Let the Games Begin

This module uses puzzles and games to introduce logical reasoning and problem-solving strategies.

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Introduction

A man was looking at a picture in a gallery. A woman standing next to him asked, "Who is the man in that picture?" The man, who loved intrigue and puzzles, thought for a moment and replied, "Brothers and sisters have I none, but this man's father is my father's son."

Who is the man in the picture? One way to answer this question is to approach the problem logically.

Activity 1

At one level, logical thinking involves the interpretation of words. Often, the shortest and most familiar words play an important role in determining the meaning of a sentence.

Mathematics Note

A **statement** is a sentence that can be determined to be either true or false, but not both. The truth or falseness of a statement is its **truth value**.

For example, consider the statement, "There are no clouds in the sky today." Its truth value can be determined by looking at the sky. If there are no clouds, the statement is true. If there are any clouds in the sky, however, the statement is false.

Two statements can be joined into a **compound statement** using the **connectives** *and* and *or*. When a friend describes a new acquaintance, for example, she might say, "Joseph has black hair and wears glasses." This compound statement consists of the statement, "Joseph has black hair," connected with the statement, "Joseph wears glasses."

In mathematics, statements are typically represented using variables. For example, if p represents the statement, "Joseph has black hair," and q represents the statement, "Joseph wears glasses," the compound statement described above can be represented by "p and q."

Exploration

In this exploration, you use a game to examine how logical connectives affect the meaning of statements. The Logic Game is very similar to the game of bingo. It involves matching the numbers on a game board with the numbers that correspond to the compound statements chosen by a caller.

A sample game board is shown in Figure **1** below. (In an actual game, each player has a different board.)

L	**O**	**G**	**I**	**C**
–8	4	–1	9	7
–2	2	3	0	3
8	1	4	–5	–6
5	–3	–4	–7	1

Figure 1: Logic Game board

The object of the Logic Game is to place a token over all the numbers in any row on the game board. The statements used in the game are listed in Table **1**.

Table 1: Statements for the Logic Game

a	The number is –9.
b	The number is less than 4.
d	The number is greater than –1.
e	The number belongs to the set {8, 9}.
f	The number belongs to the set {–9, –8, …, 0, 1}.
h	The number is greater than 2.
j	The number is even.
k	The number is less than –4.
m	The number is odd.
n	The number is negative.
p	The number belongs to the set {5, 6, 7, 8, 9}.
q	The number is positive.

Please read Parts **a–f** before beginning play.

a. Select one person to be the caller and one to be the recorder. Everyone else is a player.

b. To prepare for the game, the caller and the recorder should complete the following steps.

1. Separate the game pieces provided by your teacher into two piles: a Statement pile and a Logic pile.
2. Make a Connector Coin by marking one side of a quarter with the word *and* and the other side with the word *or*.
3. Save the Record Sheet provided for later use.

c. To prepare for the game, each player should complete the steps below.

1. Generate 20 random integers between –9 and 9, inclusive.
2. Write one of the random integers in each of the unshaded regions of the Logic Game board provided by your teacher.

d. The following list describes the caller's duties in the Logic Game.

1. The caller randomly picks two statements from the Statement pile, then flips the Connector Coin to select a logical connective. The caller uses the statements and the logical connective to form a compound statement.
2. The caller randomly selects a letter from the Logic pile, announces the column to which the compound statement applies, then reads the statement.

   For example, consider the two statements k and m, or "The number is less than –4" and "The number is odd," respectively. If the letter selected from the Logic pile is **L** and the logical connective is *and*, then the caller should announce "Under column **L**, cover any number that is both less than –4 and is odd."
3. The caller returns the selected items to the appropriate piles.
4. The caller continues selecting and announcing compound statements until one player wins.

e. For each announcement by the caller, the recorder writes the variables and logical connectives that correspond with the compound statement on the Record Sheet. In the example given in Part **d**, the recorder would write "k and m" in the first row under column **L**. A copy of the Record Sheet is shown in Figure **2**.

	L	O	G	I	C
Rule 1	*k* and *m*				
Rule 2					
Rule 3					
Rule 4					
Rule 5					
Rule 6					
Rule 7					
Rule 8					
Rule 9					
Rule 10					
Rule 11					
Rule 12					
Rule 13					
Rule 14					
Rule 15					
Rule 16					
Rule 17					
Rule 18					
Rule 19					
Rule 20					

Figure 2: Record sheet

f. The following list describes the rules players must follow when marking their game boards:

1. Players place a token over each number that satisfies the announcement given by the caller. In the example given in Part **d**, players could place a token over each number in column **L** that is both less than –4 and odd.
2. For each number marked by a token, players should write the corresponding compound statement in the appropriate shaded rectangle. In the example given in Part **d**, players would write "*k* and *m*" below each marked number.
3. To win the game, a player must have tokens covering all the numbers in one row of the game board. When this occurs, that player declares "Logic!"
4. The other players then verify that each number in the row satisfies the compound statement written below it. If so, that player is officially declared the winner. If any numbers do not correctly fit a rule, that player is disqualified until the next game and play continues until a winner is declared.

g. If time allows, play the Logic Game again using a new game board, caller, and recorder.

Discussion

a. When does a number on a Logic Game board satisfy a compound statement that uses the connective *and*? Use an example to support your response.

b. When does a number on the game board satisfy a compound statement that uses the connective *or*? Use an example to support your response.

c. How is the intersection of two sets related to a compound statement formed using the connective *and*?

d. How is the union of two sets related to a compound statement formed using the connective *or*?

Mathematics Note

Venn diagrams are mathematical models that show relationships among different sets of data.

The **intersection** of two sets is the set of all elements common to both sets. For example, the shaded region in the Venn diagram in Figure **3a** shows the intersection of sets A and B, denoted by $A \cap B$.

The **union** of two sets is the set of all elements in one, the other, or both sets. For example, the shaded region in Figure **3b** shows the union of sets A and B, denoted by $A \cup B$.

Disjoint sets have no elements in common. For example, Figure **3c** shows two disjoint sets, B and C.

A set with no elements is the **empty set** or **null set**. A symbol for the empty set is $\varnothing$. For example, the intersection of sets B and C is the empty set. This could be denoted by $B \cap C = \varnothing$.

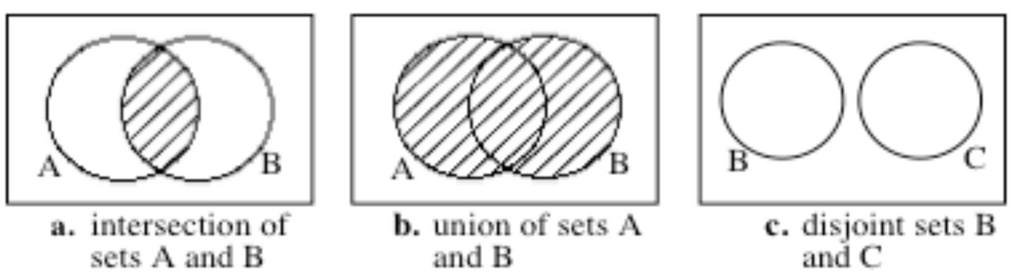

Figure 3: Venn diagrams

For example, suppose set A = $\{1, 2, 3, 4\}$, set B = $\{2, 4, 6\}$, and set C = $\{1, 3, 5\}$. The intersection of set A and set B is $\{2, 4\}$, as shown in Figure **4a**. The union of sets A and B is $\{1, 2, 3, 4, 6\}$, as shown in Figure **4b**. Sets B and C are disjoint sets because they have no numbers in common (see Figure **4c**). The intersection of set B and set C is the null set, $\varnothing$.

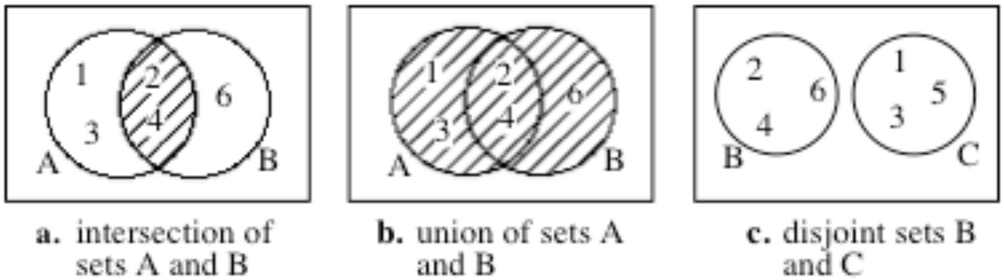

Figure 4: Sample Venn diagrams

e. How could you use Venn diagrams to help you play the Logic Game?

Assignment

Use the statements in the Logic Game from Table **1** to complete Problems **1.1–1.3**.

1.1 Identify the set of integers described by each statement in the Logic Game. Use the corresponding uppercase letter to label each set. For example, label the set of integers described by statement *a* as set A.

1.2 During one Logic Game, a recorder expressed each rule as a union or intersection of sets, labeling each set as in Problem **1.1**. Some players noticed that the recorder had made some mistakes.

Rule 1: $A \cap B = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, -9\}$
Rule 2: $F \cap J = \{-8, -6, -4, -2, 0\}$
Rule 3: $N \cup Q = \varnothing$
Rule 4: $P \cap Q = \{5, 6, 7, 8, 9\}$
Rule 5: $E \cap K = \varnothing$
Rule 6: $B \cap K = \{-4, -5, -6, -7, -8, -9\}$
Rule 7: $D \cap K = \{0, 1, 2, 3\}$
Rule 8: $M \cup J = \{-9, -8, -7, \ldots, 7, 8, 9\}$

a. Draw a Venn diagram that corresponds with each set operation in the table, including the appropriate integers from –9 to 9, inclusive, in each set. Shade the appropriate portion of each Venn diagram.

b. Assuming that the rules were recorded correctly, identify all the mistakes in the table and describe the correct sets.

1.3 At the conclusion of another game of Logic, the recorder had made the following entries on the Record Sheet:

Rule 1: $B \cap$ ___ = {0, 1, 2, 3}
Rule 4: ___ $\cup Q$ = {–8, –6, –4, –2, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9}
Rule 7: P ___ M = {5, 7, 9}
Rule 8: ___ $\cup$ ___ = {–9, 3, 4, 5, 6, 7, 8, 9}
Rule 9: D ___ B = {–9, –8, –7, …, 7, 8, 9}
Rule 10: ___ ___ ___ = {–9, –8, –7, –6, –5, 3, 4, 5, 6, 7, 8, 9}

As you can see, the recorder forgot to write down some information. Assuming that the listed numbers correctly satisfy each rule, fill in the missing sets and operations. Verify your responses using Venn diagrams.

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1.4 Triangles are classified by their sides (scalene, isosceles, or equilateral) and by their angles (acute, equiangular, right, or obtuse). Use combinations of classifications, such as "isosceles right triangle," to name all the triangles that satisfy each of the compound statements below.

a. The triangle has two complementary angles and the triangle has two congruent sides.

b. The triangle has three 60° angles or the triangle has no congruent sides.

c. The triangle has at least two congruent sides and the triangle has no obtuse angle.

1.5 Write a compound statement that describes each of the following sets.

a. {1, 9, 25, 49, 81, …}

b. {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 14, 16, 18, 20}

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Activity 2

In the Logic Game, you used connectives to describe the intersection or the union of sets. Other types of words can also affect the meaning of statements. For example, the word *not* can be used to change a statement's truth value. In this activity, you use tables to examine the truth values of compound statements.

Exploration

As basketball season approaches, the coach at your school decides that the team needs more talent. When asked to describe the ideal recruit, the coach replies, "The person is fast and the person is tall." While considering all the potential players you know, you make the following notes:

- Smith is slow but tall.
- Lopez is tall and fast.
- Yellowtail is fast but not tall.
- McMurphy is tall but slow.
- Ali is short but not slow.
- Schmidt is slow and short.

a. Using the information given above, complete a copy of Table **2** with the appropriate truth values (true or false).

Table 2: Recruit information

Person	The person is fast.	The person is tall.	The person is fast and the person is tall.
Smith	F	T	F
Lopez			
Yellowtail			
McMurphy			
Ali			
Schmidt			

b. A compound statement that uses the connective *and* is a **conjunction**. Given two statements, there are four different ways in which their truth values may be combined: TF, FT, FF, and TT.

Using the information in Table **2**, which of these ways results in a conjunction with a truth value of true? Which results in a conjunction with a truth value of false?

c. Suppose the coach had said, "The person is fast or the person is tall." Repeat Part **a** using this description of the ideal recruit.

d. A compound statement that uses the connective *or* is a **disjunction**. Repeat Part **b** for a disjunction.

e. A **truth table** shows the truth values of a compound statement for all possible truth values of its individual statements. Using your responses in Parts **b** and **d**, complete a copy of Table **3** to show the truth values for the conjunction "q and r" and the disjunction "q or r."

Table 3: Truth table for "*q* and *r*" and "*q* or *r*"

q	***r***	***q* and *r***	***q* or *r***

Discussion

a. How did you determine who would be a good basketball recruit:

1. when the coach used the connective *and*?

2. when the coach used the connective *or*?

b. Using Table **3**, when is the compound statement "q or r" true? When is it false?

c. When is the compound statement "q and r" true? When is it false?

Mathematics Note

The compound statement "p and q" has a truth value of true only when statement p and statement q are both true.

For example, the compound statement "Joseph has black hair and wears glasses" is true only when the statements "Joseph has black hair" and "Joseph wears glasses" are both true.

The compound statement "p or q" has a truth value of false only when statement p and statement q are both false.

For example, the compound statement "Joseph has black hair or wears glasses" is false only when the statements "Joseph has black hair" and "Joseph wears glasses" are both false.

A **negation** of a statement p is described as "not p," also denoted by $\sim p$. When p is true, its negation $\sim p$ must be false. When p is false, $\sim p$ must be true.

For example, consider the statement, "My teacher wears glasses." If this statement is false, then its negation—"My teacher does not wear glasses"—is true. If this statement is true, then its negation is false.

d. **1.** Suppose that q represents the statement, "The person is fast," and r represents the statement, "The person is tall." Write a compound statement that could be represented by "$\sim q$ or r."

2. Compare the sentence you wrote with those of others in your class. Do they all have the same meaning?

e. When is the compound statement "$\sim q$ or r" true? When is it false?

f. In the introduction to this module, the man in the gallery says, "Brothers and sisters have I none … ." Use your knowledge of logical connectives to interpret this phrase.

Assignment

2.1 Suppose that p represents the statement, "I love math class," and q represents the statement, "I have a pet."

a. Create a truth table for the conjunction "p and q" and the disjunction "p or q."

b. Without using mathematical symbols, write sentences that correspond to each of the four different ways in which the truth values of the individual statements can be combined in the conjunction "p and q."

2.2 **a.** Create a truth table for the compound statement "$\sim p$ and q."

b. Create a truth table for the compound statement "$\sim p$ or q."

c. Suppose that p represents the statement, "The light is on," and q represents the statement, "The door is open." Without using mathematical symbols, write sentences that correspond to the compound statements in Parts **a** and **b**.

2.3 Consider statement p, "The light is on," and statement q, "The door is open."

a. Without using mathematical symbols, write a sentence that corresponds to the compound statement $\sim(p \text{ and } q)$.

b. Create a truth table that shows the possible truth values of $\sim(p \text{ and } q)$ using the following column headings:

p	q	p and q	$\sim(p$ and $q)$

c. Create a truth table that shows the possible truth values of "$\sim p$ or $\sim q$" using the following column headings:

p	q	$\sim p$	$\sim q$	$\sim p$ or $\sim q$

d. Two statements are **logically equivalent** when they have exactly the same truth values. Are the statements ~(p and q) and "~p or ~q" logically equivalent? Defend your response using Venn diagrams.

e. Using the words *off* and *closed*, write a sentence that has the same meaning as the sentence you wrote in Part **a**.

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2.4 **a.** Create a truth table that shows the possible truth values of "(p or q) and r " using the following column headings.

p	***q***	***r***	***p* or *q***	**(*p* or *q*) and *r***

b. Verify your results in Part **a** using Venn diagrams.

2.5 **a.** Determine a logical equivalent to ~(p or q).

b. Verify your response to Part **a** using a truth table or Venn diagram.

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Activity 3

In this activity, you examine compound statements known as **conditionals**. A knowledge of conditionals can help you evaluate statements and develop strategies for solving logical problems.

Mathematics Note

A **conditional statement** is a compound statement that can be written in "if-then" form. A conditional consists of two parts: the **hypothesis** and the **conclusion**. The hypothesis is the "if" part of the conditional. The conclusion is the "then" part. A conditional statement can be represented symbolically by "if p, then q," or by $p \rightarrow q$ (read "p implies q").

For example, given the hypothesis p, "I have my hand raised," and the conclusion q, "I see a red card," the conditional statement $p \rightarrow q$ is "If I have my hand raised, then I see a red card."

Exploration

In this exploration, you play a logic game called Color Card. To start the game, players must be seated so that each one can see all the others. Each player draws one card from a deck of ordinary playing cards, in which each card is either red or black, but not both. Without looking at it, each player holds his or her card face out toward the rest of the players. All players then respond according to the following rules:

- A player who sees a red card raises one hand.
- A player who does not see a red card does not raise a hand.

After observing the number of raised hands, players try to determine the colors of their own cards. Each player writes down one of the following conclusions: “red,” “black,” or “color cannot be determined.” Players then present logical arguments to defend their decisions.

a. Play Color Card with two players. Play several rounds, drawing new cards from the deck each time.

b. List all the possible combinations of red and black cards for two players.

c. Determine whether or not it is always possible for both players to correctly determine the colors of their cards.

d. Repeat Parts **a–c** using three players.

Discussion

a. In a two-person game of Color Card, what conditional statements describe the logic that players should use to determine the colors of their cards?

b. In a three-person game of Color Card, what combinations of colors make it possible for all players to determine the colors of their cards? Explain your reasoning.

c. If the combination of colors in a three-person game is two black and one red, what conditional statements describe the logic that players should use to determine the colors of their cards?

d. The truth value of a conditional statement depends on the truth values of its hypothesis and its conclusion. Table **4** below defines the truth values for a conditional $p \rightarrow q$. Use the table to describe all the cases in which a conditional statement is true.

Table 4: Truth table for a conditional statement

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

e. Give an example of a conditional statement that illustrates each row in the truth table in Table **4**. Justify your responses.

Assignment

3.1 Two players are playing Color Card according to the rules described in the exploration. Given that the hypothesis is true, determine whether each of the conditional statements below is true or false. Justify your responses.

a. If my card is not red, then it is black.

b. If my card is red, then the other player has a hand up.

c. If the other player does not have a hand up, then my card is not red.

d. If my card is not red, then the other player has a hand up.

e. If my card is black, then the other player has a hand up.

f. If the other player does not have a hand up, then my card is black.

g. If my card is not red, then it is not black.

3.2 Three players are playing Color Card. Two of them have red cards and one has a black card. Can any of them identify the colors of their own cards? Explain your reasoning.

3.3 In a three-person game of Color Card, all three players have red cards. Following the rules described in the exploration, all three raise their hands. One player says, "I cannot determine the color of my card." Based on this statement, the other two players identify the colors of their cards. Write a conditional statement that explains the logic used by the two players who identified their cards.

3.4 Three playing cards from an ordinary deck have been placed face down in a row. Use the following clues to identify the face value and suit of each card.

- To the right of a jack there is at least one ace.
- To the left of an ace there is at least one ace.
- To the left of a club there is at least one diamond.
- To the right of a diamond there is at least one diamond.

Mathematics Note

The **contrapositive** of a conditional statement is formed by interchanging the hypothesis and conclusion and negating both of them. The contrapositive of $p \to q$ can be represented as $\sim q \to \sim p$, or "if not q, then not p."

For example, consider the conditional statement, "If it is hot outside, then the snow outside is melting." The contrapositive of this conditional is, "If the snow outside is not melting, then it is not hot outside."

3.5 Write the contrapositive of each of the following statements.

a. If I study hard, then I earn good grades.

b. If I drive carefully, then I do not get into accidents.

c. If I do not use sugar on my cereal, then I stay calm.

d. If I do not lie, then I do not feel guilty.

3.6 **a.** Create a truth table for the contrapositive $\sim q \rightarrow \sim p$.

b. Explain how Table **4** in Part **d** of the previous discussion and the truth table in Part **a** above could be used to demonstrate that a conditional and its contrapositive are logically equivalent.

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3.7 In the riddle described in the introduction to this module, a man in a picture gallery says, "Brothers and sisters have I none, but this man's father is my father's son."

Use conditional statements to solve this riddle.

3.8 Suppose that p represents the statement, "The light is on," and q represents the statement, "The door is open."

a. Use the appropriate symbols to express the conditional statement, "If the light is on, then the door is open."

b. What are all the possible circumstances that would make the conditional statement in Part **a** false?

c. Complete a truth table for the conditional statement, "If the light is on, then the door is open."

d. How does your truth table in Part **c** compare to the truth table for "$\sim p$ or q" shown below?

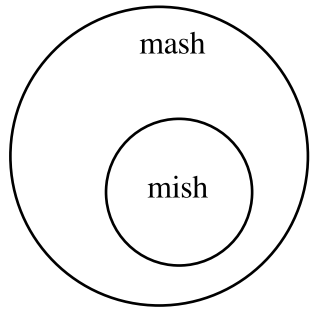
p	q	$\sim p$	$\sim p$ **or** q
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

3.9 **a.** Create a truth table for the conditional statement "If $x = 3$, then $x < 5$."

b. Create a truth table for the contrapositive of the statement in Part **a**.

c. How do the truth tables indicate that the conditional in Part **a** and its contrapositive in Part **b** are logically equivalent?

3.10 Conditionals are sometimes illustrated using Venn diagrams. For example, the Venn diagram for the true conditional, “If an object is a mish, then it is a mash,” is shown below. The inner circle represents the objects that satisfy the hypothesis, while the outer circle represents objects that satisfy the conclusion.



Use the Venn diagram above to determine a logical conclusion for each of the following hypotheses.

a. If Micah is a mish, . . .

b. If Fermi is a mash, . . .

c. If Pretty Eagle is not a mish, . . .

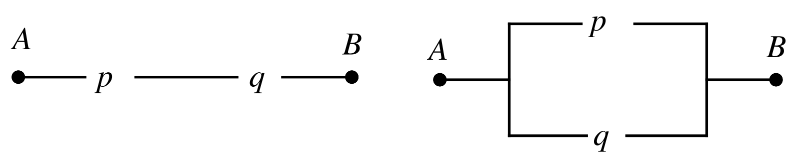
d. If Vetrovsky is not a mash, . . .

* * * * * * * * * *

Summary Assessment

1. The ability to reason logically is an important asset in many professions. Electricians, for example, often face complicated wiring problems that would be difficult to solve without logical reasoning skills.

There are two basic types of circuits, **series** and **parallel**, as shown in the diagram below. The figure on the left represents part of a series circuit. The one on the right represents part of a parallel circuit.



series circuit **parallel circuit**

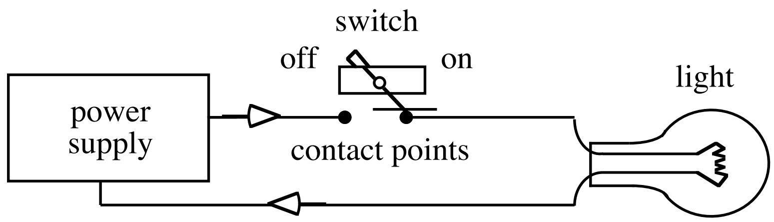
In both figures, p and q represent switches that can be turned either on or off. For electricity to flow from point A to point B, it must have an uninterrupted path between the two points.

a. Let the number 1 represent a switch in the "on" position and the number 0 represent a switch in the "off" position. Create a table that describes all the possible positions for the two switches in the series circuit and shows whether or not electricity can flow from A to B. (In this situation, a "1" in the table indicates that electricity can flow from A to B, while a "0" indicates that it cannot.)

b. Repeat Part **a** for the parallel circuit.

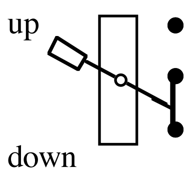
c. Which type of circuit corresponds with the connective *and* and which corresponds with the connective *or*? Use truth tables to justify your response.

2. The diagram below shows a light bulb controlled by a single switch.

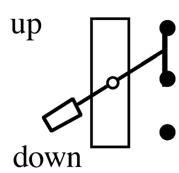


The switch in this diagram is in the "off" position. The bulb is not lit because electricity cannot flow through the switch. The bulb lights only when electricity has an uninterrupted path from the power supply to the bulb and back again. Flipping the switch to the "on" position connects the two contact points and completes the circuit.

In many homes, a single light can be controlled from two different switches. The light can be turned on or off from either switch, regardless of the position of the other switch. Because these switches have three contact points, they are called three-way switches. In each switch, there is an "up" position and a "down" position, as shown below.

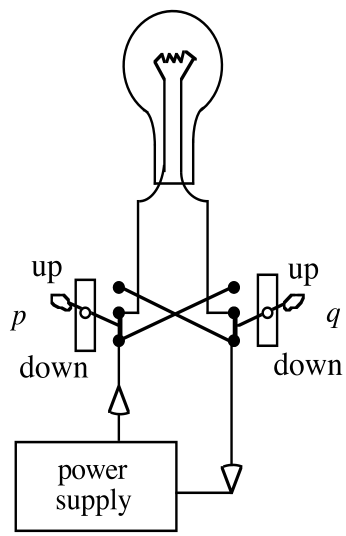


The switch is up.



The switch is down.

The diagram below shows a circuit in which the light can be controlled from either three-way switch. The large "X" in the diagram represents the wires that connect the two switches.



a. Draw a picture of all the possible sets of switch positions and determine whether the light is on or off in each one.

b. Write a conditional statement that describes the positions of switches p and q necessary to turn the light on.

c. As in Problem **1**, create a table that describes all the possible positions for the two switches and shows whether or not the light is on in each case.

d. Is the circuit shown above logically equivalent to a parallel circuit, a series circuit, or neither? Justify your response.

Module Summary

- A **statement** is a sentence that can be determined to be either true or false, but not both. The truth or falseness of a statement is its **truth value**.
- Two statements can be joined into a **compound statement** using the **connectives** *and* and *or*.
- A compound statement that uses the connective *and* is a **conjunction**.
- A compound statement that uses the connective *or* is a **disjunction**.
- **Venn diagrams** are mathematical models that show relationships among different sets of data.
- The **intersection** of two sets is the set of all elements common to both sets. The intersection of set A and set B can be denoted by $A \cap B$.
- The **union** of two sets is the set of all elements in either set or in both sets. The union of set A and set B can be denoted by $A \cup B$.
- **Disjoint sets** have no elements in common.
- The **empty set** or **null set** is a set that contains no elements. The symbol for the empty set is $\varnothing$.
- A **truth table** shows the truth values of a compound statement for all possible truth values of its individual statements.
- The compound statement "*p* and *q*" has a truth value of true only when statement *p* and statement *q* are both true. The truth table for "*p* and *q*" is shown below.

p	***q***	***p* and *q***
T	T	T
T	F	F
F	T	F
F	F	F

- The compound statement "*p* or *q*" has a truth value of false only when statement *p* and statement *q* are both false. The truth table for "*p* or *q*" is shown below.

p	***q***	***p* or *q***
T	T	T
T	F	T
F	T	T
F	F	F

- A **negation** of a statement p is described as "not p," also denoted by $\sim p$. When p is true, its negation $\sim p$ must be false. When p is false, $\sim p$ must be true. The truth table for "not p" is shown below.

p	**not** p
T	F
F	T

- Two statements are **logically equivalent** when they have exactly the same truth values.
- A **conditional statement** is a compound statement that can be written in "if-then" form. A conditional consists of two parts: the **hypothesis** and the **conclusion**. The hypothesis is the "if" part of the conditional. The conclusion is the "then" part. A conditional statement can be represented symbolically by "if p, then q," or by $p \rightarrow q$ (read "p implies q"). The truth table for the conditional $p \rightarrow q$ is shown below.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

- The **contrapositive** of a conditional statement is formed by interchanging the hypothesis and conclusion and negating both of them. The contrapositive of $p \rightarrow q$ can be represented as $\sim q \rightarrow \sim p$, or "if not q, then not p."
- A conditional statement and its contrapositive are logically equivalent.
- A conditional statement $p \rightarrow q$ has the same truth value as the compound statement $(\sim p)$ or q. In other words, $(\sim p)$ or q is false exactly when p is true and q is false.

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