How Long Is This Going to Take?

When building a house, how does a contractor estimate the time required to complete the job? In this module, you explore how to design a schedule for a complicated project.

Russ Killingsworth • Verne Schlepp • Dan West

How Long Is This Going to Take?

Introduction

Imagine that your school is participating in a community service program that builds affordable housing. Your mathematics class has been asked to help plan a major construction project. There are many phases to this project, including obtaining materials and scheduling contractors, as well as the actual construction itself. In this module, you examine some organizational strategies to help minimize the time and money required to complete the project.

Activity 1

Three local firms—United Drywall, Empire Plumbing, and J&C Hardware—have donated materials for the project. To haul these materials to the building site, a freight company has offered the use of a flatbed truck for one day. Your only expense is fuel for the truck, at a cost of $0.18 per kilometer.

Exploration 1

In this exploration, your job is to find the shortest possible route to the three firms. You must start at the building site, visit each firm, then return to the site. A **weighted graph** of the four locations is shown in Figure **1**. (Recall that in a weighted graph, each edge is assigned a numerical value). In this graph, *B* represents the building site, *U* represents United Drywall, *E* represents Empire Plumbing, and *J* represents J&C Hardware. The distances given are in kilometers.

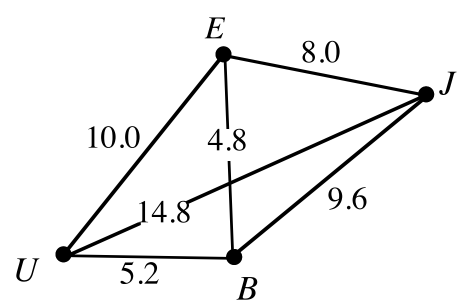


Figure 1: Graph of four locations

a. Using the graph in Figure **1**, determine one route that could be used to pick up all the materials.

b. Recall that a **Hamiltonian circuit** is a closed path that starts at one vertex, visits every other vertex in a graph exactly once, and returns to the starting vertex. Determine the total number of Hamiltonian circuits possible in Figure **1**.

c. List all the possible Hamiltonian circuits that start at the building site.

d. Considering all the circuits you listed in Part **c**, find the shortest route that could be used to pick up all the materials. Compare this route to the one you identified in Part **a**.

e. Determine the minimum cost of picking up the donated materials from the three firms.

Discussion 1

a. A **complete graph** is a graph in which each pair of vertices is connected by exactly one edge. Is the graph in Figure **1** a complete graph? Explain your response

b. When selecting a route for picking up materials, you may or may not have considered the direction of travel. In what types of situations might direction be important?

Exploration 2

Another local firm, AAA Lumber, also has agreed to donate materials to the building project. Now you must add another destination to your route. Figure **2** shows a complete graph of the four locations from Exploration **1**, along with AAA Lumber, represented by vertex *A*. As in Figure **1**, distances are given in kilometers.

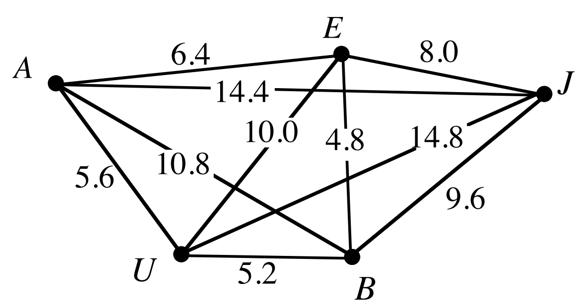


Figure 2: Graph of five locations

To spend as little as possible on fuel, you would like to minimize the total distance traveled. However, identifying the total distance for each possible route from the building site to the four companies could be very time consuming. To save time when designing routes to multiple locations, planners may use an algorithm, such as the **nearest neighbor algorithm**.

Using such an algorithm may not identify the shortest route possible. Nevertheless, it does provide an efficient method of determining a reasonably short route.

Mathematics Note

Figure **3** below shows a weighted graph with four vertices. The following steps describe the use of the **nearest neighbor algorithm** to identify a reasonably efficient Hamiltonian circuit for this graph.

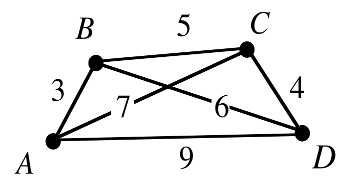


Figure 3: A weighted graph

- Starting with any vertex, select the edge to its "nearest" vertex. In other words, select the edge that has the least weight. In Figure **4**, for example, the starting vertex is *A*. Since the edge connecting *A* and *B* has a weight less than those connecting *A* to *C* or to *D*, it is selected.

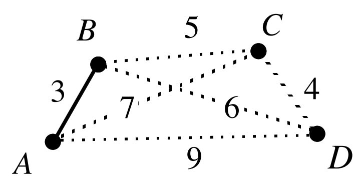


Figure 4: Selecting an edge between the starting vertex and its "nearest" vertex

- Continue this process from the second vertex, selecting the edge to the next nearest vertex not yet visited, and so on, until all vertices have been visited. For example, Figure **5a** shows the selection of the edge from *B* to *C*, while Figure **5b** shows the selection of the edge from *C* to *D*.

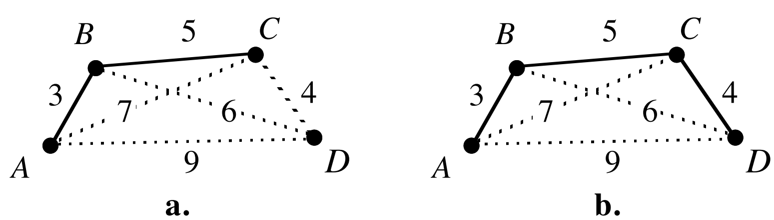


Figure 5: Continuing the nearest neighbor algorithm

- To complete a Hamiltonian circuit, return to the original vertex. For example, Figure **6** shows the edge marked from *D* to *A*.

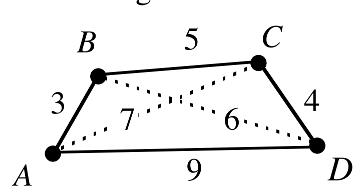


Figure 6: Completing a Hamiltonian circuit

a. Determine the number of Hamiltonian circuits possible for the graph in Figure **2.**

b. Starting at the building site (vertex *B*), use the nearest neighbor algorithm to find a Hamiltonian circuit for the graph in Figure **2**.

c. If the cost of fuel is \$0.18 per kilometer, calculate your expenses for the route identified in Part **b**.

d. Repeat Parts **b** and **c** using each vertex in the graph as the starting vertex.

Discussion 2

a. On a given graph, will the nearest neighbor algorithm always produce a circuit that uses the same edges, regardless of the starting point? Explain your response.

b. The circuit that starts at AAA Lumber, then visits United Drywall, the building site, Empire Plumbing, and J&C Hardware, before returning to AAA Lumber, is 38 km long. Is it possible to use this route when starting at the building site? Explain your response.

c. Describe some advantages and disadvantages of using the nearest neighbor algorithm to determine an efficient route.

d.
1. Compare the numbers of possible Hamiltonian circuits for a complete graph with four vertices and a complete graph with five vertices.
2. Suppose that 20 businesses wished to donate building materials to the school project. Would it be feasible to determine the shortest possible route connecting all these businesses? Explain your response.

Mathematics Note

The nearest neighbor algorithm is just one of many that can be used to select a reasonably efficient Hamiltonian circuit. In the **cheapest link algorithm**, the cheapest (or shortest) action is taken at each step, regardless of starting and stopping points. Individual, disconnected edges may occur at various stages. If the cheapest remaining action completes a circuit that does not visit all the vertices in the graph, then the next best action is taken. When a Hamiltonian circuit is found, the algorithm is complete.

For example, consider the complete weighted graph in Figure **7** below.

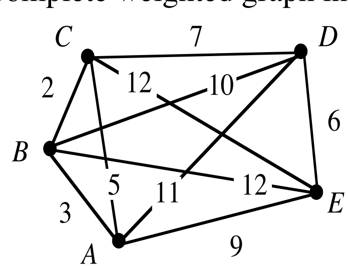


Figure 7: A weighted graph with five vertices

Figure **8** shows the first two steps in drawing a Hamiltonian circuit for this graph using the cheapest link algorithm. Since the edge between *B* and *C* has the least weight of any edge in the graph, it is selected first.

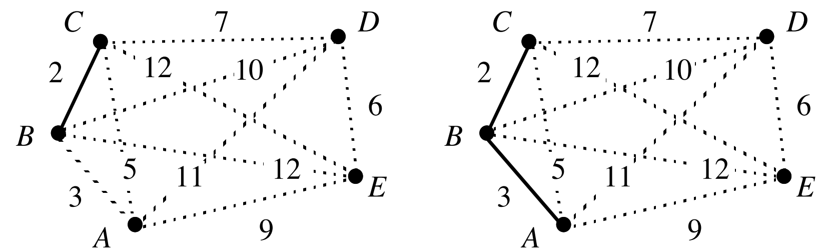


Figure 8: Using the cheapest link algorithm

Figure **9** shows the next step in drawing a Hamiltonian circuit for the complete graph in Figure **7**. Notice that the edge between *D* and *E* is selected even though the edge between *C* and *A* has a lesser weight. This occurs because selecting the edge between *A* and *C* would complete a circuit that does not visit all the vertices.

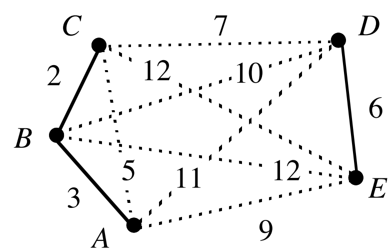


Figure 8: Continuing the cheapest link algorithm

Figure **9** shows the final two steps necessary to complete a Hamiltonian circuit for the entire graph.

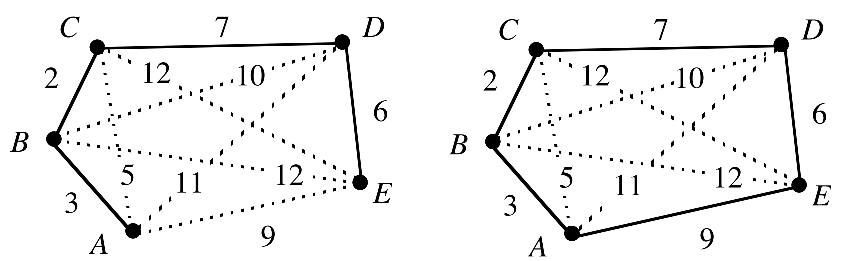


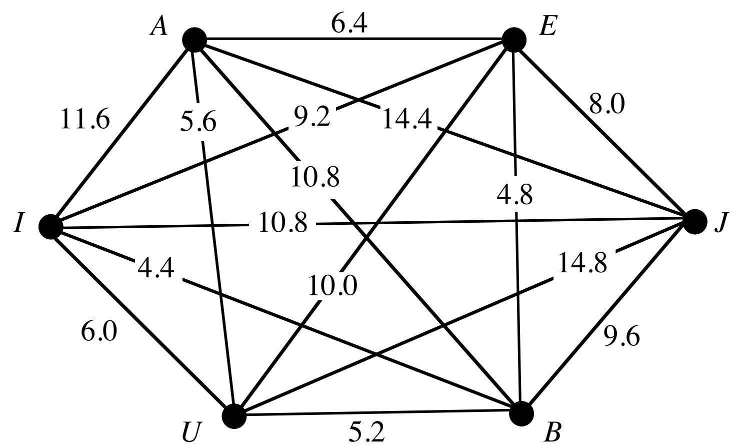
Figure 9: Completing a Hamiltonian circuit

e. **1.** Describe how to use the cheapest link algorithm to determine a route among the five locations described in Figure **2**.

2. How does the resulting route compare to the ones you found using the nearest neighbor algorithm?

Assignment

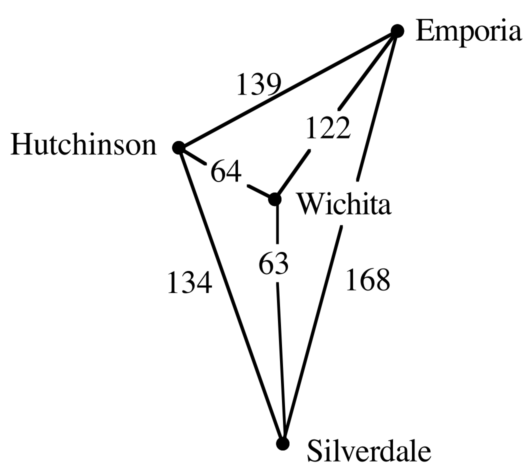
1.1 The Independent Roofing Company also offers to donate materials to the building project. You must now pick up materials from five locations before returning to the building site. The figure below shows a complete weighted graph of all these locations, with distances given in kilometers.



a. Find an efficient route for picking up materials from the five businesses. Describe the method you used to select the route.

b. What is the cost of picking up the materials using your route?

1.2 A small airline has just purchased a new plane. The company plans to use this plane to offer daily flights connecting four cities in Kansas: Emporia, Hutchinson, Silverdale, and Wichita. Since the company's headquarters are in Wichita, they would like each day's flight schedule to begin and end there. The following weighted graph shows the air distance in kilometers between the four cities.

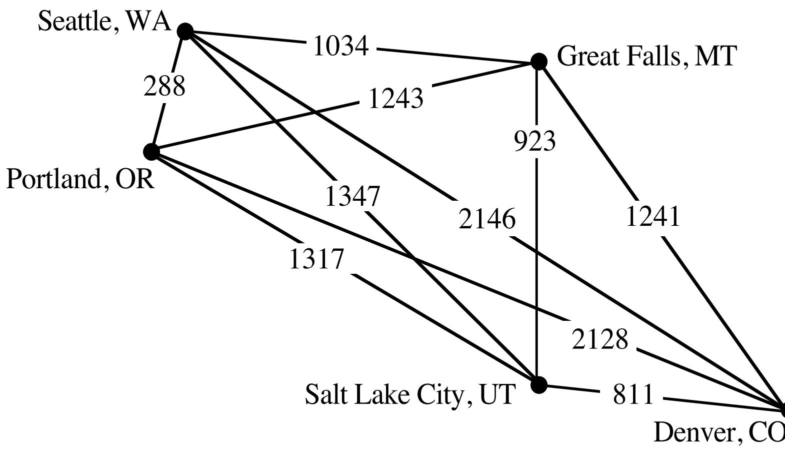


Find a Hamiltonian circuit that could help the airline design a schedule for the plane using each of the following algorithms.

a. the nearest neighbor algorithm

b. the cheapest link algorithm

1.3 Imagine that you own a trucking company based in Seattle, Washington. Your company delivers goods to four other cities in the western United States. The map below shows the distances in kilometers between each city.

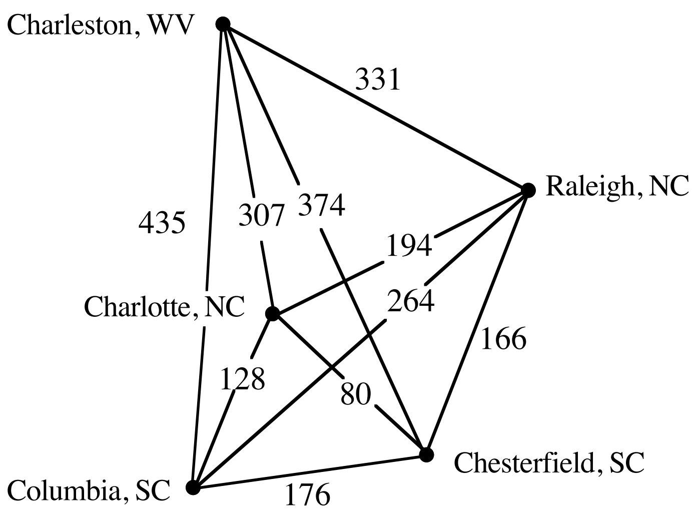


a. Use the cheapest link algorithm to find a Hamiltonian circuit that connects the five cities.

b. Use the nearest neighbor algorithm to find a Hamiltonian circuit that connects the five cities.

c. Can you be sure that either of the circuits you found in Parts **a** and **b** represents the shortest possible circuit connecting the five cities? Explain your response.

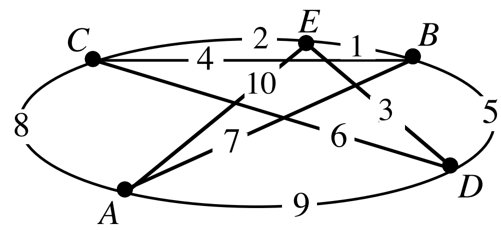
1.4 **a.** The following weighted graph shows the air distances in kilometers between five U.S. cities. Use any method you wish to find an efficient Hamiltonian circuit for the graph.



b. Calculate the total air distance for the circuit found in Part **a**.

* * * * *

1.5 Consider the following weighted graph.



a. Use the nearest neighbor algorithm to identify a Hamiltonian circuit for this graph.

b. Use the cheapest link algorithm to identify a Hamiltonian circuit for this graph.

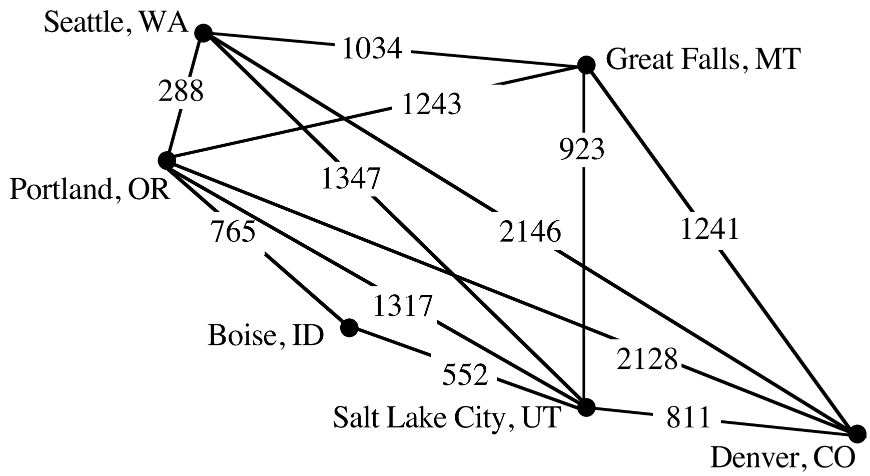
1.6 Imagine that you are creating a computer network with four workstations—A, B, C, and D—and one network server. The network's cable must connect all the computers in a closed circuit. The following table shows the distances among the workstations and the server. An X in the table means that no cable can be connected between these two stations.

	server	**D**	**C**	**B**
A	5 m	10 m	X	3 m
B	4 m	X	4 m	
C	2 m	9 m		
D	8 m			

a. Draw a graph to represent the computer network.

b. Since the connecting cable is expensive, you would like to keep the total length of cable used in the network reasonably low. How much cable should you purchase? Explain your response.

1.7 The trucking company in Problem **1.3** decides to add Boise, Idaho, to its delivery route. The map below shows the distances in kilometers between the original cities, as well as the distances between Boise and Portland and Boise and Salt Lake City.



a. Does the map above represent a complete weighted graph? Explain your response.

b. Use the cheapest link algorithm to find a Hamiltonian circuit that connects the six cities.

c. Use the nearest neighbor algorithm to find a Hamiltonian circuit that connects the six cities.

* * * * * * * * * *

Activity 2

Now that the materials are at the building site, the actual construction can begin. A local contractor has given your class a list of the jobs necessary to proceed with the project. Table **1** shows each task, its approximate completion time in hours, and any prerequisite tasks.

Table 1: Tasks in a construction project

	Task	**Hours**	**Prerequisite Tasks**
A	construct floor joists and subfloor	20	none
B	frame and raise walls	16	A
C	install trusses and roof	27	B
D	install siding, windows, and doors	26	B
E	partition walls	6	C
F	install wiring and plumbing	16	E
G	finish floor	12	E
H	install insulation	6	D
I	install drywall	19	F

Exploration

In this exploration, you investigate how to organize the construction schedule in the most efficient manner.

a. **1.** Cut nine 1-cm wide strips from a piece of stiff paper.

2. Using an appropriate scale, cut and label a strip to represent the time required for each task in Table **1**.

b. Use your paper strips to create a model that represents the maximum possible completion time for the project. Record the maximum completion time.

c. To complete the project more quickly, some tasks can be done at the same time. Use your strips of paper to create a model that represents a more efficient schedule.

Before starting any particular task, make certain that all prerequisite tasks are finished. For example, job B must follow job A; while jobs C and D must both follow B. This portion of the schedule can be modeled by placing the strips as shown in Figure **10** (not drawn to scale).

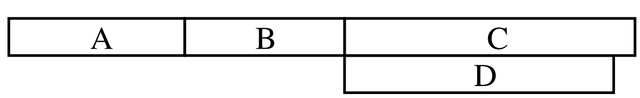
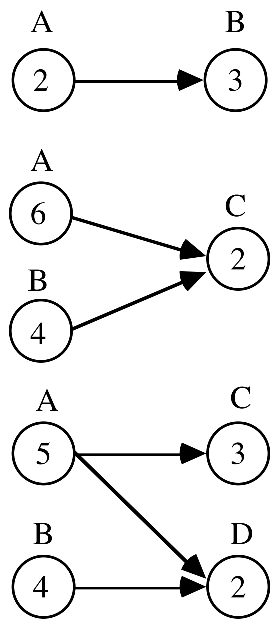


Figure 10: A paper-strip model

d. Determine the minimum completion time for the project. Draw a sketch of your model.

Mathematics Note

A **network diagram** (or order requirement digraph) can be used to represent a scheduling problem. In a network diagram, prerequisite tasks are connected with arrows. For example, Figure **11** shows three different network diagrams and their interpretations. The direction of the arrows shows the order in which the tasks must be completed. The number in each circle represents the time required to complete the task.



Task A, requiring 2 hrs, must be completed before Task B, requiring 3 hrs, can begin.

Tasks A (6 hr) and B (4 hr) can be done concurrently, but both must be completed before Task C (2 hr) can begin.

Tasks A (5 hr) and B (4 hr) can be done concurrently, but both must be completed before Task D (2 hr) can begin. Tasks C (3 hr) and D can be done concurrently, but Task A must be completed before Task C can begin.

Figure 11: Three network diagrams

The **critical path** is the longest set of prerequisite-linked tasks in a project. The length of the critical path is the minimum time required for the project. When a project is represented in a network diagram, the critical path is the longest path in the diagram. In the network diagram shown in Figure **12**, for example, the critical path P–Q–S is 21 units long.

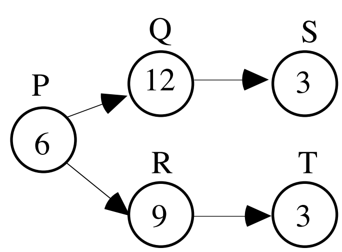


Figure 12: A network diagram

e. **1.** Draw a network diagram based on the tasks listed in Table **1**.

2. Determine the critical path for your network diagram.

Discussion

a. Describe how you used your model to determine the minimum completion time for the tasks in Table **1**.

b. Compare your results in Part **d** of the exploration with those of others in the class. Are all the schedules with the same completion time exactly the same?

c. How does the sketch of your paper-strip model compare to the network diagram?

d. How does the minimum time you found in Part **d** of the exploration compare to the critical path of your network diagram? Explain why this relationship occurs.

e. In Figure **12**, what is indicated by the lack of arrows between Q and T ?

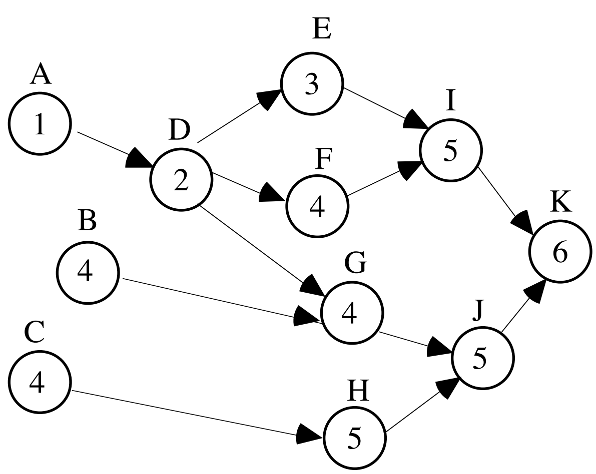
Assignment

2.1 After considering the tasks listed in Table **1**, the contractor realizes that one important job was forgotten: the installation of the heating and cooling system. This task will take about 11 hr. It cannot be started until the floor joists and subfloor are done and must be completed before the floor can be finished and the drywall installed.

a. Draw a diagram of a new work schedule for the project.

b. Determine the minimum time to complete the entire project, including this new task.

2.2 Find the length of the critical path in the network diagram below.



2.3 Imagine that you are the chief of ground operations for Fly-by-Night Airlines. During the time the company's planes are at the gate, 10 major tasks must be completed. These tasks are listed in the table below.

	Task	Time (min)	Prerequisite Tasks
A	unloading passengers	17	none
B	unloading cargo	25	none
C	loading passengers	23	A, J, F
D	loading cargo	25	B
E	refuel plane	15	none
F	stock food supplies	12	A
G	flush holding tank	6	none
H	wash cockpit windows	4	none
I	pre-flight check	12	E
J	clean plane interior	19	A

One of your responsibilities is to minimize the time each plane is on the ground. Create a schedule that completes all 10 tasks in the shortest amount of time.

2.4 After the building's shell is finished, the students working on the building will be divided into two teams for the remaining tasks. The contractor has set the following rules for the two work crews.

- The two crews cannot share a task.
- Once a crew begins a task, it must finish that task before starting another one.
- No crew can be idle while a task is available.

The table below lists each task in the final stage of the construction project, its prerequisites, and its completion time. Determine a schedule for the two crews that minimizes the time needed to complete all the tasks.

	Task	Time (hr)	Prerequisite Tasks
A	interior painting	20	none
B	exterior painting	15	none
C	pour sidewalks	21	none
D	finish carpentry	11	A
E	finish kitchen	6	A
F	wallpaper	5	D, E
G	landscaping	25	B, C, J
H	install carpeting	12	F
I	install linoleum	7	F
J	build deck	12	none

2.5 **a.** By adding one more student to each crew, the contractor believes that the time required for each task in Problem **2.4** can be reduced by 1 hr. Create a new work schedule for these larger crews. Record the hours required to complete the work.

b. As an alternative to the larger crews, one student suggests working on the deck and the landscaping at the same time. Which option would finish the project more quickly: adding one more student to each crew or building the deck during landscaping? Explain your response.

2.6 **a.** In another attempt to save time, one of the volunteers suggests dividing the students into three work crews. Assuming that the time required to complete each task remains the same as described in Problem **2.4**, write a work schedule that minimizes the time required for three crews to complete all the tasks.

b. Suppose that all the tasks must be completed in 45 hr, no matter what the cost. How many crews would it take to meet this deadline? Explain your response.

* * * * *

2.7 Zach, Vin, and Signe work at a company that makes winter coats. Zach takes 20 min to sew the inside lining of each coat. Vin requires 30 min to sew the outside of each coat. After the inside and outside are completed, Signe requires 40 min to sew the two parts together and inspect the finished product.

a. Assuming that each employee works only at a single task, create a network diagram of this situation. Identify the critical path and determine the time required to complete 5 coats.

b. Determine the time to complete 5 coats if Signe reduces the time required to finish her task by 50%.

2.8 Stacie, Chris, and Kodjo work for a company that prepares bulk mailings for businesses. At 1:00 P.M., they receive orders from 13 different customers. The time required to process each customer's mailing is listed in the following table. In order for all the mailings to be delivered on time, the processing must be finished by 3:00 P.M. that same day.

Business	Time (min)	Business	Time (min)
A	12	H	18
B	26	I	25
C	45	J	35
D	24	K	40
E	15	L	30
F	25	M	20
G	10		

The manager assigns Stacie to businesses A, D, G, J, and M; Chris to businesses B, E, H, and K; and Kodjo to businesses C, F, I, and L.

a. Can all the mail be processed in time using the manager's work schedule? Explain your response

b. Stacie, Chris, and Kodjo would like a schedule that allows them to take a 10-min break and still finish the work on time. Write a schedule that will satisfy the employees.

2.9 The copy service for the Plainville School District promises to deliver all orders received before 8:00 A.M. by the following day.

The table below shows the number of copies requested by each school in the district.

School	Copies Requested
A	1000
B	2200
C	430
D	5300
E	6500
F	200
G	600
H	700
I	4500
J	5000
K	800

a. The copy service's lone photocopier can produce 1 copy per second. What is the minimum time required to complete these orders?

b. Suppose that the copy service purchases a second photocopier that operates at the same speed. Write a schedule that will allow the two copiers to finish the orders in the least amount of time.

c. After finishing a copying job, workers need about 10 min to package and label the copies, then prepare the photocopier for the next job. Repeat Part **b** given these new conditions.

* * * * * * * * * *

Activity 3

Because the schedule that your class developed for the construction project worked so well, the landscaping crew also has asked for your advice. To create borders for trees, bushes, and flower beds, they will need landscaping timbers of many different lengths, as shown in Table **2**. However, the lumberyard is only willing to donate timbers in 10-ft lengths.

Table 2: Quantities and measurements for landscaping timbers

Quantity	Length of Each (ft)	Quantity	Length of Each (ft)
3	7	2	6
6	4	1	8
2	9	3	2
1	5	3	10
2	3		

This situation can be analyzed in terms of fitting items of different lengths into a bin of a particular size. Such situations are often called **bin-packing** problems.

Exploration

In this exploration, you use a bin-packing approach to determine how many 10-ft timbers are required for the landscaping project.

a. **1.** What is the total length of all the landscaping timbers in Table **2**?

2. Is it possible to cut all the timbers from 10-ft lengths without wasting any wood?

b. Use a diagram or a paper-strip model to determine the minimum number of 10-ft timbers needed to complete the landscaping.

c. Draw a diagram that illustrates how each timber should be cut.

Discussion

a. How does the number of 10-ft timbers you determined in Part **b** of the exploration compare with those of the rest of the class?

b. Compare your diagram from Part **c** of the exploration with a classmate who obtained the same minimum number of timbers. Describe any differences you observe.

c. Describe the strategies you used to determine the minimum number of timbers required. Which strategy appears to be most efficient?

d. How does the total length of the timbers in Table **2** compare with the total length of 10-ft timbers needed to complete the landscaping?

Assignment

3.1 Suppose that the lumberyard offered to donate landscaping timbers only in 16-ft lengths.

a. How many 16-ft timbers would you need to satisfy the requirements listed in Table **2**? Use a diagram to justify your response.

b. How many feet of timber would remain unused?

3.2 Suppose that the school decided to purchase a combination of 10-ft and 12-ft timbers.

a. How many of each length would you need to satisfy the requirements listed in Table **2**? Use a diagram to justify your response.

b. How many feet of waste would there be in your order?

c. If landscaping timbers cost $0.60 per foot, what would be the total bill for your order?

3.3 The last task in finishing the construction project is building a deck. The students assigned to this job have created a design which requires boards of several different lengths. The quantities and measurements needed are shown in the table below.

Quantity	Length of Each (ft)
6	7
12	6
30	3

The lumber can be obtained in lengths of either 8 ft, 12 ft, 16 ft, or 20 ft. However, the lumberyard that is donating the wood has requested that the boards all be the same length. Determine which size will result in the least amount of waste.

* * * * *

3.4 In addition to his full-time job, Jake owns a small T-shirt printing shop. Five days before the opening of the county fair, he receives an emergency order. Because their regular shirt printer has unexpectedly gone out of business, the fair needs the shirts listed in the table below.

Print color	Quantity
black	300
gray	300
yellow	150
red	200
purple	100
navy	250
orange	150
green	100

Jake can print 60 shirts per hour. Because of his other job, however, he can only work 6 hr per day at the shirt shop. He prints one ink color at a time and does not start printing a color unless he can finish it that day.

a. Design a schedule that allows Jake to finish the shirts in 5 days.

b. The fair's purchasing agent calls Jake back to tell him that they also will need 50 shirts printed in pink and 75 shirts in teal. Jake promises that he can finish the entire order before the fair begins. Can Jake print the additional shirts in time? Justify your response.

3.5 Imagine that you are an advertising executive for a television station. Your responsibilities include scheduling the advertising for an upcoming sports event. Your clients have purchased two 60-sec ads, four 45-sec ads, four 30-sec ads, and seven 15-sec ads. All the commercial breaks must be the same length, and no longer than 90 sec. What is the minimum number of commercial breaks needed to air these advertisements?

3.6 There are several well-known algorithms used to pack items into bins efficiently. Two examples are the next-fit and first-fit algorithms. Use each method, as described below, to determine the number of 10-ft timbers needed to provide the materials listed in the following table.

Quantity	Length of Each (ft)	Quantity	Length of Each (ft)
2	10	2	5
3	9	4	4
2	8	2	3
4	7	7	2
4	6		

a. In the next-fit algorithm, each item is placed in the first bin that has room for it. Once a bin has been passed over, it is not used again. In the case of timbers, once a cut piece is too short for the next piece in the list, the cut piece is discarded.

b. In the first-fit algorithm, each item is placed in the first bin that has room for it. If no bin has room for an item, a new bin is opened. Unlike the next-fit algorithm, no bin is closed until all items are packed. In the case of timbers, no cut piece is discarded until all pieces are used.

c. Compare your results in Parts **a** and **b**.

* * * * * * * * * *

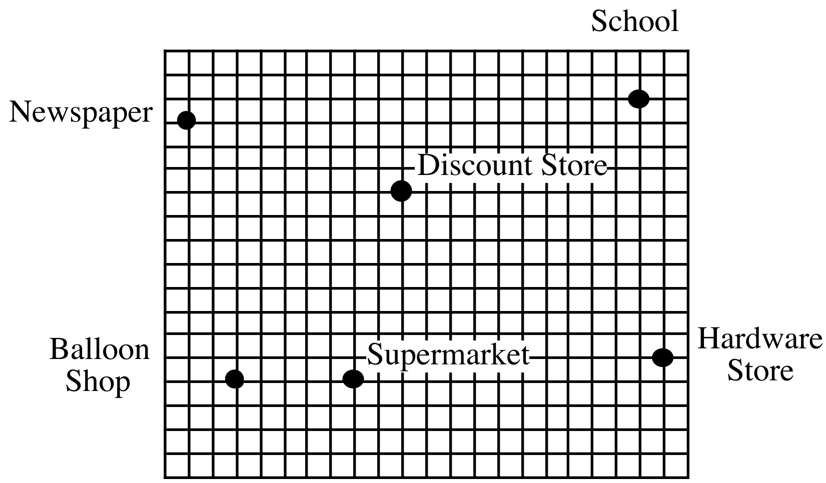
Research Project

The nearest neighbor and cheapest link algorithms are just two of many so-called **greedy algorithms**. Graphs and greedy algorithms are often used by communications and transportation companies to determine how to minimize costs or distances. One such algorithm, first suggested by Joseph Kruskal of AT&T Bell Laboratories, results in a subset of a complete graph called a **spanning tree**. Find out more about the algorithm that produces a spanning tree. Your report should include a description of the algorithm as well as some examples of how spanning trees are used by businesses.

Summary Assessment

1. The theme of this year's spring dance is "Egyptian Oasis." As chair of the student dance committee, your job is to make sure that the gym is decorated before the morning of the dance.

To pick up the decorating materials, you must stop at each location on the following map. The scale of the map is = 1 city block.



a. Draw a weighted graph to represent these locations. **Note:** All paths must follow the grid lines.

b. Find an efficient route that begins at school, visits each business, and returns to school. Describe the method you used to determine the route.

2. You have three evenings to decorate the gym. The school administration has given the dance committee access to the gym from 4:00 P.M. to 11:00 P.M. on Wednesday and Thursday and from 4:00 P.M. to midnight on Friday.

The following table shows the list of required tasks, along with the average time to complete each one using a crew of five students.

	Task	**Hours**
A	inflate 300 balloons	3
B	build frames for pyramids	3
C	decorate pyramids with crepe paper	2
D	build large cardboard sphinx	2
E	apply gold foil to sphinx	2
F	decorate ceiling with balloons and streamers	6
G	drape plastic sheeting at end of gym	3
H	cover bleachers with plastic sheeting	3
I	decorate stage with balloons and install sphinx	2
J	draw and cut out designs for walls	6
K	build palm trees	4
L	set up platform for disc jockey	2
M	decorate walls and plastic sheeting	2
N	set up palm trees on floor	3
O	set up snack bar	1
P	set up tables and chairs	3

a. Create a table that lists each task along with its prerequisites.

b. If no tasks are begun before Wednesday at 4:00 P.M., is it possible for two crews of five students each to complete the decorations by midnight on Friday? Justify your response.

c. If necessary, tasks J and K can be completed outside the gym before Wednesday evening. Design a schedule that allows two crews to complete the decorations by midnight on Friday.

3. During one part of the dance, the disc jockey plans to play music in 5-min portions. The songs she has picked have the following lengths (in sec): 60, 60, 60, 90, 90, 90, 120, 120, 120, 150, 150, 180, 180, 180, 180, 180, 210, 210, 210, 240, 240, 240, and 270.

What is the minimum number of 5-min time slots needed? Justify your response.

Module Summary

- A **complete graph** is a graph in which each pair of vertices is connected by exactly one edge.
- In a **weighted graph**, each edge is assigned a numerical value.
- A **Hamiltonian circuit** is a closed path that starts at one vertex, visits every other vertex in a graph exactly once, and returns to the starting vertex.
- To use the **nearest neighbor algorithm** to draw a Hamiltonian circuit on a weighted graph, start with any vertex and select an edge to its nearest vertex. In other words, select the edge that has the least weight. Continue this process from the second vertex to the next nearest vertex not yet visited, and so on, until all vertices have been visited. To complete a Hamiltonian circuit, return to the original vertex.
- In the **cheapest link algorithm**, the cheapest (or shortest) action is taken at each stage, regardless of starting and stopping points. When using this algorithm to draw a Hamiltonian circuit on a weighted graph, individual, disconnected edges may occur at various stages. If the cheapest remaining action completes a circuit that is not Hamiltonian, then the next best action is taken. When a Hamiltonian circuit is formed, the algorithm is complete.
- A **network diagram** (or order requirement digraph) can be used to represent a scheduling problem. In a network diagram, prerequisite tasks are connected with arrows. The direction of the arrows shows the order in which the tasks must be completed. The number in each circle represents the time required to complete the task.
- The **critical path** is the longest set of prerequisite-linked tasks in a project. The length of the critical path is the minimum time required for the project. When a project is represented in a network diagram, the critical path is the longest path in the diagram.
- A **bin-packing** approach to a problem analyzes the situation in terms of fitting items of different lengths into a bin of a particular size.

Selected References

Consortium for Mathematics and its Applications (COMAP). *For All Practical Purposes: Introduction to Contemporary Mathematics*. New York: W. H. Freeman and Co., 1991.

Graham, R. L. "The Combinatorial Mathematics of Scheduling." *Scientific American* 238 (March 1978): 124–133.

Steen, L. A., ed. *Mathematics Today*. New York: Springer-Verlag, 1978.