Risky Business

Why buy automobile insurance? The rising costs of accidents—and an examination of their probabilities—can help explain why people choose to share the economic risks of driving their cars.

Randy Carspecken • Todd Fife

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Introduction

Since cars and driving play prominent roles in our society, automobile accidents are everyone's concern. The annual cost of auto accidents amounts to billions of dollars. Often, the cost of even a single accident can be too high for the average family budget. Insurance companies can be used to help with these costs.

Business Note

Insurance companies provide protection against the costs of accidents in exchange for a fee. The fee for this service is an insurance **premium**, and the contract with the insurance company is an **insurance policy**. The people who pay insurance premiums are **policyholders**.

Figure **1** shows an example of a premium renewal notice for an automobile driven by an individual in the 16–20 age category.

SIMMS Insurance Company

Car Policy Number: 542–45983325

Continuous Renewal Policy				
Effective		Expires	Classification	1121
05-08	to	11-08	Term (Month)	06
12:01 A.M. Standard Time			Acc/Viol points	0/0
			Territory	14

Coverage is subject to the terms of the policy*

Coverage	**Limit of Liability**		**Premium**
Bodily Injury Liability	Each Person	$100,000	$95.30
	Each Occurrence	$300,000	
Property Damage Liability	Each Occurrence	$ 50,000	$46.20
Medical Payments	Each Person	$ 5,000	$23.60
Comprehensive	Actual cash value / $100 deductible		$40.50
Collision	Actual cash value / $250 deductible		$95.20
Uninsured Motor Vehicle	Each Person	$ 25,000	$15.70
Bodily Injury	Each Accident	$ 50,000	

Total Premium	$316.50
Previous Balance	$ 0.00
Premium Charge	$316.50
Payment Credit	$ 0.00
Current Balance	$316.50
	*************

*Your premium is based on the following: Principal driver under 21.
If not correct, please contact your agent.

Figure 1: Premium renewal notice

Discussion

a. How would you calculate the annual premium of the policy in Figure **1**?

b. Does this premium seem reasonable?

c. How do you think this premium would compare with the premium for a driver in the 35–44 age category?

d. If the policyholder who received this renewal notice has an accident, do you think the premium will change on the next renewal notice?

e. What factors do you think are considered by an insurance company when determining an individual's premium?

Business Note

Bodily injury liability insurance pays for any individual who is injured as a result of the negligent operation of the insured vehicle.

Collision insurance covers the cost of repairing the insured vehicle when the damage is due to its negligent operation.

Comprehensive insurance covers natural damage (by floods, hail, or storms), theft, or vandalism to the insured car.

Property damage liability insurance covers the cost of damages to another person's property caused by the negligent operation of the insured vehicle.

f. Give an example of an accident that would be covered by each type of insurance described in the business note above.

g. What additional information appears on the renewal notice? What do you think this information means?

Activity 1

Driving an automobile involves risk. Billboards, news stories, and magazine articles provide daily reminders of the hazards of the road. In this activity, you use statistics and probability to simulate the risks of driving.

Exploration

To determine the probability of a policyholder having an accident, insurance companies use accident statistics like those in Table **1**. In this exploration, you investigate the importance of considering the number of policyholders when making predictions based on such probabilities.

Table **1** summarizes U.S. automobile accident statistics for 1992. The entries in the table are given per 100,000 licensed drivers. For example, the entry in the right-hand column of the third row indicates that there were 6951 accidents for every 100,000 licensed drivers in the 25–34 age category. Of this total, 2531 were the bodily-injury category, and 4420 were the property-damage-only category.

Table 1: Accidents per 100,000 licensed drivers in 1992

Age	Bodily Injury (fatal and non-fatal)	Property Damage Only	Total
16–20	5753	10,022	15,775
21–24	3688	6687	10,375
25–34	2531	4420	6951
35–44	1945	3580	5525
45–54	1757	3432	5189
55–64	1322	2587	3909
65–69	1235	2119	3354
older than 69	1408	2353	3761

Source: National Highway Traffic Safety Administration, 1992.

a. Use the data in Table **1** to estimate the probability that a randomly selected driver in the 16–20 age category was involved in an automobile accident in 1992. Round the probability to the nearest hundredth.

b. Use a random number generator to design a simulation that predicts the driving record of a person in the 16–20 age category for one year.

c. Suppose an insurance company has *n* policyholders in the 16–20 age category. While it is not possible to predict exactly how many of these drivers will be involved in an accident during the year, insurance companies need to know the approximate percentage of their policyholders who will have accidents.

1. Use the simulation developed in Part **b** to model the driving records of different numbers of policyholders for 1 year. Record the numbers of accidents in a table with headings like in Table **2**.

Table 2: Simulation results

Number of Policyholders	Number of Accidents	Accident Rate
5		
25		
50		
100		
250		
500		
1000		

2. For each row in Table **2**, determine the ratio of the number of accidents to the number of policyholders. This is the accident rate. Record these ratios, in decimal form, in Table **2**.

d. The percentage of drivers in a specific age group who have accidents will vary from year to year. Assume that each class member's simulation represents a different year for the same age group.

To see how the accident rate can vary, collect the class data and determine the difference between the smallest and largest accident rates obtained for each value of n.

Discussion

a. Why is it reasonable to use the data in Table **1** to determine the probability that a randomly selected driver in the 16–20 age category has an accident?

b. Based on the data in Table **1**, how do the driving records of groups in the various age categories compare?

c. What are some limitations of the simulation you developed in the exploration for modeling actual driving records?

d. Compare the accident rates for different numbers of policyholders obtained in Part **c** of the exploration.

e. Describe the trend in the differences between the smallest and largest accident rates obtained in Part **d** of the exploration as the number of policyholders increases.

f. Compare the accident rates for each number of policyholders with the probability calculated in Part **a** of the exploration.

g. If you continued this simulation using larger numbers of policyholders, what number do you think the accident rate would approach?

Mathematics Note

The **law of large numbers** indicates that, if a large random sample is taken from a population, the sample proportion has a high probability of being very close to the population proportion.

For example, it would not be uncommon for a random sample of 10 children from a population with 50% females to contain 8 females and 2 males (a sample proportion of 0.8). On the other hand, it is highly unlikely that a random sample of 10,000 children from this population would contain 8000 females and 2000 males. For large sample sizes, it is much more likely that the proportion of females in the sample would be close to 0.5, the population proportion.

h. Would it be practical for an insurance company to use statistics based on a sample of 100 drivers to determine the probability of 1 driver being involved in an accident?

i. Based on the data in Table **1**, the probability that a randomly selected driver in the 16–20 age category had an accident during 1992 is approximately 0.16. Do you think this is a reliable statistic? Explain your response.

Assignment

1.1 Assume that the probability that a randomly selected newborn baby is a girl is 0.5.

a. Design a simulation that predicts the gender of a randomly selected newborn baby.

b. Use your simulation to complete the following table. Express the proportion of girls to total births to the nearest hundredth.

Number of Births	Proportion of Girls
10	
20	
40	
80	
100	
200	
300	
400	
500	

c. 1. Create a connected scatterplot of the proportion of girls versus the number of births.

2. Graph the line $y = 0.5$ on the same coordinate system. This line represents the probability that a randomly selected newborn is a girl.

d. Does your graph in Part **c** illustrate the law of large numbers? Explain your response.

1.2 An actuary is a mathematician employed by an insurance company to predict events on which policy premiums are based.

a. Why is the accident rate important when determining premiums?

b. Why would an actuary consider a larger group a better basis for making predictions than a smaller group?

1.3 Insurance companies compile accident data from previous years. This historical data is used to determine the probabilities of drivers being involved in accidents in the future. To make predictions, insurance companies treat these probabilities as theoretical probabilities for the current year.

What are some limitations of using the previous year's accident statistics to make predictions about the current year?

1.4 **a.** The following table shows the probabilities that a driver in the 16–20 age category has a bodily-injury accident, a property-damage-only accident, or no accident. All the values in the first row are based on information contained in Table **1**. Explain how these probabilities were calculated.

Accident Probabilities				
Age	**Bodily Injury**	**Property Damage Only**	**No Accident**	**Total**
16–20	0.06	0.10	0.84	1
21–24				
25–34				
35–44				
45–54				
55–64				
65-69				
older than 69				

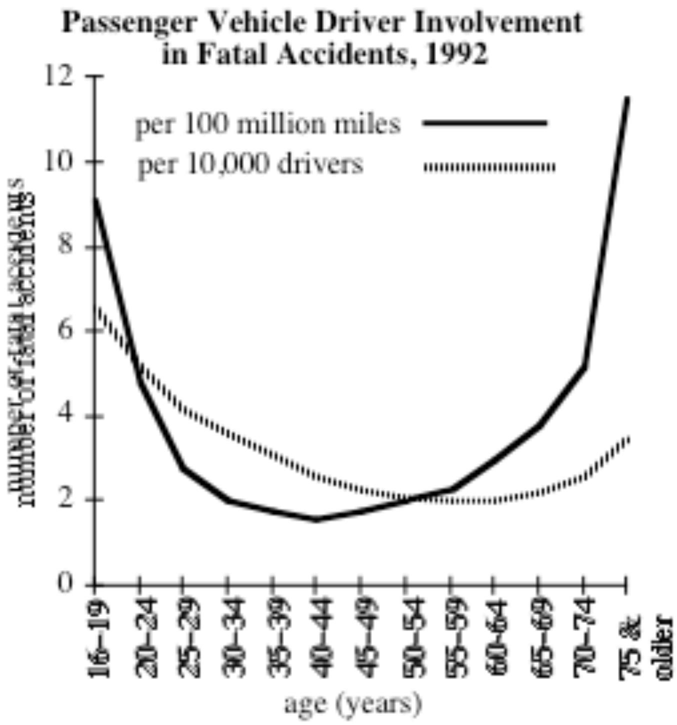
b. What is represented by the 1 in the column with the heading "Total"?

c. Copy and complete the table using the data from Table **1.**

d. Write a paragraph comparing the probabilities of having a bodily-injury or property-damage-only accident as the age of the driver increases. Discuss some possible reasons for the differences in these probabilities.

1.5 As shown in the following graph, the number of fatal automobile accidents in 1992 varied among age categories. One curve displays the number of fatal accidents for every 100 million miles driven. The other illustrates the number of fatal accidents for every 10,000 licensed drivers.

For example, drivers in the 16–19 age category had about 9 fatal accidents for every 100 million miles driven, and about 6 fatal accidents for every 10,000 licensed drivers. In the 65–69 age category, there were about 4 fatal accidents for every 100 million miles driven, and about 2 fatal accidents per 10,000 drivers.



Source: Insurance Institute for Highway Safety, 1992.

a. What statistic in the graph seems to indicate that drivers in the "75 and older" category are better drivers than teenage drivers?

b. What statistic in the graph seems to indicate that teenage drivers are better drivers than those in the "75 and older" category?

c. **1.** Depending on the statistics you select, there are two different age categories with the best driving records. Identify those categories and explain why this is possible.

2. Based on the information in the graph, which age category do you believe has the best driving skills? Explain your response.

d. Using the statistics in the graph, describe the risks encountered over a lifetime of driving. Your answer should include some possible explanations for the changes in risk over time.

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1.6 **a.** Use the following statistics to estimate the probability that a randomly selected member of the population from which the samples were taken will have the given characteristic.

1. In a random sample of 1084 adults, 813 indicated that they believed there is too much violence on television.

2. In a nationwide survey of vehicle accidents, a random sample of 64,000 accidents included 41,600 involving property damage of $800 or less.

3. Among a sample of 90 randomly selected hospital patients, 41 had type O blood.

4. In a random sample of 750 taxpayers with incomes under \$100,000, the Internal Revenue Service had audited 25.

b. **1.** Which estimate in Part **a** do you believe is the most reliable? Explain your response.

2. Which estimate do you believe is the least reliable? Explain your response.

1.7 **a.** If a coin was flipped 5 times and came up heads 4 times, would you suspect that the coin was unfair? Explain your response.

b. Create a simulation that models 1000 flips of a coin. Run this simulation 10 times and record the number of times 800 or more heads are counted.

c. If the coin from Part **a** was flipped 10,000 times and 8000 of the tosses were heads, would you suspect the coin was unfair? Explain your response.

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Activity 2

The cost of an automobile accident can range from a few dollars to well over a million. Typically, insurance companies pay part of these costs for insured drivers. To obtain benefits, policyholders must file an insurance **claim**. A claim reports the details and costs of an accident and requests payment. The claim value is the amount paid by the company.

If you were involved in an accident, how much might you expect your claim value to be? To answer this question, you will first examine some historical data.

Exploration 1

Insurance companies consider all claims made in a year and use the information from these claims to set premium costs for future years. In this exploration, you examine a method used by insurance companies to compile this information.

a. Table **3** below shows 40 bodily injury claims filed with an insurance company in 1990.

Table 3: Insurance claims in 1990, in dollars

3,300	15,236	15,461	570
562	175	4,589	1,131
7,200	1,802	4,200	250
888	52	3,563	111
3,900	3,250	2,252	23,898
1,402	290	700	8,550
89	4,300	4,602	388
12,889	5,300	1,244	20,500
20,336	995	13,202	94
410	23,555	4,001	1,452

1. Sort these claim values from least to greatest.
2. Divide the sorted claim values into 10 sets of 4 claim values each. These 10 sets are **deciles**.

b. Calculate the mean of each decile. Record the means in a table with headings like those in Table **4** below.

Table 4: Mean claim amounts for 1990, by decile

Decile	Claim Value ($)
1	
2	
3	
⋮	
10	

c. From 1985 to 1990, the consumer price index increased at an average annual rate of 4.1%. Assuming that this trend continues, estimate the mean claim amounts, by decile, for 1992.

Discussion 1

a. Suppose that an insurance company compiles data for 25 million claims in a given year. Describe the process of finding the mean decile values for that year.

b. If each decile represented 1 million claims, how could you determine the total number of claims in the data set?

c. Describe the model you used to estimate the mean claim amounts for 1992.

Exploration 2

After collecting and analyzing historical data, insurance companies must determine the amount in claims they can expect per policyholder.

a. The information in Table **5** summarizes data from approximately 25 million bodily-injury claims and 120 million property-damage claims.

Table 5: Mean claim amounts in 1992, by decile

Decile	Bodily Injury ($)	Property Damage ($)
1	125	47
2	378	150
3	753	266
4	1400	389
5	2300	527
6	3550	699
7	5400	936
8	8000	1313
9	13,250	2061
10	24,380	3462

1. Determine the number of property-damage claims represented in each decile.

2. Determine the probability that a randomly selected bodily-injury claim falls in the third decile.

Mathematics Note

A **random variable** *X* is a variable that takes on each of its possible values with a specific probability. Given possible values for *X* of x_1, x_2, …, x_k, each has its corresponding probability $p_1, p_2, \ldots, p_k$. The sum of these probabilities is 1.

For example, consider an insurance company that pays $6000 for a bodily-injury accident, $1000 for a property-damage-only accident, and $0 for no accident. A random variable *X* could be used to represent claim values as follows: $x_1 = \$6000$, $x_2 = \$1000$, and $x_3 = \$0$.

A **probability distribution** for a random variable *X* assigns probabilities $p_1, p_2, \ldots, p_k$ to the values x_1, x_2, …, x_k for *X*.

For example, if the probabilities for a bodily-injury accident, property-damage-only accident, and no accident in a given year are 0.08, 0.02, and 0.90, respectively, then the probability distribution for *X* is shown in Table **6**.

Table 6: Probability distribution for *X*

Outcome	Bodily Injury	Property Damage	No Accident
Value of X	\$6000	\$1000	\$0
Probability	0.08	0.02	0.90

The **expected value** or **mean** of a random variable *X*, denoted $E(X)$, is the sum of the products of each possible value of *X* and its respective probability:

$$E(X) = x_1p_1 + x_2p_2 + \cdots + x_kp_k$$

Using the probability distribution in Table **6**, for example, the expected value of the claims per insured driver during one year can be found as follows:

$$E(X) = \$6000(0.08) + \$1000(0.02) + \$0(0.90) = \$500$$

b. Table **7** shows the probabilities of a bodily-injury accident, a property-damage-only accident, or no accident for a driver in the 16–20 age category. (These are the values you found in Problem **1.4** using the information from Table **1**.)

Table 7: Accident probabilities for the 16-20 age category

Bodily Injury	Property Damage Only	No Accident
0.06	0.10	0.84

A claim value selected at random is equally likely to fall in any given decile. Therefore, the probability of a driver having a claim value for a bodily-injury accident in any given decile is $0.06(1/10) = 0.006$. The probability of a driver having a claim value for a property-damage-only accident in any given decile is $0.10(1/10) = 0.01$.

Using the 1992 claim values from Table **5**, create a probability distribution for the random variable *C*, where *C* is assigned to the set of claim values for a driver in the 16–20 age category. As noted in Table **8** below, the claim value is \$0 if the driver does not have an accident.

Table 8: Probability distribution for *C*

	Bodily Injury		Property Damage	
Decile	**Claim (\$)**	**Probability**	**Claim (\$)**	**Probability**
1		0.006		0.01
2		0.006		0.01
⋮		⋮		⋮
10		0.006		0.01
	No Accident			
	\$0	0.84		

c. Determine the expected value of *C*.

Discussion 2

a. Describe the significance to an insurance company of the value found in Part **c** of the exploration.

b. In Part **b** of Exploration **2**, how many possible values are there for the random variable C?

c. Explain how an insurance company might use the expected value of C to determine an annual premium for drivers in a given age category.

d. A weighted mean is a representative value for a set of numbers in which each number may be assigned a different relative importance.

For example, suppose that a teacher has decided that each of three research projects should count for $1/5$ of a student's final grade, while each of four test scores determine $1/10$ of the grade. If a student receives scores of 76, 87, and 92 on the research projects, and 88,75, 82, and 96 on the tests, the final grade could be calculated as follows:

$$0.2(76)+0.2(87)+0.2(92)+0.1(88)+0.1(75)+0.1(82)+0.1(96)\approx 85$$

Compare the expected value of a random variable to a weighted mean.

Assignment

2.1 **a.** The data in Table **5** summarizes approximately 25 million bodily-injury claims and 120 million property-damage claims. Estimate the total value of these claims.

b. In 1992, the U.S. population was approximately 256 million. Use your estimate from Part **a** to determine the cost per person represented by the claims in Table **5**.

2.2 **a.** From 1992 to 1997, the consumer price index increased at an average annual rate of 2.6%. Given this fact, determine a model you could use to predict claim values in future years.

b. Use your model and the data in Table **5** to predict mean decile values for the current year. **Note:** Save your work for use in Problems **2.3** and **2.4**.

c. The table below shows the probabilities of a bodily-injury accident, a property-damage-only accident, or no accident for a driver in the 21–24 age category.

Bodily Injury	Property Damage Only	No Accident
0.04	0.07	0.89

Use this table and the predicted values from Part **b** to create a probability distribution, then find the expected claim value for a driver in the 21–24 age category.

2.3 **a.** Use the predicted decile values from Problem **2.2b** to find a mean bodily-injury claim value and a mean property-damage-only claim value for the current year.

b. Let the random variable C represent the claim values for a driver in the 21–24 age category during a given year. Use the mean claim values from Part **a** to complete the probability distribution for C.

Probability Distribution for *C*			
Outcome	**Bodily Injury**	**Property Damage**	**No Accident**
Value of *C*			\$0
Probability	0.04	0.07	0.89

c. Determine the expected value of C and describe what it represents to an insurance company.

2.4 In Problem **2.3c**, expected value was found by multiplying each of the decile values by its corresponding probability. Each of these probabilities was determined by dividing a given probability by 10, the number of equally likely outcomes in that category. This method can be represented using the following equation:

$$\begin{aligned} E(C) &= x_1 \bullet \left(\frac{p_1}{10}\right) + x_2 \bullet \left(\frac{p_1}{10}\right) + \cdots + x_{10} \bullet \left(\frac{p_1}{10}\right) \\ &\quad + x_{11} \bullet \left(\frac{p_2}{10}\right) + x_{12} \bullet \left(\frac{p_2}{10}\right) + \cdots + x_{20} \bullet \left(\frac{p_2}{10}\right) \\ &\quad + 0 \bullet p_3 \end{aligned}$$

In Problem **2.3c**, expected value was found by averaging the claim values for each category, then multiplying each mean by the given probability. This method can be represented using the following equation:

$$E(C) = \left(\frac{x_1 + x_2 + \cdots + x_{10}}{10} \bullet p_1\right) + \left(\frac{x_{11} + x_{12} + \cdots + x_{20}}{10} \bullet p_2\right) + 0 \bullet p_3$$

Explain why these two methods yield the same value.

2.5 In a given year, the probability that a driver in the 65–69 age category is involved in a bodily-injury accident is about 0.01. The probability that a driver in the same age category has a property-damage-only accident is about 0.02. The probability that a driver is not involved in an accident is 0.97.

a. Use the predicted mean values from Problem **2.3a** to create a probability distribution table for drivers in the 65–69 age category.

b. Let the random variable C represent the claim values for a driver in the 65–69 age category during a given year. Determine the expected value of C and describe what it represents to an insurance company.

c. Use your results in Problems **2.3c** and **2.5b** to explain why drivers in the 21–24 age category typically pay higher insurance premiums than drivers in the 65–69 age category.

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2.6 Consider a baseball player with a lifetime batting average of 0.315.

a. Assume that the probability this player gets a hit on the next at-bat is $P(\text{H}) = 0.315$. Determine the probability that the player does *not* get a hit, or $P(\text{N})$.

b. In an upcoming game, the player will have four turns at bat. One possible outcome in this situation is HNHH. List all the possible outcomes for the four at-bats and, assuming that they are independent events, determine the corresponding probabilities.

c. Complete the following probability distribution table for the player's next four at-bats.

No. of Hits	Probability
0	
1	
2	
3	
4	

d. How many hits do you predict that this player will get in the next four at-bats? Explain your response.

2.7 According to one math teacher's grading system, group work counts for 20% of the final grade, projects 30%, and tests 50%.

a. A student's scores for the class are shown in the table below. Use this information to determine the final grade.

Group Work	Projects	Tests
78	85	92
95	75	88
82	99	95
75	89	79
98	82	
83	65	
85		
90		
74		

b. Compare this teacher's method of determining final grades to the calculation of expected value.

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Activity 3

Whenever you choose to drive, there is a risk of having an accident. The potential costs of an accident—as well as the law, in many states—motivates most drivers to insure themselves. By insuring a large number of drivers, insurance companies provide a service that allows policyholders to share these financial risks with others.

Exploration

By simulating the operation of an insurance company, you may gain insight into some issues concerning insurance premiums.

Suppose that your class decides to create its own insurance company in which every member of the class is insured for both bodily injury and property damage.

Using statistics from previous years, the class determines that the probability of having a bodily-injury claim for a policyholder in the 16–20 age category is 0.06. The probability of having a claim for property damage only is 0.1. Since the total probability of filing a claim is $0.06 + 0.1 = 0.16$, the probability of not filing a claim is $1 - 0.16 = 0.84$.

To simplify the operation of the company, the class has decided to limit claim payments to the 20 values listed in Table **9**. If no claim is filed, the claim value is $0.

Table 9: Probability distribution table for *C*

	Bodily Injury		**Property Damage Only**	
Decile	**Claim Value ($)**	**Probability**	**Claim Value ($)**	**Probability**
1	208	0.006	22	0.01
2	572	0.006	88	0.01
3	1105	0.006	196	0.01
4	2050	0.006	319	0.01
5	3079	0.006	453	0.01
6	4501	0.006	615	0.01
7	6861	0.006	835	0.01
8	9771	0.006	1189	0.01
9	15,266	0.006	1909	0.01
10	26,642	0.006	3567	0.01
	No Accident			
	0	0.84		

a. The class wants to keep insurance premiums as low as possible. To analyze this situation, they use the random variable C to represent the 21 possible claim values.

Since the expected value of C represents the mean annual claim per policyholder, the class decides to use this value as the annual premium. Determine $E(C)$.

b. Design a simulation of the class insurance company for a one-year period. For each policyholder, your simulation should complete the following sequence of steps:

1. Determine if the policyholder files a claim during the year.
2. If a claim is filed, determine if it is for bodily injury or for property damage only. Hint: Consider the ratios $0.06/0.16 = 0.375$ and $0.1/0.16 = 0.625$.
3. If the claim is for bodily injury, determine which of the 10 claim amounts the company will pay. Similarly, if the claim is for property damage only, determine which of the 10 claim amounts the company will pay.
4. Record the cost of the driver's claim, if any, for the year.

c. Run the simulation once to simulate your driving record for the year.

d.

1. Collect the class data and determine the sum of the claim values for the entire class.
2. Calculate the claims cost per policyholder for the year.
3. Using the annual premium from Part **a**, determine the profit or loss per policyholder for the year.
4. Determine the total profit or loss for the year for the class insurance company.

e. Repeat Part **c** nine more times, to obtain data for a total of 10 years. Compile the class data for each year. Determine the mean value of each of the following for the 10 simulations:

1. the claims cost per policyholder
2. the profit or loss per policyholder
3. the total profit or loss for the year

Note: Save your work for use in the assignment.

f. Suppose the number of policyholders increases to 100. To examine how this might affect premiums, use your simulation to model the claims of 100 policyholders over 10 years. For each year, determine the mean values described in Part **e** above.

Note: Save the results for use in the assignment.

Discussion

a. What are some of the limitations of your simulation in terms of modeling the operation of an actual insurance company?

b. **1.** Why is it not reasonable for an insurance company to set its premiums equal to the expected value of claims per policyholder?

2. Would it be reasonable for the company to set premiums much higher than the expected value of claims?

c. How did your company's ability to pay claims for the class compare with its ability to pay claims for 100 policyholders?

d. A typical insurance company insures a relatively large group of people. What are the advantages and disadvantages of insuring small groups like those in the exploration?

e. Compare the means calculated for the two different groups in Parts **e** and **f** of the exploration. How would you expect this statistic to change for 10,000 policyholders?

f. How does the law of large numbers relate to the data you collected in the exploration?

g. Based on your results in the exploration, what annual premium would you recommend to insure your class only? to insure a group of 100 policyholders?

Assignment

3.1 Imagine that you are employed by an insurance company. The chief executive officer (CEO) requests a report on the predictability of insurance claims. Write a report for the CEO, using the results of the exploration to support your explanation.

3.2 Insurance companies typically use 65% of the premiums they collect to cover claim costs. This part of the premium is the *pure premium*. Another 30% covers operating costs, or *overhead*. The remaining 5% of the premiums collected represents profit.

a. Explain why the annual premium used in your simulation is lower than what an actual insurance company would charge.

b. What premium would a typical insurance company charge in a situation in which the claim cost per policyholder was $512?

c. Estimate the annual profit for a typical insurance company providing coverage for your class.

3.3 An insurance company's *loss ratio* is defined as follows:

$$\frac{\text{total claims paid}}{\text{total premiums collected}}$$

a. Would an insurance company want its loss ratio to be large or small? Explain your response.

b. Use your responses to Problem **3.2** to explain why an insurance company's loss ratio is typically 65%.

c. Use the results of the simulations in the exploration and an annual premium of $788 to find the largest and smallest loss ratio for each of the following:

1. your class

2. 100 people

3. 1 million people (estimated).

d. Use the law of large numbers and your responses from Part **c** to describe some considerations insurance companies may have regarding numbers of policyholders.

3.4 Consider an insurance company with 3500 policyholders. The average annual claim is $707. The probability that a policyholder will file a claim is 9%. Considering the percentages described in Problem **3.2**, suggest an appropriate premium for this insurance.

3.5 The following table shows the annual number of accidents per 100,000 licensed drivers of eight different age groups.

Number of Accidents per 100,000 Drivers			
Age	**Bodily Injury**	**Property Damage Only**	**Total**
16–20	5753	10,022	15,775
21–24	3688	6687	10,375
25–34	2531	4420	6951
35–44	1945	3580	5525
45–54	1757	3432	5189
55–64	1322	2587	3909
65–69	1235	2119	3354
older than 69	1408	2353	3761

The table below shows predicted decile values for insurance claims in the same population.

Decile	Bodily Injury Claim ($)	Property Damage Claim ($)
1	150	100
2	442	255
3	873	361
4	1622	474
5	2591	613
6	3925	795
7	5975	1049
8	8725	1450
9	14,125	2225
10	25,415	3513

a. An insurance company plans to offer coverage for both bodily injury and property damage to this population. Use the data given, along with the percentages in Problem **3.2**, to suggest an annual insurance premium for a 17-year-old driver. Describe how you determined your response.

b. Repeat Part **a** for a 67-year-old driver.

c. Compare the premium of the 67-year-old with the premium of the 17-year-old.

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3.6 A life insurance company sells term insurance policies to 20-year-old males that pay $40,000 if the policyholder dies within the next 5 years. The company collects an annual premium of $200 from each policyholder.

The following table shows the probability distribution for X, the random variable that represents the company's income (or loss) per policyholder. In this case, x_1 represents the income if the policyholder dies during the first year (at age 20), while x_6 represents the income if the policyholder dies after the policy expires (at age 26 or older).

Income (or Loss)	Probability
$x_1 = -\$39,800$	0.00175
$x_2 = -\$39,600$	0.00181
$x_3 = -\$39,400$	0.00184
$x_4 = -\$39,200$	0.00189
$x_5 = -\$39,000$	0.00191
$x_6 = \$1000$	p_6

a. Determine the probability that the policyholder will die after the policy expires.

b. Calculate $E(X)$, the company's expected income per policyholder.

c. **1.** Explain why it might be risky for this company to insure only one policyholder.

2. Explain why it is not as risky for the company to insure 10,000 policyholders.

3. How much income would the company expect from insuring 10,000 policyholders?

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Research Project

Many insurance companies give discounts for drivers or cars that fit certain criteria. For example, since a car with airbags is considered safer than a comparable car without airbags, a company might offer lower premiums for this feature.

a. Compile a list of the discounts that might apply to you as a driver.

b. Determine the amount that premiums are typically reduced for each type of discount in Part **a**.

c. Comment on the relationship between each discount and the probability of having a claim or the potential value of the claim.

d. Determine the annual premium you would pay—including any applicable discounts—for comprehensive coverage on each of the following types of vehicles:

1. a sports car

2. a luxury car

3. a family car.

Summary Assessment

Cars are not the only possessions that can be expensive to repair or replace. As a result, many people buy insurance for their homes, jewelry, and appliances. Because of the potential for water damage, some even buy insurance for their waterbeds. In fact, it's possible to insure almost anything—from a champion race horse to a concert pianist's hands.

For example, contact lenses are especially easy to lose or damage. Imagine that you own an insurance business that provides coverage for contact lens. Before setting your premiums, you do some research on contact lens claims. Here are the statistics:

- An average of 10 out of every 100 people with contact lens insurance files a claim for damage or loss each year.
- The first decile of claims averages $75.00, deciles 2–7 average $125.00, and deciles 8–10 average $300.

Continue your analysis by completing the following steps.

1. Develop a simulation for annual contact lens claims. Your simulation should utilize a random variable and a probability distribution table. It also should identify mean claim values and be able to model different numbers of policyholders.

2. Use your simulation to compare the mean claim cost per policyholder, as well as the range in claim costs per policyholder, for different numbers of policyholders.

3. Suggest a pure premium and a full premium for this insurance coverage and describe how you determined these values.

4. Use the results of your simulation and the law of large numbers to describe how the number of policyholders affects your ability to earn a predictable profit.

Module Summary

- The **law of large numbers** indicates that, if a large random sample is taken from a population, the sample proportion has a high probability of being very close to the population proportion.
- A **random variable** *X* is a variable that takes on each of its possible values with a specific probability. Given possible values for *X* of $x_1, x_2, \ldots, x_k$, each has its corresponding probability $p_1, p_2, \ldots, p_k$. The sum of these probabilities is 1.
- A **probability distribution** for a random variable *X* assigns probabilities $p_1, p_2, \ldots, p_k$ to the values $x_1, x_2, \ldots, x_k$ for *X*.
- The **expected value** or **mean** of a random variable *X*, denoted as $E(X)$, is the sum of the products of each value of *X* and its respective probability:

$$E(X) = x_1p_1 + x_2p_2 + \cdots + x_kp_k$$

- Insurance companies provide protection against part of the cost of an accident in exchange for a fee. The fee for this service is an insurance **premium**, and the contract with the company is called an **insurance policy**. The people who pay insurance premiums are **policyholders**.
- **Bodily injury liability insurance** pays for any individual who is injured as a result of the negligent operation of the insured vehicle.
- **Collision insurance** covers the cost of repairing the insured vehicle when the damage is due to its negligent operation.
- **Comprehensive insurance** covers natural damage (by floods, hail, or storms), theft, or vandalism to the insured car.
- **Property damage liability insurance** covers the cost of damages to another person's property caused by the negligent operation of the insured vehicle.
- To obtain benefits, policyholders must file an insurance **claim**. The claim reports the details and costs of an accident and requests payment.
- The percentage of insurance premiums used to cover claim costs is the **pure premium**. The operating costs of a business are its **overhead**.
- An insurance company's **loss ratio** is defined as

$$\frac{\text{total claims paid}}{\text{total premiums collected}}$$

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*For the latest statistics on highway accidents, contact The Highway Loss Data Institute, Washington, DC.