Here We Go Again!

What do teeter-totters, pendulums, and trumpet notes have in common? These real-life phenomena can all be modeled by the same type of functions.

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Introduction

Breathe in and out, in and out, in and out. Your breathing is an event that repeats itself, over and over. A person inhaling and exhaling air, a satellite orbiting Earth, a Ferris wheel in motion, and a city bus following the same route all have at least one thing in common: they involve events that repeat over time.

Discussion

a. Describe several other events that repeat over time.

b. What other characteristics do the events you described in Part **a** have in common?

Activity 1

A repeating event often has a definite beginning and a definite ending. Each swing of a pendulum, for example, has a beginning and an ending. How can this characteristic help you model a pendulum's motion? In the following exploration, you use sand to trace the movement of a pendulum and model its motion.

Exploration

a. Construct a pendulum using a paper cup and string, as shown in Figure **1**. Suspend the pendulum from a solid support arm so that the bottom of the cup is no more than 3 cm above the base.

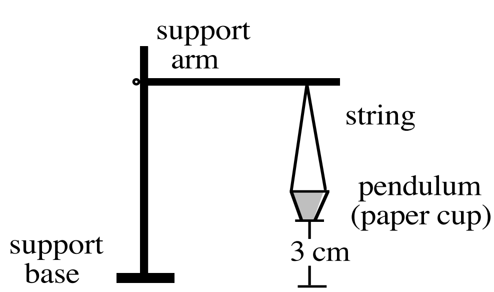


Figure 1: A pendulum

b. **1.** As shown in Figure **2**, tape eight pieces of graph paper together along the longer sides to create a single, long sheet.

2. Create a coordinate system with its x-axis along the length of the sheet and its y-axis along the width of the sheet.

3. Place one end of the sheet underneath the pendulum, so that the pendulum's motion will be parallel to the y-axis.

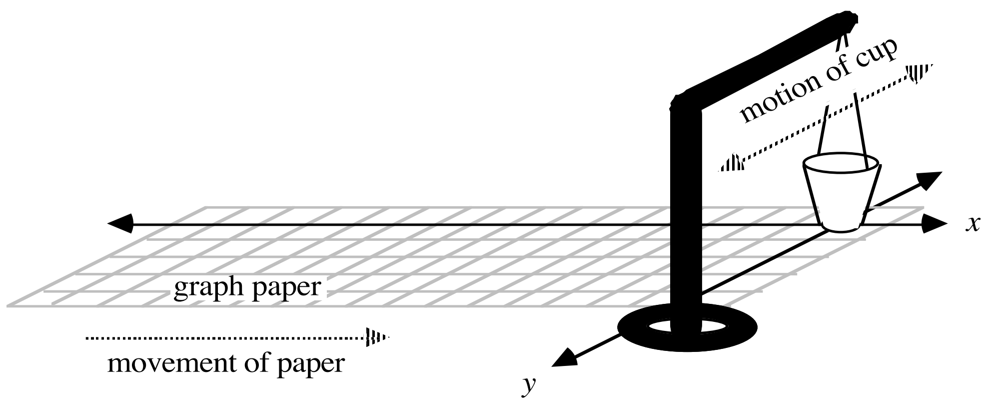


Figure 2: Pendulum and graph paper

c. Make a small hole in the bottom of the paper cup. Cover the hole with your finger, then fill the cup with sand.

d. You are now ready to create a sand graph of the pendulum's motion. While one member of your group uncovers the hole in the cup and gently starts the cup swinging, another should simultaneously begin to move the sheet of graph paper at a constant speed underneath the moving pendulum.

Mathematics Note

A **periodic function** is a function in which values from the range repeat at equal intervals across the domain.

The **period** is the smallest interval of the domain over which the function repeats. A **cycle** is the portion of the function included in one period.

If a periodic function has a maximum M and a minimum m, its **amplitude** is defined as:

$$\left|\frac{M - m}{2}\right|$$

For example, Figure **3** shows a periodic function with a period of 3. This indicates that the function completes 1 cycle every 3 units.

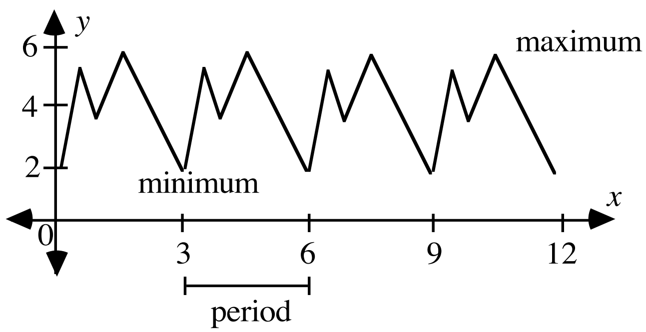


Figure 3: A periodic function

Because the maximum is 6 and the minimum is 2, the amplitude of the function is

$$\left|\frac{6 - 2}{2}\right| = 2$$

e. Determine the period and amplitude of your sand graph.

f. Determine the coordinates of several points on the sand graph.

g. Enter these data points in a graphing utility and create a scatterplot that models the sand graph.

Mathematics Note

There are many types of periodic functions. One example is the sine function, $g(x) = \sin x$. In fact, all functions of the form below, where $a \neq 0$, are periodic functions.

$$f(x) = a\sin[b(x-h)] + k$$

For all periodic functions of this form, the amplitude is $|a|$, the number of cycles in 2π radians is b, and the period is $2\pi/b$. The value of h describes the horizontal translation of the graph from the parent function $g(x) = \sin x$, while the value of k describes the vertical translation from the parent.

For example, the function $f(x) = 4\sin 2(x-3) + 7$ has 2 cycles in 2π radians, a period of π, and an amplitude of 4 units. Its graph is translated 3 units to the right and 7 units up from the graph of $g(x) = \sin x$.

Figure **4** shows the graphs of $f(x)$ and $g(x)$. Note that point *P*, the *y*-intercept of $g(x)$, is translated 3 units to the right and 7 units up to the point P' on $f(x)$.

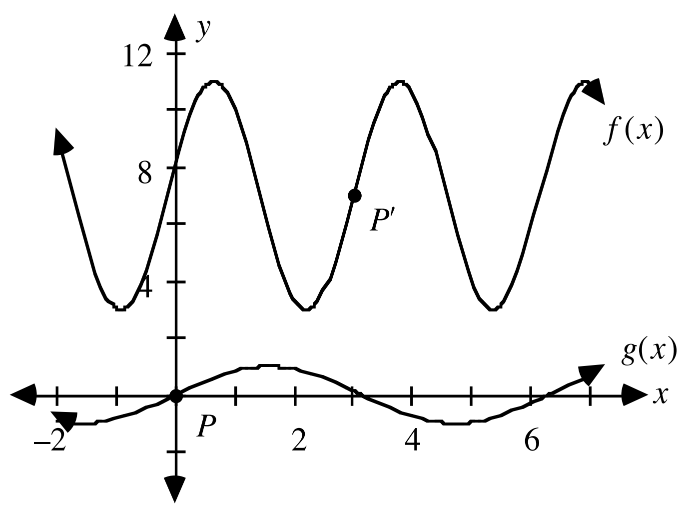


Figure 4: Graphs of $f(x) = 4\sin[2(x-3)] + 7$ and $g(x) = \sin x$

h. Determine a function of the form $f(x) = a\sin[b(x-h)] + k$ that models the data points in the pendulum experiment.

i. Figure **4** shows a graph of the function $f(x) = 4\sin[2(x-3)] + 7$. Find another sine function that has the same graph.

Discussion

a. How does the sand graph compare to the graph of $g(x) = \sin x$?

b. **1.** In the pendulum experiment, what is the greatest possible value for the amplitude?

2. What is the smallest possible value for the amplitude?

c. How do you think you could change the period of the pendulum?

d. Did the function you wrote in Part **h** of the exploration include a horizontal or vertical translation of the function $g(x) = \sin x$? If so, describe these translations.

e. The sine function is one example of a periodic function. Name some other periodic functions.

f. **1.** Can the function $f(x) = 0$ be considered a periodic function? Explain your response.

2. Is the greatest integer function a periodic function?

g. How does each constant in the function $f(x) = 6\cos[0.5(x+5)] - 2$ affect its graph in comparison to the graph of $g(x) = \cos x$?

h. How could you determine the value of b in $g(x) = \sin(bx)$ by examining its graph? Hint: The period of $f(x) = \sin x$ is 2π.

i. The graph in Figure **5** is a transformation of $f(x) = \sin x$. Describe how to determine a possible value for the horizontal translation in this transformation.

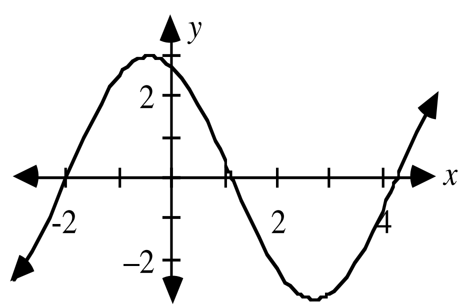
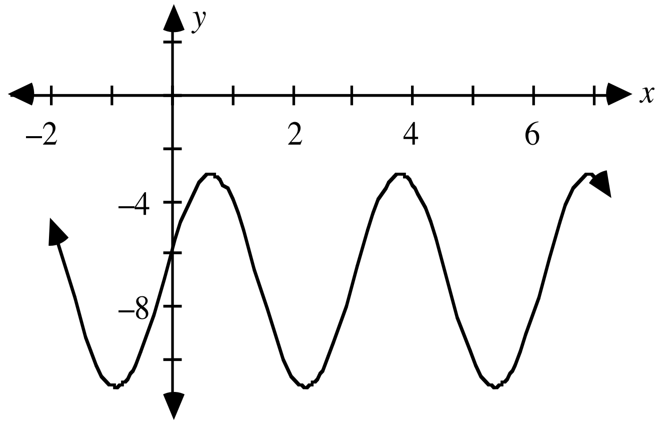


Figure 5: A transformation of $f(x) = \sin x$

j. How many different equations can be written whose graphs are the ones shown in Figure **5**?

Assignment

1.1 The graph shown below is a transformation of the function $f(x) = \sin x$.

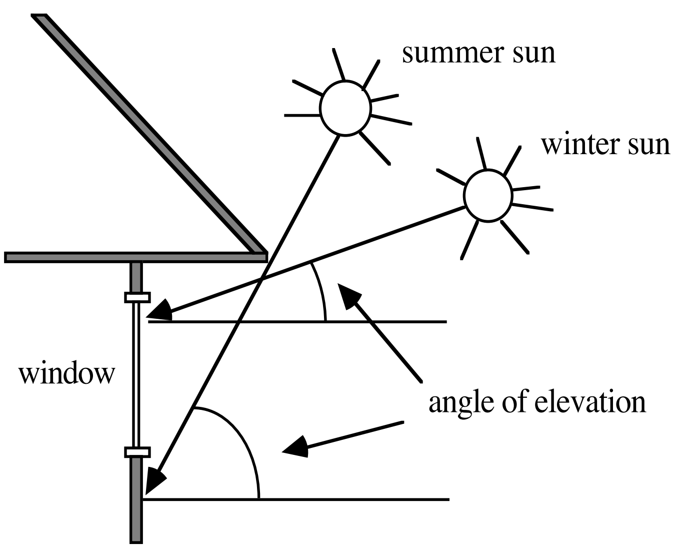


a. Describe how to determine each of the following for this graph:

1. the period
2. the amplitude
3. the vertical shift
4. the horizontal shift

b. Find two functions that model the curve.

1.2 When designing a solar house, the sun's angle of elevation above the horizon at midday is an important concern. As shown below, the sun's angle of elevation is greater in the summer than in the winter.



Imagine that you work for a construction company called Solar Sensation. To improve heating and cooling efficiency, the company designs the eaves on its houses so that windows receive full sun during the winter months, but are shaded during the summer months.

a. The table below shows the sun's angle of elevation at midday for the 15th day of each month in 1994 for Spokane, Washington. Graph this data in a scatterplot.

Date	Angle of Elevation	Date	Angle of Elevation
January 15	21.60°	July 15	63.77°
February 15	29.92°	August 15	56.08°
March 15	40.87°	September 15	44.97°
April 15	52.72°	October 15	33.45°
May 15	61.65°	November 15	23.63°
June 15	65.83°	December 15	19.13°

b. Find a periodic function that models the scatterplot.

c. Use your equation from Part **b** to estimate the sun's angle of elevation (to the nearest degree) on each of the following dates:

1. June 1, 1994

2. October 31, 1995.

d. How could you use your equation to predict the sun's midday angle of elevation for any day of any year?

1.3 **a.** Make a prediction about the relationship between the sun's angle of elevation and mean monthly temperature.

b. The following table shows the mean monthly temperature in degrees Celsius for Spokane, Washington. Graph this data in a bar graph.

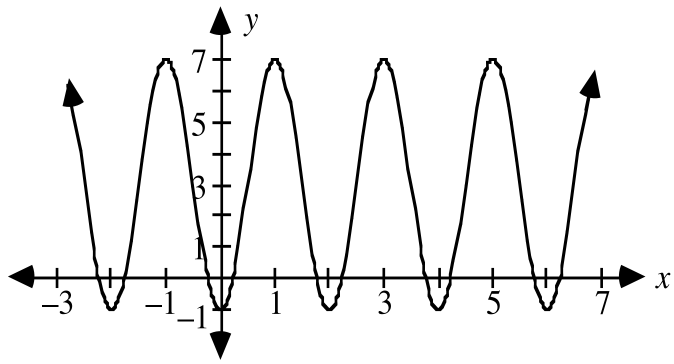
Month	Mean Temperature	Month	Mean Temperature
January	–6.0°	July	20.1°
February	–2.5°	August	19.4°
March	0.7°	September	13.6°
April	6.4°	October	8.6°
May	11.7°	November	1.0°
June	16.4°	December	–4.5°

c. **1.** Locate the midpoint of the segment at the top of each bar in the bar graph and place a dot there. Then connect the dots.

2. Compare the shape formed with the scatterplot of the sun's angle of elevation from Problem **1.2a**.

3. Do the graphs support your prediction from Part **a**? Explain your response.

1.4 The following graph shows a transformation of the parent function $f(x) = \cos x$. Write two possible equations for this function.



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1.5 **a.** A yo-yo rises and falls from a player's hand. Suppose that the distance from the yo-yo to the player's hand can be modeled with a transformation of the sine function.

1. In this situation, what measurement(s) correspond to the amplitude of the function?

2. What measurement corresponds to the period of the function?

b. Suppose that the length of a yo-yo string is 79 cm, and that it takes 1.2 sec to complete 1 cycle. Assume that at 0 sec, the yo-yo is 0 cm from the player's hand.

1. Sketch the graph of a function that could model this relationship.

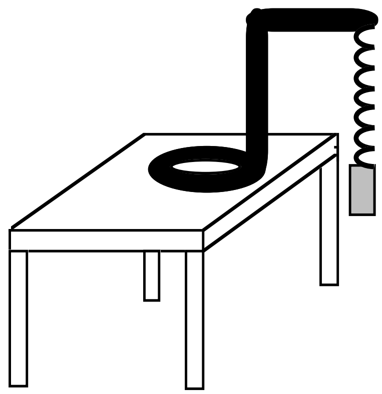
2. Find a transformation of the sine function that models the motion of this yo-yo.

c. Use the periodic function from Part **b** to estimate the distance from the yo-yo to the player's hand after 2 sec.

d. At what times before 3 sec will the yo-yo be 60 cm from the player's hand?

1.6 Consider an object attached to a spring, as shown in the following diagram. The object, after being pulled straight down and released, bounces up and down. Approximately 0.5 sec after the object is released, it reaches its highest point, 50 cm above the plane of the tabletop. It reaches its lowest point, 30 cm below the plane of the tabletop, 1.5 sec after its release.

By ignoring friction (which would eventually stop the motion of the spring), a transformation of the cosine function can be used to model the object's distance above or below the plane of the tabletop versus time.



a. Sketch a graph of a function that models the object's distance above or below the plane of the table top versus time.

b. Write an equation that models the graph in Part **a**.

c. Where is the object located 3.2 sec after its release?

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Activity 2

In Activity **1**, you used simple periodic functions to model physical phenomena such as a swinging pendulum and the action of a spring. Other physical phenomena can be modeled by periodic functions that are much more complex. For example, the sound waves produced by musical instruments can be modeled by sums of simple trigonometric functions.

Exploration

In this exploration, you investigate the behavior of functions that are formed by adding sine and cosine functions.

a.

1. Determine the period and amplitude of $f(x) = \sin(3x)$ and $g(x) = 2\sin(3x)$.

2. Define $h(x) = f(x) + g(x)$. Compare the period of $h(x)$ to the periods of $f(x)$ and $g(x)$.

3. Compare the amplitude of $h(x)$ to the amplitudes of $f(x)$ and $g(x)$.

b. Let $f(x) = a\sin(bx)$, $g(x) = c\sin(bx)$, and $h(x) = f(x) + g(x)$. Repeat Part **a** for two different sets of values for a, b, and c.

c. Let $f(x) = a\sin(bx)$, $g(x) = c\cos(bx)$, and $h(x) = f(x) + g(x)$. Repeat Part **a** for two different sets of values for a, b, and c.

d. **1.** Determine the period and amplitude of $f(x) = \sin(2x)$ and $g(x) = 2\sin(3x)$.

2. Assuming that $x = 0$ is the beginning of a cycle, list several positive values of x at which each function will start a new cycle. Identify the smallest positive value common to both lists. This is the shortest interval between common starting points for the cycles of these two functions.

3. Define $h(x) = f(x) + g(x)$. How is the period of $h(x)$ related to the periods of $f(x)$ and $g(x)$ and your response to Step **2**?

4. Compare the amplitude of $h(x)$ to the amplitudes of $f(x)$ and $g(x)$.

e. Let $f(x) = a\sin(bx)$, $g(x) = c\sin(dx)$, and $h(x) = f(x) + g(x)$. Repeat Part **d** for two different sets of values for a, b, c, and d.

f. Let $f(x) = a\sin(bx)$, $g(x) = c\cos(dx)$, and $h(x) = f(x) + g(x)$. Repeat Part **d** for two different sets of values for a, b, c, and d.

Discussion

a. Based on what you discovered in the exploration, if $f(x)$, $g(x)$, and $f(x) + g(x)$ are all periodic functions, when will the period of $f(x) + g(x)$ equal the period of $f(x)$?

b. For two periodic functions $f(x)$ and $g(x)$, when does the period of $f(x) + g(x)$ equal the least common multiple of the periods of the two original functions?

c. Describe how to determine the period of $h(x) = f(x) + g(x)$, where $f(x) = \cos(3x)$ and $g(x) = 5\sin(2x)$.

d. Do you think that the function $g(x) = \sin(2x) + \sin(\pi x)$ could have a period of 2π? Explain your response.

e. How does the sum of the amplitudes of $f(x)$ and $g(x)$ compare with the amplitude of $f(x) + g(x)$?

f. Give an example of two non-periodic functions whose sum is a periodic function.

Assignment

2.1 **a.** Find the amplitude and period of the functions $f(x) = 3\cos(4\pi x)$, $g(x) = 5\cos(4\pi x)$, and $f(x) + g(x)$.

b. Find the amplitude and period of the functions $f(x) = 2\cos(3x)$, $g(x) = 4\cos(5x)$, and $f(x) + g(x)$.

c. When cosine curves are added, do you think that the effects on amplitude and period will be similar to those that occur when sine curves are added? Explain your response.

2.2 One electronic test signal is produced by adding the terms of a series based on the sine function. The test signal can be modeled by $h(x)$, where $h(x)$ is the sum of the following five functions:

$$f_1(x) = -\sin(x),\ f_2(x) = -\frac{1}{2}\sin(2x),\ f_3(x) = -\frac{1}{3}\sin(3x),$$

$$f_4(x) = -\frac{1}{4}\sin(4x),\ f_5(x) = -\frac{1}{5}\sin(5x)$$

a. Graph $h(x)$ and describe the shape of the graph.

b. Determine the period and amplitude of $h(x)$.

c. List the next three terms in the series.

d. Graph the sum of the first eight terms of the series.

e. Describe any differences you observe between the graphs in Parts **a** and **d**.

2.3 The force required to stretch or compress a spring is proportional to the distance that spring is stretched or compressed from its natural length. The constant of proportion is referred to as the spring constant.

Ignoring friction, the movement of an object attached to the spring can be modeled with periodic functions. The displacement (in meters) at any given time can be modeled by the following function:

$$d(t) = A\cos\left(t\sqrt{k/m}\right) + B\sin\left(t\sqrt{k/m}\right)$$

where t represents time in seconds, m is the mass of the object in kilograms, and k is the spring constant.

The velocity (in meters per second) at any given time can be modeled by the function below:

$$v(t) = -A\left(\sqrt{k/m}\right)\sin\left(t\sqrt{k/m}\right) + B\left(\sqrt{k/m}\right)\cos\left(t\sqrt{k/m}\right)$$

where t represents time in seconds, m is the mass of the object in kilograms, and k is the spring constant.

a. Consider a mass of 10 kg attached to a spring which has a spring constant of 1.6 N/m. At $t = 0$, the initial displacement of the object is 25 cm, while its initial velocity is 2 m/sec. Determine the values of A and B in this situation.

b. Determine the object's displacement and velocity at $t = 3.6$ sec.

c. The object's displacement over time is described by a sum of two periodic expressions. Represent this sum as a single periodic expression.

d. The object's velocity over time also is described by a sum of two periodic expressions. Represent this sum as a single periodic expression.

e. Substitute $t = 3.6$ into your equations from Parts **c** and **d** and compare the results with your responses to Part **b**.

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2.4 Consider the functions $f(x) = a\sin x$ and $g(x) = b\cos x$.

a. Select values for a and b. Create a graph of $f(x) + g(x)$.

b. Determine the domain and range of the function $h(x) = f(x) + g(x)$.

c. It is possible to represent the sum $f(x) + g(x)$ as a single function of the form $h(x) = c\sin(x + d)$. Determine a function of this form whose graph matches the graph of the sum created in Part **a**.

2.5 Some people believe that biological rhythms influence our personal lives. In its simplest form, the biorhythm theory states that, from birth to death, each of us is influenced by three internal cycles: the physical, the emotional, and the intellectual. The 23-day physical cycle affects a broad range of bodily functions, including resistance to disease and strength. The 28-day emotional cycle governs creativity, sensitivity, and mood. The 33-day intellectual cycle regulates memory, alertness, receptivity to knowledge, and other mental processes.

According to this theory, each of the cycles starts at a neutral baseline or zero point on the day of birth. From that zero point, the three cycles enter a rising, positive phase, during which the energies and abilities associated with each are high, and then gradually decline. Each cycle crosses the zero point midway through its period, entering a negative phase in which physical, emotional, and intellectual capabilities are somewhat diminished, and energies are recharged.

Since the three cycles have different periods, the highs, lows and baseline crossings rarely coincide. As a result, the theory predicts that people are usually subject to a mix of biorhythms and that their behavior—from physical endurance to academic performance—is a composite of these varying influences.

a. On the same set of axes, use sine curves to model the three internal cycles for the first 60 days after birth. Represent time on the *x*-axis and energy level on the *y*-axis. Let the energy level vary from 2 to –2.

b. According to biorhythm theory, the most vulnerable days occur when each cycle crosses the baseline, switching from positive to negative or vice versa. These are "critical days." Identify the coordinates of the first critical days after birth for each of the three cycles on the graph.

c. Determine your biological rhythms for today.

d. In an average human life span of 70 years, when will the three cycles coincide on the baseline?

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Research Project

Sound waves are created by vibrations. The changes in pressure created by these vibrations can be modeled by a sine function, as shown in Figure **6**.

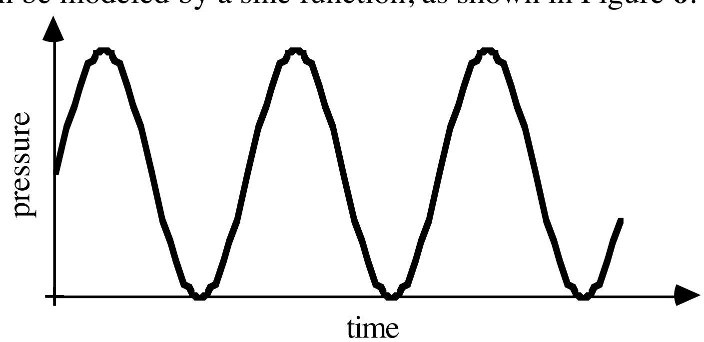


Figure 6: Graph of pressure versus time for a sound wave

The **frequency** of a sound wave describes the number of cycles per unit time (the reciprocal of the period). Frequency is usually measured in hertz (Hz), where 1 Hz equals 1 cycle per second. For example, a sound wave with a frequency of 5 Hz (or 5 cycles/sec) has a period of $1/5$ sec.

The **fundamental frequency** of an object is the lowest frequency at which it can vibrate. **Harmonics** are whole-number multiples of the fundamental frequency. For example, if an object has a fundamental frequency of 40 Hz, the third harmonic is 3 times the fundamental frequency, or 120 Hz.

Find out more about harmonics and write a report explaining how they affect the quality of sounds produced by musical instruments. Include graphs of the first, second, and third harmonics, and explain why the graph of a sound created by a musical instrument might represent a sum of two or more periodic functions.

Activity 3

As you have seen in Activity **2**, periodic functions can be used to model things as diverse as seasonal temperature changes and sound waves. Some periodic functions are also important as parts of larger mathematical systems. In this activity, you investigate the reciprocals of some familiar trigonometric functions.

Exploration 1

Models for periodic phenomena can be written in several different forms by using identities. Recall that a **trigonometric identity** is an equation involving trigonometric functions that is true for all real numbers in its domain.

a.

1. Graph $f(x) = \sin x$ and $g(x) = \cos x$ on the same coordinate system.

2. Determine a transformation of the graph of $g(x)$ for which the image is $f(x)$.

3. Use your response to Step **2** to write an identity involving the sine and cosine functions.

4. Test your answer to Step **3** by graphing the two expressions in the identity.

5. Given $h(x) = 3\sin(2x)$, use your identity to write an equivalent expression for $h(x)$ involving the cosine function.

b.

1. Determine a transformation of the graph of $f(x) = \sin x$ for which the image is $g(x) = \cos x$.

2. Use your response to write an identity involving the cosine and sine functions.

3. Test your answer by graphing the two expressions in the identity.

c.

1. Graph the functions $f(x) = \sin x$ and $g(x) = \sin(x + \pi)$ on the same coordinate system.

2. Determine a relationship between the two graphs and use that relationship to write an identity involving $\sin(x + \pi)$ and $\sin x$.

3. Test your answer by graphing the two expressions.

Discussion 1

a. Use the identities you determined in Exploration **1** to describe an equivalent expression for each of the following:

1. $\sin(\pi/3)$

2. $\cos(\pi/4)$

b. Consider the function $f(x) = \cos(x) + \sin(x + \pi)$.

1. How can this function be written in terms of sine only?

2. How can it be written in terms of cosine only?

Exploration 2

In this exploration, you investigate the reciprocals of some familiar trigonometric functions.

Mathematics Note

The **cosecant** of x, denoted $\csc x$, is the reciprocal of $\sin x$, or $\csc x = 1/\sin x$, where $\sin x \neq 0$. For example, Table **1** shows some sample values for $f(x) = \sin x$ and $g(x) = \csc x$.

Table 1: Sample values for the sine and cosecant functions

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$f(x) = \sin x$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$g(x) = \csc x$	undefined	$2/1$	$2/\sqrt{2}$	$2/\sqrt{3}$	1

The **secant** of x, denoted $\sec x$, is the reciprocal of $\cos x$, or $\sec x = 1/\cos x$, where $\cos x \neq 0$. For example, if $\cos x = 1/2$, then

$$\sec x = \frac{1}{1/2} = 2$$

The **cotangent** of x, denoted $\cot x$, is the reciprocal of $\tan x$, or $\cot x = 1/\tan x$, where $\tan x \neq 0$. For example, if $\tan x = \sqrt{3}/2$, then

$$\cot x = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}$$

a. Graph at least three periods of the function $f(x) = \sin x$.

b. Graph $g(x) = \csc x$ on the same coordinate system as in Part **a**.

c. **1.** Determine any points of intersection for the graphs of $f(x)$ and $g(x)$.

2. Verify that the points identified in Part **c** satisfy both functions.

d. Determine the equations of the vertical asymptotes for the graph $g(x)$.

e. Given a function of the form $f(x) = a\sin(bx)$, you know that a affects the amplitude of the graph and b affects the period. In the following steps, you examine the effects of a and b on a function of the form $g(x) = a\csc(bx)$.

1. Select values other than 0 and 1 for both a and b. Graph at least three periods of the function $f(x) = a\sin(bx)$.
2. On the same coordinate system—and using the same values selected above for a and b—graph $g(x) = a\csc(bx)$.
3. Determine any points of intersection for the graphs of $f(x)$ and $g(x)$.
4. Verify that these points satisfy both functions.
5. Determine the equations of the vertical asymptotes for the graph of $g(x)$.

Discussion 2

a. The functions $f(x) = \sin x$ and $g(x) = \csc x$ are reciprocals. What other information about $f(x)$ helps to verify that $g(x)$ is undefined at the asymptotes found in Part **d** of Exploration **2**?

b. How does the domain of $f(x) = \sin x$ compare to the domain of $g(x) = \csc x$?

c. How does the range of $f(x) = \sin x$ compare to the range of $g(x) = \csc x$?

d. What is the period of $g(x) = \csc x$? Defend your response.

e. Will the period of $g(x) = a\csc(bx)$ always be the same as the period of $f(x) = a\sin(bx)$? Defend your response.

Assignment

3.1. Consider a graph of a function of the form $g(x) = a\csc(bx)$, where $a \neq 0$. Each branch that lies between consecutive asymptotes intersects the graph of $f(x) = a\sin(bx)$ in exactly one point.

Describe the significance of these points of intersection.

3.2. Find an equivalent expression in terms of the sine function for each of the following.

a. $\sin(x - \pi)$

b. $\sin(\pi - x)$

c. $\sin\left(x - (3\pi/2)\right)$

d. $\sin\left(x + (3\pi/2)\right)$

e. $\sin(-x)$

3.3. For each expression given in Problem **3.2**, find an equivalent expression in terms of the cosine function.

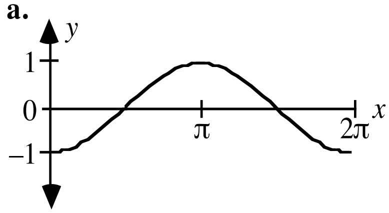
3.4 Use a graph of each of the following equations to rewrite it as an identity.

a. $y = \cos^2 x + \sin^2 x$

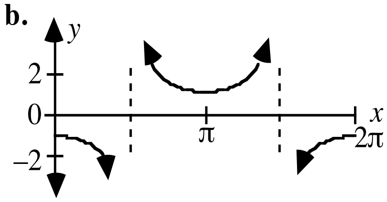
b. $y = \dfrac{1 - \cos^2 x}{\sin x}$; $\sin x \neq 0$

3.5 Which of the following graphs best illustrates the graph of $g(x) = \sec x$ over the domain $[0, 2\pi]$? Defend your choice.

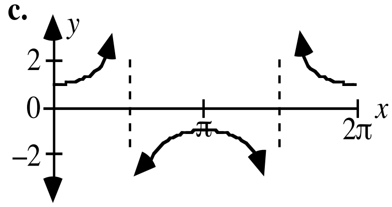
a.



b.



c.



3.6 **a.** Graph the following two functions on the same coordinate system.

$$f(x) = 5\cos\left(\frac{\pi}{2}x\right) \text{ and } g(x) = 5\sec\left(\frac{\pi}{2}x\right)$$

b. Determine the domain and range of each function.

c. Is the secant function periodic? Defend your response.

d. Graph $g(x)$ and $1/f(x)$ on the same coordinate system. Describe any relationships you observe between the two graphs.

e. In general, what is the relationship between $1/f(x)$, where $f(x) = a\cos(bx)$, and $g(x) = a\sec(bx)$?

3.7 **a.** The ratio for the tangent, $\tan\theta = \sin\theta/\cos\theta$, is true for individual values of θ. If this relationship is true for all values of θ, then it is an identity. Is the tangent ratio an identity? Justify your response.

b. Demonstrate algebraically that $\tan x = 1/\cot x$.

c. Consider the functions $f(x) = \tan x$ and $g(x) = \cot x$. Determine the domain and range of each function.

d. Is the cotangent function periodic? Explain your response.

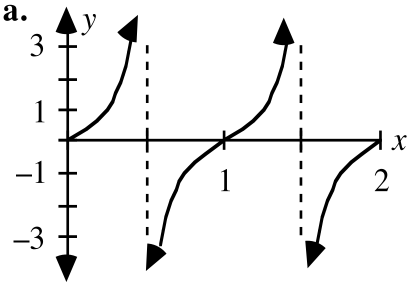
* * * * *

3.8 **a.** Determine the relationship between $f(x) = \cos x$ and $g(x) = \cos(x + \pi)$. Express this relationship as an identity.

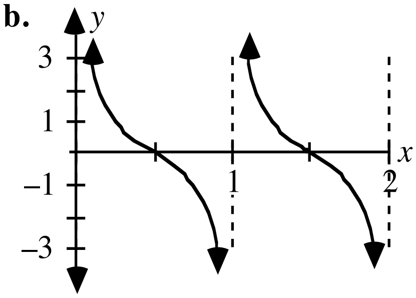
b. Determine the relationship between $f(x) = \cos(x - \pi)$ and $g(x) = \cos(x + \pi)$. Express this relationship as an identity.

3.9 Which of the following graphs best illustrates the graph of $f(x) = \cot(\pi x)$ over the domain $[0, 2]$? Defend your choice.

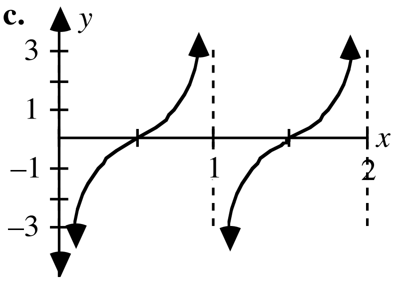
a.



b.



c.



3.10 Use reciprocal properties and identities to simplify each of the following expressions:

a. $\sin x \bullet \sec x \bullet \cot x$

b. $\csc x \bullet \tan x \bullet \cos x$

* * * * * * * * * *

Summary Assessment

1. Imagine that you are a wildlife biologist for the state of North Dakota. As part of an ecosystem restoration project, you have been assigned to reintroduce a colony of prairie dogs to a 50-acre wildlife preserve.

   The preserve already has a resident population of voles, which feed on the same plants as prairie dogs. It is also home to a population of owls, which prey on both voles and prairie dogs. The most recent data on these populations is shown in the table below.

Owl	**Population**	**Date**
Maximum	37	July 1, 1992
Minimum	11	July 3, 1993
Vole	**Population**	**Date**
Maximum	1752	January 1, 1993
Minimum	976	July 1, 1994

   In order to give the prairie dogs a good chance at establishing themselves, you would like to introduce them when both competition from voles and predation from owls are at a minimum.

   The fluctuations in the owl and vole populations can be modeled using transformations of the sine function. Use the information above to determine the best times and the worst times to introduce the prairie dogs between January 1992 and January 1999. Write a report on your findings. Use graphs and functions to support your reasoning.

2. In the northern hemisphere, the earliest sunset occurs near December 21 each year, while the latest sunset occurs near June 21. (The actual dates for the earliest and latest sunsets may vary by a few days from December 21 and June 21, respectively.)

   Assume that a graph of the time of sunset versus the day of the year can be modeled with a transformation of the sine or cosine curves.

   a. Sketch a curve that models a graph of the time of sunset in your town versus the day of the year.

   b. Write two different equations that model the graph from Part **a**.

   c. Use one of your models to predict the time the sun will set today and compare your prediction to the actual time of sunset.

   d. Do you think that your model could accurately estimate the time of sunset on any given day? Justify your response.

Module Summary

- A **periodic function** is a function in which values repeat at constant intervals.
- The **period** is the smallest interval of the domain over which the function repeats.
- A **cycle** is the portion of the function included in one period.
- If a periodic function has a maximum *M* and a minimum *m*, its **amplitude** is defined as

$$\left|\frac{M-m}{2}\right|$$

- For all periodic functions of the form $f(x) = a\sin[b(x-h)] + k$, where $a \neq 0$, the amplitude is $|a|$, the number of cycles in 2π radians is *b*, and the period is $2\pi/b$.

  The value of *h* describes the horizontal translation of the graph from the parent function $g(x) = \sin x$, while the value of *k* describes the vertical translation from the parent.
- A **trigonometric identity** is an equation involving trigonometric functions that is true for all real numbers in its domain.
- The **cosecant** of *x*, denoted $\csc x$, is the reciprocal of $\sin x$, or $\csc x = 1/\sin x$, where $\sin x \neq 0$.
- The **secant** of *x*, denoted $\sec x$, is the reciprocal of $\cos x$, or $\sec x = 1/\cos x$, where $\cos x \neq 0$.
- The **cotangent** of *x*, denoted $\cot x$, is the reciprocal of $\tan x$, or $\cot x = 1/\tan x$, where $\tan x \neq 0$.

Selected References

Giancoli, D. *Physics*. Englewood Cliffs, NJ: Prentice Hall, 1991.

Hewitt, P. *Conceptual Physics*. Menlo Park, CA: Addison-Wesley, 1987.