Everyone Counts

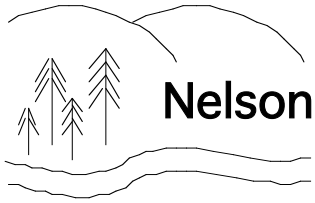
The grand opening of a shopping mall promises lots of new choices for customers. How many choices? In this module, you investigate some different ways to count your options.

Art Perleberg • Deanna Turley • Dan West

Everyone Counts

Introduction

After months of intense work, real estate developer Nadia Nelson is exhausted but happy. All construction is complete and spaces have been leased to a dozen businesses. Riverside Mall is ready to open. As owner and manager of the mall, Nadia welcomes her new tenants with the following memo.



Nelson Real Estate Property Management

To: Members of the Riverside Store Managers' Association
From: Nadia Nelson
Re: Opening of Riverside Mall

Congratulations! Your businesses were selected for the mall after a survey of Riverside shoppers. I would like to welcome each of you to Riverside Mall.

Last Chance Beauty Parlor	Les Arts Magnifique
Kuzlowski's Pizza	Almost Human Pets
Riverside Read and Feed	Music Mania
Trendz Clothing	The Zone Sporting Goods
Toddler's Haven Toys	Planet Alpha Arcade
Zuzu's Petals and Gifts	Flix Theaters

Please keep me informed of all activities planned for the mall's grand opening. And if you have any questions, don't hesitate to contact me.

Activity 1

As part of the grand opening festivities, the managers' association is planning a promotion called "Crack the Safe." A large safe will be filled with merchandise contributed by each store. The shopper who manages to open the safe will take home all the prizes.

Nadia Nelson likes the idea, but wonders about the manner in which the promotion will be conducted. She expresses her concerns in another memo.

Nelson Real Estate Property Management

To: Members of the Store Managers' Association
From: Nadia Nelson
Re: "Crack the Safe" Promotion

I am excited about the "Crack the Safe" promotion. You have created an ingenious way to attract shoppers to the mall. I do have some questions, however. Three types of locks are available for the safe: rotary combination, keyed pin, and digital combination. Which type should we use? And how will we make sure that the promotion lasts for several days, yet gives shoppers a reasonable chance to win? Please send your recommendations for these challenges as soon as possible. Opening day is coming!

Exploration

a. As shown in Figure **1**, one type of rotary combination lock consists of a dial with 40 tick marks representing the numbers 0 through 39. The lock is opened by turning the dial to the right, then to the left, then back to the right, stopping each time at the corresponding number of a three-number code.

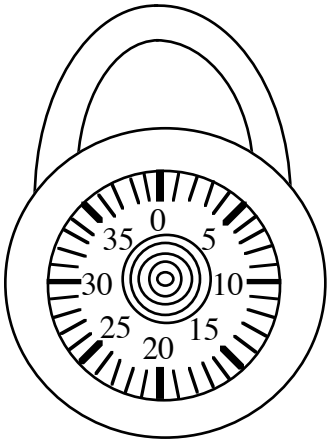


Figure 1: Rotary combination lock

1. Estimate how many different three-number combinations are possible for this type of lock.
2. Using your estimate in Step **1**, determine the probability that a combination chosen at random will open the lock.
3. Develop a systematic plan to find the actual number of different combinations.

Mathematics Note

The **fundamental counting principle** states that if one event can occur in h ways, and for each of these ways a second event can occur in k ways, then the number of different ways in which the two events can occur is $h \bullet k$.

For example, suppose a store sells soft drinks in 2 different sizes and 5 different flavors. In this situation, one event is the choice of size. The second event is the choice of flavor. Using the fundamental counting principle, there are a total of $2 \bullet 5$ or 10 different selections from which to choose.

The fundamental counting principle can be extended to situations involving more than two events. For example, if the store also sells 4 different brands of soft drinks, each of which comes in 2 different sizes and 5 different flavors, the total number of possible selections is $2 \bullet 5 \bullet 4$ or 40.

b. **1.** Use the fundamental counting principle to determine the number of possible combinations.

2. Calculate the probability that any one combination will open the lock.

c. The second type of lock under consideration by the managers' association requires a key. Figure **2** shows a key for a standard pin lock. Depending on the manufacturer, such keys can have either 5 or 6 valleys. Each valley can have from 6 to 10 different depths.

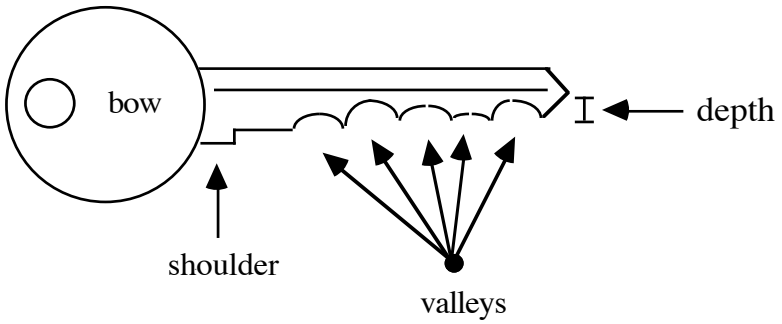


Figure 2: Key for a standard pin lock

1. Draw a key with 5 valleys. Label these valleys with the letters A–E. If 6 different depths are available for each valley in this key, how many different keys are possible?

2. If 10 different depths are available for each valley in a key with 6 valleys, how many different keys are possible?

3. Using the number of keys found in Step **2**, determine the probability that a key chosen at random will open the mall contest lock.

Discussion

a. **1.** Describe the strategy you devised in Part **a** of the Exploration for determining the number of possible combinations for the lock.

2. Did your strategy consider combinations with repeated digits? Explain your response.

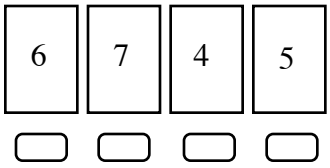
3. Why would it be impractical to use a tree diagram to determine the number of possibilities for the lock?

b. When a visitor to the mall opens the safe, the promotion is over. If you wanted the promotion to last as long as possible, which type of lock would you choose: rotary combination or keyed pin? Discuss the advantages and disadvantages of your choice.

c. Is it possible that the first person who tries to open the safe will be successful?

Assignment

1.1 The third type of lock being considered for the mall promotion is a digital combination lock. As illustrated below, one kind of digital combination lock features four electronic displays, each of which can show a number from 0 to 9. To enter a four-digit code, users press the buttons underneath the displays until the desired number is obtained.



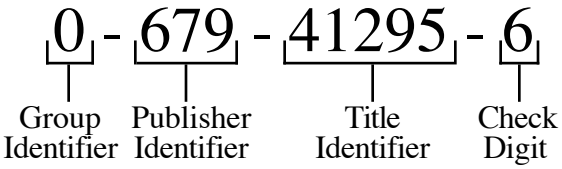
a. Use the fundamental counting principle to determine the number of possible combinations.

b. Determine the probability that a combination chosen at random will open the lock.

1.2 Considering all three types of locks, which would you recommend for the mall's "Crack the Safe" promotion? Justify your choice.

1.3 In addition to selling books and espresso, the manager of Riverside Read and Feed plans to publish works by members of the local community. Before the store can publish its first book, the manager must apply for a Publisher Identifier number for the International Standard Book Number (ISBN).

In most industrialized nations, an ISBN is assigned to each published book. It consists of 10 digits divided into 4 different areas. The Group Identifier represents the language in which the book is written. The Publisher Identifier represents the publishing company. The Title Identifier represents the title of the book. The Check Digit is used to detect errors before shipping. A sample ISBN is shown below.



The Group Identifier and Check Digit each consist of 1 digit. The numbers of digits in the Publisher and Title Identifiers may vary, but must represent a total of 8 digits.

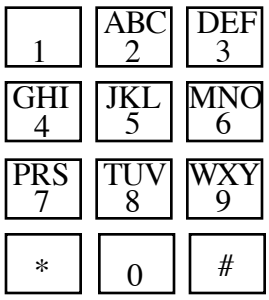
a. The manager of Riverside Read and Feed hopes to publish 20 books during her first year in business. What is the minimum number of digits needed in the Title Identifier? Explain your response.

b. Considering your response to Part **a**, how many different possibilities would the store have for its Publisher Identifier? Explain your response.

c. Do you think a large publishing company would want more or less digits designated for its Publisher Identifier? Justify your answer.

d. Suppose Riverside Read and Feed receives a three-digit Publisher Identifier. What is the maximum number of books it could publish?

1.4 Businesses often use the letters that correspond to a telephone number to help attract customers. The number for the Coiffure Salon, for example, is 244-4247, or BIG HAIR. As shown below, most touch-tone telephones have 24 letters on the keypad.



a. Which letters are missing from the keypad above?

b. The first three digits of a local telephone number represent its numerical prefix. In the past, the letters that correspond with these numbers also represented a prefix. For example, a resident of the Parkway neighborhood might have had the telephone number PAR-2374.

1. Use the fundamental counting principle to determine how many 3-letter prefixes can be formed using the letters on this keypad.

2. Using the number keys that also contain letters, how many 3-digit prefixes can be formed?

3. Compare your answers to Parts **b1** and **b2**. Explain any differences you observe.

c. **1.** Suppose that no letter may be repeated in a prefix. In this case, how many 3-letter prefixes can be formed from the keypad?

2. Write your response to Part **c1** as a ratio of factorials.

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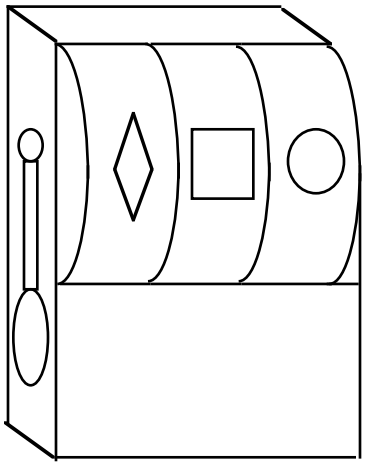
1.5 Some telephone keypads now display all 26 letters. To accommodate the extra letters, two buttons—the 7 and the 9 —correspond with 4 letters each.

a. If letters may be repeated, how many 3-letter prefixes can be formed from this keypad?

b. How many 3-letter prefixes can be formed if letters may not be repeated?

1.6 The toy store in the mall sells an imitation slot machine with 3 dials. Each dial contains the same sequence of 6 different symbols.



a. How many different arrangements of 3 symbols are possible?

b. What is the probability of getting the same symbol on all 3 dials?

1.7 In 1997, the state of Alaska used two different numbering and lettering systems for its automobile license plates.

a. One system used three letters followed by three digits. Assuming that all digits and letters may be repeated, how many different license plates are possible using this system?

b. The other system consisted of four digits followed by two letters. Assuming that all digits and letters may be repeated, how many different license plates are possible using this system?

c. Suppose that the state planned to use only one numbering and lettering system for the next 10 years. Which one would you recommend? Defend your choice.

d. Would either of these numbering and lettering systems work in the state of California? Write a paragraph explaining your response.

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Activity 2

Excited by the initial success of Riverside Mall, Nadia Nelson has begun planning a new addition. As shown in Figure **3**, the new wing will house seven more shops.

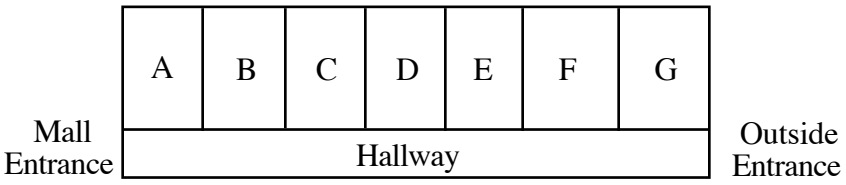

Figure 3: Riverside Mall addition

Exploration

Two business owners are already trying to reserve prime locations in the new wing. In this exploration, you help Ms. Nelson determine the number of different ways in which the stores can be arranged.

a. Draw a tree diagram to determine how many different ways the two businesses can be placed in the seven available locations.

b. Use the fundamental counting principle to verify your Part **a** response.

c. Once the first two stores have been placed, five spaces remain. Determine how many different arrangements are possible for the five stores that plan to rent these spaces.

d. Using the fundamental counting principle, find the total number of possible arrangements if locations are assigned as described in Parts **a** and **c**.

e. **1.** Determine the number of possible arrangements if locations are assigned to seven stores at one time.

2. Compare this number with your response to Part **d**.

f. Using your responses to Parts **c** and **e**, write the number of arrangements you determined in Part **a** as a ratio.

g. Use the process described in Parts **b–f** to write a ratio that represents the number of arrangements of r stores in n available spaces, given that $n > r$.

Discussion

a. The numerator of the fraction you wrote in Part **g** of the exploration can be expressed as $n \bullet (n-1) \bullet (n-2) \bullet \cdots \bullet 3 \bullet 2 \bullet 1$. How can this expression be written using factorial notation?

b. How could you use factorial notation to express the denominator of the fraction you wrote in Part **g** of the exploration?

c. How could you use factorial notation to describe the number of ways that r stores can be arranged in n available spaces?

Mathematics Note

A **permutation** is an ordered arrangement of symbols or objects. The number of permutations of n different symbols or objects taken r at a time, denoted by $P(n,r)$, is given by the following formula:

$$P(n,r) = n(n-1)(n-2)\cdots(n-r+1) = \frac{n!}{(n-r)!}$$

Another commonly used notation for permutations is ${}_nP_r$.

For example, suppose that a license plate consists of 7 digits from 1 to 9 in which no digit can be repeated. The number of different license plates that can be made using this system is the number of permutations of 9 symbols taken 7 at a time, or $P(9,7)$. Using the formula given above,

$$P(9,7) = \frac{9!}{(9-7)!} = \frac{9!}{2!} = \frac{9 \bullet 8 \bullet 7 \bullet 6 \bullet 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1}{2 \bullet 1} = 9 \bullet 8 \bullet 7 \bullet 6 \bullet 5 \bullet 4 \bullet 3 = 181{,}440$$

d. In the formula for $P(n,r)$ given in the mathematics note, why is $n(n-1)(n-2)\cdots(n-r+1)$ equivalent to the following ratio?

$$\frac{n!}{(n-r)!}$$

e. The permutation of n objects arranged n at a time, $P(n,n)$, is $n!$ Use this fact and the formula for $P(n,r)$ to suggest a definition for $0!$

Assignment

2.1 Before a game of slow-pitch softball, each team must submit a batting order—a list of 10 players that identifies the order in which they bat.

a. Does a batting order represent a permutation? Explain your response.

b. The Mall Misfits have 10 players on their team. How many different batting orders are possible?

c. In how many ways can the first, second, and third batters be chosen from the group of 10 players?

2.2 Music Mania's sound technician is meeting with a local band to discuss the arrangement of songs on the band's upcoming demo tape.

a. Explain why the arrangement of songs on a tape is a permutation.

b. The demo tape will contain 11 songs, 6 on side A and 5 on side B. In how many ways can 6 of the songs be arranged on side A? Write your answer both as a number and in the form $P(n,r)$.

c. After the songs for side A have been selected, in how many ways can the remaining 5 songs be arranged on side B? Write your answer both as a number and in the form $P(n,r)$.

d. The band also plans to issue a compact disc (CD). A CD has only one "side." In how many ways can the 11 songs be arranged on a CD? Write your answer both as a number and in the form $P(n,r)$.

e. Multiply your answers to Parts **b** and **c** and compare the product to your answer to Part **d**.

f. Use your results from Parts **c** and **d** to write an equation for $P(11,6)$.

2.3 As part of the mall's grand opening, Almost Human Pets is holding a prize drawing. The first 400 visitors to the store each receive one ticket. At the end of the week, three ticket numbers will be drawn at random. The person who holds the first ticket drawn wins a St. Bernard. The person who holds the second ticket drawn wins a hamster. The person who holds the third ticket drawn wins a guppy.

a. In how many ways can 3 winners be selected from the 400 ticket holders?

b. After the St. Bernard and the hamster are given away, what is the probability that any one of the remaining ticket holders will win the guppy? Explain your response.

2.4 Each November, the citizens of Riverside hold elections for city council. The council has 3 members. In this year's election, 6 candidates, including all 3 incumbents, are vying for the 3 positions.

Although the candidates' names are supposed to be listed in random order on the ballot, the first 3 names on this year's ballot are all incumbents.

a. In how many different ways can the 3 incumbents be listed in the first 3 positions on the ballot?

b. In how many different ways can the 3 challengers be listed in the last 3 positions on the ballot?

c. How many different ballot arrangements are possible if the incumbents appear in the first 3 positions and the challengers appear in the last 3 positions?

d. How many different arrangements are possible if all 6 names are placed randomly on the ballot?

e. Studies of voter behavior have shown that when there is no strong preference for candidates, those names that appear first on a ballot are more likely to be selected. One of the challengers complains that the ballot was "fixed" to favor the incumbents. Use probability to argue for or against this claim.

2.5 The Store Managers' Association has decided to use a rotary combination lock with 40 numbers for their "Crack the Safe" promotion. One manager suggests telling customers that the secret combination consists of 3 different numbers.

a. Without this hint, how many 3-number combinations are possible?

b. How many are possible with this hint?

c. What percentage of the possible combinations is eliminated by the hint?

d. How is the probability that someone will open the safe with one try affected by knowing the hint? Explain your response.

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2.6 Although the Riverside Mall has 12 stores, there are only 8 parking spaces reserved for store managers. If the 8 spaces are always full, in how many ways can they be occupied by the managers' cars?

2.7 In slow-pitch softball, a "rover" or "short fielder" is added to the traditional trio of outfielders. This results in a total of 10 positions on the field. Although Jean would accept any position on the Mall Misfits, he would like to play catcher.

a. Before the season begins, 14 players try out for the Mall Misfits. In how many ways can the team fill the 10 positions, if there are no restrictions on who plays a given position?

b. If Jean must play catcher, in how many ways can the remaining 9 positions be filled?

c. If all 10 positions are filled at random, what is the probability that Jean will be the catcher?

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Research Project

Prepare a report on the system of numbering used by the U.S. Social Security Administration. Your report should include answers to the following questions:

a. How does the Social Security Administration assign numbers to individuals?

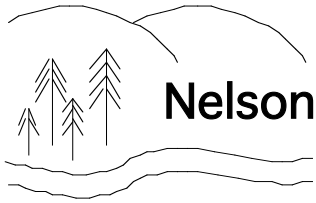
b. Can two people be assigned the same number?

c. How many different social security numbers are possible?

d. At current rates of population growth, are there enough numbers to last until the year 2000? until the year 2500?

Activity 3

As the winter holiday season approaches, Nadia Nelson begins thinking of ways to attract more shoppers to the mall. The decorations, she decides, should be both elaborate and eye-catching. In order to involve the store managers in the decorating process, she circulates the following memo.



Nelson Real Estate Property Management

To: Members of the Store Managers' Association
From: Nadia Nelson
Re: Selection of Decorating Committee

Congratulations again! Although Riverside Mall has been open for only three months, business has exceeded all initial predictions. Just a reminder that the holidays will be here soon and we must begin planning the mall decorations. My hope is that these decorations will reflect the diversity of your membership and provide a strong enticement to holiday shoppers.

The decorations committee consists of three members. Five managers have volunteered to serve. According to the association bylaws, this committee must be selected randomly from all volunteers. The drawing will take place Tuesday evening at the association's regular meeting. See you there!

Exploration

The last names of the 5 volunteers for the decorating committee are Letasky, Milligan, Novotney, Oliphant, and Payne. In this exploration, the name of each volunteer will be represented by its first letter. For example, Letasky will be represented by the letter L.

a. In Activity **2**, you learned that the number of permutations of n distinct items taken r at a time is given by $P(n,r)$. Determine the number of permutations of 5 volunteers taken 3 at a time.

b. In this committee, the order of selection does not matter. In other words, a committee of Letasky, Oliphant, and Payne (LOP) is the same as a committee of Oliphant, Payne, and Letasky (OPL).

1. Determine the number of 3-letter arrangements from Part **a** that include L, O, and P.

2. Express this number of arrangements using permutation notation.

c. All the 3-letter arrangements that include L, O, and P can be considered as one committee. Similarly, all the 3-letter arrangements that include L, M, and O represent another committee. How many different 3-member committees can be selected from the 5 volunteers?

d. Use the fundamental counting principle and your responses to Parts **b** and **c** to write a product that equals the number of permutations of 5 letters chosen 3 at a time, or $P(5,3)$.

e. A collection of symbols or objects in which order is not important is called a **combination**. For example, the number of 3-member committees that can be selected from a group of 5 volunteers is the combination of 5 items taken 3 at a time. This can be written as $C(5,3)$.

Write an equation that relates the permutation of 5 items taken 3 at a time to the combination of 5 items taken 3 at a time.

f. Write $C(5,3)$ in terms of $P(5,3)$ and $P(3,3)$.

Discussion

a. How did the selection process for the arrangements of stores in Activity **2** differ from the selection process for the decorating committee?

b. Explain why the permutation of 5 items taken 3 at a time is not appropriate for counting the number of committees of 3 from a group of 5.

c. What does the permutation of 3 items taken 3 at a time represent in this context?

d. As mentioned in Part **e** of the exploration, a combination is a collection of symbols or objects in which order is not important. Besides its mathematical definition, the word *combination* has several other meanings.

Earlier in this module, for example, you determined the number of different combinations possible for a digital lock. What two elements of the mathematical definition show that the *combination* in a combination lock is not a mathematical one?

Mathematics Note

The number of combinations of n different symbols or objects taken r at a time, denoted by $C(n,r)$, is given by the following formula:

$$C(n,r) = \frac{P(n,r)}{P(r,r)} = \frac{P(n,r)}{r!} = \frac{1}{r!} \bullet \frac{n!}{(n-r)!} = \frac{n!}{r!(n-r)!}$$

Two other common notations for combinations are ${}_nC_r$ and

$$\binom{n}{r}$$

For example, the number of combinations of 7 letters—A, B, C, D, E, F, and G—taken 3 at a time, can be found as shown below:

$$C(7,3) = \frac{P(7,3)}{P(3,3)} = \frac{7!}{3!(7-3)!} = \frac{7!}{3!(4!)} = \frac{7 \bullet 6 \bullet 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1}{(3 \bullet 2 \bullet 1)(4 \bullet 3 \bullet 2 \bullet 1)} = 35$$

e. The process described in Parts **a–d** of the exploration leads to the general equation $P(n,r) = C(n,r) \bullet P(r,r)$. Describe how this equation can be used to find the formula for $C(n,r)$ given in the mathematics note.

Assignment

3.1 Using the letters H, I, J, and K to represent four people, find the number of committees of 2 that can be selected from a group of 4. In other words, determine the combination of 4 items taken 2 at a time. Verify your response by listing the committees.

3.2 In the formula for a combination, what does $r!$ represent?

3.3 The narrators for the mall's daily radio ads are selected from a pool of 12 employees, one from each store. As part of management's efforts to build positive relationships among employees, new teams are chosen each day. Ideally, each employee should work with several different teams during the course of a year.

a. What size teams would you recommend for the daily radio ads? Justify your response.

b. James from the pet store and Alice from the flower and gift shop are friends. If the managers' association decides to use teams of 2 for the radio ads, how often can they expect to work as a team?

3.4 The mall's toy store, Toddler's Haven, plans to sell coloring kits that contain 4 crayons, each one a different color. The crayons are available in a total of 12 different colors. Should the store package every possible assortment of 4 crayons or create assortments as they are ordered? Explain your response.

3.5 One of the prizes in the "Crack the Safe" promotion was a deck of cards. Each card in the deck contained the description of an item from one of the stores. After randomly selecting 30 cards from the deck, the winner received each item described on those cards.

In one advertisement for the promotion, store managers claimed "There are more prize combinations possible than there are molecules of air in the mall!"

a. The mall is a rectangular prism 50 m wide, 120 m long, and 5 m high. One cubic centimeter ($1\ cm^3$) contains approximately $3 \bullet 10^{19}$ molecules of air. About how many molecules of air are there in the mall?

b. If the managers' claim was true, what was the minimum number of cards in the deck?

3.6 Kuzlowski's Pizza offers 3 sizes of pizza, 3 types of crust, and 10 different toppings. Create an advertisement for Kuzlowski's that emphasizes the variety of pizzas available. Make your claims more believable by including the following:

a. the number of choices possible considering crust and size only

b. the number of combinations possible when choosing from 0 to 10 toppings [Hint: Find the number of ways that 0 toppings can be selected from 10, then the number of ways that 1 topping can be selected from 10, and so on.]

c. the total number of pizza choices.

3.7 The figure below shows the first five rows of a pattern you may recall from other modules: Pascal's triangle.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

a. Extend the triangle until it includes 10 rows.

b. Find the sum of the terms in the 10th row.

c. Describe how the sum of the terms in the 10th row of Pascal's triangle is related to your response to Problem **3.6b**.

d. What is the relationship between the combinations possible for each number of toppings in Problem **3.6** and the individual terms in the 10th row of Pascal's triangle?

e. Write the 10th row of Pascal's triangle using combination notation.

f. Write the first 5 rows of Pascal's triangle using combination notation.

3.8 **a.** Find the sum of the terms in each row of Pascal's triangle for at least the first 5 rows. Use your results to develop a formula for finding these sums.

b. Use your formula from Part **a** to verify the sum of the terms of the 10th row of Pascal's triangle from Problem **3.7b**.

c. Describe the significance of each number in your formula for the sum of the terms of the 10th row of Pascal's triangle as it applies to pizzas and their toppings.

d. Explain why your formula is directly related to the fundamental counting principle.

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3.9 **a.** Predict a pattern for determining how many distinct subsets exist for a finite set.

b. If R = {1, 3, 8, 11}, how many distinct subsets of R exist?

c. List the subsets of R.

d. Verify your answer to Part **b** by writing the number as a sum of combinations.

3.10 Given 8 coplanar points, no 3 of which are collinear, how many different triangles can you draw?

3.11 In how many ways can a committee of 5 be selected from 20 people?

3.12 In how many ways can a 5-card hand be dealt from a 52-card deck?

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Summary Assessment

1. Compact disc technology has changed dramatically in the past few decades. The first compact disc (CD) players accepted only one disc at a time and could play music only in the order in which it was recorded.

a. Manufacturers soon provided an option that allowed users to hear songs in random order. Assuming all orders are possible and no songs are repeated, in how many ways could you play a CD with 15 songs?

b. The next generation of CD players could hold more than one disc. Some early models allowed users to play up to 5 discs. Imagine that 2 of these 5 discs each contain 13 songs, while the other 3 discs each contain 16 songs. Assuming all orders are possible and no songs are repeated, in how many ways could you play all the selections on the 5 discs?

2. Riverside Read and Feed plans to stock coffee mugs personalized with first and last initials.

a. How many different mugs would it take to stock every possible arrangement of initials?

b. Would you recommend that the store also stock mugs that include a middle initial? Explain your response.

3. As part of an advertising campaign, the Zone Sporting Goods sponsors a drawing. Each customer is allowed one entry in the drawing, and each entrant can win only one prize. A total of 800 people enter the drawing.

a. Each of the first 20 winners receives the same prize: a discount coupon for 20% off all purchases made on the following Monday. In how many ways can these winners be selected?

b. After the first 20 winners have been selected, each of the next 30 winners also receives an identical prize: 30% off all purchases made on the following Tuesday. In how many ways can these winners be selected?

c. After the first 50 winners have been selected, a final drawing is made from the remaining entrants for a single grand prize: 50% off all purchases for the next year. In how many ways can the grand-prize winner be selected?

d. Imagine that your name is entered in the drawing. You have waited patiently as the first 50 prizes were announced, without winning anything. What is the probability that you will win the grand prize.

Module Summary

- The **fundamental counting principle** provides a method for determining the total number of ways a task can be performed. If an event that can occur in m ways is followed by an event that can occur in n ways, then the total number of ways that the two events can occur is $m \bullet n$.
- A **permutation** is an ordered arrangement of symbols or objects. The number of permutations of n different symbols or objects taken r at a time, denoted by $P(n,r)$, is given by the following formula:

$$P(n,r) = n(n-1)(n-2)\cdots(n-r+1) = \frac{n!}{(n-r)!}$$

  Another commonly used notation for permutations is ${}_nP_r$.

- A collection of symbols or objects in which order is *not* important is a **combination**. The number of combinations of n different symbols or objects taken r at a time, denoted by $C(n,r)$, is given by the following formula:

$$C(n,r) = \frac{P(n,r)}{P(r,r)} = \frac{P(n,r)}{r!} = \frac{1}{r!} \bullet \frac{n!}{(n-r)!} = \frac{n!}{r!(n-r)!}$$

  Two other common notations for combinations are ${}_nC_r$ and

$$\binom{n}{r}$$

Selected References

Brualdi, R. A. *Introductory Combinatorics*. New York: Elsevier North-Holland, 1977.

Lindgren, B. W., G. W. McElrath, and D. A. Berry. *Introduction to Probability and Statistics*. New York: Macmillan Publishing Co., 1978.

Malkevitch, J., G. Froelich, and D. Froelich. "Codes Galore." The History of Mathematics and Its Applications (HistoMAP) Project. Module 18. Arlington, MA: COMAP, 1991.

Sutherland, D. C. "Error-Detecting Identifications Codes for Algebra Students." *School Science and Mathematics* 90(April 1990): 283–290.

U.S. Department of Health and Human Services. "Your Social Security Number." Social Security Administration Publication 05-10002. Washington, DC: U.S. Government Printing Office, 1993.