c** are equivalent.

3.6 In Parts **a–c** below, determine a function of the form $f(x) = a(x - c)^2 + d$ or $g(x) = a(x - c)^3 + d$ whose graph is represented by the bold curve.

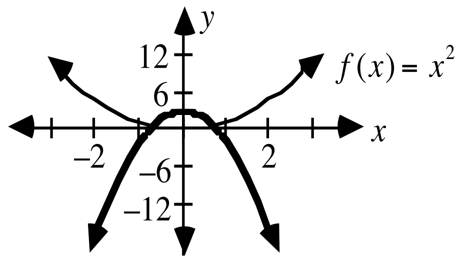
a. Hint: The point (1,4) on the graph is the image of the point (1,1).



b. Hint: The point (–2,–3) on the graph is the image of the point (0,0).



c. Hint: The vertex is at (0,3) and the point (–2,–11) is the image of the point (–2,4).



3.7 The distance-time data in the table below can be modeled by a quadratic function.

Time (sec)	Distance (m)	Time (sec)	Distance (m)
0.0	3.0	0.8	7.8
0.2	4.8	1.0	8.0
0.4	6.2	1.2	7.8
0.6	7.2	1.4	7.2

a. Create a scatterplot of the data.

b. Determine a quadratic function that models the data and graph it on the same coordinate system as in Part **a**.

c. Write the function in Part **b** in the general form of a polynomial.

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3.8 **a.** Write the function whose graph results in a translation 2 units to the left, a translation 6 units upward, and a reflection in the *x*-axis of the function $g(x) = x^3$.

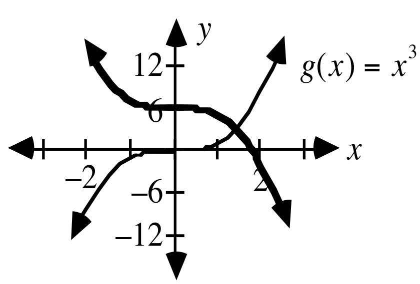
b. To verify your response, graph $g(x) = x^3$ and the function from Part **a** on the same coordinate system.

c. Rewrite the function in Part **a** in the general form of a polynomial.

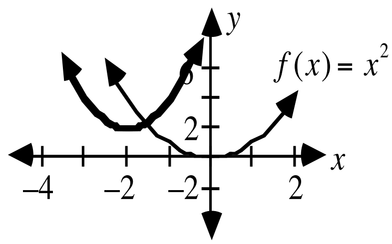
d. Demonstrate that the functions found in Parts **a** and **c** are equivalent.

3.9 In Parts **a** and **b** below, determine a function of the form $f(x) = a(x-c)^2 + d$ or $g(x) = a(x-c)^3 + d$ whose graph is represented by the bold curve.

a. Hint: The point (–1,7) is the image of point (1,1).



b. Hint: The point (–1,3) is the image of point (1,1).



3.10 The data in the table below can be modeled by a cubic function.

x	$f(x)$	x	$f(x)$
0.0	7.00	0.8	2.04
0.2	4.56	1.0	2.00
0.4	3.08	1.2	1.96
0.6	2.32	1.4	1.68

a. Create a scatterplot of the data.

b. Determine a cubic function that models the data and graph it on the same coordinate system as in Part **a**.

c. Rewrite the function in Part **b** in the general form of a polynomial.

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Activity 4

According to legend, Isaac Newton "discovered" gravity after watching an apple fall from a tree. In this activity, you explore how the **acceleration** due to gravity affects the distance-time graphs of freely falling objects.

Science Note

Acceleration is the rate of change in velocity with respect to time.

For example, consider a car driving along a straight section of highway. Over time, the velocity of the car can increase, decrease, or remain the same. When the car's velocity increases, its acceleration is positive. When the car's velocity decreases, its acceleration is negative. If the car's velocity remains constant, its acceleration is 0.

The average acceleration of an object over a particular time interval can be determined by dividing the change in velocity by the change in time. For example, consider a model rocket launched straight into the air. At $t = 3$ sec, its velocity is 48.65 m/sec. At $t = 5$ sec, its velocity is 29.33 m/sec. The rocket's average acceleration during this period can be estimated as follows:

$$\frac{29.33 \text{ m/sec} - 48.65 \text{ m/sec}}{5 \text{ sec} - 3 \text{ sec}} = -9.65 \text{ m/sec}^2$$

This means that, during the time interval [3, 5], the rocket's velocity decreased by an average of 9.65 m/sec for every second that passed.

Discussion 1

a. When you rolled a ball along a level track in Activity **2**, its velocity remained almost constant over time. If one end of the track were raised, and the ball rolled down the incline, do you think that its velocity also would remain constant? Explain your response.

b. As the ball continues down the track, how would the distances traveled in equal time intervals compare?

c. What do you think a graph of the distance-time data collected for a ball rolling down an inclined track will look like?

Exploration

In this exploration, you collect distance-time data for a ball rolling down an incline. You then use polynomial functions to model this data.

a. Obtain a track, a ball, and the range-finder apparatus from your teacher. Set up the track and range finder as in Activity **2**, then use books or blocks to raise the end of the track with the range finder on it.

b. Place the ball approximately 0.5 m from the range finder and gently release it. (Do not push the ball down the track.) You should begin collecting distance-time data just before the ball is released.

c. Repeat Part **b** several times, then select a data set and graph that you think accurately describes the motion of the ball down the track.

d.
1. Edit your data set so that it contains only information collected as the ball was actually moving, beginning with the moment of its release.
2. Determine a quadratic equation that appears to model this data set.

e. Recall that a **residual** is the difference between an observed value and the corresponding value predicted by a model, and that the sum of the squares of the residuals can be used to evaluate how well a model fits a data set.

Calculate the sum of the squares of the residuals for your model. Adjust the equation until the sum of the squares of the residuals indicates that the model closely approximates the data. **Note:** Save your data set, graph, and equation for use later in the module.

f. In Activity **2**, you used technology to generate linear models called linear regression equations. Many calculators and computer software programs also can generate other types of regression equations, including exponential, power, and polynomial regressions.

Use technology to determine a quadratic regression equation for your data set in Part **d**.

g. If the track were extremely steep, the ball's motion would be virtually a **free fall**. In physics, the term *free fall* refers to an object falling without air resistance and affected only by the force of gravity.

Remove the range finder from the track and point it at the floor from a height of approximately 60 cm. Hold the ball directly beneath the range finder at a height of about 10 cm. Release the ball.

You should begin collecting distance-time data just before the ball is released and stop after the ball hits the ground. Repeat this experiment several times, then select a data set and graph that you think accurately describes the motion of the ball.

h. Repeat Parts **d–f** using the data for the falling ball.

Discussion 2

a.
1. How did raising one end of the track affect the speed of the ball over time?
2. How is this effect displayed on the distance-time graphs?

b.
1. Rewrite the quadratic functions you found in Parts **d** and **h** of the exploration in the general form of a polynomial function.
2. Compare each of these equations to the quadratic regression equation for the same data set.

c. How do the distance-time graphs and equations you found in this exploration compare with those you used to model a ball rolling on a level track in Activity **2**?

d. Use your graphs from this exploration and the one in Activity **2** to answer the following questions.

1. Describe the shape of a distance-time graph when an object's acceleration is 0.
2. What influence does an object's acceleration have on the shape of its distance-time graph?
3. How does the magnitude of the acceleration affect the equations used to model the distance-time data?

Science Note

The acceleration due to gravity is a constant typically denoted by g. On earth's surface, the acceleration due to gravity is about 9.8 m/sec^2 in a direction toward the earth's center. For comparison, the acceleration due to gravity on the moon's surface is about 1.6 m/sec^2.

When an object is acted on only by gravity, its distance from the ground is described by the following function:

$$d(t) = -\frac{1}{2}gt^2 + v_0 t + d_0$$

where $d(t)$ represents the object's distance from the ground after t sec, g is the acceleration due to gravity, v_0 is the object's velocity in the vertical direction at $t = 0$, and d_0 is the object's distance above the ground at $t = 0$.

For example, consider a tennis ball dropped from a height of 10 m. Since the ball is dropped and not thrown, its initial velocity in the vertical direction is 0, or $v_0 = 0$. Since its initial distance above the ground is 10 m, $d_0 = 10$. On earth, the value of g is about 9.8 m/sec^2. To calculate the ball's height above the ground after 1 sec, these values can be substituted into the equation for $d(t)$ as follows:

$$\begin{aligned} d(1 \text{ sec}) &= -\frac{1}{2}\left(9.8 \text{ m/sec}^2\right)(1 \text{ sec})^2 + (0 \text{ m/sec})(1 \text{ sec}) + 10 \text{ m} \\ &= -4.9 \text{ m} + 0 \text{ m} + 10 \text{ m} \\ &= 5.1 \text{ m} \end{aligned}$$

e.

1. Using the general formula described in the previous science note, write a quadratic function that should describe the distance from the ground over time of the falling ball in Part **h** of the exploration.
2. Compare this function to the ones you determined in the exploration. Why do you think there are differences in these equations?

Assignment

4.1 The table below shows the space shuttle's distance above the earth at various times after liftoff.

Time (sec)	Distance (m)	Time (sec)	Distance (m)
0	0	72	15,539
24	1791	96	27,920
48	7274		

Source: Johnson Space Center, Houston, Texas.

a. Create a distance-time graph of this data.

b. Find an equation that closely models the data. Graph this equation on the same coordinate system as in Part **a**.

c. Use your model to estimate the shuttle's distance above the earth at each of the following times:

1. 50 sec

2. 600 sec

d. This shuttle will orbit the earth at an altitude of approximately 160 km. Given this fact, do your predictions in Part **c** seem reasonable? Explain your response.

e. Determine the approximate number of seconds required for the shuttle to reach an altitude of 160 km.

4.2 The data in the following table was generated using a ball, a ramp, and a range finder.

Time (sec)	Distance (m)	Time (sec)	Distance (m)
0	0.022	1.0	0.426
0.2	0.022	1.2	0.733
0.4	0.022	1.4	1.124
0.6	0.071	1.6	1.600
0.8	0.206	1.8	2.164

a. Determine an equation that models the data collected while the ball was rolling.

b. Predict the ball's position 2 sec after its release.

c. If the ramp were long enough, how long would it take the ball to reach a position 4 m from the range finder?

4.3 Use the data given in Problem **4.2** to complete Parts **a–c** below.

a. Estimate the ball's instantaneous velocity at each of the times listed in the following table.

Time (sec)	**Velocity (m/sec)**
0.5	
0.7	
0.9	
1.1	
1.3	

b. Use the values found in Part **a** to estimate the ball's average acceleration during each of the intervals listed in the table below.

Time Interval (sec)	**Acceleration $\left(\text{m/sec}^2\right)$**
[0.5, 0.7]	
[0.7, 0.9]	
[0.9, 1.1]	
[1.1, 1.3]	

c. How does the acceleration of the ball appear to change over time?

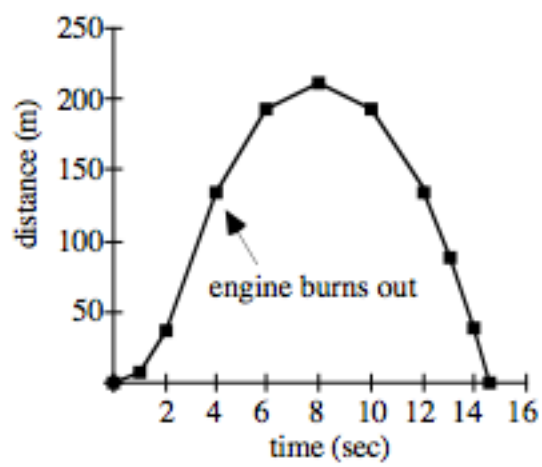
4.4 Sketch a distance-time graph that could represent each of the situations described below.

a. A ball moves away from a range finder with an acceleration of 0.

b. A ball moves away from a range finder with a positive acceleration.

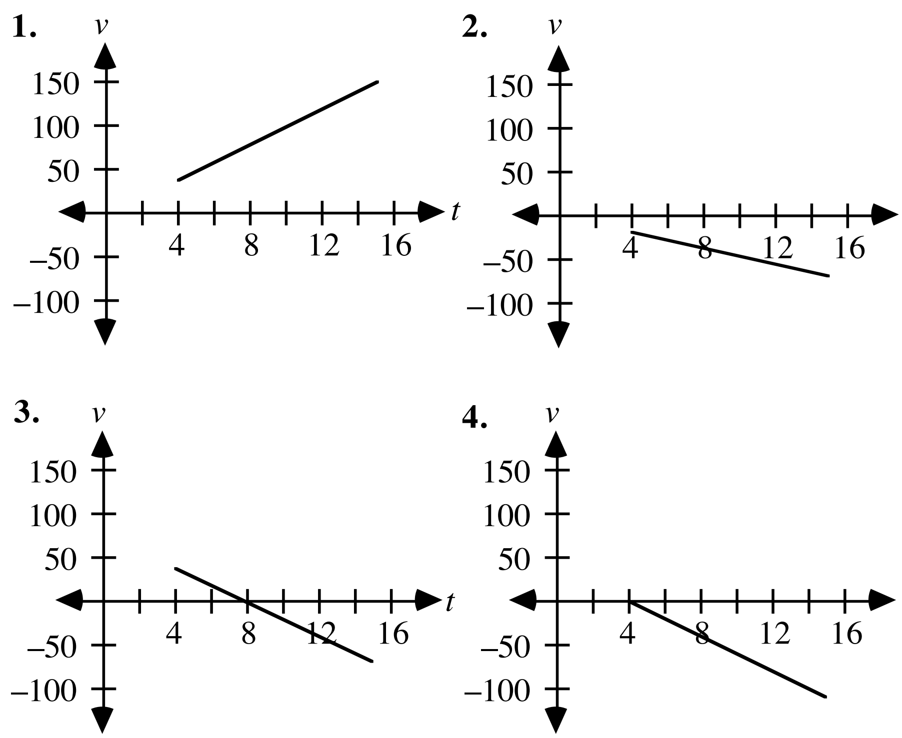
c. A ball moves away from a range finder with a negative acceleration.

4.5 The distance-time graph below shows data collected during the flight of a model rocket. After its engine burns out, the primary force acting on the rocket is gravity.



a. Identify the locations on the graph where the velocity of the rocket is positive, zero, or negative.

b. Based on your responses to Part **a**, which of the graphs below represents a graph of velocity versus time for the interval [4, 14]? Justify your choice.



c. Using the graph you selected in Part **b**, describe a graph of the rocket's acceleration versus time over the same interval. Justify your response.

4.6 The table below shows the distance above the ground at various times for a bouncing ball.

Time (sec)	Distance (m)	Time (sec)	Distance (m)
0.00	0.0	0.48	0.933
0.08	0.301	0.56	0.878
0.16	0.549	0.64	0.762
0.24	0.736	0.72	0.583
0.32	0.865	0.80	0.342
0.40	0.929	0.88	0.0

a. Find a quadratic equation that closely models this data.

b. According to your model, when does the ball reach its highest point?

c. What is the velocity of the ball at the time it reaches this point?

d. What is the velocity of the ball at $t = 0$?

4.7 When the acceleration of a rocket increases at a constant rate, then the rocket's distance above the ground with respect to time can be modeled by a cubic equation. The data in the table below shows the height above the ground of a model rocket launched straight into the air. Find a third-degree polynomial that closely fits this data.

Time (sec)	Distance (m)	Time (sec)	Distance (m)
0.0	0.0	1.5	17.9
0.5	2.6	2.0	36.9
1.0	7.4		

* * * * *

4.8 Suppose that you have used a range finder to collect distance-time data for a freely falling object. Your data can be modeled by the function $d(t) = 4.9t^2$, where $d(t)$ represents distance in meters and t represents time in seconds.

a. Describe the motion of the object, including its initial velocity and initial distance from the range finder.

b. Use the model equation to estimate when the object was 4 m from the range finder.

4.9 Sir Isaac Newton once said that if he had seen farther than other scientists and mathematicians, this was because he had "stood on the shoulders of giants." One of those giants was Galileo Galilei (1564–1642), who died in the year of Newton's birth. In fact, Newton's first law of motion was actually a variation on Galileo's concept of inertia.

a. Besides describing inertia, Galileo theorized that, in the absence of air resistance, two objects of different sizes and weights dropped from the same height would reach the ground at the same time. What does Galileo's theory predict about the motions of an apple falling from a tree and the ball you dropped in the exploration?

b. On one of the Apollo missions to the moon, an astronaut demonstrated Galileo's theory by dropping a hammer and a feather from the same height. Given that the acceleration due to gravity on the moon is $1/6$ that on earth, what function could be used to describe the two objects' distance from the lunar surface with respect to time?

c. If the hammer and feather were dropped from a height of 2 m, how long would it take them to reach the lunar surface?

d. If this demonstration were conducted on earth, how long would it take the hammer to reach the ground?

4.10 The data in the table below shows a rocket's distance above the ground at various times after launch. At $t = 2$ sec, the rocket's engine burned out. After this time, gravity is the primary force acting on the rocket.

Time (sec)	Distance (m)	Time (sec)	Distance (m)
2	36.9	8	212.3
4	134.2	10	192.9
6	192.9	11	170.8

a. Find a polynomial equation that closely fits the data.

b. Interpret the significance of each coefficient in your equation.

4.11 The table below shows a space shuttle's distance above the ground during the first seconds after liftoff. Find an equation that models this data. Explain why you think your model fits the data well.

Time (sec)	Distance (m)	Time (sec)	Distance (m)
0.00	0.00	7.68	153.31
1.92	6.10	9.60	249.63
3.84	33.53	11.52	371.86
5.76	81.69	13.44	519.99

Source: Johnson Space Center, Houston, Texas.

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Research Project

Select one of the following topics.

a. In addition to his three laws of motion, Isaac Newton also proposed a law of universal gravitation. Together, these few principles revolutionized the sciences of physics and astronomy. Write a report on Newton's contributions to the study of motion, including an explanation of the relationship among force, mass, and acceleration.

b. Scientists at NASA closely analyze each launch of the space shuttle and make a wealth of information available to the public. Contact the Johnson Space Center regarding a future shuttle flight. Write a report on the planned launch, including an analysis of the flight-path data.

Summary Assessment

1. The distance-time data shown below was obtained by moving a book toward and away from a range finder taped to a desk.

Time (sec)	Distance (m)	Time (sec)	Distance (m)
0.0	1.393	2.4	1.856
0.4	1.145	2.8	1.838
0.8	0.851	3.2	1.549
1.2	0.682	3.6	1.308
1.6	0.859	4.0	0.841
2.0	1.328		

a. Create a distance-time graph of this data.

b. Describe what happens to the velocity of the book during the interval from 0 sec to 4 sec.

c. Identify at least three different time intervals in which the book's average velocity is 0.

d. During which 0.4-sec interval is the book moving the fastest?

e. Find an equation that models the motion of the book during each of the following intervals:

1. from 0 sec to 0.8 sec

2. from 0.4 sec to 2.0 sec.

2. The following distance-time data was recorded for a falling ball:

Time (sec)	Distance (m)	Time (sec)	Distance (m)
0.00	0.42	0.45	0.84
0.05	0.42	0.50	1.00
0.10	0.42	0.55	1.19
0.15	0.42	0.60	1.40
0.20	0.42	0.65	1.63
0.25	0.44	0.70	1.89
0.30	0.50	0.75	2.17
0.35	0.59	0.80	2.06
0.40	0.70		

a. Create a distance-time graph for this data.

b. Describe the time interval for which the ball was actually falling.

c. Find an equation that models the distance-time graph for this interval.

d. Explain how the terms of the equation you found in Part **c** relate to the movement of the ball.

e. Calculate the average velocity of the ball during each of the following intervals:

1. $[0.25, 0.35]$

2. $[0.65, 0.75]$

f. Explain why the two average velocities you found in Part **e** are different.

Module Summary

- A **distance-time graph** displays the distance between two objects as a function of time.
- **Displacement** is a change in the position of an object. It has both magnitude and direction.
- **Velocity** is the rate of change in position with respect to time.
- The **average velocity** of an object can be calculated by dividing its displacement by the change in time.
- The **instantaneous velocity** of an object is its velocity at a particular instant in time. To estimate the instantaneous velocity at a given instant t, one can use the average velocity of the object during a small interval containing t.
- A **force** is a physical quantity that can affect the motion of an object. Two familiar forces are gravity and friction.
- According to Newton's **first law of motion**, an object in a state of rest or moving in a straight line at a constant speed will continue in that state unless acted on by a force.
- A **polynomial** in one variable, x, is an algebraic expression of the general form:

  $$a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x^1 + a_0$$

  where n is a whole number and the **coefficients** a_i are real numbers for $i = 0, 1, 2, \ldots, n$.
- The **degree** of a polynomial is equal to the greatest exponent of the variable in the expression with a non-zero coefficient. A polynomial in the general form shown above has a degree of n, provided that $a_n \neq 0$.
- A function f is a **polynomial function** if $f(x)$ is defined as a polynomial in x. The degree of a polynomial function is the same as the polynomial it contains.
- Second-degree polynomial functions also are known as **quadratic functions**. The graph of a quadratic function is a **parabola**.
- Third-degree polynomial functions also are known as a **cubic functions**.
- **Acceleration** is the rate of change in velocity with respect to time.
- The acceleration due to gravity is a constant typically denoted by g. On earth's surface, the acceleration due to gravity is about 9.8 m/sec^2 in a direction toward the earth's center.

- When an object is acted on only by gravity, its distance from the ground is described by the following function:

$$d(t) = -\frac{1}{2}gt^2 + v_0t + d_0$$

where $d(t)$ represents the object's distance from the ground after t sec, g is the acceleration due to gravity, v_0 is the object's velocity in the vertical direction at $t = 0$, and d_0 is the object's distance above the ground at $t = 0$.

Selected References

Frautschi, S., R. Olenick, T. Apostol, and D. Goodstein. *The Mechanical Universe*. Cambridge: Cambridge University Press, 1986.

Halliday, D., and R. Resnick. *Physics*. New York: John Wiley & Sons, 1978.

Murphy, J., J. Hollon, and P. Zitzewitz. *Physics: Principles and Problems*. Toronto: Charles Merrill Publishing, 1972.

For more information about the space shuttle program, contact the NASA Teacher Resource Room, Mail Code AP–4, Johnson Space Center, Houston, TX 77058; 713–483–8696.