Log Jam

What do earthquakes, noise levels, and upset stomachs have in common? Data about these phenomena can be difficult to interpret when graphed on a linear scale. In this module, you use logarithms to investigate another type of scale.

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Log Jam

Introduction

Sheila can't sleep because of tomorrow's job interview. What questions will she be asked? How should she respond? As Sheila tosses and turns, her stomach grows uneasy. She climbs out of bed and heads for the medicine cabinet.

Her pain may be due to an irritation of the stomach lining caused by an increase in gastric acid. By neutralizing the excess acid with nonprescription antacids, Sheila can temporarily relieve the pain.

Acidity is commonly measured on the pH scale. The *p* comes from the German word for power; the *H* from the chemical symbol for hydrogen. The **pH scale** describes the relative strength of an acid or base in solution by measuring the concentration of hydrogen ions in a solution.

Figure **1** shows the range of common pH values. On the pH scale, a lower value indicates a more acidic solution. The pH of the gastric juices in your stomach is normally about 2.5. Water, which is neutral, has a pH of 7. Any solution which has a pH less than 7 is acidic; any solution which has a pH greater than 7 is basic. A pH of 0 indicates an extremely acidic solution. A pH of 14 indicates an extremely basic solution.

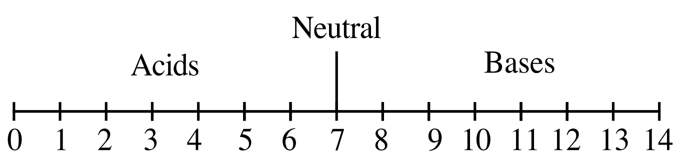

Figure 1: The pH scale

Science Note

An **ion** is an atom (or group of atoms) that has a net positive or negative charge.

An **acidic solution** contains an excess of hydrogen ions (H^+). Many familiar fruits and vegetables, such as lemons, apples, and tomatoes, are weakly acidic. Strong acids, like those found in car batteries, are dangerously corrosive and extremely poisonous.

A **basic solution** contains an excess of hydroxide ions (OH^-). Antacids, some water softeners, and some laxatives contain weak bases. Strong bases, such as those found in lye and drain cleaners, are very caustic and can burn skin.

A solution that is neither acidic nor basic is **neutral**.

Exploration

a. Predict whether each of the following solutions is acidic or basic.

1. a carbonated soft drink

2. milk

3. a solution of baking soda and water

4. vinegar

5. orange juice

b. Blue litmus paper turns pink when dipped in an acidic solution; pink litmus paper turns blue when dipped in a basic solution. Use litmus paper or a pH probe to test your predictions from Part **a**.

Discussion

a. **1.** Which of the solutions in the exploration are acidic?

2. What other common solutions are acidic?

b. **1.** Which of the solutions in the exploration are basic?

2. What other common solutions are basic?

c. What would you expect to happen when an acid and a base are mixed?

Activity 1

After reading the label on the package, Sheila swallows the recommended dosage of antacid and waits for the pain to subside. How does a manufacturer decide how much medication is necessary to relieve the typical symptoms of acid indigestion?

When a basic solution is added to an acidic solution, the excess hydroxide ions react with the excess hydrogen ions to form water. In the following exploration, you observe how the pH of an acidic solution changes with the addition of measured units of a base.

Exploration

a. **1.** Pour about 150 mL of a carbonated soft drink into a 250-mL beaker. Stir or swirl the liquid until most of the carbonation disappears.

2. Measure the pH and record its value.

b. In another beaker, dissolve 2 level teaspoons of baking soda in about 75 mL of water. **Note**: If the baking soda does not completely dissolve, use only the clear solution for Parts **c–d**.

c. **1.** Fill an eye dropper with about 1 mL of the baking soda solution.

2. Add the contents of the dropper to the beaker of soft drink and stir.

3. Measure the pH of this new solution and record its value.

d. Repeat Part **c** 8 to 15 more times.

e. Create a spreadsheet with headings similar to the ones in Table **1** below.

Table 1: Results of pH experiment

Droppers of Baking Soda Solution	pH	Change in pH
0		
1		
2		
3		
⋮		

f. Enter your data in the spreadsheet and determine the change in pH for each consecutive pair of pH values.

g. Determine the number of droppers of basic solution that must be added to the soft drink to produce a change in pH approximately equal to the change in pH obtained when the second dropper was added.

h. Create a scatterplot of pH vs. droppers of basic solution added.

Discussion

a. What happened to the pH as each dropper of baking soda solution was added to the soft drink?

b. How was the change in pH for each consecutive pair of pH values affected as the number of droppers increased?

c. Compare your result in Part **g** of the exploration with those of your classmates. What reasons might there be for any variations in this number?

d. Figure **2** contains a graph of data collected while performing the experiment in the exploration using a sensitive pH monitor. Because this experiment allowed continuous monitoring of the soft-drink solution as droppers of basic solution were added, the graph shows pH versus time in seconds.

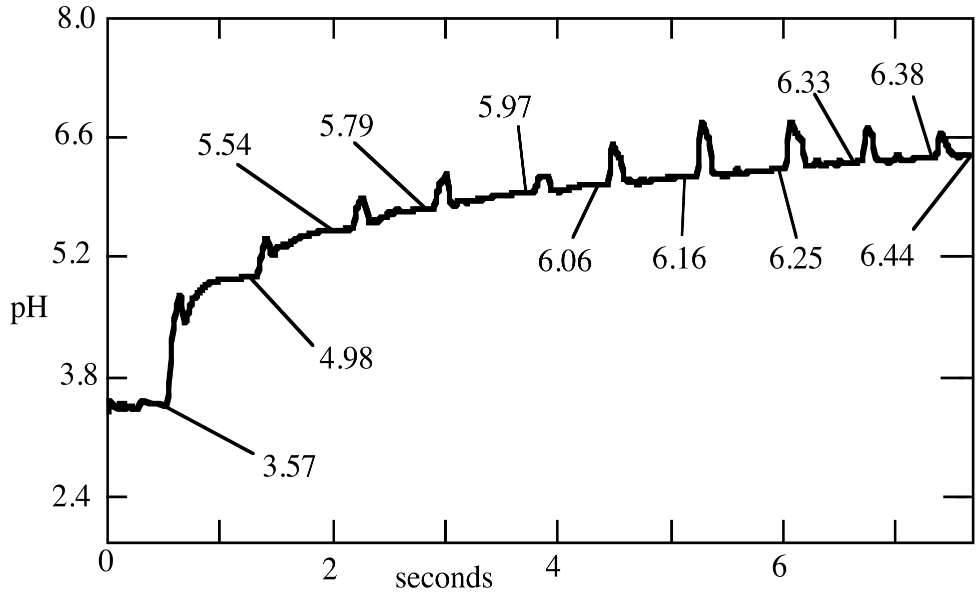

Figure 2: Data from pH experiment

1. What do the spikes on the graph in Figure **2** represent?
2. How does the scatterplot you created in Part **h** of the exploration compare to the graph in Figure **2**?
3. In the graph in Figure **2**, the change in pH produced by the first dropper of basic solution was 1.41. How many more droppers do you think would have to be added to raise the pH another 1.41 units?

Science Note

Acidity can be reported as the concentration of hydrogen ions (H^+) in a solution. On the pH scale, a decrease in 1 unit indicates a change in the concentration of hydrogen ions by a power of 10. For example, the concentration of H^+ in a solution with a pH of 3 is 10 times the concentration in a solution with a pH of 4.

The number of hydrogen ions in a liter of water is about $6 \bullet 10^{16}$. Because the quantities of ions in ordinary volumes of solution are so great, chemists often express concentration in moles per liter. A **mole** of any substance always contains the same number of particles: about $6.02 \bullet 10^{23}$. This number is known as Avogadro’s number, in honor of Amadeo Avogadro (1776-1856), an Italian chemist and physicist.

For example, the concentration of hydrogen ions in water is about $1 \bullet 10^{-7}$ moles/L. The relationship between the number of hydrogen ions in water and their concentration in moles per liter is shown below:

$$\frac{6 \bullet 10^{16} \text{ ions}}{1 \text{ L}} \bullet \frac{1 \text{ mole}}{6.02 \bullet 10^{23} \text{ ions}} \approx \frac{1 \bullet 10^{-7} \text{ moles}}{1 \text{ L}}$$

e. Considering the example given in the mathematics note, how does the exponent of the hydrogen ion concentration appear to be related to pH?

f. Which has a greater concentration of hydrogen ions: water or a solution with a pH of 13?

g. Scientific notation is used to express very large or small quantities as the product of a power of 10 and a number greater than or equal to 1 and less than 10. Describe how to simplify each of the following expressions using the properties of exponents.

1. $(1 \bullet 10^{-7})(6.02 \bullet 10^{23})$
2. $(5.4 \bullet 10^{5})/(2.2 \bullet 10^{-2})$

h. What is the change in hydrogen ion concentration from a pH of 3 to a pH of 7? Explain your response using a property of exponents.

i. The concentration of hydrogen ions in a solution with a pH of 2.5 can be described as $1 \bullet 10^{-2.5}$ moles/L. What is the mathematical meaning of $10^{-2.5}$?

j.
1. Decimal values for pH correspond with hydrogen ion concentrations expressed using decimal exponents. Compare the mathematical meanings of $10^{5.97}$ and $10^{597/100}$.
2. What problems arise when treating $10^{1/3}$ as $10^{(0.333\ldots)}$?
3. Since $10^{(0.333\ldots)}$ is equal to $10^{(0.3+0.03+0.003+\cdots)}$, it can be treated as shown below:

$$10^{0.3} \bullet 10^{0.03} \bullet 10^{0.003} \bullet \cdots$$

Given this fact, describe a relationship between $10^{1/3}$ and $10^{(0.333\ldots)}$.

k. Consider an irrational number n (such as π) that cannot be expressed in the form a/b. How could you evaluate 10^{n}?

Assignment

1.1
a. What is the concentration of hydrogen ions in a solution with a pH of 4?

b. What is their concentration in a solution with a pH of 11?

c. If a solution contains a hydrogen ion concentration of $1 \bullet 10^{-12}$ moles/L, what is its pH?

1.2
a. Determine the approximate number of hydrogen ions in 1 L of a solution with:

1. a pH of 3
2. a pH of 8
3. a pH of n.

b. If 1 L of a solution contains 10^{-n} moles of hydrogen ions, what is its pH?

1.3 **a.** Use ratios to compare the concentration of hydrogen ions in solutions with a pH of 3, a pH of 4, and a pH of 8, respectively.

b. How is the decrease in pH related to the increase in the concentration of hydrogen ions?

1.4 Create a table with the headings shown below.

pH	Concentration of Hydrogen Ions (Moles/L)	No. of Hydrogen Ions per Liter
1		
2		
3		
⋮		
14		

a. Complete the table for pH values from 1 to 14.

b. Make a scatterplot of concentration of hydrogen ions versus pH.

c. Make a scatterplot of number of hydrogen ions per liter versus pH.

d. Compare the shapes of the two graphs from Parts **b** and **c**.

e. Given only the graphs in Parts **b** and **c** (without the table of values), how could you estimate the concentration or number of ions per liter for solutions with a pH greater than 3? Are these graphs useful? Explain your response.

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1.5 The Richter scale is used to measure the intensity of earthquakes. An increase in magnitude of 1 unit on this scale corresponds to a 10-fold increase in intensity. According to the Richter scale, an earthquake of magnitude 4, although mild, can be felt by most people. About how many times stronger than an earthquake of magnitude 4 is an earthquake of magnitude 7?

1.6 The magnitude of the historic San Francisco earthquake of 1906 is estimated at 8.3 on the Richter scale.

a. How does its intensity compare to the 1989 San Francisco quake, which measured 6.9 on the Richter scale?

b. Explain this difference in intensity using the following property of exponents: $a^m \bullet a^n = a^{m+n}$.

c. **1.** Give a mathematical interpretation of $10^{8.3}$ using a radical.

2. Give a mathematical interpretation of $10^{8.3}$ using the property of exponents described in Part **b**.

3. Describe how to use a calculator to evaluate each interpretation in Steps **1** and **2**.

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Activity 2

The numbers of atoms, molecules, or ions involved in chemical analysis often cover an extremely wide range of values. As a result, it can be difficult to interpret such data graphically using a familiar linear scale. In this activity, you examine another useful type of scale: a **logarithmic** scale.

Exploration 1

In the Level 2 module "Atomic Clocks Are Ticking," you investigated some of the properties of exponents. In this exploration, you use technology to reexamine decimal exponents.

a. Create a graph of the function $y = 10^x$.

b. Determine the value of *y* that corresponds to each of the following values of *x*: 0, 1, 2, and 3.

c. Use the trace feature to find the value of *x* (to the nearest 0.01) that corresponds to each value of *y* listed in Table **2**.

Table 2: Values of *x* and *y* for the equation $y = 10^x$

x	*y*
0	1
	2
	3
	4
	5
	6
	7
	8
	9
1	10

d. Using the data in Table **2**, create a scatterplot of the *y*-values versus the corresponding *x*-values.

e. Using the data in Table **2**, create a scatterplot of the *x*-values versus the corresponding *y*-values.

Discussion 1

a. What is the relationship between the graph you created in Part **d** of the exploration and the one you created in Part **e**?

b. Is there a value of x such that $10^x = 0$? Why or why not?

c. A **linear scale** is based on an arithmetic sequence of units in which the difference between successive units is always the same.

1. What type of sequence is formed by the set of y-values in Table **2**?
2. Would a scale based on the x-values in Table **2** form a linear scale? Explain your response.

Mathematics Note

The inverse of the exponential equation $y = b^x$ is the logarithmic equation $x = \log_b(y)$. In other words, a relationship of the form $b^n = m$ can also be expressed as follows: the **logarithm** of m to the base b is n. This can be denoted symbolically as $\log_b(m) = n$. The base b must be greater than 0 and unequal to 1.

On a **logarithmic scale**, the number m corresponding to the number n on a linear scale is the power of the base such that $b^n = m$.

For example, Figure **3** shows a logarithmic scale in which the base is 10. On this scale, 10 corresponds to 1 on the linear scale because $10^1 = 10$ or $\log_{10}(10) = 1$. Similarly, 1 on the logarithmic scale corresponds to 0 on the linear scale because $10^0 = 1$ or $\log_{10}(1) = 0$.

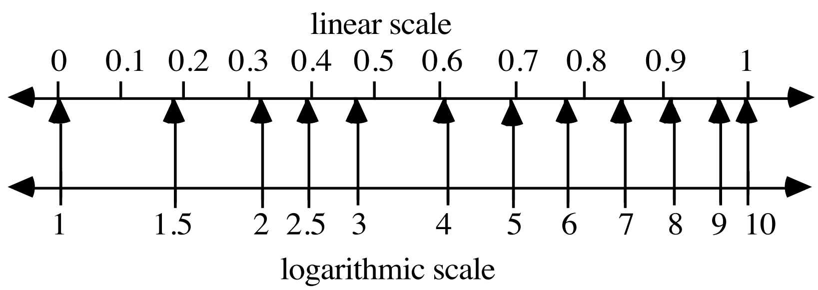

Figure 3: Logarithmic and linear scales

Logarithms of base 10 are referred to as **common logarithms**. The common logarithm of x may be written as $\log_{10} x$, but is usually condensed to $\log x$.

A coordinate system with only one axis marked in a logarithmic scale is a **semilog coordinate system**. When an exponential function of the form $y = b^x$ is graphed on a semilog coordinate system in which the vertical axis is a logarithmic scale, the graph is a straight line.

When both axes are marked in logarithmic scales, they constitute a **log-log coordinate system**.

d. As described in the previous mathematics note, the logarithm of x to base b can be written as $\log_b x$ and has restrictions of $b > 0$, $b \neq 1$, and $x > 0$.

1. Explain why the number 1 cannot be a base.

2. Explain why the base b cannot be negative.

e. Why don't negative numbers have logarithms?

f. The relationship $2^3 = 8$ can be expressed as $\log_2 8 = 3$. Describe how to express each of the following using logarithms:

1. $25 = 5^2$

2. 3^3

3. 81

Exploration 2

a. In Figure **4** below, a logarithmic scale has been labeled with successive powers of 10. Determine the common logarithm of each power of 10.

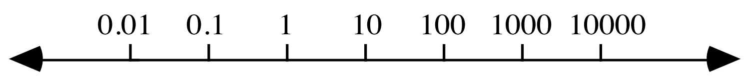

Figure 4: A logarithmic scale

b. The concentration of hydrogen ions in the normal range of pH values may vary from 1 mole/L to $1 \bullet 10^{-14}$ moles/L. Figure **5** shows a scatterplot of this data on a coordinate system that uses linear scales on both axes.

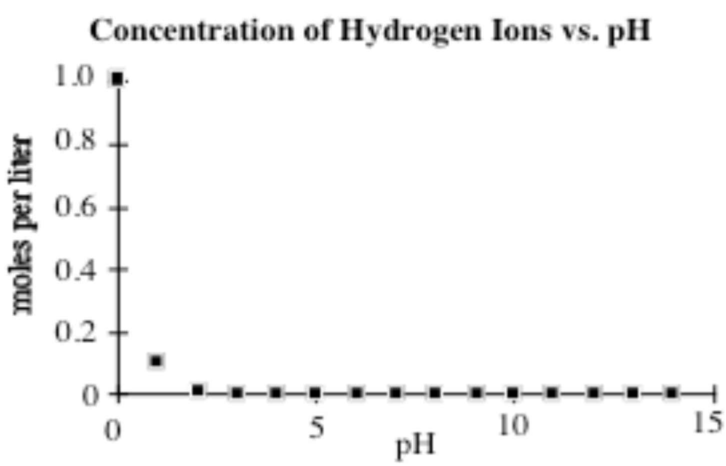

Figure 5: Concentration of hydrogen ions versus pH

How do you think this graph would be affected by changing the scale on the vertical axis to the logarithmic scale in Figure **4**? Record your prediction.

c. Table **3** shows the concentrations of hydrogen ions, in moles per liter, that correspond with pH values from 0 to 14. **Note:** The data in the middle column is a mixture of scientific and regular notation. All data could be written in either form.

Table 3: pH and concentration of hydrogen ions

pH	Concentration of Hydrogen Ions (Moles per Liter)	log(Concentration of Hydrogen Ions)
0	1	
1	0.1	
2	0.01	
3	0.001	
4	0.0001	
5	0.00001	
6	0.000001	
7	0.0000001	
8	0.00000001	
9	$1 \bullet 10^{-9}$	
10	$1 \bullet 10^{-10}$	
11	$1 \bullet 10^{-11}$	
12	$1 \bullet 10^{-12}$	
13	$1 \bullet 10^{-13}$	
14	$1 \bullet 10^{-14}$	

1. Complete the right-hand column of Table **3** using the common log of the concentration of hydrogen ions for pH values from 0 to 14.
2. Create a semilog coordinate system using a base 10-logarithmic scale like the one in Figure **4** for the *y*-axis.
3. Using the semilog coordinate system from Step **2**, create a scatterplot of the concentration of hydrogen ions versus pH.

Discussion 2

a. Compare the graph in Figure **5** with the graph of the same information plotted on a semilog coordinate system in Part **c** of Exploration **2**.

b. The ratio of the concentrations of hydrogen ions in solutions with a pH of 5 and a pH of 10 is 10^5. How is this ratio illustrated by the graph in Figure **5**? How is this ratio illustrated by the graph in Part **c**?

c. Describe some advantages to graphing the data in Table **3** on a semilog coordinate system.

d. Consider the logarithmic equation $3 = \log(x + 2)$. How would you determine the value of x that makes this a true statement?

Assignment

2.1 Why is the number 10 a convenient base for a semilog coordinate system?

2.2 Rewrite each of the following equations using logarithmic notation:

a. $6^4 = 1{,}296$

b. $5^7 = 78{,}125$

c. $2^{10} = 1{,}024$

2.3 Find each of the following common logarithms:

a. $\log\left(10^{-2}\right)$

b. $\log 10{,}000$

c. $\log 0.0001$

2.4 The notation commonly used in chemistry to represent the concentration of hydrogen ions in moles per liter is $[H^+]$. Using this notation, which one of the following formulas expresses pH in terms of hydrogen ion concentration? Explain why you believe your choice is correct.

a. $\text{pH} = \log\left(-\left[H^+\right]\right)$

b. $\text{pH} = \log[H^+]$

c. $\text{pH} = -\log[H^+]$

2.5 Use technology and the relationship between hydrogen ion concentration and pH you identified in Problem **2.4** to calculate the pH of these common liquids.

	Solution	**Concentration of Hydrogen Ions (Moles per Liter)**
a.	lemon juice	0.005
b.	vinegar	0.0016
c.	carbonated water	0.001
d.	milk	$2.51 \bullet 10^{-7}$
e.	blood	$3.98 \bullet 10^{-8}$

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2.6 Imagine that a country's annual steel production, in millions of tons, can be modeled by the equation $y = 96\log(x+3)$, where $x = 0$ represents the year 1955.

a. Graph this logarithmic function on a graphing utility, using appropriate intervals for the domain and range. Describe the shape of the graph.

b. Use the graph to estimate the country's steel production in 1980 and in 1995.

c. Using this model, during what year do you predict the steel production will reach 200 million tons?

2.7 A greeting card company spends $300,000 per year on advertising. The marketing department has determined that if the amount of money spent on advertising is increased by *p* dollars, the total sales *S* can be modeled by the equation $S = 132{,}000\log(p + 300{,}000)$.

a. Graph this logarithmic function on a graphing utility, using appropriate intervals for the domain and range. Describe the shape of the graph.

b. Determine the company's current total sales.

c. Use this model to predict total sales if the company doubles its advertising budget.

d. Based on your results in Parts **b** and **c**, do you think the company should double its advertising budget? Explain your response.

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Activity 3

In previous activities, you used graphs to discover the logarithmic relationship between the pH of a solution and its hydrogen ion concentration. In this activity, you explore some algebraic methods for solving problems involving logarithms.

Exploration

Because logarithms are exponents, it is possible to apply your knowledge of exponents to problems involving logarithms. In this exploration, you use technology to help determine a property of logarithms that corresponds to each of the following properties of exponents:

- $b^x \bullet b^y = b^{x+y}$
- $b^x / b^y = b^{x-y}$
- $(b^x)^y = b^{x \bullet y}$

a. Create a spreadsheet with headings similar to the ones shown in Table **5**.

Table 5: Logarithmic formulas

x	y	**log$x$**	**log$y$**	**log(x • y)**
1	25			
2	24			
3	23			
⋮	⋮			
24	2			
25	1			

b. Complete the spreadsheet for $\log x$, $\log y$, and $\log(x \bullet y)$. Round these values to the nearest 0.001.

c. Note any relationships you observe between values in the column labeled "$\log(x \bullet y)$" and the values in any of the other four columns.

d. **1.** Add a column to your spreadsheet to calculate $\log(x/y)$.

2. Repeat Part **c** for $\log(x/y)$.

e. Repeat Part **d** for $\log x^y$, $\log 10^x$, and $10^{\log x}$.

f. Repeat Parts **a–e** using a base other than 10.

Discussion

a. Do you think the relationships you found for common logarithms are true for logarithms of any base? Justify your response.

Mathematics Note

The following properties are true for logarithms of any base b, where $b > 0$ and $b \neq 1$.

- product property: $\log_b(x \bullet y) = \log_b x + \log_b y$, for $x > 0$ and $y > 0$
- quotient property: $\log_b(x/y) = \log_b x - \log_b y$, for $x > 0$ and $y > 0$
- power property: $\log_b x^y = y\log_b x$, for $x > 0$
- inverse property: $\log_b b^x = x$ and $b^{\log_b x} = x$, for $x > 0$

For example, using the product property of logarithms:

$$\log_2\left(2^2 \bullet 2^{0.5}\right) = \log_2\left(2^2\right) + \log_2\left(2^{0.5}\right) = 2 + 0.5 = 2.5$$

Using the quotient property, $\log_2\left(2^2/2^{0.5}\right) = \log_2\left(2^2\right) - \log_2\left(2^{0.5}\right) = 2 - 0.5 = 1.5$.

Using the power property, $\log_2\left(2^3\right) = 3\log_2(2) = 3$.

b. Compare the properties of logarithms described in the mathematics note to the properties of exponents.

c. When is $\log_b x = \log_b y$?

Assignment

3.1 Use the properties of logarithms to determine the value of *x* that makes each of the following statements true.

a. $6^x = 234$

b. $5^x = 1000$

c. $2^x = 50$

3.2 When graphed on a semilog coordinate system, the relationship between hydrogen ion concentration $[H^+]$ in moles per liter and pH can be modeled by the equation:

$$[H^+] = \frac{1}{10^{pH}}$$

Use the properties of logarithms to show that this equation is equivalent to $pH = -\log[H^+]$.

3.3 Imagine that you are a cell biologist. For your next experiment, you must grow 100,000 bacteria of a particular type. You begin the bacterial culture with 1 cell. Through a process known as mitosis, that cell divides into 2. The 2 new cells split again, creating 4 cells. The bacterial population quadruples every day.

a. Predict how many days it will take to produce enough cells for the experiment.

b. Write an equation that represents the relationship between the number of days and the number of cells.

c. Using appropriate technology, estimate the number of days (to the nearest 0.1 days) required for the cell population to reach 100,000. Explain the process you used to find this approximation.

d. Using algebraic methods, determine the number of days required for the cell population to reach 100,000. Compare this solution with your estimate in Part **c**.

e. Use a symbolic manipulator to verify your solution in Part **d**.

3.4 A *Los Angeles Times* article reported that the intensity of the 1971 Sylmar quake which rocked the northern San Fernando Valley was one-fourth that of the 1989 San Francisco quake. Since the magnitude of the 1989 quake was 6.9 on the Richter scale, the equation that models this situation is $1 \bullet 10^{6.9} = 4 \bullet 10^x$, where *x* represents the magnitude of the Sylmar quake. Find the magnitude of the Sylmar quake.

3.5 If log n represents the magnitude of an earthquake on the Richter scale, then $\log(2n)$ represents the magnitude of an earthquake with twice the intensity.

a. **1.** Use the properties of logarithms to determine the increase in magnitude when the intensity of an earthquake is doubled.

2. Does the original intensity affect the increase in magnitude? Justify your response.

b. What is the increase in magnitude on the Richter scale when the intensity of an earthquake is tripled? Justify your response.

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3.6 The properties of logarithms can be used to solve many problems involving exponents. For example, the formula for calculating the total amount in an interest-bearing savings account is:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

In this formula, A is the total in the account, P is the original investment or principal, r is the annual interest rate in decimal form, n is the number of times per year that the interest is calculated or compounded, and t is the time in years that the account is active.

a. Imagine that you have deposited \$100 in a savings account at 6% annual interest, compounded monthly. Estimate how long the principal and interest must be left in the account for the total to reach \$450.

b. Using appropriate technology, determine a more precise answer to Part **a**. Describe the process that you used.

c. To determine an answer to Part **a** algebraically, you must substitute the known values into the formula above and solve for t. The solution follows Steps **1–10** below. Describe what has been done in each step.

1. $450 = 100\left(1 + (0.06/12)\right)^{12t}$

2. $450 = 100(1.005)^{12t}$

3. $450/100 = (1.005)^{12t}$

4. $4.5 = (1.005)^{12t}$

5. $\log 4.5 = \log(1.005)^{12t}$

6. $\log 4.5 = 12t \log(1.005)$

7. $\log 4.5/\log 1.005 = 12t$

8. $301.57 \approx 12t$

9. $301.57/12 \approx t$

10. $t \approx 25.13$

d. Which property of logarithms is necessary in this solution? In which step is the property used?

e. Why is it necessary to use the property identified in Part **d**?

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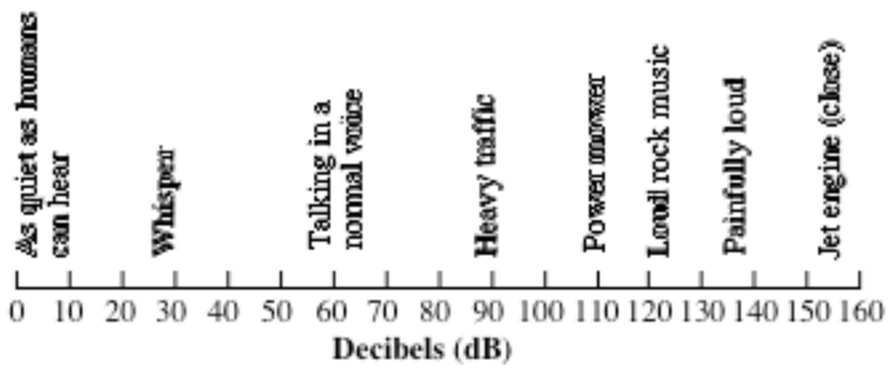
Research Project

Several large earthquakes occur each year around the world. Find an article in a magazine or newspaper that describes one of these quakes. In your report, identify the location and magnitude of the earthquake and compare it to the 1989 San Francisco earthquake. Your comparison should mention the differences in magnitude, intensity, and amount of damage done by each quake and explain any reasons for those differences.

Summary Assessment

1. One measure of the relative intensity of sound is the decibel (dB). The decibel chart below shows the magnitude of sound generated by some common objects or situations.



Extremely loud sounds can exert enough pressure on the human ear to cause pain. The following table gives the amount of pressure on the ear, measured in newtons per square meter (N/m^2), at various decibel levels. The pressure at 0 dB, a sound too quiet to be detected by the human ear, is 0.00002 N/m^2. This value is the reference pressure, P_r.

Decibel Level (dB)	Pressure on Ear (N/m^2)
0	0.00002
10	0.0002
20	0.002
30	0.02
40	0.2
50	2
60	20
70	200
80	2,000
90	20,000
100	200,000
110	2,000,000
120	20,000,000
130	200,000,000
140	2,000,000,000
150	20,000,000,000
160	200,000,000,000

a. Use the data given to find a relationship between the decibel level (d) of a sound and the pressure (P) created by that sound.

b. Show that the following equation is equivalent to the relationship you found in Part **a**, where d is the decibel level, P is the pressure created by the sound, and P_r is the reference pressure of 0.00002 N/m^2:

$$d = 10\log\left(\frac{P}{P_r}\right)$$

c. Using the properties of logarithms, solve the equation you found in Part **a** for pressure, P.

2. **a.** What is the pressure on the eardrum caused by music at a loud rock concert (118 dB)? How many times greater is this pressure than the pressure created by normal conversation (54 dB)?

b. A sound creates a pressure on the eardrum of $1 \bullet 10^3 \text{ N/m}^2$. Is this sound audible to the human ear? If so, is this sound painful?

3. Swimming underwater also affects the pressure on your eardrums. For each 1 m of depth, the pressure increases by $10,000 \text{ N/m}^2$.

a. At what depth is the pressure on your ears equivalent to the pressure created by the sound of heavy traffic?

b. If you dove to a depth of 100 m, what would be the pressure on your ears? What is the magnitude of the sound (in decibels) required to create an equivalent amount of pressure?

Module Summary

- An **ion** is an atom (or group of atoms) that has a net positive or negative charge.
- A **basic solution** contains an excess of hydroxide ions (OH^-).
- An **acidic solution** contains an excess of hydrogen ions (H^+).
- A solution that is neither acidic nor basic is **neutral**.
- A **mole** of any substance always contains the same number of particles: about $6.02 \bullet 10^{23}$. The number $6.02 \bullet 10^{23}$ is known as **Avogadro's number**.
- The **pH scale** describes the relative strength of an acid or base in solution. The pH of a solution is equal to the negative log of the hydrogen ion concentration in moles per liter.
- The inverse of the exponential equation $y = b^x$ is the logarithmic equation $y = \log_b(x)$. In other words, a relationship of the form $b^n = m$ can also be expressed as follows: the **logarithm** of *m* to the base *b* is *n*. This can be denoted symbolically as $\log_b(m) = n$. The base *b* must be greater than 0 and unequal to 1.
- Logarithms of base 10 are referred to as **common logarithms**. A common logarithm may be written as $\log_{10} x$, but is usually condensed to $\log x$.
- A **linear scale** is based on an arithmetic sequence of units in which the difference between successive units is always the same.
- On a **logarithmic scale**, the number *m* corresponding to the number *n* on a linear scale is the power of the base such that $b^n = m$.
- A coordinate system with only one axis marked in a logarithmic scale is a **semilog coordinate system**. When both axes are marked in logarithmic scales, they constitute a **log-log coordinate system**.
- The following properties are true for logarithms of any base *b*, where $b > 0$ and $b \neq 1$.

product property:	$\log_b(x \bullet y) = \log_b x + \log_b y$, for $x > 0$ and $y > 0$
quotient property:	$\log_b(x/y) = \log_b x - \log_b y$, for $x > 0$ and $y > 0$
power property:	$\log_b x^y = y \log_b x$, for $x > 0$
inverse property:	$\log_b b^x = x$ and $b^{\log_b x} = x$, for $x > 0$

Selected References

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